DIFFRACTION BY A WIDE SLIT AND COMPLEMENTARY STRIP. II

By R. F. MILLAR

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ABSTRACT. The notions developed in Part I in connexion with the diffraction of an E-polarized wave are here applied to the case of H-polarization. Induced current densities, aperture and far fields, and the transmission coefficient are again found in the form of infinite series in inverse powers of the slit-width to wavelength ratio. By means of Babinet's principle, the solution for diffraction by a strip is obtained.

A detailed comparison is made with values of the transmission coefficients for both polarizations found previously and by the present method. It appears that this theory provides accurate information when the slit-width is greater than about a wavelength.

1. *Introduction.* The solution to the problem of diffraction of an E-polarized plane electromagnetic wave by a wide slit in a perfectly conducting plane screen has been developed in the foregoing paper, henceforth referred to as I. In the present paper, the corresponding analysis will be outlined for the case of H-polarization. The procedure is essentially the same as that followed in I, but a differential-integral equation, rather than a pure integral equation, must be solved.

The diffraction of an E- or H-polarized wave by a strip is briefly treated, and comparisons are made with values of the transmission coefficients deduced by Skavlem (11) and Morse and Rubenstein (9).

2. *The solution for H-polarization.* 2·1. *Formulation of the problem in terms of a pair of differential-integral equations.* As in the treatment for the case of E-polarization, it is assumed that the screen and slit occupy the plane $x = 0$ (see Fig. 1 in I). Then the magnetic vector in the incident wave is parallel to the axis of z

$$\mathbf{H}^i = (0, 0, 1)\, e^{ik(x\cos\alpha + y\sin\alpha)}. \tag{1}$$

Since H_z is, by definition, the only non-vanishing component of the magnetic field, it follows that a current, of density $J_y(y;\alpha)$ flows on the screen in the direction of the y-axis. The total magnetic field at a point (x_0, y_0) is then determined by the following expression:

$$H_z(x_0, y_0) = e^{ik(x_0\cos\alpha + y_0\sin\alpha)} - \frac{i}{4}\frac{\partial}{\partial x_0}\left\{\int_{-\infty}^{-h} + \int_0^{\infty}\right\} J_y(y;\alpha)\, H_0^{(2)}(khr)\, dy, \tag{2}$$

in which hr is defined by equation (3) of I and h is equal to the slit width.

The vanishing of $E_y(x_0, y_0)$ at all points on the screen yields a differential-integral equation for the current density

$$\frac{\partial^2}{\partial x_0^2}\left\{\int_{-\infty}^{-h} + \int_0^{\infty}\right\} J_y(y;\alpha)\, H_0^{(2)}(khr)\, dy = 4k\cos\alpha\, e^{iky_0\sin\alpha} \tag{3}$$

for $y_0 > 0$ and $y_0 < -h$, with $x_0 = 0$. The expression on the right-hand side is unknown for $-h < y_0 < 0$.

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It is not permissible to perform the indicated differentiation under the sign of integration, for the resulting integrals are divergent. It has been shown by Baker and Copson ((1), p. 184), and Bouwkamp ((2), (3)) that these divergencies may be easily eliminated when it is observed that

$$\left\{\int_{-\infty}^{-h}+\int_{0}^{\infty}\right\}J_\nu(y;\alpha)\,H_0^{(2)}(khr)\,dy$$

is a solution of the two-dimensional wave equation. Therefore

$$\frac{\partial^2}{\partial x_0^2}\left\{\int_{-\infty}^{-h}+\int_{0}^{\infty}\right\}J_\nu(y;\alpha)\,H_0^{(2)}(khr)\,dy=-\left(k^2+\frac{\partial^2}{\partial y_0^2}\right)\left\{\int_{-\infty}^{-h}+\int_{0}^{\infty}\right\}J_\nu(y;\alpha)\,H_0^{(2)}(khr)\,dy,$$

and, since the integral is continuous across the screen, the differential-integral equation (3) is equivalent to

$$\left(\frac{d^2}{dy_0^2}+k^2\right)\left\{\int_{-\infty}^{-h}+\int_{0}^{\infty}\right\}J_\nu(y;\alpha)\,H_0^{(2)}(k\,|y-y_0|)\,dy=-4k\cos\alpha\,e^{iky_0\sin\alpha} \tag{4}$$

for $y_0>0$ and $y_0<-h$.

If $$J^+(y;\alpha)=J_\nu(y;\alpha),\quad J^-(y;\alpha)=J_\nu(-y-h;\alpha),$$

with $y>0$, equation (4) may be replaced by a pair of simultaneous differential-integral equations

$$\left.\begin{aligned}\left(\frac{d^2}{dy_0^2}+k^2\right)\left\{\int_0^\infty J^+(y;\alpha)\,H_0^{(2)}(k\,|y-y_0|)\,dy+\int_0^\infty J^-(y;\alpha)\,H_0^{(2)}[k(y+y_0+h)]\,dy\right\}\\ =-4k\cos\alpha\,e^{iky_0\sin\alpha},\\ \left(\frac{d^2}{dy_0^2}+k^2\right)\left\{\int_0^\infty J^-(y;\alpha)\,H_0^{(2)}(k\,|y-y_0|)\,dy+\int_0^\infty J^+(y;\alpha)\,H_0^{(2)}[k(y+y_0+h)]\,dy\right\}\\ =-4k\cos\alpha\,e^{-ik(y_0+h)\sin\alpha}\end{aligned}\right\} \tag{5}$$

for $y_0>0$.

2·2. *Solution of the differential-integral equations.* The general solutions of the differential equations defined by equations (5) are readily determined

$$\left.\begin{aligned}\int_0^\infty J^+(y;\alpha)\,H_0^{(2)}(k\,|y-y_0|)\,dy+\int_0^\infty J^-(y;\alpha)\,H_0^{(2)}[k(y+y_0+h)]\,dy\\ =-4\frac{e^{iky_0\sin\alpha}}{k\cos\alpha}+A_1(\alpha)\,e^{-iky_0}+B_1(\alpha)\,e^{iky_0},\\ \int_0^\infty J^-(y;\alpha)\,H_0^{(2)}(k\,|y-y_0|)\,dy+\int_0^\infty J^+(y;\alpha)\,H_0^{(2)}[k(y+y_0+h)]\,dy\\ =-4\frac{e^{-ik(y_0+h)\sin\alpha}}{k\cos\alpha}+A_2(\alpha)\,e^{-iky_0}+B_2(\alpha)\,e^{iky_0}\end{aligned}\right\} \tag{6}$$

for $y_0>0$.

Since the diffracted field which arises through the presence of the slit must represent outgoing waves at large distances from the origin, the functions $B_1(\alpha)$ and $B_2(\alpha)$ are of necessity identically zero.

Because the current flow is in a direction normal to the edges of the slit, it will be assumed that $J^\pm(y;\alpha)$ tend to zero as y approaches zero. It follows subsequently that

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$J^{\pm}(y;\alpha)$ do, in fact, tend to zero as $y^{\frac{1}{2}}$, in accordance with the well-known edge condition of Meixner (see for example, Bouwkamp (3)). However, the somewhat weaker form stated above proves adequate for the present purposes.

Before proceeding with the determination of $A_1(\alpha)$ and $A_2(\alpha)$, it is once more necessary to employ Fox's formula (equation (5) of I). The equations (6) then take the form

$$\left.\begin{aligned}\int_0^\infty [J^+(y;\alpha)+j^-(y;\alpha)]\,H_0^{(2)}(k\,|y-y_0|)\,dy &= -\frac{4\,e^{iky_0\sin\alpha}}{k\cos\alpha}+A_1(\alpha)\,e^{-iky_0},\\ \int_0^\infty [J^-(y;\alpha)+j^+(y;\alpha)]\,H_0^{(2)}(k\,|y-y_0|)\,dy &= -\frac{4e^{-ik(y_0+h)\sin\alpha}}{k\cos\alpha}+A_2(\alpha)\,e^{-iky_0},\end{aligned}\right\} \tag{7}$$

where $y_0 > 0$, and

$$j^+(y;\alpha)=\frac{1}{\pi}\int_0^\infty J^+(z;\alpha)\,\frac{e^{-ik(y+z+h)}}{y+z+h}\left(\frac{z+h}{y}\right)^{\frac{1}{4}}dz,$$

$$j^-(y;\alpha)=\frac{1}{\pi}\int_0^\infty J^-(z;\alpha)\,\frac{e^{-ik(y+z+h)}}{y+z+h}\left(\frac{z+h}{y}\right)^{\frac{1}{4}}dz$$

for $y > 0$.

Equations 7 are of the Wiener–Hopf type and have solutions

$$\left.\begin{aligned}J^+(y;\alpha)+j^-(y;\alpha) &= -\frac{4\psi(y;\alpha)}{k\cos\alpha}+A_1(\alpha)\,\psi(y;-\tfrac{1}{2}\pi),\\ J^-(y;\alpha)+j^+(y;\alpha) &= -4e^{-ikh\sin\alpha}\,\frac{\psi(y;-\alpha)}{k\cos\alpha}+A_2(\alpha)\,\psi(y;-\tfrac{1}{2}\pi),\end{aligned}\right\} \tag{8}$$

where the function $\psi(y;\alpha)$ is defined by equation (7) of I.

It is evident that $j^{\pm}(y;\alpha)$, together with the right-hand sides of equations (8), are of order $y^{-\frac{1}{2}}$ for small values of y, while by the edge condition, $J^{\pm}(y;\alpha)$ must vanish in the limit as y tends to zero. If, then, equations (8) are multiplied by $y^{\frac{1}{2}}$, and the limit formed as y tends to zero, it is found that $A_1(\alpha)$ and $A_2(\alpha)$ are uniquely determined by the relations

$$\left.\begin{aligned}A_1(\alpha) &= \frac{4\sin(\frac{1}{4}\pi-\frac{1}{2}\alpha)}{(z+h)^{\frac{1}{2}}}-4\sqrt{2}\,\frac{e^{\frac{1}{4}\pi i}}{\sqrt{\pi}}\,k^{\frac{1}{2}}\cos\alpha\int_0^\infty J^-(z;\alpha)\,\frac{e^{-ik(z+h)}}{(z+h)^{\frac{1}{2}}}\,dz,\\ A_2(\alpha) &= \frac{4\sin(\frac{1}{4}\pi+\frac{1}{2}\alpha)}{k\cos\alpha}+4\sqrt{2}\,\frac{e^{i\frac{1}{4}\pi}}{\sqrt{\pi}}\,k^{\frac{1}{2}}\cos\alpha\int_0^\infty J^+(z;\alpha)\,\frac{e^{-ik(z+h)}}{(z+h)^{\frac{1}{2}}}\,dz.\end{aligned}\right\} \tag{9}$$

The equations (8) are thereby reduced to the following pair:

$$\left.\begin{aligned}J^+(y;\alpha)-\frac{1}{\pi}\int_0^\infty J^-(z;\alpha)\,\frac{e^{-ik(y+z+h)}}{y+z+h}\left(\frac{y}{z+h}\right)^{\frac{1}{2}}dz &= \chi(y;\alpha),\\ J^-(y;\alpha)-\frac{1}{\pi}\int_0^\infty J^+(z;\alpha)\,\frac{e^{-ik(y+z+h)}}{y+z+h}\left(\frac{y}{z+h}\right)^{\frac{1}{2}}dz &= e^{-ikh\sin\alpha}\,\chi(y;-\alpha),\end{aligned}\right\} \tag{10}$$

in which

$$\chi(x;\alpha)=-\frac{4e^{\frac{1}{4}\pi i}}{\sqrt{\pi}}\,e^{ikx\sin\alpha}\,f[\sqrt{(2kx)}\sin(\tfrac{1}{4}\pi+\tfrac{1}{2}\alpha)] \tag{11}$$

and $f(x)$ is defined by equation (15) of I.

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From equations (10) it is apparent that the current densities are of order $y^{\frac{1}{2}}$ for small y, in agreement with the Meixner edge condition. It is also readily verified that $\chi(y;\alpha)$ is equal to the density of the current which flows on a half-plane $x=0$, $y\geqslant 0$ when irradiated by an H-polarized plane wave incident at an angle α to the positive x-axis.

The pair of integral equations for $J^{+}(y;\alpha)$ and $J^{-}(y;\alpha)$ leads to series solutions of the form

$$J^{\pm}(y;\alpha)=\sum_{n=0}^{\infty}J_n^{\pm}(y;\alpha), \tag{12}$$

where
$$J_n^{\pm}(y;\alpha)=\frac{1}{\pi}\int_0^{\infty}J_{n-1}^{\mp}(z;\alpha)\frac{e^{-ik(y+z+h)}}{y+z+h}\left(\frac{y}{z+h}\right)^{\frac{1}{2}}dz \tag{13}$$

for $n\geqslant 1$ and $\quad J_0^{+}(y;\alpha)=\chi(y;\alpha),\quad J_0^{-}(y;\alpha)=e^{-ikh\sin\alpha}\chi(y;-\alpha).$

With the aid of this series solution, the asymptotic expansions of the current densities will be calculated in the following section.

2·3. *The asymptotic expansion of the solution.* Since the asymptotic expansions of the current densities are obtained in a manner entirely analogous to that discussed in connexion with E-polarization, many details may be omitted from the subsequent analysis.

The normalized equations have the form

$$J_n^{\pm}(hY;\alpha)=\frac{1}{\pi}e^{-i\kappa}e^{-i\kappa Y}\,Y^{\frac{1}{2}}\int_0^{\infty}J_{n-1}^{\mp}(hZ;\alpha)\frac{e^{-i\kappa Z}}{Y+Z+1}\frac{dZ}{(1+Z)^{\frac{1}{2}}} \tag{14}$$

for $n\geqslant 1$, and

$$J_0^{+}(hY;\alpha)=-4\pi^{-\frac{1}{2}}e^{\frac{1}{4}\pi i}e^{i\kappa Y\sin\alpha}f[(2\kappa Y)^{\frac{1}{2}}\sin(\tfrac{1}{4}\pi+\tfrac{1}{2}\alpha)], \tag{15}$$

$$J_0^{-}(hY;\alpha)=e^{-i\kappa\sin\alpha}J_0^{+}(hY;-\alpha). \tag{16}$$

It is apparent that the integrals which must now be evaluated bear a marked resemblance to several considered in I, the chief difference being the inversion of the factors $(1+Z)^{\frac{1}{2}}Z^{-\frac{1}{2}}$ and $(1+Z)^{\frac{1}{2}}$ which occur in the latter.

If
$$K_3(Y;p,q)=\int_0^{\infty}f[(\kappa qZ)^{\frac{1}{2}}]\frac{e^{-i\kappa pZ}}{Y+Z+1}\frac{dZ}{(1+Z)^{\frac{1}{2}}}, \tag{17}$$

then a glance at the corresponding analysis which concerns $K_2(Y;p,q)$ in § 2·3 of I suffices to indicate that

$$K_3(Y;p,q)=-e^{\frac{1}{4}\pi i}\int_0^{\infty}G[(\kappa qv)^{\frac{1}{2}}]\frac{e^{-\kappa pv}}{Y+1-iv}\frac{dv}{(1-iv)^{\frac{1}{2}}}. \tag{18}$$

Let
$$l(v,Y;\kappa p)=\int_v^{\infty}\frac{e^{-\kappa px}}{Y+1-ix}\frac{dx}{(1-ix)^{\frac{1}{2}}}. \tag{19}$$

Then
$$l(v,Y;\kappa p)=e^{-\kappa pv}L(v,Y;\kappa p),$$

where
$$L(v,Y;\kappa p)=\int_0^{\infty}\frac{e^{-\kappa pt}}{Y+1-i(v+t)}\frac{dt}{[1-i(v+t)]^{\frac{1}{2}}}. \tag{20}$$

If
$$(Y+1-iv)^{-1}(1-iv)^{-\frac{1}{2}}=\sum_{n=0}^{\infty}c_n(Y)v^n \tag{21}$$

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for $|v|$ sufficiently small, then

$$K_3(Y;p,q) \sim -\tfrac{1}{2}\sqrt{\pi}\, e^{\frac{1}{4}\pi i} L(0,Y;\kappa p) + \tfrac{1}{2}\sqrt{\pi}\, e^{\frac{1}{4}\pi i} \sum_{n=0}^{\infty} c_n(Y)\rho_n(p,q;\kappa), \tag{22}$$

where $\rho_n(p,q;\kappa)$ is defined by equation (26) of I. Then

$$\left.\begin{aligned} J_1^+(hY;\alpha) &\sim 2i\pi^{-1} e^{-i\kappa p} e^{-i\kappa Y} Y^{\frac{1}{2}} [L(0,Y;\kappa p) - \sum_{n=0}^{\infty} c_n(Y)\rho_n(p,q;\kappa)], \\ J_2^+(hY;\alpha) &\sim \frac{e^{-\frac{1}{4}\pi i}}{\sqrt{2\pi^2}} \frac{e^{-i2\kappa}}{\kappa^{\frac{3}{2}}} e^{-i\kappa Y} Y^{\frac{1}{2}} \sum_{n+m=0}^{\infty}{}' \frac{\Gamma(m+\frac{3}{2})}{(2\kappa)^m} [d_m(Y;\kappa q) - c_{nm}(Y)\rho_n(q,p;\kappa)], \\ J_3^+(hY;\alpha) &\sim O(\kappa^{-3}); \end{aligned}\right\} \tag{23}$$

and

$$J_n^-(hY;\alpha) = e^{-i\kappa \sin\alpha} J_n^+(hY;-\alpha). \tag{24}$$

It may be observed that because of the presence of the factor $Z^{\frac{1}{2}}$ in the integrands associated with the present polarization, the first terms in the expansions of $J_2^{\pm}$ are already of order $\kappa^{-\frac{3}{2}}$. For the present purposes, knowledge of only the first three terms in the series (12) is sufficient.

The coefficients $c_n(Y)$ are given by equation (21) and

$$\frac{c_n(-iv)}{(Y+1-iv)(1-iv)^{\frac{1}{2}}} = \sum_{m=0}^{\infty} c_{nm}(Y) v^m, \tag{25}$$

$$\frac{L(0,-iv;\kappa q)}{(Y+1-iv)(1-iv)^{\frac{1}{2}}} = \sum_{m=0}^{\infty} d_m(Y;\kappa q) v^m. \tag{26}$$

It readily follows that

$$\left.\begin{aligned} c_0(Y) &= (Y+1)^{-1}, \quad c_1(Y) = i(Y+1)^{-2} + \tfrac{1}{2}i(Y+1)^{-1}, \quad c_{00}(Y) = (Y+1)^{-1}, \\ d_0(Y;\kappa q) &= 2i\{1 - 2i(\kappa q)^{\frac{1}{2}} e^{i\kappa q} F[(\kappa q)^{\frac{1}{2}}]\} c_0(Y), \\ d_1(Y;\kappa q) &= -\tfrac{2}{3}\{1 - 2i\kappa q - 4(\kappa q)^{\frac{3}{2}} F[(\kappa q)^{\frac{1}{2}}]\} c_0(Y) \\ &\qquad + 2i\{1 - 2i(\kappa q)^{\frac{1}{2}} e^{i\kappa q} F[(\kappa q)^{\frac{1}{2}}]\} c_1(Y). \end{aligned}\right\} \tag{27}$$

It has been shown that, when the incident field is E-polarized, the current density on the screen tends to zero as α approaches $\frac{1}{2}\pi$. On the contrary, in the case of H-polarization, the induced current is not zero in the limit of grazing incidence. The necessity for this is made evident by a consideration of the complementary problem in which an E-polarized wave falls at grazing incidence onto a strip. In order that the boundary condition on the strip be satisfied, the existence of a scattered field is essential.

The asymptotic behaviour of the current density on each half of the screen when $\alpha = \frac{1}{2}\pi$ may be readily deduced from equations (15), (16), (23) and (24)

$$\left.\begin{aligned} J^+(hY;\tfrac{1}{2}\pi) &\sim -2e^{i\kappa Y} + \frac{4e^{\frac{1}{4}\pi i}}{\sqrt{\pi}} e^{i\kappa Y} F[(2\kappa Y)^{\frac{1}{2}}] \\ &\qquad + \frac{e^{-\frac{1}{4}\pi i}}{2\sqrt{2\pi^{\frac{3}{2}}}} \frac{e^{-i2\kappa}}{\kappa^{\frac{3}{2}}} e^{-i\kappa Y} \frac{Y^{\frac{1}{2}}}{Y+1} \left[2i - \frac{3}{2\kappa}\left(1 + \frac{1}{Y+1}\right)\right] + O(\kappa^{-3}), \\ J^-(hY;\tfrac{1}{2}\pi) &\sim -\frac{4e^{-i\kappa}}{\pi} e^{-i\kappa Y} \tan^{-1} Y^{\frac{1}{2}} - \frac{i}{2\pi} \frac{e^{-i\kappa}}{\kappa} e^{-i\kappa Y} \frac{Y^{\frac{1}{2}}}{Y+1} \\ &\qquad + \frac{3}{16\pi} \frac{e^{-i\kappa}}{\kappa^2} e^{-i\kappa Y} \frac{Y^{\frac{1}{2}}}{Y+1} \left(\frac{1}{2} + \frac{1}{Y+1}\right) + O(\kappa^{-3}). \end{aligned}\right\} \tag{28}$$

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The presence in $J^-(hY;\ \frac{1}{2}\pi)$ of a term which tends to $-2e^{-i\kappa(1+Y)}$ as Y tends to infinity (and which is of order $Y^{\frac{1}{2}}$ as Y tends to zero) might be expected on physical grounds, since for large Y the current distribution must approximate to that which flows on an infinite plane.

2·4. *The field in the slit.* Reference has been made at an earlier point to the fact that the distribution of tangential electric field is essentially the only unknown quantity in the aperture. In the present instance, the electric field in the aperture is determined in a manner analogous to that employed in the case of E-polarization

$$E_y(x_0, y_0) = i\frac{Z_0}{k}\frac{\partial}{\partial x_0}H_z(x_0, y_0), \tag{29}$$

where $H_z(x_0, y_0)$ is defined by equation (2).

A repetition of the arguments which led to the differential-integral equations (5) now indicates that $E_y(0, -hY_0)$ may be written in the following manner

$$E_y(0, -hY_0) = \sum_{n=0}^{\infty} E_n(0, -hY_0), \tag{30}$$

where

$$Z_0^{-1}E_n(0, -hY_0) = -\tfrac{1}{4}\kappa\left[1+\frac{d^2}{d(\kappa Y_0)^2}\right]\left\{\int_0^\infty J_n^+(hY;\alpha)\,H_0^{(2)}[\kappa(Y+Y_0)]\,dY + \int_0^\infty J_n^-(hY;\alpha)\,H_0^{(2)}[\kappa(Y+1-Y_0)]\,dY\right\} \tag{31}$$

for $n \geqslant 1$, while $E_0(0, -hY_0)$ is once again the incident field plus the field scattered by the two isolated half-planes. This electric field may be determined from the corresponding magnetic component

$$\begin{aligned} H_0(x_0, y_0) = &-e^{ik(x_0\cos\alpha + y_0\sin\alpha)} + \frac{e^{\frac{1}{4}\pi i}}{\sqrt{\pi}}\{e^{i\kappa r_0\cos(\beta-\alpha)}F[(2\kappa r_0)^{\frac{1}{2}}\cos\tfrac{1}{2}(\beta-\alpha)] \\ &+ e^{-i\kappa r_0\cos(\beta+\alpha)}F[(2\kappa r_0)^{\frac{1}{2}}\sin\tfrac{1}{2}(\beta+\alpha)] \\ &+ e^{-i\kappa\sin\alpha}(e^{i\kappa r_1\cos(\gamma+\alpha)}F[(2\kappa r_1)^{\frac{1}{2}}\cos\tfrac{1}{2}(\gamma+\alpha)] \\ &+ e^{-i\kappa r_1\cos(\gamma-\alpha)}F[(2\kappa r_1)^{\frac{1}{2}}\sin\tfrac{1}{2}(\gamma-\alpha)])\}, \end{aligned} \tag{32}$$

so that

$$\left.\begin{aligned} Z_0^{-1}E_0(0, -hY_0) &= -\cos\alpha\, e^{-i\kappa Y_0\sin\alpha} + 2\pi^{-\frac{1}{2}}e^{\frac{1}{4}\pi i}[B_0(Y_0, -\alpha) + e^{-i\kappa\sin\alpha}B_0(1-Y_0, \alpha)], \\ Z_0^{-1}E_1(0, -hY_0) &= -\frac{i\kappa}{2\pi}e^{-i\kappa}\left[1+\frac{d^2}{d(kY_0)^2}\right][e^{-i\kappa\sin\alpha}B_1(Y_0, \alpha) + B_1(1-Y_0, -\alpha)], \\ Z_0^{-1}E_2(0, -hY_0) &= -\frac{e^{-\frac{1}{4}\pi i}}{4\sqrt{2\pi^2}}\frac{e^{-i2\kappa}}{\kappa^{\frac{1}{2}}}\left[1+\frac{d^2}{d(\kappa Y_0)^2}\right][B_2(Y_0, -\alpha) + e^{-i\kappa\sin\alpha}B_2(1-Y_0, \alpha)], \\ Z_0^{-1}E_3(0, -hY_0) &= O(\kappa^{-\frac{7}{2}}), \end{aligned}\right\} \tag{33}$$

where

$$\left.\begin{aligned} B_0(Y_0, -\alpha) &= \cos\alpha\, e^{-i\kappa Y_0\sin\alpha}F[(\kappa q Y_0)^{\frac{1}{2}}] + \tfrac{1}{2}ip^{\frac{1}{2}}e^{-i\kappa Y_0}(\kappa Y_0)^{-\frac{1}{2}}, \\ B_n(Y_0, \alpha) &= L_n(Y_0, \alpha) + D_n(Y_0, \alpha) \end{aligned}\right\} \tag{34}$$

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for $n \geqslant 1$, and

$$\left.\begin{aligned} L_1(Y_0,\alpha) &\sim \int_0^\infty e^{-i\kappa Y}\, Y^{\frac{1}{2}} L(0, Y; \kappa p)\, H_0^{(2)}[\kappa(Y+Y_0)]\, dY, \\ L_2(Y_0,\alpha) &\sim \sum_{m=0}^{\infty} \frac{\Gamma(m+\frac{3}{2})}{(2\kappa)^m} \int_0^\infty e^{-i\kappa Y}\, Y^{\frac{1}{2}} d_m(Y; \kappa p)\, H_0^{(2)}[\kappa(Y+Y_0)]\, dY, \\ D_1(Y_0,\alpha) &\sim -\sum_{n=0}^{\infty} \rho_n(p,q;\kappa) \int_0^\infty e^{-i\kappa Y}\, Y^{\frac{1}{2}} c_n(Y)\, H_0^{(2)}[\kappa(Y+Y_0)]\, dY, \\ D_2(Y_0,\alpha) &\sim -\sum_{n+m=0}^{\infty} \frac{\Gamma(m+\frac{3}{2})}{(2\kappa)^m} \rho_n(p,q;\kappa) \int_0^\infty e^{-i\kappa Y}\, Y^{\frac{1}{2}} c_{nm}(Y)\, H_0^{(2)}[\kappa(Y+Y_0)]\, dY. \end{aligned}\right\} \quad (35)$$

Evidently it is necessary to consider the expansions of integrals of the forms

$$Q_n(Y_0) = \int_0^\infty \frac{e^{-i\kappa Y}}{(Y+1)^n}\, Y^{\frac{1}{2}} H_0^{(2)}[\kappa(Y+Y_0)]\, dY, \quad (36)$$

and $L_1(Y_0,\alpha)$. It is not difficult to show (see § 2·4 of I) that

$$Q_n(Y_0) = \frac{2e^{-\frac{1}{4}\pi i}}{\pi} e^{-i\kappa Y_0} \int_0^\infty \frac{e^{-i\kappa Y_0 u}}{[u(u+2)]^{\frac{1}{2}}}\, du \int_0^\infty \frac{e^{-\kappa(u+2)v}}{(1-iv)^n}\, v^{\frac{1}{2}} dv.$$

Since

$$\int_0^\infty \frac{e^{-\kappa(u+2)v}}{(1-iv)^n}\, v^{\frac{1}{2}} dv \sim \sum_{m=0}^{\infty} \frac{r_{nm}\,\Gamma(m+\frac{3}{2})}{[\kappa(u+2)]^{m+\frac{3}{2}}},$$

where several of the coefficients r_{nm} are given by equations (56) of I, it follows that

$$Q_n(Y_0) \sim -\frac{2i}{\pi\kappa^{\frac{3}{2}}} \sum_{m=0}^{\infty} \frac{r_{nm}\,\Gamma(m+\frac{3}{2})}{\kappa^m} F_{m+2}(\kappa Y_0), \quad (37)$$

where $F_m(u)$ is defined by equation (50) of I. Also,

$$L_1(Y_0,\alpha) = \frac{2e^{-\frac{1}{4}\pi i}}{\pi} e^{-i\kappa Y_0} \int_0^\infty \frac{e^{-i\kappa Y_0 u}}{[u(u+2)]^{\frac{1}{2}}}\, du \int_0^\infty e^{-\kappa(u+2)v}\, v^{\frac{1}{2}} L(0, -iv; \kappa p)\, dv,$$

so that

$$L_1(Y_0;\alpha) \sim -\frac{2i}{\pi\kappa^{\frac{3}{2}}} \sum_{n=0}^{\infty} \frac{l_n(\kappa p)\,\Gamma(n+\frac{3}{2})}{\kappa^n} F_{n+2}(\kappa Y_0), \quad (38)$$

where

$$\int_0^\infty e^{-\kappa(u+2)v}\, v^{\frac{1}{2}} L(0, -iv; \kappa p)\, dv \sim \sum_{n=0}^{\infty} \frac{l_n(\kappa p)\,\Gamma(n+\frac{3}{2})}{[\kappa(u+2)]^{n+\frac{3}{2}}}. \quad (39)$$

The coefficients $l_n(\kappa p)$ may be obtained from the series expansion of $L(0, -iv; \kappa p)$ near $v = 0$:

$$L(0, -iv; \kappa p) = \sum_{n=0}^{\infty} l_n(\kappa p)\, v^n. \quad (40)$$

Then (see equation (20)),

$$\left.\begin{aligned} l_0(\kappa p) &= 2i + 4i(\kappa p)^{\frac{1}{2}} e^{i\kappa p} F[(\kappa p)^{\frac{1}{2}}], \\ l_1(\kappa p) &= -\tfrac{2}{3}\{1 - 2i\kappa p - 4(\kappa p)^{\frac{3}{2}} e^{i\kappa p} F[(\kappa p)^{\frac{1}{2}}]\}. \end{aligned}\right\} \quad (41)$$

From equations (21), (26) and (40), it is seen that

$$d_\nu(Y; \kappa q) = \sum_{n+m=\nu} l_n(\kappa q)\, c_m(Y). \quad (42)$$

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Then

$$\left.\begin{aligned} B_1(Y_0,\alpha) &\sim -\frac{i}{\sqrt{\pi}}\frac{l_0(\kappa p)}{\kappa^{\frac{3}{2}}}F_2(\kappa Y_0)+\frac{i}{\sqrt{\pi}\kappa^{\frac{5}{2}}}\left[\frac{1}{p}\left(1-\frac{q^{\frac{1}{2}}}{\sqrt{2}}\right)F_2(\kappa Y_0)-\tfrac{3}{2}l_1(\kappa p)\,F_3(\kappa Y_0)\right]+O(\kappa^{-\frac{7}{2}}),\\ B_2(Y_0,\alpha) &\sim -\frac{il_0(\kappa p)}{2\kappa^{\frac{3}{2}}}F_2(\kappa Y_0)+O(\kappa^{-\frac{5}{2}}). \end{aligned}\right\} \tag{43}$$

2·5. *The far-field behind the slit.* When β is sufficiently distant from both $\frac{1}{2}\pi$ and $\frac{3}{2}\pi$, the far-field may be determined most readily by the direct asymptotic evaluation of equation (2). A number of results determined in this manner will be quoted later. For arbitrary α and β, the present case is best treated from the point of view of the tangential electric field in the aperture. The appropriate two-dimensional form of the Rayleigh formula now gives

$$H_z(x_0,y_0)=-\frac{\kappa}{2Z_0}\int_0^1 E_y(0,-hY)\,H_0^{(2)}(\kappa r)\,dY, \tag{44}$$

where $x_0\leqslant 0$ and r is once more given by equation (62) of I. $H_z(x_0,y_0)$ may be written as

$$H_z(x_0,y_0)=\sum_{n=0}^{\infty}H_n(x_0,y_0), \tag{45}$$

where

$$H_n(x_0,y_0)=-\frac{\kappa}{2Z_0}\int_0^1 E_n(0,-hY)\,H_0^{(2)}(\kappa r)\,dY \tag{46}$$

for $n\geqslant 0$. In the region of the far field

$$H_n(x_0,y_0)\sim -\frac{\kappa}{\sqrt{(2\pi)}}e^{\frac{1}{4}\pi i}\frac{e^{-i\kappa r_0}}{(\kappa r_0)^{\frac{1}{2}}}\frac{1}{Z_0}\int_0^1 E_n(0,-hY)\,e^{-i\kappa Y\sin\beta}\,dY+O(r_0^{-\frac{3}{2}}). \tag{47}$$

If

$$\theta_n(\alpha,\beta)=\int_0^1 e^{-i\kappa Y\sin\beta}\left[1+\frac{d^2}{d(\kappa Y)^2}\right]B_n(Y,\alpha)\,dY \tag{48}$$

for $n\geqslant 1$, and

$$\theta_0(\alpha,\beta)=\int_0^1 B_0(Y,\alpha)\,e^{-i\kappa Y\sin\beta}\,dY, \tag{49}$$

then

$$\left.\begin{aligned} H_0(x_0,y_0) &\sim -\frac{e^{-\frac{1}{4}\pi i}}{\sqrt{(2\pi)}}\kappa\cos\alpha\frac{e^{-i\kappa r_0}}{(\kappa r_0)^{\frac{1}{2}}}\left[\frac{e^{-i\kappa(\sin\alpha+\sin\beta)}-1}{\kappa(\sin\alpha+\sin\beta)}\right]\\ &\qquad -\frac{\sqrt{2}\,i}{\pi}\kappa\frac{e^{-i\kappa r_0}}{(\kappa r_0)^{\frac{1}{2}}}[\theta_0(-\alpha,\beta)+e^{-i\kappa(\sin\alpha+\sin\beta)}\theta_0(\alpha,-\beta)],\\ H_1(x_0,y_0) &\sim -\frac{e^{-\frac{1}{4}\pi i}}{2\sqrt{2}\,\pi^{\frac{3}{2}}}\kappa^2 e^{-i\kappa}\frac{e^{-i\kappa r_0}}{(\kappa r_0)^{\frac{1}{2}}}[e^{-i\kappa\sin\alpha}\theta_1(\alpha,\beta)+e^{-i\kappa\sin\beta}\theta_1(-\alpha,-\beta)],\\ H_2(x_0,y_0) &\sim \frac{\kappa^{\frac{1}{2}}e^{-2\kappa}}{8\pi^{\frac{5}{2}}}\frac{e^{-i\kappa r_0}}{(\kappa r_0)^{\frac{1}{2}}}[\theta_2(-\alpha,\beta)+e^{-i\kappa(\sin\alpha+\sin\beta)}\theta_2(\alpha,-\beta)], \end{aligned}\right\} \tag{50}$$

and

$$H_3(x_0,y_0)\sim O(\kappa^{-3}).$$

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In the above equations the following functions appear:

$$\left.\begin{aligned}\theta_0(-\alpha,\beta) &= \cos\alpha\,\phi_0(-\alpha,\beta)+\tfrac{1}{2}ip^{\frac{1}{2}}I_0(\beta),\\ \theta_1(\alpha,\beta) &= -\frac{i}{\sqrt{\pi}}\frac{l_0(\kappa p)}{\kappa^{\frac{3}{2}}}I_2'(\beta)+\frac{i}{\sqrt{\pi}\,\kappa^{\frac{5}{2}}}\left[\frac{1}{p}\left(1-\frac{q^{\frac{1}{2}}}{\sqrt{2}}\right)I_2'(\beta)-\tfrac{3}{2}l_1(\kappa p)\,I_3'(\beta)\right]+O(\kappa^{-4}),\\ \theta_2(\alpha,\beta) &= -\frac{il_0(\kappa p)}{2\kappa^{\frac{3}{2}}}I_2'(\beta)+O(\kappa^{-3}),\end{aligned}\right\} \tag{51}$$

where $\phi_0(-\alpha,\beta)$ is defined by equation (68) of I and

$$\left.\begin{aligned}I_0(\beta) &= \frac{1}{\sqrt{\pi}}\int_0^1 e^{-i\kappa Y\sin\beta}F_0(\kappa Y)\,dY,\\ I_n'(\beta) &= \int_0^1 e^{-i\kappa Y\sin\beta}\left[1+\frac{d^2}{d(\kappa Y)^2}\right]F_n(\kappa Y)\,dY\end{aligned}\right\} \tag{52}$$

for $n = 2, 3$.

$I_0(\beta)$ is readily integrated to give

$$I_0(\beta) = \frac{2}{\kappa^{\frac{1}{2}}}\left\{\frac{\frac{1}{2}\sqrt{\pi}\,e^{-\frac{1}{4}\pi i}-F[(\kappa n)^{\frac{1}{2}}]}{(\kappa n)^{\frac{1}{2}}}\right\}, \tag{53}$$

while a double differentiation of equation (51) of I leads to

$$\left(1+\frac{d^2}{du^2}\right)F_{m+2}(u) = 2F_{m+1}(u)-F_m(u),$$

so that
$$I_2'(\beta) = 2I_1(\beta)-\sqrt{\pi}\,I_0(\beta),\quad I_3'(\beta) = 2I_2(\beta)-I_1(\beta), \tag{54}$$

where $I_1(\beta)$ and $I_2(\beta)$ are given by equations (69) and (70) respectively, of I.

2·6. *The transmission coefficient.* The transmission coefficient per unit length of the slit for H-polarized waves incident at an angle α will be denoted by $t_H(\alpha;\kappa)$. If the far field in the direction of propagation is written as

$$H_z(x_0,y_0) \sim \frac{e^{\frac{1}{4}\pi i}}{\sqrt{(2\pi)}}\frac{e^{-i\kappa r_0}}{(\kappa r_0)^{\frac{1}{2}}}T_H+O(r_0^{-\frac{3}{2}}), \tag{55}$$

then
$$t_H(\alpha;\kappa) = \frac{1}{\kappa}\Re T_H \tag{56}$$

and
$$T_H = \sum_{n=0}^{\infty}T_H^{(n)}, \tag{57}$$

where, from equations (50), it is seen that

$$\left.\begin{aligned}\kappa^{-1}T_H^{(0)} &\sim \cos\alpha-2\pi^{-\frac{1}{2}}e^{\frac{1}{4}\pi i}[\theta_0(-\alpha,\pi+\alpha)+\theta_0(\alpha,-\pi-\alpha)],\\ \kappa^{-1}T_H^{(1)} &\sim \tfrac{1}{2}i\kappa\pi^{-1}e^{-i\kappa}[e^{-i\kappa\sin\alpha}\theta_1(\alpha,\pi+\alpha)+e^{i\kappa\sin\alpha}\theta_1(-\alpha,-\pi-\alpha)],\\ \kappa^{-1}T_H^{(2)} &\sim \frac{e^{-\frac{1}{4}\pi i}}{4\sqrt{2\pi^2}}\frac{e^{-i2\kappa}}{\kappa^{\frac{1}{2}}}[\theta_2(-\alpha,\pi+\alpha)+\theta_2(\alpha,-\pi-\alpha)],\end{aligned}\right\} \tag{58}$$

while $\kappa^{-1}T_H^{(3)} \sim O(\kappa^{-4})$.

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For near-normal incidence, $\kappa p \gg 1, \kappa q \gg 1$, and it is found that $t_H(\alpha; \kappa)$ is of the form

$$t_H(\alpha; \kappa) \sim \cos\alpha + \frac{\cos^2\alpha}{2\sqrt{(2\pi)}\,\kappa^{\frac{5}{2}}}\left\{\frac{\cos[\kappa(1+\sin\alpha)+\frac{1}{4}\pi]}{(1+\sin\alpha)^3} + \frac{\cos[\kappa(1-\sin\alpha)+\frac{1}{4}\pi]}{(1-\sin\alpha)^3}\right\} + O(\kappa^{-\frac{7}{2}}), \tag{59}$$

while if $\alpha = 0$,

$$t_H(0; \kappa) \sim 1 + \frac{\cos(\kappa+\frac{1}{4}\pi)}{\sqrt{(2\pi)}\,\kappa^{\frac{5}{2}}} + O(\kappa^{-\frac{7}{2}}). \tag{60}$$

Higher-order terms are more readily found for the case of near-normal incidence by the alternative method to which reference has been made. The following result is quoted from (8)

$$t_H(0; \kappa) \sim 1 + \frac{\cos(\kappa+\frac{1}{4}\pi)}{\sqrt{(2\pi)}\,\kappa^{\frac{5}{2}}} + \frac{27}{8\sqrt{(2\pi)}}\frac{\cos(\kappa-\frac{1}{4}\pi)}{\kappa^{\frac{7}{2}}} - \frac{\cos 2\kappa}{8\pi\kappa^4} + O(\kappa^{-\frac{9}{2}}). \tag{61}$$

The expression (59) has been obtained by Levine (7). The two leading terms in the expansion for $t_H(0; \kappa)$ have been given previously by Clemmow (4).

If on the other hand $\alpha = \frac{1}{2}\pi$, so that the plane wave is at grazing incidence, then the terms of lowest order in equations (58) give

$$t_H(\tfrac{1}{2}\pi; \kappa) \sim 2(\pi\kappa)^{-\frac{1}{2}} + O(\kappa^{-\frac{3}{2}}).$$

Higher-order terms may be found from equations (58), but it is possible to determine them in the following more simple manner.

Let the total magnetic field in the illuminated half-space ($x_0 \geqslant 0$) in the absence of the slit be H_z^0. In the presence of the slit the total field in $x_0 > 0$ is $H_z^0 - H_z^p(-x_0, y_0)$, and in $x_0 < 0$, $H_z^p(x_0, y_0)$, where the function $H_z^p(x_0, y_0)$ is defined only for $x_0 \leqslant 0$. Then the transmission coefficient $t_H(\frac{1}{2}\pi; \kappa)$ may be deduced from knowledge of $\lim_{x_0 \to 0-} H_z^p(x_0, y_0)$ when $\alpha = \frac{1}{2}\pi$.

Now the discontinuity in the total magnetic field across the screen is numerically equal to the current density. Since

$$H_z^0 = 2e^{-i\kappa}e^{-i\kappa Y}$$

on the half of the screen corresponding to $\beta = \frac{3}{2}\pi$, it follows that

$$\lim_{x_0 \to 0-} H_z^p(x_0, y_0) = e^{-i\kappa}e^{-i\kappa Y} + \tfrac{1}{2}J^-(hY; \alpha),$$

for $\beta = \frac{3}{2}\pi$.

With $J^-(hY; \frac{1}{2}\pi)$ determined by equation (28), it is seen that for $\alpha = \frac{1}{2}\pi$ and large Y,

$$\lim_{x_0 \to 0-} H_z^p(x_0, y_0) \sim \left[\frac{2}{\pi} - \frac{i}{4\pi\kappa} + \frac{3}{64\pi\kappa^2} + O(\kappa^{-3})\right] Y^{-\frac{1}{2}}e^{-i\kappa(1+Y)} + O(Y^{-\frac{3}{2}}),$$

and therefore

$$t_H(\tfrac{1}{2}\pi; \kappa) \sim \frac{2}{\sqrt{\pi}\,\kappa^{\frac{1}{2}}} - \frac{1}{4\sqrt{\pi}\,\kappa^{\frac{3}{2}}} + \frac{3}{64\sqrt{\pi}\,\kappa^{\frac{5}{2}}} + O(\kappa^{-\frac{7}{2}}). \tag{62}$$

The first two terms in this series have been found by Levine (7). It is evident that the neglected terms will contribute an oscillatory fine structure to the asymptotic form of $t_H(\frac{1}{2}\pi; \kappa)$, but the effect will be quite small.

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3. *Diffraction by a wide strip.* By virtue of Babinet's principle (see Bouwkamp (3)), the solution to the problem of diffraction by a perfectly conducting strip of width h follows immediately from the foregoing deductions. In the present section, the current density on the strip will be found when the incident field is E- or H-polarized.

3·1. *The solution for E-polarization.* If the incident field is E-polarized and given by equation (1) of I, then the complementary field is H-polarized and is represented by equation (1). It follows that the tangential components of scattered magnetic field on the surfaces of the strip are

$$\mp H_y^s(0, -hY_0) = E_y(0, -hY_0),$$

where the upper and lower signs refer to the illuminated and dark sides of the strip respectively, and $E_y(0, -hY_0)$ is defined by equations (30) and (31).

If the current density on the strip is denoted by $I_z(-hY_0; \alpha)$, then

$$I_z(-hY_0; \alpha) \sim -2 \sum_{n=0}^{\infty} E_n(0, -hY_0), \tag{63}$$

where the first few terms in the series are given by equations (33), (34) and (43).

The scattering coefficient of the strip is equal to $2t_H(\alpha; \kappa)$.

3·2. *The solution for H-polarization.* When the incident field is H-polarized (equation (1)), the complementary field is E-polarized, and is the negative of equation (1) of I. Then the tangential components of scattered magnetic field on the surfaces of the strip are given by

$$\mp H_z^s(0, -hY_0) = -E_z(0, -hY_0),$$

where $E_z(0, -hY_0)$ is defined by equations (37), (38) and (39) of I, and the $\mp$ sign is chosen according to the above convention.

If the current density on the strip is denoted by $I_y(-hY_0; \alpha)$, then

$$I_y(-hY_0; \alpha) \sim -2 \sum_{n=0}^{\infty} E_n(0, -hY_0). \tag{64}$$

The first few terms in this series are given by equations (42), (43) and (60) of I.

The scattering coefficient of the strip is equal to $2t_E(\alpha; \kappa)$.

4. *Discussion and concluding remarks.* As an indication of the utility of the present theory, it is worth while to make a few comparisons with the published data of Skavlem (11) and Morse and Rubenstein (9).

By employing the method of separation of variables of the wave equation, Skavlem was able to calculate values of the transmission coefficient for slits of up to about four wavelengths in width. In Table 1 are compared the values of $t_E(0; \kappa)$ calculated by Skavlem, and from equation (79) of I. The effect of each term in this equation is made evident by writing $t_E(0; \kappa)$ in the form

$$t_E(0; \kappa) \sim t_E^{(1)} + t_E^{(2)} + t_E^{(3)} + t_E^{(4)} + O(\kappa^{-\frac{7}{2}}), \tag{65}$$

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where

$$\left.\begin{aligned} t_E^{(1)} &= 1 - \frac{2\sqrt{2}}{\sqrt{\pi}} \frac{\cos(\kappa - \frac{1}{4}\pi)}{\kappa^{\frac{3}{2}}}, \\ t_E^{(2)} &= \frac{2}{\pi} \frac{\cos 2\kappa}{\kappa^2}, \\ t_E^{(3)} &= -\frac{\sqrt{2}}{\pi^{\frac{3}{2}}} \left[\frac{\cos(3\kappa + \frac{1}{4}\pi) - \frac{7}{4}\pi \cos(\kappa + \frac{1}{4}\pi)}{\kappa^{\frac{5}{2}}} \right], \\ t_E^{(4)} &= -\frac{1}{\pi^2} \left[\frac{\sin 4\kappa - \frac{5}{2}\pi \sin 2\kappa}{\kappa^3} \right]. \end{aligned}\right\} \tag{66}$$

Table 1

κ	t_E (Skavlem)	$t_E^{(1)}$	$t_E^{(1)} + t_E^{(2)}$	$t_E^{(1)} + t_E^{(2)} + t_E^{(3)}$	$t_E^{(1)} + t_E^{(2)} + t_E^{(3)} + t_E^{(4)}$
2·0	0·54540	0·80326	0·69923	0·42854	0·34073
2·8	1·11719	1·14627	1·20925	1·13192	1·11355
4·0	1·18426	1·19894	1·19315	1·18859	1·20134
6	0·97202	0·94773	0·96266	0·97367	0·97211
8	0·94244	0·95792	0·94839	0·94088	0·94033
10	1·04992	1·04935	1·05194	1·05037	1·05102
12	0·99559	0·99166	0·99353	0·99596	0·99558
14	0·97174	0·97572	0·97259	0·97131	0·97141
16	1·02332	1·02196	1·02403	1·02337	1·02346
18	1·00199	1·00134	1·00109	1·00214	1·00200
20	0·98224	0·98333	0·98227	0·98206	0·98215

Table 2

κ	t_H(Skavlem)	$t_H^{(1)}$	$t_H^{(1)} + t_H^{(2)}$	$t_H^{(1)} + t_H^{(2)} + t_H^{(3)}$
2·0	0·99085	0·93390	0·97540	0·97703
2·8	0·98126	0·97253	0·95679	0·95629
4·0	0·99478	1·00091	1·00000	1·00003
6	—	1·00397	1·00519	1·00517
8	0·99896	0·99823	0·99879	0·99879
10	—	0·99974	0·99932	0·99932
12	—	1·00078	1·00083	1·00083
14	—	0·99967	0·99978	0·99978
16	0·99976	0·99982	0·99974	0·99974
18	—	1·00029	1·00029	1·00029
20	—	0·99992	0·99996	0·99996

It is apparent that equation (65) represents with considerable accuracy the behaviour of the transmission coefficient for $\kappa \gtrsim 4$, and, in fact, $t_E^{(1)}$ alone is quite adequate in this range. The effect of the higher-order terms is shown in an improvement of the approximation for the larger values of κ, but the range in which the asymptotic expression is valid does not appear to be extended by an appreciable amount towards $\kappa = 0$ by the additional terms.

A similar comparison between the asymptotic and exact values of the transmission coefficient for H-polarized waves may be found in Table 2. In analogy with equation (65), the expression (61) has been written in the form

$$t_H(0; \kappa) \sim t_H^{(1)} + t_H^{(2)} + t_H^{(3)} + O(\kappa^{-\frac{9}{2}}), \tag{67}$$

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where

$$\left.\begin{aligned} t_H^{(1)} &= 1+\frac{\cos(\kappa+\frac{1}{4}\pi)}{\sqrt{(2\pi)}\,\kappa^{\frac{5}{2}}}, \\ t_H^{(2)} &= \frac{27}{8\sqrt{(2\pi)}}\frac{\cos(\kappa-\frac{1}{4}\pi)}{\kappa^{\frac{7}{2}}}, \\ t_H^{(3)} &= -\frac{\cos 2\kappa}{8\pi\kappa^4}. \end{aligned}\right\} \tag{68}$$

The difference in behaviour between $t_E(0;\kappa)$ and $t_H(0;\kappa)$ is immediately apparent. The small amplitude in the oscillations of $t_H(0;\kappa)$ is a consequence of the relatively weak interaction between the half-planes when the incident field is H-polarized.

When $\kappa \gtrsim 4$, the behaviour of $t_H(0;\kappa)$ is predicted well by equation (67), and the term $t_H^{(1)}$ alone provides a good approximation in this range.

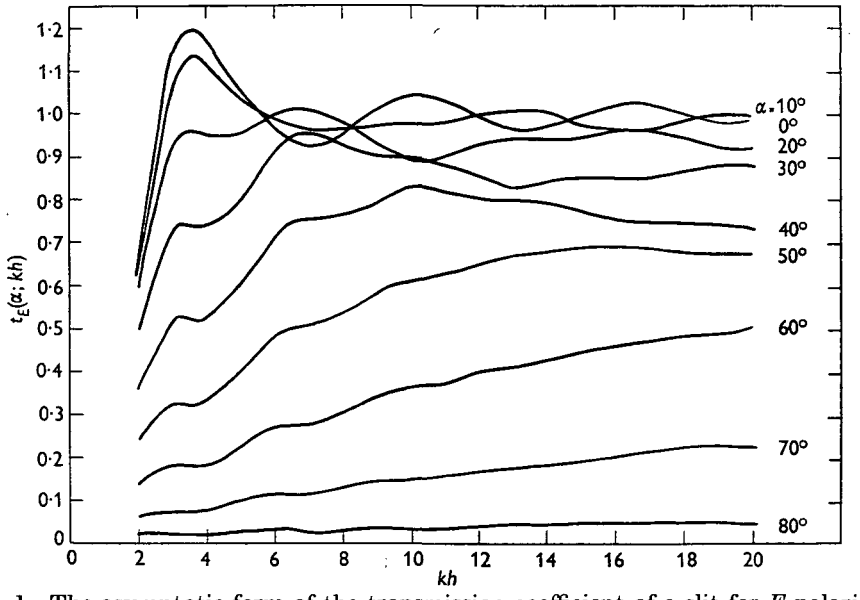


Fig. 1. The asymptotic form of the transmission coefficient of a slit for E-polarized plane waves as a function of kh, for various angles of incidence (α).

Figs. 1 and 2 illustrate the variation of the transmission coefficients with the angle of incidence, α, and κ. Fig. 1 is based on the first two equations (75) of I, including only the term $2h_0(\kappa p)\,\kappa^{-\frac{1}{2}}I_1(\beta)$ in $\phi_1(\alpha,\beta)$, while Fig. 2 corresponds to $\kappa^{-1}T_H^{(0)}$ in equations (58). There is qualitative agreement with the results of Morse and Rubenstein over the range of κ in which both are valid, although the peaks in Fig. 1 do not appear to be so pronounced as found by these authors.

It is likely that the correspondence could be made closer by the consideration of higher-order terms in the present development, especially in the case of H-polarization. That this is true is illustrated in Fig. 3 where values of $t_H(\frac{1}{2}\pi;\kappa)$ determined from equation (62) are compared with points scaled from the cited paper of Morse and Rubenstein. It is evident that extremely close agreement obtains even for κ less than 2.

The current induced on a ribbon by an E-polarized incident field has been calculated by Moullin and Phillips (10) from the formulae of Morse and Rubenstein. Similar

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calculations based on equation (63) of the present work have been performed elsewhere (Millar (8)). It has been found that certain of the curves computed by Moullin and Phillips do not agree with the results of the present theory, but the reason for

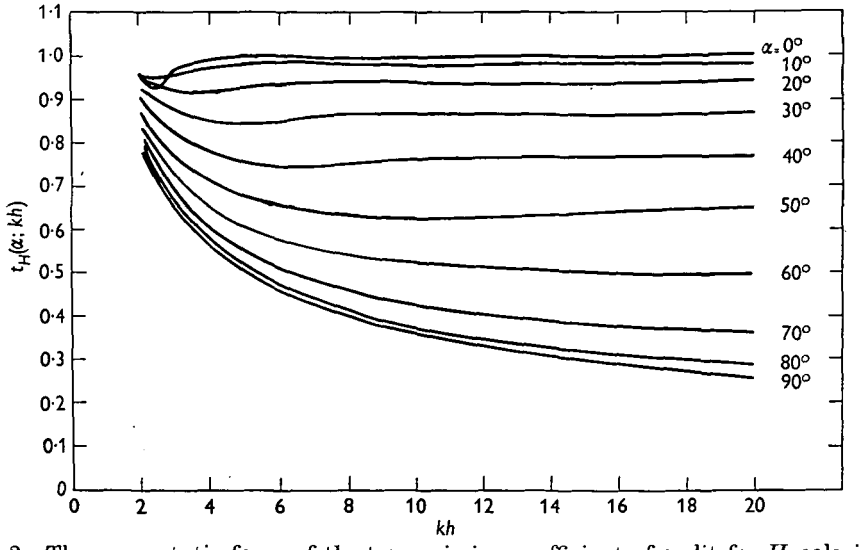


Fig. 2. The asymptotic form of the transmission coefficient of a slit for H-polarized plane waves as a function of kh, for various angles of incidence (α).

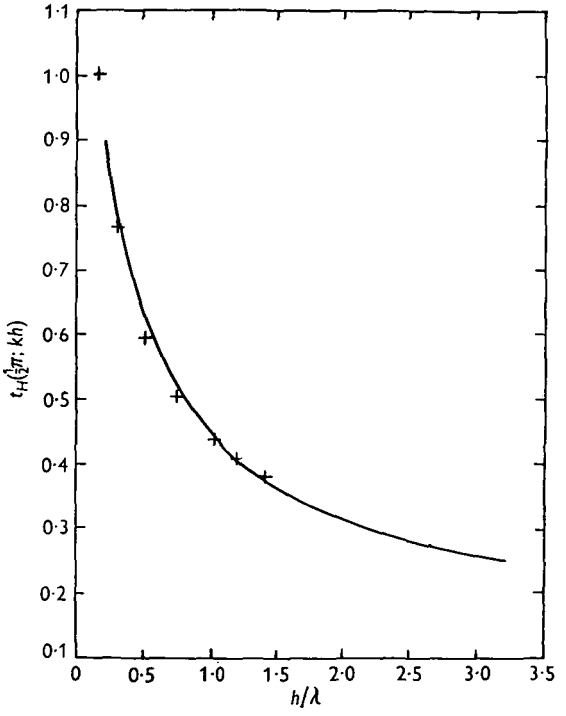


Fig. 3. A comparison between the asymptotic form of the transmission coefficient of a slit for H-polarized plane waves at grazing incidence and the results of Morse and Rubenstein. +, Morse and Rubenstein; curve, present theory. $k=2\pi/\lambda$.

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this discrepancy is not clear. It appears that, in general, the first term in equation (63) approximates closely to the density distribution, and only near the edge $Y_0 = 1$ (that is, the edge 'farther' from the incident wave if $0 < \alpha \leqslant \frac{1}{2}\pi$) may the higher-order terms play a significant role.

Evidently the approximate solutions of Karp and Russek ((5), (6)) and Clemmow (4), to which reference has been made in I, correspond to the present solution for $\kappa(1 \pm \sin\alpha) \gg 1$. In both it has been assumed that the excitation of one half-plane by the other may be represented as a series in inverse powers of $[kr(1 \pm \sin\alpha)]^{\frac{1}{2}}$, where r is the distance from the edge of the scattering half-plane. This series arises through the asymptotic expansion of the Fresnel integrals which occur in the half-plane solution.

However, when α approaches $\frac{1}{2}\pi$, one of the excited half-planes lies near the shadow boundary of the other, as well as near the edge of the wave reflected by the exciting surface. If the condition $\kappa(1 \pm \sin\alpha) \gg 1$ does not obtain, the exciting field in the important region near the edge of the other half-plane is not of the above form, although this representation will remain valid for sufficiently large values of r if $\alpha \neq \frac{1}{2}\pi$. In the above circumstances, the exciting field near the edge would have to be found by power series expansion of the Fresnel integral, and the diffraction of such a field by a half-plane has yet to be considered.

These difficulties do not arise in the present development, which yields a solution uniformly valid for all α. Of possible note is the occurrence of the function $\tan^{-1} Y^{\frac{1}{2}}$ which is associated with the current density induced by an H-polarized wave at grazing incidence. It has been remarked that the presence of a function with similar properties might have been expected on physical grounds; however, the precise form does not appear to have been previously determined.

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