J. Plasma Physics (1999), *vol.* 62, *part* 1, *pp.* 53–64. Printed in the United Kingdom
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Parametric excitation of electromagnetic waves in a magnetized plasma

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(Received 22 July 1998 and in revised form 15 October 1998)

Abstract. The parametric interaction of an upper-hybrid pump wave with kinetic Alfvén and electromagnetic waves that propagate in parallel and perpendicular directions to the ambient magnetic field is investigated on the basis of two-fluid magnetohydrodynamics. A nonlinear dispersion relation describing three-wave interaction and instability growth rates are found. The theoretical results are used for the interpretation of satellite observations in the magnetospheric plasma. It is shown that as a result of the decay of an upper-hybrid wave, electromagnetic waves propagating in both parallel and perpendicular directions to the ambient magnetic field are generated. The instability growth rate is much higher in the case of left-polarized electromagnetic wave generation than in the case of ordinary electromagnetic wave generation. The nonlinear parametric processes studied here could also take place during powerful bursts on the Sun, and in the magnetosphere of Jupiter.

1. Introduction

Direct satellite measurements of wave fields have revealed a wide spectrum of different oscillations and waves in the near-Earth and interplanetary plasma. Gurnett and Shaw (1973) and Kurth et al. (1979) reported the detection of low-frequency electromagnetic waves with frequency slightly higher than the local electron Langmuir frequency. It was also found that a source emitting electrostatic waves with frequency close to the upper-hybrid resonance frequency existed in the area of the electromagnetic emission. Important results concerning wave processes in the ionospheric plasma were obtained during heating experiments (Stubbe et al. 1984, 1994). The most interesting of these processes is the secondary electromagnetic emission caused by powerful electromagnetic waves in the ionospheric plasma. Satellite measurements show that upper-hybrid waves (UHW) are the most stable electrostatic waves in the magnetosphere (Oya et al. 1990). Therefore, it is reasonable to assume that UHW could be a possible source of electromagnetic emission in the magnetospheric plasma. Cascade decay processes involving UHW decay and generation are possible during ionospheric heating experiments, and may lead to secondary electromagnetic emission as well as to cascading of stimulated electromagnetic emission. Powerful sources of UHW also exist in the solar atmosphere. Charged particles are accelerated to high energies during powerful solar bursts. Energetic particle flows

https://doi.org/10.1017/S0022377899007709 Published online by Cambridge University Press

are launched into the corona and then into interplanetary space. These flows can generate UHW, which will generate electromagnetic emission. Parametric instability is a possible source of millisecond solar radio bursts (Wang and Li 1991). It is now widely accepted that *s*-radioemission of Jupiter is related to different plasma instabilities of the particle flows in the Io–Jupiter magnetic tube (Smith 1978). UHW can be generated as a result of flow instability evolution in the Io–Jupiter magnetic tube. Then, the UHW can decay and generate electromagnetic emission. Thus the transformation of UHW into electromagnetic emission is a widespread phenomenon in space plasmas.

Nonlinear parametric processes in space plasmas are always of great interest, and there are a number of publications devoted to them (Murtaza and Shukla 1984; Saleem et al. 1985; Yukhimuk et al. 1992; Stenflo and Shukla 1992, 1995; Stenflo 1994; Yukhimuk and Yukhimuk 1994; Shukla and Stenflo 1995). Murtaza and Shukla (1984) considered the transformation of UHW into electromagnetic waves. This paper on parametric wave–wave interactions has been quoted by many authors, and the results have been used for interpretation of observations in the ionosphere and the magnetosphere. In the complementary paper by Saleem et al. (1985), an additional decay channel for extraordinary-wave generation by pump UHW in particular, including shear Alfvén waves, was considered.

These successful applications of UHW–electromagnetic wave interaction to the particular problems of space physics lead us to consider once again a UHW decay with the participation of KAW. Three-wave parametric interaction of UHW with ordinary electromagnetic wave was studied by Yukhimuk et al. (1992) and Yukhimuk and Yukhimuk (1994).

In the present paper we study the decay of an upper-hybrid (UH) pump wave into a kinetic Alfvén wave (KAW) and an electromagnetic wave propagating in perpendicular directions to the ambient magnetic field and a left-hand circular, polarized electromagnetic wave. This problem has not been considered before.

A nonlinear dispersion relation for the electromagnetic waves that includes the nonlinear current generated by the beating of the UHW and the KAW is obtained. The electromagnetic wave field with the addition of the UH pump wave field generates a low-frequency force that leads to KAW growth.

A homogeneous magnetized plasma ($\mathbf{B}_0 = B_0\mathbf{e}_z$) is considered. The UH pump wave

$$\mathbf{E}_0 = (E_{0x}\mathbf{e}_x + E_{0z}\mathbf{e}_z)\exp[i(-\omega_0 t + k_{0x}x + k_{0z}z)] + c.c., \tag{1.1}$$

with $k_{0z} \ll k_{0x}$, propagates and decays into electromagnetic waves with frequencies ω_j and wave vectors $\mathbf{k}_j$ and a KAW with frequency ω and wave vector $\mathbf{k}$. It is assumed that the following wave synchronism condition is satisfied:

$$\omega_0 = \omega + \omega_j, \qquad \mathbf{k}_0 = \mathbf{k} + \mathbf{k}_j, \tag{1.2}$$

where $j = 1, 2$ (with '1' and '2' corresponding to the ordinary and the left-hand polarized electromagnetic waves respectively). We assume that all wave vectors are situated in the (x, z) plane.

2. Basic equations

For studying three-wave parametric interaction, we use two-fluid magnetohydrodynamics (MHD):

$$\frac{\partial \mathbf{v}_\alpha}{\partial t} = \frac{1}{m_\alpha}(e_\alpha \mathbf{E} + \mathbf{F}_\alpha) + (\mathbf{v}_\alpha \times \boldsymbol{\omega}_{B\alpha}) - \frac{T_\alpha}{m_\alpha n_\alpha}\boldsymbol{\nabla} n_\alpha, \tag{2.1}$$

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$$\frac{\partial n_\alpha}{\partial t} = -\nabla \cdot (n_\alpha \mathbf{v}_\alpha), \tag{2.2}$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}, \tag{2.3}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \tag{2.4}$$

$$\nabla \cdot \mathbf{E} = 4\pi\rho, \tag{2.5}$$

where

$$\mathbf{j} = e(n_i \mathbf{v}_i - n_e \mathbf{v}_e),$$

$$\rho = e(n_i - n_e),$$

$$\mathbf{F}_\alpha = \frac{e_\alpha}{c}(\mathbf{v}_\alpha \times \mathbf{B}) - m_\alpha(\mathbf{v}_\alpha \cdot \nabla)\mathbf{v}_\alpha.$$

The index $\alpha = i, e$ corresponds to ions and electrons respectively. The electron density and velocity and the electric and magnetic fields are written in the forms

$$n_e = n_0 + \tilde{n}_0 + \tilde{n}_{eA}, \tag{2.6a}$$

$$\mathbf{v}_e = \mathbf{v}_0 + \mathbf{v}_j + \mathbf{v}_A, \tag{2.6b}$$

$$\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_j + \mathbf{E}_A, \tag{2.6c}$$

$$\mathbf{B} = B_0 \mathbf{e}_z + \mathbf{B}_j + \mathbf{B}_A, \tag{2.6d}$$

where n_0 is the average value of the plasma density, $\tilde{n}_0$ is the electron density perturbation caused by the UH pump wave, indexes 0 and A in the expressions for $\mathbf{v}_e$ and $\mathbf{E}$ correspond to the UH pump wave and the KAW respectively. The index $j = 1, 2$ corresponds to the electromagnetic waves.

It should be mentioned that the electromagnetic radiation cannot produce a density perturbation because we are studying the case of parallel and perpendicular (respectively to the ambient magnetic field) propagation.

3. Generation of the ordinary electromagnetic wave

The source of radiation of the ordinary electromagnetic wave is the nonlinear current produced as a result of resonant interaction of the UH pump wave with the low-frequency KAW. The electromagnetic wave field with the addition of the UH pump wave field generates a low-frequency force leading to KAW amplification. A nonlinear dispersion relation for the KAW and a dispersion relation for the ordinary electromagnetic wave will be obtained later. Using these relations, we shall find a nonlinear dispersion relation describing three-wave interaction.

3.1. Nonlinear dispersion relation for KAW

To obtain the dispersion relation for the low-frequency perturbation, we use the plasma approximation

$$\tilde{n}_{eA} = \tilde{n}_{iA}, \tag{3.1}$$

where $\tilde{n}_{eA}$ and $\tilde{n}_{iA}$ are the perturbations of electron and ion densities respectively.

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From the parallel component of the equation of motion (2.1) and the continuity equation (2.2), we find the expression for $\tilde{n}_{eA}$ related to the KAW:

$$\frac{\tilde{n}_{eA}}{n_0} = \left(1 - \frac{v_{\rm ph}^2}{v_{T_e}^2}\right)^{-1} \frac{e}{T_e}\left[\varphi - A + \frac{k_x}{k_z^2}\frac{\omega}{\omega_{Be}}\frac{1}{e}\left(i\frac{\omega}{\omega_{Be}}F_x + F_y\right) + \frac{F_z}{iek_z}\right], \qquad (3.2)$$

where

$$v_{T_e} = \left(\frac{T_e}{m_e}\right)^{1/2}, \qquad v_{\rm ph} = \frac{\omega}{k_z}, \qquad A = \frac{\omega}{k_z c}A_z,$$

and φ and A_z are the scalar and vector potentials of the KAW electric field respectively. From the equation of motion and the continuity equation for ions, an expression for the ion density is found:

$$\frac{\tilde{n}_{iA}}{n_0} = -\frac{e}{T_i}\frac{\mu_i}{1+\mu_i}\left(\varphi + \frac{k_z^2}{k_x^2}\frac{\omega_{Bi}^2}{\omega^2}A\right), \qquad (3.3)$$

where

$$\mu_i = k_x^2\rho_i^2, \qquad \rho_i = \frac{v_{Ti}}{\omega_{Bi}}, \qquad V_{T_i} = \left(\frac{T_i}{m_i}\right)^{1/2}.$$

Next we exclude the potential A_z from the Ampère equation and the equation of motion for electrons. Taking into account that the electrical current along the magnetic field for a plasma with $\beta \ll 1$ is determined mostly by the electrons, we obtain

$$\frac{\tilde{n}_{eA}}{n_0} = \frac{e}{T_e}\left[\varphi - \left(1 + k_x^2\delta_e^2\right)A + \frac{F_z}{iek_z}\right], \qquad (3.4)$$

where $\delta_e = c/\omega_{pe}$ is the electron inertial length. Using (3.1) and the expressions (3.2) and (3.4) for the perturbations of electron and ion density, we find the KAW dispersion relation:

$$\varepsilon_A\varphi = Q_{NL}, \qquad (3.5)$$

where

$$\varepsilon_A = \omega^2 - k_z^2V_A^2\frac{1 + (1+t)\,\mu_i}{1+\kappa_e},$$

$$\kappa_e = k_x^2\delta_e^2, \qquad t = \frac{T_e}{T_i},$$

$$Q_{NL} = \frac{k_z^2V_A^2(1+\mu_i)}{\kappa_e}\left[\frac{k_x\omega}{ek_z^2\omega_{Be}}\left(i\frac{\omega}{\omega_{Be}}F_x + F_y\right) + \frac{\kappa_eF_z}{iek_z(1+\kappa_e)}\right],$$

where the components of the ponderomotive force

$$F_{x,y} = -\frac{e}{c}(\mathbf{v}_e \times \mathbf{B})_{x,y} - m_e(\mathbf{v}_e \cdot \boldsymbol{\nabla})v_{ex,y}, \qquad (3.6)$$

are determined by the interaction of the UH pump wave with the ordinary electromagnetic wave, and V_A is the Alfvén speed. For $\beta \ll m_e/m_i$, the effects produced by the finite electron inertia length are dominant, but for $m_e/m_i \ll \beta \ll 1$ finite-Larmor-radius effects are more important. Therefore the finite-Larmor-radius effects are more important in the regions of the magnetosphere considered here.

Next we consider the ordinary electromagnetic wave propagating along the x axis, for which the wave electric field is parallel to the z axis. Then the equation of

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motion for an electron in the field of the ordinary electromagnetic wave is given by

$$\frac{\partial v_{1z}}{\partial t} = -\frac{eE_{1z}}{m_e}, \tag{3.7}$$

from which we find v_{1z}:

$$v_{1z} = -i\frac{eE_{1z}}{m_e\omega_1}. \tag{3.8}$$

From the equation of motion and the continuity equation we find the electron velocity components and the electron density perturbation related to UH pump-wave propagation:

$$V_{0x} = -v_{T_e}\frac{\omega_0 k_{0x} v_{T_e}}{\omega_0^2 - \omega_{Be}^2}\Phi_0, \tag{3.9a}$$

$$V_{0y} = -iv_{T_e}\frac{\omega_{Be} k_{0x} v_{T_e}}{\omega_0^2 - \omega_{Be}^2}\Phi_0, \tag{3.9b}$$

$$V_{0z} = -v_{T_e}\frac{k_{0z} v_{T_e}}{\omega_0}\Phi_0, \tag{3.9c}$$

$$\frac{\tilde{n}_0}{n_0} = -\left(\frac{k_{0x}^2 v_{T_e}^2}{\omega_0^2 - \omega_{Be}^2} + \frac{k_{0z}^2 v_{T_e}^2}{\omega_0^2}\right)\Phi_0, \tag{3.9d}$$

where $\Phi_0 = e\varphi_0/T_e$. Using (3.8) and (3.9), the dispersion relation (3.5) can be written in the form

$$\varepsilon_A \varphi = \mu_A \varphi_0 E_{1z}^*, \tag{3.10}$$

where μ_A is the coupling coefficient, given by

$$\mu_A = -i\frac{e}{m_e}\left(\frac{V_A}{c}\right)^2 \frac{\omega}{\omega_1}\frac{k_{0z}k_{0x}}{k_x}(1+\mu_i)\left(1 + \kappa_e \frac{k_z}{k_x}\frac{k_{0z}}{k_{0x}}\frac{\omega_{pe}^2}{\omega_0\omega}\right).$$

3.2. Dispersion relation for the ordinary electromagnetic wave

An ordinary electromagnetic wave propagating along the x axis with electric field parallel to the z axis is considered. Excluding the magnetic field from the Maxwell's equations (2.3) and (2.4), we obtain the equation for the electric field of the ordinary electromagnetic wave:

$$\varepsilon_1 E_{1z} = -\frac{\omega_{pe}^2}{e}F_{1z} + 4\pi i\omega_1 e(nv_z)_{NL}, \tag{3.11}$$

where $\varepsilon_1 = \omega_1^2 - k_1^2c^2 - \omega_{pe}^2$ and

$$(nv_z)_{NL} = \tilde{n}_0 v_z^* + \tilde{n}_e^* v_{0z}. \tag{3.12}$$

The velocity components in the electric field of the UH pump wave are defined by (3.9), and the velocity components in the field of the KAW are found from the equation of motion and the continuity equation for the electrons:

$$v_x = v_s\mu_s\left(\frac{\omega}{\omega_{Be}}\right)\Phi, \qquad v_y = iv_s\mu_s\Phi, \tag{3.13a,b}$$

$$v_z = -V_A(1+t)\mu_i\Phi, \qquad \frac{\tilde{n}_e}{n_0} = -(1+t)\mu_i\Phi, \tag{3.13c,d}$$

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where

$$v_s = \left(\frac{T_e}{m_i}\right)^{1/2}, \qquad \mu_s = k_x\rho_s, \qquad \rho_s = \frac{v_s}{\omega_{Bi}}, \qquad \Phi = \frac{e\varphi}{T_e}.$$

From Maxwell's equations (2.3) and (2.4), we find the relation between the potentials φ and A_z:

$$A_z = \frac{\omega}{k_z V_A}\frac{c}{V_A}\varphi.$$

Using this relation, an expression for the KAW magnetic field perturbation is found:

$$B_y = -ik_x\frac{c}{V_A}\frac{T_e}{e}\Phi. \tag{3.14}$$

From (3.11)–(3.14), we find the dispersion relation for the ordinary electromagnetic wave:

$$\varepsilon_1 E_{1z} = \mu_1\varphi_0\varphi^*, \tag{3.15}$$

where the coefficient μ_1 is given by

$$\mu_1 = -i\frac{e}{m_e}\omega_0\frac{k_x k_{0x}}{V_A}\left[1 - \mu_s^2\frac{V_A^2}{v_{Te}^2}\frac{k_{0x}}{k_x}\frac{\omega_1}{\omega_0}\left(1 + \frac{k_{0z}}{k_{0x}}\frac{\omega_{pe}^2}{V_A\omega_0 k_{0x}}\right)\right].$$

3.3. Nonlinear dispersion relation

From (3.10) and (3.15), we find the nonlinear dispersion relation describing three-wave interaction (decay of the UH pump wave into a KAW and an ordinary electromagnetic wave):

$$\varepsilon_A\varepsilon_1^* = \mu_A\mu_1^*|\varphi_0|^2. \tag{3.16}$$

Assuming in (3.16) that

$$\omega = \omega_r + i\gamma_1,$$

$$\omega_1 = \omega_{1r} + i\gamma_1,$$

(where $|\gamma| \ll \omega_r, \omega_{1r}$) and decomposing ε_A and ε_1 into series in the small parameter $i\gamma_1$, we obtain an expression for the instability growth rate:

$$\gamma_1 = \left.\frac{\mu_A\mu_1^*|\varphi_0|^2}{\dfrac{\partial\varepsilon_A}{\partial\omega}\dfrac{\partial\varepsilon_1}{\partial\omega_1}}\right|_{\omega=\omega_r,\omega_1=\omega_{1r}}, \tag{3.17}$$

where ω_r and ω_{1r} can be found from the equations

$$\varepsilon_A(\omega_r, \mathbf{k}) = 0, \qquad \varepsilon_1(\omega_1, \mathbf{k}_1) = 0.$$

Substituting the values of the derivatives

$$\frac{\partial\varepsilon_A}{\partial\omega} = 2\omega, \qquad \frac{\partial\varepsilon_1}{\partial\omega_1} = 2\omega_1$$

and the coefficients μ_A and μ_1 into (3.17), we obtain

$$\gamma_1 = \left(\frac{W}{2}\right)^{1/2}\frac{v_{Te}}{c}\left(\frac{\omega}{\omega_1}\right)^{1/2}\omega_{pe}\left(1 - \rho_s^2\frac{V_A^2}{v_{Te}^2}\frac{\omega_1}{\omega_0}k_{0x}^2\frac{k_x}{k_{0x}}\right)^{1/2}\left(1 + \delta_e^2 k_z k_{0z}\frac{\omega_{pe}^2}{\omega_0\omega}\frac{k_x}{k_{0x}}\right)^{1/2}, \tag{3.18}$$

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where

$$W = \frac{|E_{0x}|^2}{4\pi n_0 T_e}.$$

4. Generation of the left-hand polarized electromagnetic wave

The source of radiation of the left-hand polarized electromagnetic wave is also the nonlinear current produced by the resonant interaction of the UH pump wave with the low-frequency KAW. But the coefficient in the nonlinear dispersion equation for the three-wave interaction is different from the coefficient obtained previously because of difference between the electron velocity components in the fields of the left-hand polarized and ordinary electromagnetic waves.

4.1. Dispersion relation for the left-hand polarized electromagnetic wave

Excluding the magnetic field from Maxwell's equations (2.3) and (2.4), we obtain an equation for the electric field of the left-hand polarized electromagnetic wave:

$$\left(\frac{\partial^2}{\partial t^2} - c^2\frac{\partial^2}{\partial z^2}\right) E_2 - 4\pi e n_0 \frac{\partial v}{\partial t} = -4\pi \frac{\partial j_{NL}}{\partial t}, \tag{4.1}$$

where the nonlinear current density is determined by the beating of the pump wave and the KAW:

$$j_{NL} = -e\left(n^* v_0 + \tilde{n}_0 v^*\right). \tag{4.2}$$

The frequency and wave vector of the left-hand polarized electromagnetic wave are related by the expression

$$\frac{k_2^2 c^2}{\omega_2^2} = 1 - \frac{\omega_{pe}^2}{\omega_2(\omega_2 + \omega_{Be})}. \tag{4.3}$$

The electron velocities and densities in the fields of the pump wave and the KAW are given by (3.9) and (3.13).

Using (3.9) and (3.13), we obtain the dispersion relation for the left-hand polarized electromagnetic wave from (4.1) and (4.2):

$$\varepsilon_2 E_{2x} = \mu_2 \varphi_0 \varphi^*, \tag{4.4}$$

where

$$\varepsilon_2 = \omega_2^2 - k_2^2 c^2 - \omega_{pe}^2 \frac{\omega_2}{\omega_2 + \omega_{Be}},$$

$$\mu_2 = i\frac{e}{m_e}\left(\frac{c}{V_A}\right)^2 \frac{\omega_0 \omega_2}{\omega_0^2 - \omega_{Be}^2} k_x^2 k_{0x},$$

and ω_2 and k_2 are the frequency and wave vector of the left-hand polarized electromagnetic wave.

4.2. Dispersion relation for the KAW

The dispersion relation for the KAW was obtained in Sec. 3.1 and given by (3.5), where the components of ponderomotive force $F_{x,y,z}$ are determined by the interaction of the UH pump wave with the left-hand polarized electromagnetic wave.

To find $F_{x,y,z}$, it is necessary to know the electron velocity components in the field of the left-hand polarized electromagnetic wave. The electron velocity components in the field of the UH pump wave are known and are given by (3.9).

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For the left-hand polarized electromagnetic wave, the electric field is given by

$$\mathbf{E}_2 = E_x(\mathbf{e}_x - i\mathbf{e}_y),$$

and the electron velocity in the wave field is given by the expression

$$\mathbf{v}_2 = -i\frac{e\mathbf{E}_2}{m_e(\omega_2 + \omega_{Be})}. \tag{4.5}$$

From (3.2), (3.3), (3.9) and (4.5), we find the nonlinear dispersion relation for the KAW:

$$\varepsilon_3\varphi = \mu_3\varphi_0 E_{2x}^*, \tag{4.6}$$

where the coupling coefficient is given by

$$\mu_3 = -i\frac{e}{m_e}\left(\frac{V_A}{c}\right)^2\frac{k_{0x}}{k_x}\frac{\omega}{\omega_2}k_{0x}\left\{1+\frac{\omega_{pe}^2}{\omega_0\omega_2}\frac{k_{0z}^2}{k_{0x}^2}\left[\frac{k_{2z}}{k_{0x}}-\frac{\kappa_e}{1+\kappa_e}\frac{k_{0x}}{k_x}\left(1-\frac{k_{2z}}{k_{0z}}\right)^2\right]\right\}. \tag{4.7}$$

4.3. Nonlinear dispersion relation

From (4.4) and (4.6), we find the nonlinear dispersion relation describing three-wave interaction (decay of the UH pump wave into a KAW and a left-hand polarized electromagnetic wave):

$$\varepsilon_2\varepsilon_3^* = \mu_2\mu_3^*|\varphi_0|^2. \tag{4.8}$$

Assuming in (4.8) that

$$\omega = \omega_r + i\gamma_2,$$

$$\omega_2 = \omega_{2r} + i\gamma_2,$$

(where $|\gamma_2| \ll \omega_r, \omega_{2r}$), we obtain an expression for the instability growth rate:

$$\gamma_2 = \left.\frac{\mu_2\mu_3^*|\varphi_0|^2}{\dfrac{\partial\varepsilon_2}{\partial\omega}\dfrac{\partial\varepsilon_3}{\partial\omega_2}}\right|_{\omega=\omega_r,\omega_2=\omega_{2r}}, \tag{4.9}$$

where ω_r and ω_{2r} can be found from the equations

$$\varepsilon_2(\omega_r, \mathbf{k}) = 0, \qquad \varepsilon_3(\omega_{2r}, \mathbf{k}_2) = 0.$$

Substituting the values of the coefficients μ_2 and μ_3 and the derivatives $\partial\varepsilon_2/\partial\omega$ and $\partial\varepsilon_3/\partial\omega_2$ into (4.9), we obtain

$$\gamma_2 = \frac{W^{1/2}}{2}k_{0x}\lambda_d\omega_{pe}\left\{1+\frac{\omega_{pe}^2}{\omega_0\omega_2}\frac{k_{0z}^2}{k_{0x}^2}\left[\frac{k_{2z}}{k_{0z}}-\frac{\kappa_e}{1+\kappa_e}\frac{k_{0x}}{k_x}\left(1-\frac{k_{2z}}{k_{0z}}\right)^2\right]\right\}^{1/2}, \tag{4.10}$$

where $\lambda_d = v_{Te}/\omega_{pe}$.

5. Modified parametric decay including KAW

The results of the previous sections are not applicable if the nonlinear increment γ_{NL} is greater than KAW frequency. In this case, we consider a modified parametric decay, where the frequency ω of low-frequency wave is determined mainly by the

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nonlinear term $\omega \gg \omega_k$:

$$\omega^2\varphi = \mu_{A,3}\varphi_0 E^*_{1,2},$$
$$(\omega^2_{1,2} - \omega^2_{1,2k})E_{1,2} = \mu_{1,2}\varphi_0\varphi^*.$$

Using the resonant condition $\omega_{1,2} = \omega_0 - \omega$ and assuming $|\omega_{1,2} - \omega_{1,2k}| \ll \omega_{1,2k}$, we get a nonlinear dispersion relation for modified instability:

$$\omega^3 = -2\omega_k\gamma^2_{NL}, \tag{5.1}$$

where

$$\gamma^2_{NL} = \frac{\mu_{1,2}\mu^*_{A,3}|\varphi_0|^2}{2\omega_k\omega_{1,2k}}. \tag{5.2}$$

Equation (5.1) has the unstable solution

$$\omega = \frac{1 + i\sqrt{3}}{2}\left(2\omega_k\gamma^2_{NL}\right)^{1/3}, \tag{5.3}$$

and therefore the growth rate of the modified parametric decay

$$\gamma_{md} = \frac{\sqrt{3}}{2^{2/3}}\left(\frac{\omega_k}{\gamma_{NL}}\right)^{1/3}\gamma_{NL}. \tag{5.4}$$

6. Application and discussion

The results obtained above are applicable to the explanation of phenomena observed in space plasmas. First let us consider magnetospheric wave phenomena. Satellite observations show the presence of electromagnetic waves with frequency about the upper-hybrid resonance frequency in the Earth's magnetosphere (Gurnett and Shaw 1973, Kurth et al. 1979). Taking into account the presence of UHW in the region of electromagnetic emission generation, we have good reason to assume that UHW are the source of this emission. To estimate the growth rate and characteristic time of the electromagnetic emission, we use the data cited by Decreau et al. (1982); according to these data, the typical values of magnetosphere plasma parameters on the GEOS-1 orbit are $n_0 \approx 10\ \mathrm{cm}^{-3}$, $T_e \approx T_i \approx 1$ eV, $E_{0x} \approx 1\ \mathrm{mV\ m}^{-1}$ and $W \approx 10^{-5}$.

First we estimate the instability growth rate and characteristic time for the case of an UH pump wave decay into an ordinary electromagnetic wave and a KAW. With $v_{T_e}/c \approx 10^{-3}$, $\omega/\omega_1 \approx 10^{-4}$, $\omega_{pe} \approx 1.7 \times 10^5\ \mathrm{s}^{-1}$, $V_A/v_{T_e} \approx 0.2$ and $\rho_s \approx 10^5$ cm, the instability growth rate, given by (3.18), attains its maximum value $\gamma_1 \approx 0.3\ \mathrm{s}^{-1}$ at $k_x/k_{0x} \approx -10$. The minus sign here indicates that the excited KAW propagates in the direction opposite to that of the pump wave. The characteristic time of instability development $\tau_1 \approx 3$ s. The generation of KAW and X-mode radiation in the perpendicular direction was considered by Saleem et al. (1985), and the increment was found to be of the order of Ω_e. Since the perpendicular O-mode increment is of the order of ω_{pe}, for wave vectors of the same order, generation of the perpendicular O-mode is ω_{pe}/Ω_e times more effective than generation of the perpendicular X mode. On the other hand, the increment of the perpendicular X mode is an increasing function of wavelength, and, at a sufficiently long wavelength, may overcome the increment of the perpendicular O-mode.

However, the perpendicular wavelength is restricted by the perpendicular inhomogeneity scale, which may be very short in a space plasma. Then, the generation

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of the perpendicular X mode is sensitive to perpendicular inhomogenety, whereas excitation of the perpendicular O mode is most efficient at fixed $\lambda_x = 0.1\lambda_{0x}$, that is $\lambda_r \approx \lambda_{0x}$.

The instability threshold may be found from $\gamma_1 > (\gamma_1^L \gamma_K^L)^{1/2}$, where $\gamma_1^L = -0.5\nu_{ei}$ is the collisional damping of the electromagnetic wave and

$$\gamma_K^L = -\omega \left(\frac{\pi}{8}\right)^{1/2} \frac{V_A}{v_{Te}} \frac{\mu_i}{1 + (1+t)\mu_i}$$

is the Landau damping of the KAW; collisional damping has been neglected for the high-frequency KAW (Hasegawa and Chen 1976). This condition is easily satisfied, because in the region of interest of the magnetosphere $\gamma_1^L \approx 10^{-3}\ \mathrm{s}^{-1}$ and the Landau damping of KAW $\gamma_K^L \approx 10^{-1}\ \mathrm{s}^{-1}$. The decay threshold is

$$E_{\mathrm{cr}} = 2\frac{m_e c}{e}\frac{\omega_0}{\omega_{pe}}\left(\frac{\omega}{\omega_0}\gamma_K^L\gamma_1^L\right)^{1/2} \approx 1\ \mathrm{mV\ m}^{-1}.$$

Now, for the same UHW parameters, we estimate the instability growth rate and characteristic time for the case of UH pump-wave decay into a left-hand polarized electromagnetic wave and a KAW. By setting $k_{0x}^2\lambda_d^2 \approx 0.1$ and $k_{0z}/k_{0x} \approx 10^{-3}$ in (4.10), we obtain the maximum value of the growth rate $\gamma_2 \approx 50\ \mathrm{s}^{-1}$ and the characteristic time of instability evolution $\tau_1 \approx 0.02$ s. As we can see, $\gamma_2 > \omega$, which contradicts our original assumption that $\gamma_2 < \omega$. Estimation of the increment of modified decay in this case gives $\gamma_{md} \approx 32\ \mathrm{s}^{-1}$.

The estimates mentioned above show that as a result of three-wave parametric interaction, generation of the left-hand polarized electromagnetic wave is much faster than that of the ordinary electromagnetic wave. We note the agreement of our results with the conclusions of Murtaza and Shukla (1984), where UHW decay into a left-hand polarized electromagnetic wave and a lower-hybrid wave.

The nonlinear three-wave parametric interaction studied in this paper could take place in the Earth's atmosphere during heating experiments (Stubbe et al. 1984, 1994), where secondary electromagnetic emission was observed. This secondary emission appeared and could be observed several milliseconds after heating of the ionosphere by a powerful electromagnetic wave. One can suggest that the following cascade process takes place: UHW are generated by powerful radioemission, and while scattering on fluctuations (KAW, lower-hybrid waves and ion acoustic waves), generate electromagnetic emission at frequencies close to the UH frequency.

The proposed electromagnetic emission mechanism could be responsible for the *s*-radioemission of Jupiter. According to observations, Jovian *s*-radioemission has frequencies lower than 39.5 MHz, and its power rapidly decreases on approaching the frequency 39.5 MHz (Korr and Desh 1979; Rjabov and Gerasimova 1990). So the upper cut-off frequency of the Jovian *s*-radioemission spectrum could be interpreted according to the proposed mechanism as the UH resonance frequency $\omega_h^2 = \omega_{pe}^2 + \omega_{Be}^2$.

Nonlinear mechanism of electromagnetic emission can also work in the solar corona during powerful bursts. Type III radiobursts are the most interesting from a theoretical point of view. They are observed in a wide frequency range, varying from tenths of kilohertz (satellite observations) up to hundreds of megahertz while the emission source is moving from the Sun to the Earth. The source is beams of particles accelerated during the bursts and propagating from the Sun. UHW generated by high-energy particle beams and scattering on KAW (and on lower-hybrid

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and ion acoustic waves) will generate electromagnetic emission with frequencies close to the UH resonance frequency.

Thus there are several possible manifestations of the considered phenomenon of UHW transformation into electromagnetic emission in space plasmas, namely in the ionosphere, in the magnetosphere, in the Io–Jupiter magnetic tube and in the solar corona.

Acknowledgement

This work was supported in part by INTAS Grant 96-530.

References

Decreau, P. M., Bechin C. and Parrot, M. 1982 Global characteristics of the cold plasma in the equatorial plasmapause region as deduced from GEOS 1 mutual impedance probe. *J. Geophys. Res.* **87**, 695–712.

Gurnett, D. A. and Shaw, R. R. 1973 Electromagnetic radiation trapped in the magnetosphere above the plasma frequency. *J. Geophys. Res.* **78**, 8136–8149.

Hasegawa, A. and Chen, L. 1976 Kinetic processes in plasma heating by resonant mode conversion of Alfvén wave. *Phys. Fluids* **30**, 1924–1934.

Korr, T. D. and Desh, M. D. 1979 New Jupiter observations in decametric and gectametric regions. In: *Jupiter: Studies of the Interior, Atmosphere, Magnetosphere and Satellites* (ed. T. Gehrels), Vol. 3, pp. 91–144. Mir, Moscow (in Russian).

Kurth, W. S., Ashour-Abdall, M., Frank, L. A., Kennel, C. F., Gurnett, D. A., Sentman, D. D. and Burck, B. G. 1979 A comparison of intense electrostatic waves near FUHR with linear instability theory. *Geophys. Res. Lett.* **6**, 487–490.

Murtaza, G. and Shukla, P. K. 1984 Nonlinear generation of electromagnetic waves. *J. Plasma Phys.* **31**, 423–436.

Oya, H., Marioka, A., Kobayashi, K., Izima, M., Ono, T., Miyaoka, H., Okada, T. and Obara, T. 1990 Plasma wave observation and sounder experiments (PWS) using the Akebano (EXOS-D) satellite-instrumentation and initial result including discovery of the high altitude equatorial plasma turbulence. *J. Geomagn. Geoelectr.* **42**, 441–442.

Rjabov, B. P. and Gerasimova, N. N. 1990 *Decametric Sporadic Radio Emission from Jupiter.* Naykova Dymka, Kyiv (in Russian).

Saleem, H., Chaudhry, M. B., Murtaza, G. and Shukla, P. K. 1975 Generation of extraordinary mode radiation by an electrostatic pump. *Phys. Fluids* **28**, 830.

Shukla, P. K. and Stenflo, L. 1995 Nonlinear Alfvén waves. *Physica Scripta*, **60**, 32.

Smith, P. A. 1978 The models of Jupiter decametric emission. In: *Jupiter: Studies of the Interior, Atmosphere, Magnetosphere and Satellites* (ed. T. Gehrels), Vol. 1, pp. 433–485. Mir, Moscow (in Russian).

Stenflo, L. 1994 Resonant three-wave interactions in plasmas. *Physica Scripta*, **50**, 15.

Stenflo, L. and Shukla, P. K. 1992 Generation of radiation by upper hybrid waves in non-uniform plasmas. *Planet. Space Sci.* **40**, 473-476.

Stenflo, L. and Shukla, P. K. 1995 Generation of radiation by upper hybrid waves. *J. Geophys. Res.* **100**, 17261.

Stubbe, P., Kopka, H., Thide, B. and Derblom, H. 1984 Stimulated electromagnetic emissions: a new technique to study the parametric decay instability in the ionosphere. *J. Geophys. Res.* **89**, 7523–7536.

Stubbe, P., Stoker, A. J., Honary, F., Robinson, T. K. and Jones, T. B. 1994 Stimulated electromagnetic emissions and anomalous HF wave absorption near electron gyroharmonics. *J. Geophys. Res.* **99**, 6233–6246.

Wang, D.-Y. and Li, D.-Y. 1991 Nonlinear parametric instability and millisecond solar radio spikes. *Sol. Phys.* **135**, 393–405.

https://doi.org/10.1017/S0022377899007709 Published online by Cambridge University Press

Yukhimuk, A. K., Kotsarenko, N. Ja. and Yukhimuk, V. A. 1992 Nonlinear interaction of Alfvén waves in solar atmosphere. In: *Study of the Solar-Terrestrial System: Proceedings of 26th ESLAB Symposium Killarney, 16–19 June*, pp. 337–341.

Yukhimuk, A. K., Yukhimuk, V. A. and Kucherenko, V. P. 1995 Nonlinear mechanism for the excitation of Alfvén waves in a magnetized plasma. *Kinem. Phys. Celest. Bodies* **11**, 57–63.

Yukhimuk, V. A. and Yukhimuk, A. K. 1994 Parametric excitation of upper-hybrid and ion-acoustic waves in the space plasma. *Kinem. Phys. Celest. Bodies* **10**, 67–73.

https://doi.org/10.1017/S0022377899007709 Published online by Cambridge University Press