A TWO-DIMENSIONAL PROBLEM FOR A ROTATING MAGNETO-THERMOELASTIC HALF-SPACE WITH VOIDS AND GRAVITY IN A TWO-TEMPERATURE GENERALIZED THERMOELASTICITY THEORY

S. Deswal N. Hooda*

Department of Mathematics
Guru Jambheshwar University of Science and Technology
Haryana,India

ABSTRACT

The present paper is concerned with an in-depth study of the effects of rotation, two-temperature parameter and voids on the magneto-thermoelastic interactions in a homogeneous, isotropic, generalized half-space with gravity field. The formulation is applied within the frame-work of two-temperature generalized thermoelasticity based on the hyperbolic heat conduction model with one relaxation time. Using normal mode analysis technique for the physical variables appearing in the governing equations, we get the analytical expressions for displacement components, stress, thermodynamic temperature, conductive temperature and change in volume fraction field. The general solution obtained is then applied to a specific problem of an infinite half-space having isothermal boundary subjected to mechanical load. Variations of the considered variables through the vertical distance are illustrated graphically.

Keywords: Two-temperature, Generalized thermoelasticity, Voids, Normal mode analysis.

1. INTRODUCTION

The dynamical interactions between the thermal and mechanical fields in solids have a lot of applications, utilitarian aspects, especially, in modern aeronautics, astronautics, nuclear reactors, high energy particle accelerators and industrial engineering *etc*. Nowadays the dynamical problems associated with an elastic body analysed in the context of thermoelasticity with second sound effect have a great role in various fields of science and technology. The classical linear uncoupled and coupled theories of thermoelasticity involve infinite speed of propagation of thermal signals, which contradicts the physical observations. Non-classical theories (Generalized theories) of thermoelasticity claim for finite speed of heat transportation *i.e.* these theories advocate the so-called "second sound" effects. Lord and Shulman [1] presented a generalized thermoelasticity theory by involving a flux-rate term into Fourier's Law of heat conduction. This theory admits finite speed of thermal signals by involvement of a hyperbolic-type heat transport equation. Another important generalization of the linear coupled thermoelasticity theory was suggested by Green and Lindsay [2]. They developed temperature-rate-dependent thermoelasticity theory by involving two relaxation times. This theory modifies all the equations of coupled theory except the heat conduction equation. It doesn't violate the classical Fourier's law of heat conduction. Hetnarski and Ignaczak [3] presented closed form solutions for thermoelastic problems in generalized theory of thermoelasticity.

Materials having small distributed pores may be called porous materials or materials with voids. For accurate study of thermoelastic interactions, we cannot neglect the presence of voids in various geological and biological materials like rocks, sponge, soils and manufactured porous materials like ceramics, pressed powders and cork *etc*. Thus classical theory is inadequate for porous materials. Therefore, researchers presented one more important generalization of the classical theory of elasticity by including void volume as one more kinematic variable. There are a number of theories regarding mechanical behaviour of porous materials. Concept of a distributed body was introduced by Goodman and Cowin [4] which represents a continuum model for granular as well as porous materials. Nunziato and Cowin [5] presented a non-linear theory for the behaviour of porous solids in which the skeletal or matrix material is elastic and the interstices are void of material. In this theory, the bulk density is written as the product of two fields, the matrix material density field and the volume fraction field. The linearized theory of elastic materials with voids is given by Cowin and Nunziato [6]. Iesan [7] established a linear theory of thermoelastic materials with voids. He presented the basic field equations and discussed about the conditions for propagation of acceleration waves in a homo-

* Corresponding author (hooda.neelam84@gmail.com)

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geneous isotropic thermoelastic material with voids. He showed that transverse wave propagates without affecting the temperature and porosity of the material. Tomar [8] presented a survey article on the dynamical problems in elastic materials with voids. Domain of influence theorem in the linear theory of elastic materials with voids was discussed by Dhaliwal and Wang [9]. Dhaliwal and Wang [10] also formulated a thermoelasticity theory for elastic materials with voids to include heat flux among the constitutive variables.

Generalized thermoelasticity theory with two temperatures is now well-known. Gurtin and Williams [11,12] suggested the second law of thermodynamics for continuous bodies in which the entropy due to heat conduction was governed by one temperature, that of the heat supply by another temperature. Based on this suggestion, Chen and Gurtin [13] and Chen *et al.* [14,15] presented a new theory including two temperatures of heat conduction in deformable bodies. This theory suggests that heat conduction comprises of two distinct temperatures, *i.e.*, the conductive temperature φ and the thermodynamic temperature θ. For time-independent situations, the difference between these two temperatures is proportional to the heat supply and in the absence of any heat supply, the two temperatures are identical. For time-dependent problems and wave propagation problems, in particular, this is not the case. The two temperatures are in general different regardless of the presence of a heat supply. The two-temperature theory involves a material parameter $a^* > 0$. The limit $a^* \to 0$ implies that $\varphi \to \theta$ and the classical theory can be recovered from two-temperature theory. Bassiouny and Youssef [16] used the theory of two-temperature generalized thermoelasticity to solve boundary value problems of one-dimensional finite piezoelectric rod with different types of heating. Youssef [17] considered the governing equations in the light of two-temperature generalized thermoelasticity for studying a problem of thermoelastic interactions in an elastic infinite medium with cylindrical cavity having a thermally shocked bounding surface subjected to moving heat source. Research work proposed by Kaushal *et al*. [18] is concerned with the time harmonic deformations in a homogeneous, isotropic, generalized thermoelastic medium with two temperatures. Ezzat *et al*. [19] introduced the model of equations of micropolar generalized magneto-thermoelasticity including the two temperature theory of generalized thermoelasticity. Ezzat and El-Karamany [20] established the model of one-dimensional equations of the two-temperature generalized magneto-thermoelasticity theory with two relaxation times in a perfect electric conducting medium. Banik and Kanoria [21] studied the thermoelastic interactions for the three-phase-lag heat conduction model in an isotropic infinite elastic body with a spherical cavity under two-temperature generalized thermoelasticity theory.

In classical theory of elasticity, gravity field is generally neglected. The effect of gravity in the problem of wave propagation in solids, particularly on the elastic globe, was first studied by Bromwich [22]. Ahmed [23] discussed the influences of gravity on the propagation of waves in granular medium. A problem on Rayleigh waves propagating in an orthotropic thermoelastic medium under gravity and initial stress is discussed by Abd-Alla and Ahmed [24]. Mahmoud [25] studied the influence of rotation, relaxation times, magnetic field, initial stress and gravity field on attenuation coefficient and Rayleigh waves velocity in an elastic half-space of granular medium.

A number of thermoelastic problems of rotating media have been investigated by many researchers so far. Roy-Choudhuri and Mukhopdhyay [26] studied the effect of rotation and relaxation time on plane waves in an infinite generalized thermo-viscoelastic solid of Kelvin-Voigt type. Ailawalia and Narah [27] analysed the effect of rotation in generalized thermoelastic solid under the influence of gravity with an overlying infinite thermoelastic fluid. Abd-Alla *et al*. [28] presented a model of Rayleigh waves on the surface of generalized magneto-thermoelastic granular medium under the influence of gravity field and initial stress. Effect of rotation in a magneto-thermoelastic problem of a non-homogeneous orthotropic hollow cylinder has been illustrated by Abd-Alla and Mahmoud [29].

The dynamical problem of magneto-thermoelasticity has received much attention in the literature during the past decade. Theory of magneto-thermoelasticity which deals with the interactions among strain, temperature and electromagnetic fields has become the centre of attraction for many researchers as it covers a large number of disciplines such as geophysics for understanding the effect of Earth's magnetic field on seismic waves, damping of acoustic waves in magnetic field, emissions of electromagnetic radiations from nuclear devices, development of highly sensitive superconduction magnetometer, electric power engineering, optics *etc*. Othman and Song [30] pointed out the impact of rotatory motion on the reflection of magneto-thermoelastic waves under thermoelasticity without energy dissipation. Sherief and Helmy [31] dealt with a two-dimensional problem for a half-space in magneto-thermoelasticity with thermal relaxation. Nayfeh and Nemat-Nasser [32] studied the propagation of plane waves in a thermoelastic solid with electric and magnetic permeability.

This research article is devoted to the study of a plane strain problem of a semi-infinite porous thermoelastic solid. Application of the present model cannot be ruled out in the physical world. Inclusion of additional parameters like magnetic field, rotatory motion, gravity field and two-temperature parameter makes our model best fitted with some geophysical situations. To relate the present study and previous work, we note that a thermoelastic solid with voids has been studied by various researchers [33-37] in the context of different generalized theories considering different modes of vibrations for the disturbances produced in the porous thermoelastic solid and by this research article, we have taken a step ahead to extend the work of various eminent researchers. The novelty of our present work resides in the fact that we have presented a quite new and different way of dependence of field variables (displacement components, normal stress, thermodynamic

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temperature, conductive temperature and change in volume fraction field) on rotatory motion, two-temperature discrepancy and void parameters. Also, the work in its present form has not been investigated so far.

The present paper is classified as follows: In section 2, we present the governing equations. Analytical two-dimensional solution is derived in section 3. Section 4 is devoted to study a specific application of mechanical loading on the isothermal boundary of the medium. Section 5 contains some special cases of interest. The numerical results are presented, displayed graphically and discussed in section 6. Finally, the conclusions are drawn in section 7.

2. MATHEMATICAL MODEL

We consider a homogeneous, isotropic, generalized thermoelastic semi-infinite solid with voids, occupying the region $z \geq 0$. The boundary $z = 0$ is assumed to be unstressed, unstrained and initially at uniform temperature T_0. The whole body is considered under the influence of gravity. Rectangular Cartesian coordinate system has been chosen with origin on the surface $z = 0$. The z-axis is pointing vertically downward into the medium. The whole body is rotating with uniform angular velocity $\vec{\Omega} = \Omega \hat{n}$, $\hat{n}$ is unit vector representing the direction of the axis of rotation. Due to accelerated frame of reference, the displacement equation of motion has two additional terms (Roy-Choudhury and Mukhopdhayay [27]): (i) Centripetal acceleration $\left(\vec{\Omega}\times(\vec{\Omega}\times\vec{u})\right)$ due to time varying motion only and (ii) Coriolis acceleration $(2\vec{\Omega}\times\dot{\vec{u}})$ due to moving reference frame, where $\vec{u} = (u, v, w)$ is the dynamic displacement vector. Generalized two-temperature theory for heat conduction has been taken into consideration. A magnetic field $\vec{H} = (0, H_0, 0)$ is acting along positive direction of y-axis. $\vec{h}$ and $\vec{E}$ are respectively the perturbed magnetic and electric fields over the constant primary magnetic field. Considering linear theory of thermoelasticity, both $\vec{h}$ and $\vec{E}$ are small in magnitude.

We assume that pattern of deformation is same in every plane parallel to x-z plane which in turn implies that all physical quantities will be independent of y-coordinate. Thus the motion is supposed to take place only in (x, z)-dimensions. All the considered functions are assumed to be bounded. The state of plane-strain parallel to x-z plane is defined by

$$u = u(x, z, t)\,,\ v = 0\,,\ w = w(x, z, t)\,.$$

The electric field intensity vector $\vec{E}$ being normal to both the magnetic intensity and the displacement vector has the components

$$E_x = E_1\,,\ E_y = 0\,,\ E_z = E_3\,.$$

The current density vector $\vec{J}$, being parallel to $\vec{E}$, has the components

$$J_x = J_1\,,\ J_y = 0\,,\ J_z = J_3\,.$$

The linearized form of Maxwell's equations of electrodynamics for a slowly moving perfectly conducting medium are

$$\nabla\times\vec{h} = \vec{J} + \varepsilon_0\frac{\partial\vec{E}}{\partial t}\,, \tag{1}$$

$$\nabla\times\vec{E} = -\mu_0\frac{\partial\vec{h}}{\partial t}\,, \tag{2}$$

$$\vec{E} = -\mu_0\left(\frac{\partial\vec{u}}{\partial t}\times\vec{H}\right), \tag{3}$$

$$\nabla\cdot\vec{h} = 0\,. \tag{4}$$

From above equations, we can obtain

$$E_1 = \mu_0 H_0\frac{\partial w}{\partial t}\,,\ E_3 = -\mu_0 H_0\frac{\partial u}{\partial t}\,, \tag{5}$$

$$h_1 = 0\,,\ h_2 = -H_0 e\,,\ h_3 = 0\,, \tag{6}$$

$$J_1 = -\varepsilon_0\mu_0 H_0\frac{\partial^2 w}{\partial t^2}\,,\ J_3 = \varepsilon_0\mu_0 H_0\frac{\partial^2 u}{\partial t^2}\,. \tag{7}$$

Lorentz's force $\vec{F}$ is given by

$$\vec{F} = \mu_0(\vec{J}\times\vec{H})\,. \tag{8}$$

From Eqs. (7) and (8), we obtain

$$F_x = -\varepsilon_0\,\mu_0^2 H_0^2\frac{\partial^2 u}{\partial t^2}\,,\ F_y = 0\,,\ F_z = -\varepsilon_0\,\mu_0^2 H_0^2\frac{\partial^2 w}{\partial t^2}\,. \tag{9}$$

Following Lord and Shulman [1] and Iesan [7], the basic governing equations in terms of displacement, change in volume fraction field and temperature field for a uniformly rotating, linear, homogeneous, isotropic, magneto-thermoelastic solid with voids under the influence of gravity in the absence of heat sources and extrinsic equilibrated body forces are

$$\rho\left[\frac{\partial^2 u}{\partial t^2} - \Omega^2 u + 2\Omega\frac{\partial w}{\partial t}\right] = (\lambda+2\mu)\frac{\partial^2 u}{\partial x^2} + \mu\frac{\partial^2 u}{\partial z^2} + (\lambda+\mu)\frac{\partial^2 w}{\partial x\partial z} - \beta\frac{\partial\theta}{\partial x} + b\frac{\partial\phi}{\partial x} + \rho g\frac{\partial w}{\partial x} - \varepsilon_0\,\mu_0^2 H_0^2\frac{\partial^2 u}{\partial t^2}\,, \tag{10}$$

$$\rho\left[\frac{\partial^2 w}{\partial t^2} - \Omega^2 w - 2\Omega\frac{\partial u}{\partial t}\right] = (\lambda+2\mu)\frac{\partial^2 w}{\partial z^2} + \mu\frac{\partial^2 w}{\partial x^2} + (\lambda+\mu)\frac{\partial^2 u}{\partial x\partial z} - \beta\frac{\partial\theta}{\partial z} + b\frac{\partial\phi}{\partial z} - \rho g\frac{\partial u}{\partial x} - \varepsilon_0\,\mu_0^2 H_0^2\frac{\partial^2 w}{\partial t^2}\,, \tag{11}$$

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$$\alpha\nabla^2\phi - b(\nabla\cdot\vec{u}) - \xi_1\phi + m\theta = \rho\chi\ddot{\phi}\,, \tag{12}$$

$$k\nabla^2\varphi = \rho c_e(\dot{\theta}+\tau_0\ddot{\theta}) + \beta T_0(\dot{e}_{kk}+\tau_0\ddot{e}_{kk}) + mT_0(\dot{\phi}+\tau_0\ddot{\phi}) \tag{13}$$

The expression relating thermodynamic temperature and conductive temperature is

$$\theta = (1-a^*\nabla^2)\,\varphi\,. \tag{14}$$

The constitutive relations are given by

$$t_{ij} = \lambda\delta_{ij}\,u_{k,k} + \mu(u_{i,j}+u_{j,i}) + b\phi\delta_{ij} - \beta\theta\delta_{ij}\,, \tag{15}$$

$$q_i + \tau_0\dot{q}_i = k\theta_{,i}\,,\ \ h_i = \alpha\phi_i\,, \tag{16}$$

$$\rho\eta = \beta e_{kk} + a'\theta + m\phi\,, \tag{17}$$

$$s = -be_{kk} - \xi_1\phi + m\theta\,. \tag{18}$$

In the above equations, the usual summation convention is used. Indices following comma indicate partial spatial derivatives. Over dot indicates partial temporal derivative.

For convenience, we will make use of the following non-dimensional quantities

$$(x', z') = \frac{\overline{\omega}}{c_1}(x, z),\quad (t', \tau_0') = \overline{\omega}(t, \tau_0)\,,$$

$$(u', w') = \frac{\rho\overline{\omega}c_1}{\beta T_0}(u, w)\,,\ \ (\theta', \varphi') = \left(\frac{\theta}{T_0}, \frac{\varphi}{T_0}\right),$$

$$(t'_{zx}, t'_{zz}) = \frac{1}{\beta T_0}(t_{zx}, t_{zz}),\ \ \phi' = \frac{\overline{\omega}^2\chi}{c_1^2}\phi,\ \ \Omega' = \frac{\Omega}{\overline{\omega}}\,. \tag{19}$$

where $\overline{\omega} = \dfrac{\rho c_e\, c_1^2}{k}$, $c_1 = \sqrt{\dfrac{\lambda+2\mu}{\rho}}$ are the characteristic frequency and longitudinal wave velocity in the medium respectively.

We assume that the displacement $\vec{u}$ is derivable from the scalar and vector potentials ψ_1 and $\vec{\psi}_2$ by the relation

$$\vec{u} = \nabla\psi_1 + \nabla\times\vec{\psi}_2\,,\ \ \vec{\psi}_2 = (0\,, \psi_2\,, 0)\,, \tag{20}$$

which reduces to

$$u = \frac{\partial\psi_1}{\partial x} - \frac{\partial\psi_2}{\partial z},\quad w = \frac{\partial\psi_1}{\partial z} + \frac{\partial\psi_2}{\partial x}\,. \tag{21}$$

Now in terms of dimensionless quantities given in Eq. (19), the Eqs. (10) ~ (14) with the aid of Eq. (21) along with some simplifications take the form (after dropping primes)

$$\gamma^2\left[\alpha_1\frac{\partial^2\psi_1}{\partial t^2} - \Omega^2\psi_1 + 2\Omega\frac{\partial\psi_2}{\partial t}\right] - \gamma^2\nabla^2\psi_1 + a_1\theta - a_2\phi - a_3\frac{\partial\psi_2}{\partial x} = 0\,, \tag{22}$$

$$\gamma^2\left[\alpha_1\frac{\partial^2\psi_2}{\partial t^2} - \Omega^2\psi_2 - 2\Omega\frac{\partial\psi_1}{\partial t}\right] - \nabla^2\psi_2 + a_3\frac{\partial\psi_1}{\partial x} = 0\,, \tag{23}$$

$$\nabla^2\phi - a_4\nabla^2\psi_1 - a_5\phi + a_6\theta - a_7\ddot{\phi} = 0\,, \tag{24}$$

$$\nabla^2\varphi - \varepsilon_1\frac{\partial}{\partial t}\left(1+\tau_0\frac{\partial}{\partial t}\right)\nabla^2\psi_1 - a_8\left(1+\tau_0\frac{\partial}{\partial t}\right)\dot{\phi} - \left(1+\tau_0\frac{\partial}{\partial t}\right)\dot{\theta} = 0\,, \tag{25}$$

$$\theta = (1-\eta_1\nabla^2)\,\varphi\,. \tag{26}$$

where all the unknowns are defined in Appendix A.

3. PLANE WAVE ANALYSIS

In this method of solution, we consider that the displacement components, displacement potentials, stress components, temperatures and volume fraction field, all are given in the following form:

$$\left[u,\, w,\, \psi_1,\, \psi_2,\, t_{ij},\, \theta,\, \varphi,\, \phi\right](x, z, t) = \left[u^*, w^*, \psi_1^*, \psi_2^*, t_{ij}^*, \theta^*, \varphi^*, \phi^*\right](z)\exp(\omega t + \iota a x)\,, \tag{27}$$

where ω is the complex time constant, $\iota = \sqrt{-1}$, a is the wave number in the x-direction and starred quantities are the amplitudes of the corresponding field quantities.

Making use of Eq. (27) in Eqs. (22) ~ (26), we obtain

$$\gamma^2(D^2 - a_9)\,\psi_1^* + (a_{10}D^2 - a_{11})\,\varphi^* + a_2\,\phi^* + a_{12}\,\psi_2^* = 0\,, \tag{28}$$

$$(D^2 - a_{13})\,\psi_2^* = a_{12}\,\psi_1^*\,, \tag{29}$$

$$a_4(D^2 - a^2)\,\psi_1^* + (a_{23}D^2 - a_{24})\,\varphi^* - (D^2 - a_{15})\,\phi^* = 0\,, \tag{30}$$

$$(a_{20}D^2 - a_{21})\,\psi_1^* - (a_{18}D^2 - a_{19})\,\varphi^* + a_{22}\,\phi^* = 0\,, \tag{31}$$

$$\theta^* = (1 + a*^2\,\eta_1)\,\varphi^* - \eta_1\frac{d^2\varphi^*}{dz^2}\,, \tag{32}$$

where $D \equiv d/dz$. All other constants are given in Appendix A.

Elimination of $\psi_2^*(z)$, $\varphi^*(z)$ and $\phi^*(z)$ among Eqs. (28) ~ (31) yields the following eighth order partial differential equation satisfied by $\psi_1^*(z)$

$$(D^8 + d_1D^6 + d_2D^4 + d_3D^2 + d_4)\,\psi_1^*(z) = 0\,, \tag{33}$$

where all d_i (i = 1, 2, 3, 4) are given in Appendix A.

In a similar manner, we can show that $\phi^*(z)$, $\varphi^*(z)$ and $\psi_2^*(z)$ also satisfy the equations

$$(D^8 + d_1D^6 + d_2D^4 + d_3D^2 + d_4)\left(\phi^*(z),\, \varphi^*(z),\, \psi_2^*(z)\right) = 0\,. \tag{34}$$

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Auxiliary equation of the above differential Eqs. (33) ~ (34) is given by

$$m^8 + d_1 m^6 + d_2 m^4 + d_3 m^2 + d_4 = 0,$$

which can be factorized as

$$(m^2 - k_1^2)(m^2 - k_2^2)(m^2 - k_3^2)(m^2 - k_4^2) = 0,$$

where k_1^2, k_2^2, k_3^2, k_4^2 are the roots of the characteristic equation $k^8 + d_1 k^6 + d_2 k^4 + d_3 k^2 + d_4 = 0$, which is a biquadratic in k^2.

Solutions of Eqs. (33) and (34), which are bounded as $z \to \infty$ can be written as:

$$\psi_1^*(z) = \sum_{i=1}^{4} R_i(a, \omega)\, e^{-k_i z}, \tag{35}$$

$$\phi^*(z) = \sum_{i=1}^{4} a_i^* R_i(a, \omega)\, e^{-k_i z}, \tag{36}$$

$$\varphi^*(z) = \sum_{i=1}^{4} b_i^* R_i(a, \omega)\, e^{-k_i z}, \tag{37}$$

$$\psi_2^*(z) = \sum_{i=1}^{4} d_i^* R_i(a, \omega)\, e^{-k_i z}. \tag{38}$$

where R_i (a, ω) are parameters depending upon a and ω. a_i^*, b_i^*, d_i^* $(i = 1, 2, 3, 4)$ are defined in Appendix A.

Introduction of Eq. (37) in Eq. (32) yields

$$\theta^*(z) = \sum_{i=1}^{4} c_i^* R_i(a, \omega)\, e^{-k_i z}, \tag{39}$$

where c_i^* $(i = 1, 2, 3, 4)$ is defined in Appendix A.

Equation (21) with the help of Eq. (27) provides

$$u^*(z) = \iota a\, \psi_1^*(z) - D\, \psi_2^*(z), \tag{40}$$

$$w^*(z) = D\, \psi_1^*(z) + \iota a\, \psi_2^*(z). \tag{41}$$

Inserting Eqs. (35) and (38) into Eqs. (40) and (41), the displacement components u and w, are evaluated as

$$u = \left(\sum_{i=1}^{4} (\iota a + k_i\, d_i^*) R_i(a, \omega)\, e^{-k_i z} \right) e^{(\omega t + \iota a x)}, \tag{42}$$

$$w = \left(\sum_{i=1}^{4} (\iota a d_i^* - k_i) R_i(a, \omega)\, e^{-k_i z} \right) e^{(\omega t + \iota a x)}. \tag{43}$$

Stress components are of the form

$$t_{zx} = \left(-\sum_{i=1}^{4} (2\iota a \delta_2\, k_i + (a^2 + k_i^2)\, \delta_2\, d_i^*) R_i\, e^{-k_i z} \right) e^{(\omega t + \iota a x)}, \tag{44}$$

$$t_{zz} = \left(\sum_{i=1}^{4} (\delta_1 (k_i^2 - a^2) + 2\delta_2 k_i^2 - 2\iota a \delta_2\, d_i^* k_i + b_1 a_i^* - \beta T_0\, c_i^*) R_i\, e^{-k_i z} \right) e^{(\omega t + \iota a x)}, \tag{45}$$

where

$$\delta_1 = \frac{\lambda}{\rho c_1^2}, \quad \delta_2 = \frac{\mu}{\rho c_1^2}, \quad \delta_3 = \frac{\mu_0 H_0^2}{\rho c_1^2}, \quad b_1 = \frac{b c_1^2}{\beta T_0 \bar{\omega}^2 \chi}.$$

4. BOUNDARY CONDITIONS

In order to determine the parameters $R_i(a, \omega)$ $(i = 1, 2, 3, 4)$, we shall consider the following boundary conditions.

4.1 Mechanical Load on the Surface of the Half-Space

The plane bounding surface $z = 0$ is taken to be isothermal and is subjected to an instantaneous mechanical load. Hence the boundary conditions in this case are

$$\begin{aligned} &t_{zz} + \bar{\tau}_{zz} = -p_0(x, t), \; t_{zx} = 0, \\ &\varphi = 0, \; \frac{\partial \phi}{\partial z} = 0 \;\text{ at }\; z = 0 \end{aligned} \tag{46}$$

where $P_0(x, t)$ is a given function of x and t and $\bar{\tau}_{zj}\,(j = x, y, z)$ is the Maxwell stress given in the form

$$\bar{\tau}_{zj} = \mu_0 [H_z h_j + H_j h_z - H_k h_k \delta_{zj}], \quad (j = x, y, z). \tag{47}$$

Using Eqs. (36) ~ (37) and Eqs. (44) ~ (45) into the boundary conditions along with Eq. (19) and $p_0' = p_0 / \beta T_0$, we obtain the boundary conditions in the dimensionless form. On suppressing the primes and invoking normal mode analysis, we obtain the following system of equations:

$$\begin{aligned} &\sum_{i=1}^{4} L_i R_i = -p_0, \; \sum_{i=1}^{4} M_i R_i = 0, \; \sum_{i=1}^{4} Q_i R_i = 0, \\ &\sum_{i=1}^{4} N_i R_i = 0. \end{aligned} \tag{48}$$

where p_0 is the magnitude of the external force applied on the boundary and expressions for L_i, M_i, Q_i, N_i $(i = 1, 2, 3, 4)$ are given in Appendix A.

Solution of the above system of Eqs. (48) can be written as

$$R_1 = \frac{\Delta_1}{\Delta}, \quad R_2 = \frac{\Delta_2}{\Delta}, \quad R_3 = \frac{\Delta_3}{\Delta}, \quad R_4 = \frac{\Delta_4}{\Delta}. \tag{49}$$

where Δ, Δ_i $(i = 1, 2, 3, 4)$ are given in Appendix A.

Making use of above equation in Eqs. (42) ~ (45), (36), (37) and (39) the expressions for displacement components, stress components, change in volume fraction field and temperature field are obtained as

$$u = \frac{1}{\Delta} \left(\sum_{i=1}^{4} (\iota a + k_i d_i^*) \Delta_i e^{-k_i z} \right) e^{(\omega t + \iota a x)} \tag{50}$$

$$w = \frac{1}{\Delta} \left(\sum_{i=1}^{4} (\iota a d_i^* - k_i) \Delta_i e^{-k_i z} \right) e^{(\omega t + \iota a x)} \tag{51}$$

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$$t_{zx} = \frac{1}{\Delta}\left(-\sum_{i=1}^{4} M_i \Delta_i e^{-k_i z}\right) e^{(\omega t + \iota a x)} \tag{52}$$

$$t_{zz} = \frac{1}{\Delta}\left(\sum_{i=1}^{4} L_i \Delta_i e^{-k_i z}\right) e^{(\omega t + \iota a x)} \tag{53}$$

$$\phi = \frac{1}{\Delta}\left(\sum_{i=1}^{4} a_i^* \Delta_i e^{-k_i z}\right) e^{(\omega t + \iota a x)} \tag{54}$$

$$\varphi = \frac{1}{\Delta}\left(\sum_{i=1}^{4} b_i^* \Delta_i e^{-k_i z}\right) e^{(\omega t + \iota a x)} \tag{55}$$

$$\theta = \frac{1}{\Delta}\left(\sum_{i=1}^{4} c_i^* \Delta_i e^{-k_i z}\right) e^{(\omega t + \iota a x)} \tag{56}$$

5. LIMITING CASES

5.1 Ignoring the Void Effect

Neglecting the material constants due to the presence of voids *i.e.* ($\alpha = b = \xi_1 = m = \chi = 0$); the expressions for displacement components, stresses and temperature distributions are obtained as

$$u = \frac{1}{\Delta^{**}}\left(\sum_{j=5}^{7} (\iota a + k_j e_j^*)\Delta_j^{**}\right) e^{(\omega t + \iota a x)}, \tag{57}$$

$$w = \frac{1}{\Delta^{**}}\left(\sum_{j=5}^{7} (-k_j + \iota a e_j^*)\Delta_j^{**}\right) e^{(\omega t + \iota a x)}, \tag{58}$$

$$t_{zx} = -\frac{1}{\Delta^{**}}\left(\sum_{j=5}^{7} \left(2\iota a \delta_2 k_j + \delta_2 e_j^* (\mathrm{a}^2 + k_j^2)\right)\Delta_j^{**} e^{-k_j z}\right) e^{(\omega t + \iota a x)}, \tag{59}$$

$$t_{zz} = \frac{1}{\Delta^{**}}\left(\sum_{j=5}^{7} \left((\delta_1 + \delta_3)(k_j^2 - a^2) + 2\delta_2 k_j^2 - 2\iota a \delta_2 e_j^* k_j - \beta T_0 g_j^*\right)\Delta_j^{**} e^{-k_j z}\right) e^{(\omega t + \iota a x)}, \tag{60}$$

$$\varphi = \frac{1}{\Delta^{**}}\left(\sum_{j=5}^{7} f_j^* \Delta_j^{**} e^{-k_j z}\right) e^{(\omega t + \iota a x)}, \tag{61}$$

$$\theta = \frac{1}{\Delta^{**}}\left(\sum_{j=5}^{7} g_j^* \Delta_j^{**} e^{-k_j z}\right) e^{(\omega t + \iota a x)}, \tag{62}$$

where k_5^2, k_6^2, k_7^2 are roots of the equation

$$k^6 + c_1 k^4 + c_2 k^2 + c_3 = 0 \tag{63}$$

with all the constants incorporated in Appendix B. Expressions for $\Delta^{**}, \Delta_j^{**}$ $(j = 5, 6, 7)$ are also present in Appendix B.

5.2 With one Temperature

By setting $a^* = 0$ and consequently $\eta_1 = 0$ in the governing equations, we get the corresponding expressions of the physical variables in the context of classical one-temperature L-S (Lord-Shulman) theory from Eqs. (50) ~ (56).

6. NUMERICAL RESULTS AND DISCUSSION

In the calculation process, we consider the following physical constants of a magnesium like material modelled as an isotropic thermoelastic solid half space.

$\lambda = 9.4\times10^{10}\,\mathrm{kgm^{-1}s^{-2}}$, $\mu = 4.0\times10^{10}\,\mathrm{kgm^{-1}s^{-2}}$,
$\rho = 1.74\times10^{3}\,\mathrm{kgm^{-3}}$, $\alpha_t = 2.36\times10^{-5}\,\mathrm{K^{-1}}$,
$c_e = 9.623\times10^{2}\,\mathrm{Jkg^{-1}K^{-1}}$, $T_0 = 293\mathrm{K}$, $k = 2.510\ \mathrm{Wm^{-1}K^{-1}}$,
$\varepsilon_0 = (10^{-9}/36\pi)F\ \mathrm{m^{-1}}$, $\mu_0 = 4\pi\times10^{-7}\ \mathrm{Hm^{-1}}$,
$H_0 = (10^{7}/4\pi)A\ \mathrm{m^{-1}}$, $g = 9.8\ \mathrm{ms^{-2}}$.

Since, we have $\omega = \omega_0 + \iota\zeta$ where ι is the imaginary unit, $e^{\omega t} = e^{\omega_0 t}(\cos\varsigma t + \iota \sin\varsigma t)$ and for small values of time we can take ω as real *i.e.* $\omega = \omega_0$.

Void parameters are (Kumar and Rani [38]):

$\alpha = 3.688\times10^{-5}\ \mathrm{N}$, $\xi_1 = 1.475\times10^{10}\ \mathrm{Nm^{-2}}$,
$\chi = 1.753\times10^{-15}\ \mathrm{m^2}$,
$b = 1.13849\times10^{10}\ \mathrm{Nm^{-2}}$, $m = 7.322\times10^{3}\ \mathrm{Nm^{-2}K^{-1}}$.

Other parameters arising in the present analysis are taken as:

$p_0 = 10$, $\tau_0 = 0.02$, $a = 1.2$, $\omega = 2$.

Considering the above physical data, we compute the numerical values of displacement, stress, thermodynamic temperature, conductive temperature and change in volume fraction field for a wide range of z at $t = 0.01$ on the plane $x = 1$ with $g = 9.8$. The results are displayed in various figures. We have considered the transient distributions for the real parts of the amplitudes of the field quantities which are represented along the vertical axis. From clarity point of view, we have divided the graphical representations into three categories. First category (Figs. 1 ~ 5) is meant for analysing the effects of non-dimensional rotational speeds which are taken as 0.0, 0.05, 0.15, 0.25 on various field variables with $\eta_1 = 0.075$. In second category (Figs. 6 ~ 10) we have explored the effects of non-dimensional two-temperature parameter taken as 0.0, 0.01, 0.05, 0.075 and 0.1 on the spatial variations of physical fields with $\Omega = 0.15$. Third category (Figs. 11 ~ 14) shows the effect of voids on various kinematic variables pertaining to our study for $\eta_1 = 0.075$, $\Omega = 0.15$.

Category I

Figure 1 depicts the space variation of the normal displacement w against distance z. Displacement field

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starts with the maximum values 2.46, 2.37, 2.1 and 1.81 at the boundary of the half-space in all the four cases. It weakens gradually to a comparatively small value and ultimately reaches to steady state after some distance $z \geq 2.8$. Displacement field has a qualitatively similar behaviour for every rotational speed with difference in magnitude. Magnitude of variations of displacement field for large values of Ω is small as compared to the case of zero value of Ω which clearly indicates that rotation has a decreasing effect.

In Fig. 2, we have compared the variations of normal stress t_{zz}. It is clear from the figure that stress is compressive for all the cases and is having a coincident starting point 2.89 (in magnitude). Absence of rotation causes to increase the numerical values of stress field. Increment in the values of non-dimensional rotational speed causes a decrement in the absolute values of normal stress. Thus normal stress is inversely proportional to rotation.

Figure 3 has been plotted to examine the effects of rotation on the pattern of thermodynamic temperature θ. For all cases, θ starts with value 1.15 initially. Qualitative behaviour of temperature distribution is same for all the four cases. θ attains its peak values near the bounding surface. Absence of rotatory motion is responsible for increasing the numerical values of temperature distribution θ. It is also manifested from the figure that increasing values of rotation are having a decreasing effect on the magnitude of temperature variations. Variations in conductive temperature distribution φ with spatial coordinate z have been displayed in Fig. 4. These variations are having a common starting point of zero magnitude which agrees with the boundary conditions. Time to reach steady state is almost same for every case of rotation. Presence of rotation decreases the magnitude of conductive temperature. Moreover, with increasing rotational speed there is a decrease in the magnitude of conductive temperature distribution. Thus conductive temperature is inversely proportional to rotation. Figure 5 reveals that rotational factor acts to decrease the magnitude of change in volume fraction field ϕ. Its values are maximum at the origin and after some range of z seem to be vanishing.

Category II

Figure 6 is plotted to show the variations of normal displacement w with distance z. Displacement field starts with values 1.27, 0.97, 0.58, 0.24 and 0.17 for the five values of η_1 taken as 0.0, 0.01, 0.05, 0.075 and 0.1, which clearly shows that the two-temperature parameter has a significant effect on the displacement profile. This plot illuminates that in the case ($\eta_1 = 0$) when conductive temperature coincides with thermodynamic temperature, magnitude of variations of displacement profile is maximum. Also increasing values of two-temperature parameter are responsible for decreasing pattern of numerical values of w. Figure 7 gives a comparison of the profile of stress distribution t_{zz} for varying values of two-temperature parameter.

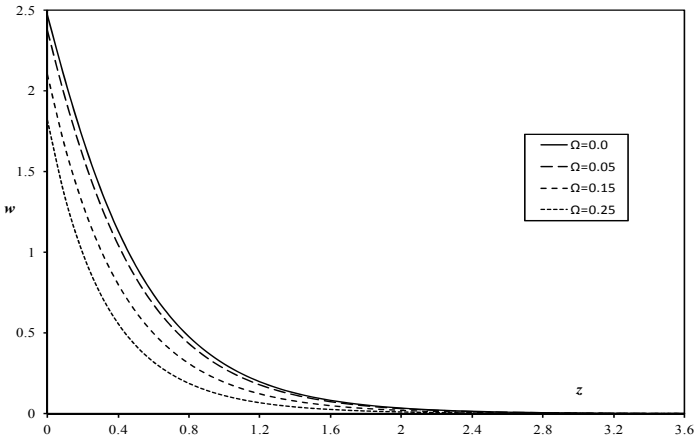


Fig. 1 Normal displacement distribution with different rotating speeds.

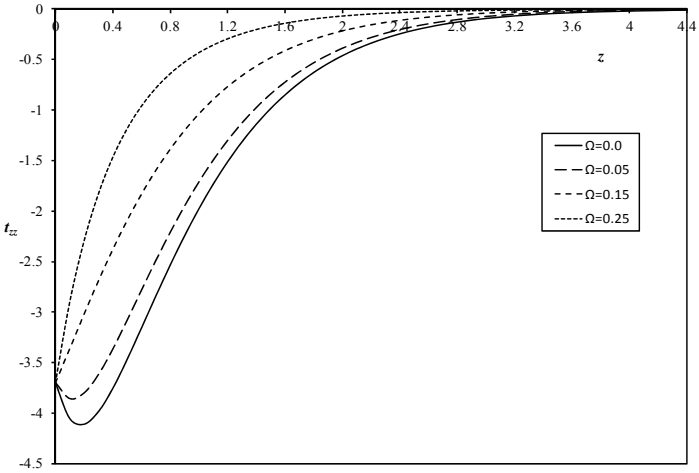


Fig. 2 Normal stress distribution with different rotating speeds.

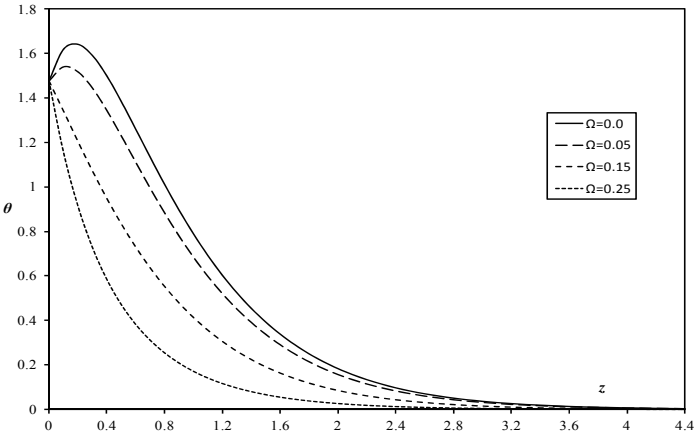


Fig. 3 Thermodynamic temperature distribution with different rotating speeds.

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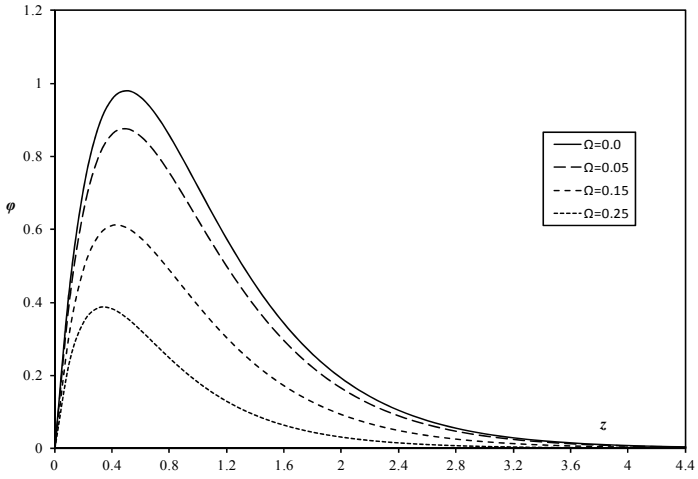


Fig. 4 Conductive temperature distribution with different rotating speeds.

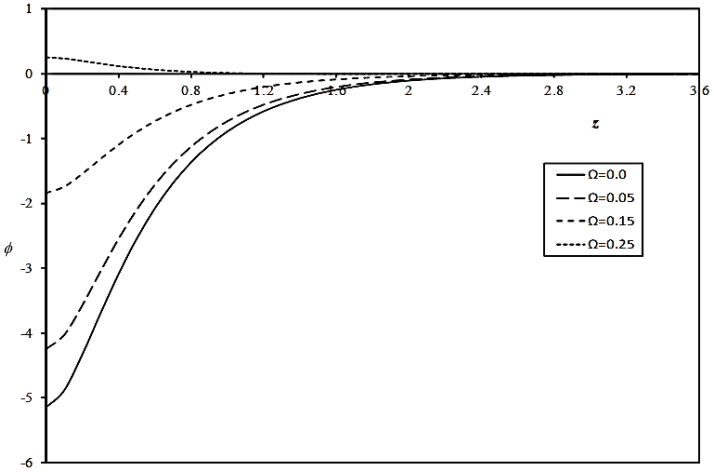


Fig. 5 Change in volume fraction field distribution with different rotating speeds.

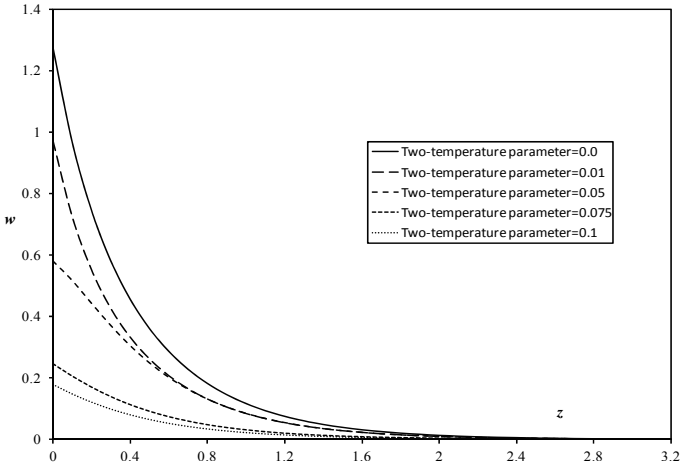


Fig. 6 Normal displacement distribution with varying two-temperature discrepancy.

Our plot indicates that when conductive temperature coincides with thermodynamic temperature, t_{zz} starts with zero value. Otherwise starting value is 3.63 (in magnitude). Pattern of variations is same for every value of two-temperature parameter. It is observed that there is an 'inversely proportional relationship' between absolute values of stress field and two-temperature parameter. Figure 8 displays the spatial distribution of thermodynamic temperature θ. Clearly, temperature distribution is inversely proportional to two-temperature parameter.

In Figure 9, we have shown the spatial variation of conductive temperature φ. Distribution of φ follows similar trend and dissimilarity lies on the ground of magnitudes. Also this difference in magnitudes becomes indistinct after some distance $z \geq 4$. Absence of two-temperature parameter causes an increment in the numerical values of φ. It is noticed that amplitude of distribution of φ decreases owing to increasing values of two-temperature discrepancy. Figure 10 exhibits its distribution of change in volume fraction field ϕ. Numerical values of ϕ are large in magnitude in generalized thermoelasticity theory (*i.e.* $\eta_1 = 0$) as compared to two-temperature generalized thermoelasticity theory (*i.e.* $\eta_1 \neq 0$) which implies that presence of two temperatures decreases the absolute values of ϕ. It is observable that increment in the two-temperature discrepancy decreases the magnitude of variations in ϕ.

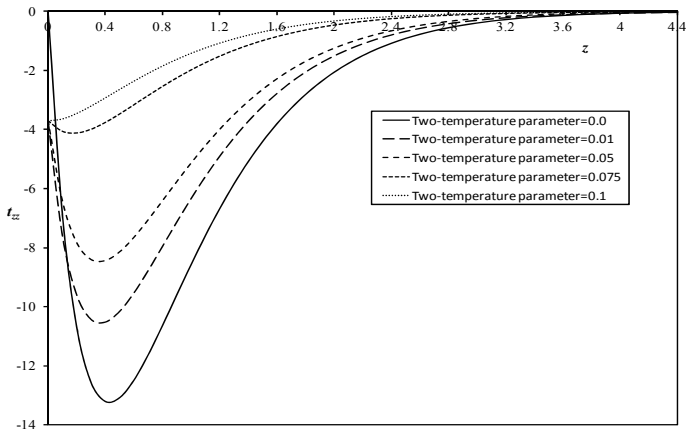


Fig. 7 Normal stress distribution with varying two-temperature discrepancy.

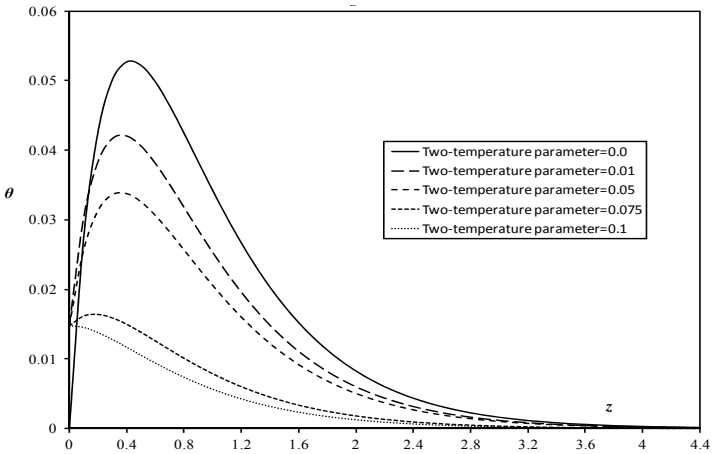


Fig. 8 Thermodynamic temperature distribution with varying two-temperature discrepancy.

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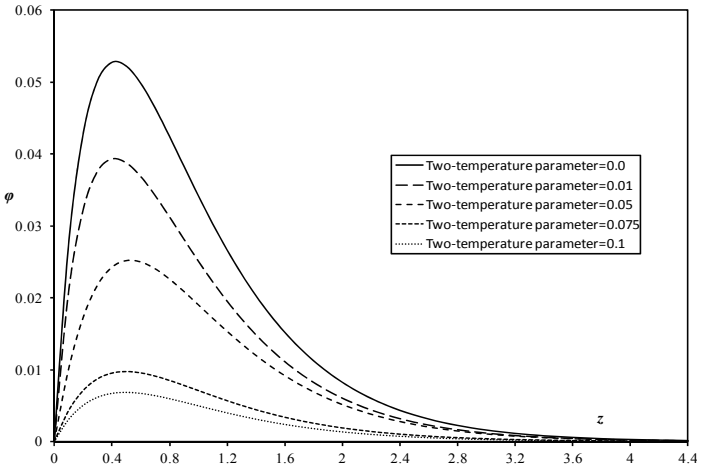


Fig. 9 Conductive temperature distribution with varying two-temperature discrepancy.

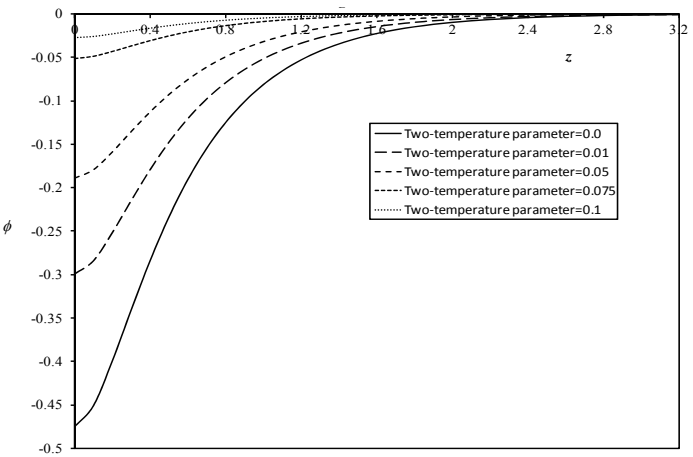


Fig. 10 Change in volume fraction field distribution with varying two-temperature discrepancy.

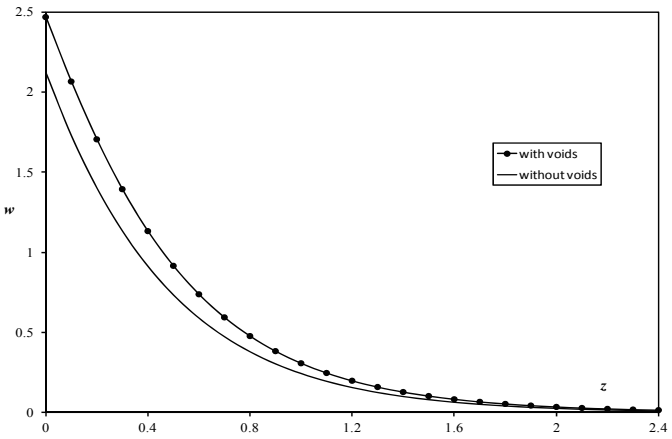


Fig. 11 Effect of voids on normal displacement distribution.

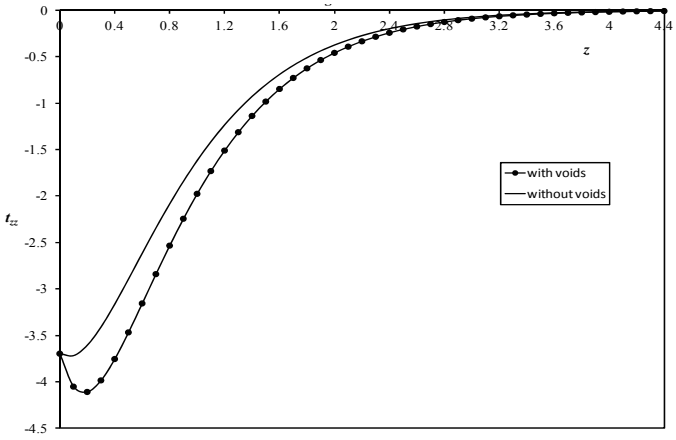


Fig. 12 Effect of voids on normal stress distribution.

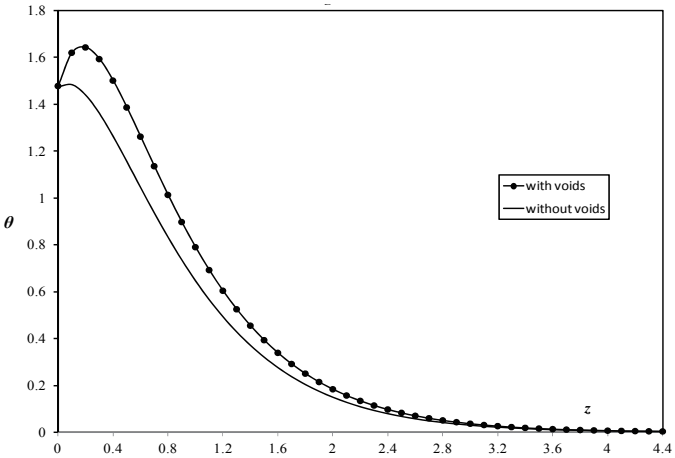


Fig. 13 Effect of voids on thermodynamic temperature distribution.

Category III

This category comprises of Figs. 11 ~ 14 to show the influence of void parameters on various field variables. Figure 11 shows the transient effect of applied mechanical load on normal displacement w for generalized thermoelasticity without voids (solid line) and generalized thermoelasticity with voids (solid line with centered symbol). Curves of variations start with the values 2.46 and 2.12 for generalized thermoelasticity with and without voids respectively. Solution curves for both cases follow same pattern of variations with the difference in magnitude of values. Figure indicates that voids significantly increase the numerical values of w.

Figure 12 illustrates pattern of normal stress distribution with respect to location z. Stress is compressive having same initial value 3.72 (in magnitude) for both cases. By comparing two solution curves it is found that absolute values of normal stress t_{zz} are higher in the presence of voids. Behaviour of stress for both cases is alike with different degree of magnitude.

Figure 13 represents thermodynamic temperature distribution with distance z. Amplitude of vibrations of solution curves for thermodynamic temperature θ gets suppressed due to absence of voids. For a small range of distance z near the bounding surface, θ varies like an increasing function of z and after that it behaves as a decreasing function till it attains steady state near $z = 4$.

Figure 14 has been plotted to observe the effect of voids on conductive temperature φ against distance z. For both cases, φ shows same nature of variations. As can be seen from the plot, both solution curves have coincident starting point with value zero. Presence of voids enhances the values of φ.

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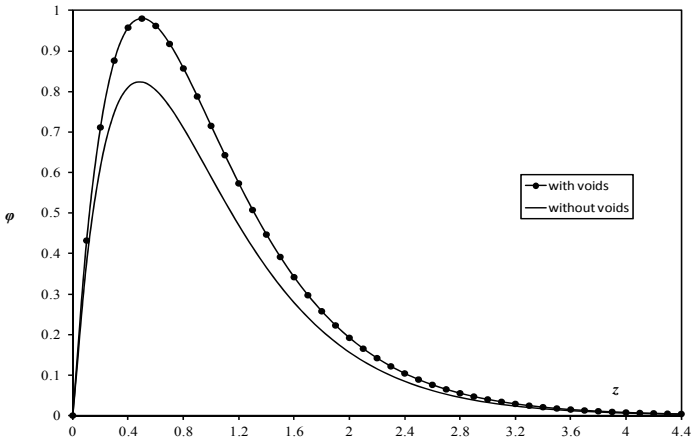


Fig. 14 Effect of voids on conductive temperature distribution.

It is clear from Figs. 11 ~ 14 that near the surface, where the boundary conditions dominate, influence of voids is markedly observed and away from the bounding surface this becomes negligible which complies the real situation. Increasing effect of voids on the numerical values of various field variables can be attributed to the cross effects arising from the coupling of the fields of temperature, strain and void volume fraction. Due to these cross effects additional fields of displacement, stresses and temperatures are generated.

7. CONCLUSIONS

Study of the transient disturbance owing to presence of voids, variations in temperature discrepancy and rotatory motion in a magneto-thermoelastic half-space under gravity is an interesting problem of two-temperature generalized thermoelasticity theory. In this study, the method of normal mode analysis is adopted which proves to be a quite effective approach in grappling with such type of problems. This mathematical technique gives exact solutions without any assumed restrictions on the actual physical quantities appearing in the governing equations of the problem considered. Following points will help to conclude our present investigation:

1. The curves obtained are highly dependent on the vertical distance from origin. The value of all physical quantities converges to zero with increase in distance z. All functions are continuous.
2. Phenomenon of finite speed of propagation is manifested in all the figures and solution curves are quite close to each other with increase in distance, which is in agreement with the generalized theories of thermoelasticity.
3. It is apparent from figures that presence of voids increases the magnitude of variations of each kinematic variable.
4. Theoretical as well as numerical results reveal that all the field variables are sensitive towards rotation and temperature discrepancy. Presence of rotatory motion and temperature discrepancy shows a decreasing effect on various field distributions.
5. From the temperature distributions, we have obtained a wave type heat propagation in the medium. Heat wavefronts move forward with a finite speed which proves that the present theory is very close to the practically possible elastic materials.

The present theoretical results derived here may provide highly beneficial and interesting information for scientists/researchers/seismologists working in the field of two-temperature generalized thermoelasticity theories.

APPENDIX A

$$\gamma^2=\frac{\lambda+2\mu}{\mu},\ \alpha_1=1+\frac{c_A^2}{c^2},\ c_A^2=\frac{\mu_0 H_0^2}{\rho},\ c^2=\frac{1}{\mu_0\varepsilon_0},$$

$$a_1=\frac{\rho c_1^2}{\mu},\ a_2=\frac{\rho b c_1^4}{\beta T_0\mu\bar{\omega}^2\chi},\ a_3=\frac{\rho g c_1}{\mu\bar{\omega}},$$

$$a_4=\frac{b\chi\beta T_0}{\alpha\rho c_1^2},\ a_5=\frac{\xi_1 c_1^2}{\bar{\omega}^2\alpha},$$

$$a_6=\frac{mT_0\chi}{\alpha},\ a_7=\frac{\rho c_1^2\chi}{\alpha},\ a_8=\frac{mc_1^4}{\bar{\omega}^3\chi k},$$

$$\varepsilon_1=\frac{\beta^2 T_0}{\rho\bar{\omega}k},\ \eta_1=\frac{a^*\bar{\omega}^2}{c_1^2},$$

$$a_9=a^2+\alpha_1\omega^2-\Omega^2,\ a_{10}=a_1\eta_1,$$
$$a_{11}=a_1+a^2a_{10},\ a_{12}=-2\Omega\omega\gamma^2+\iota a a_3,$$

$$a_{13}=a^2+\gamma^2(\alpha_1\omega^2-\Omega^2),\ a_{14}=a^2+a_5+a_7\omega^2,$$
$$a_{16}=\omega(1+\tau_0\omega),\ a_{17}=1+a^2\eta_1,$$

$$a_{18}=1+a_{16}\eta_1,\ a_{19}=a^2+a_{16}a_{17},$$
$$a_{20}=\varepsilon_1\omega(1+\omega),\ a_{21}=a^2\varepsilon_1\omega(1+\tau_0\omega),$$
$$a_{22}=a_8\omega(1+\tau_0\omega),\ a_{23}=a_6\eta_1,\ a_{24}=a_6a_{17},$$

$$a_{25}=a_9+a_{13},\ a_{26}=a_9a_{13}+a_{12}^2,\ a_{27}=a_{11}+a_{10}a_{13},$$

$$a_{28}=a_{11}a_{13},\ a_{29}=a_{12}^2,$$

$$a_{30}=a_4a_{10}-\gamma^2a_{23},$$
$$a_{31}=\gamma^2a_{23}a_{25}+\gamma^2a_{24}-a_4a_{27}-a^2a_4a_{10},$$

$$a_{32}=a^2a_4a_{27}+a_4a_{28}-\gamma^2a_{23}a_{26}-\gamma^2a_{24}a_{25},$$
$$a_{33}=\gamma^2a_{24}a_{26}-a^2a_4a_{28},$$

$$a_{34}=a_2a_{23}-a_{10}a_{14}-a_{27},$$
$$a_{35}=a_{28}+a_{14}a_{27}-a_2a_{24}-a_2a_{13}a_{23},$$

$$a_{36}=a_2a_{13}a_{24}-a_{14}a_{18},$$
$$a_{37}=\gamma^2a_{25}+\gamma^2a_{15}-a_2a_4,$$

$$a_{38}=a_2a_4(a^2+a_{13})+\gamma^2(a_{15}a_{25}-a_{26}),$$
$$a_{39}=\gamma^2a_{15}a_{26}-a^2a_2a_4a_{13},$$

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$$a_{40} = a_2 a_{23} - a_{10} a_{15} - a_{27} ,$$
$$a_{41} = a_{28} + a_{15} a_{27} - a_2 a_{24} - a_2 a_{13} a_{23} ,$$
$$a_{42} = a_2 a_{13} a_{24} - a_{15} a_{28} ,$$

$$d_1 = \frac{f_1}{f_0}, \; d_2 = \frac{f_2}{f_0}, \; d_3 = \frac{f_3}{f_0}, \; d_4 = \frac{f_4}{f_0} ,$$
$$f_0 = a_{10} a_{20} + \gamma^2 a_{18} ,$$

$$f_1 = -a_{10} a_{21} - a_{20} a_{27} - a_{10} a_{14} a_{20} + a_2 a_{20} a_{23} - \gamma^2 a_{18} a_{25} ,$$
$$- \gamma^2 a_{14} a_{18} + a_2 a_4 a_{18} - \gamma^2 a_{19} + a_4 a_{10} a_{22} - \gamma^2 a_{22} a_{23} ,$$

$$f_2 = a_{20} a_{28} + a_{14} a_{20} a_{27} - a_2 a_{20} a_{24} - a_2 a_{13} a_{20} a_{23} + a_{21} a_{27} ,$$
$$+ a_{10} a_{14} a_{21} - a_2 a_{21} a_{23} - a^2 a_2 a_4 a_{18} - a_2 a_4 a_{13} a_{18} + \gamma^2 a_{18} a_{26} .$$
$$+ \gamma^2 a_{14} a_{18} a_{25} + \gamma^2 a_{19} a_{25} + \gamma^2 a_{14} a_{19} - a_2 a_4 a_{19}$$
$$+ \gamma^2 a_{22} a_{23} a_{25} + \gamma^2 a_{22} a_{24} - a_4 a_{22} a_{27} - a^2 a_4 a_{10} a_{22} ,$$

$$f_3 = a_2 a_{13} a_{20} a_{24} - a_{14} a_{20} a_{28} - a_{21} a_{28} - a_{14} a_{21} a_{27} + a_2 a_{21} a_{24}$$
$$+ a_2 a_{13} a_{21} a_{23} - \gamma^2 a_{14} a_{18} a_{26} + a^2 a_2 a_4 a_{13} a_{18} + a^2 a_2 a_4 a_{19}$$
$$+ a_2 a_4 a_{13} a_{19} - \gamma^2 a_{19} a_{26} - \gamma^2 a_{14} a_{19} a_{25} - \gamma^2 a_{22} a_{23} a_{26}$$
$$- \gamma^2 a_{22} a_{24} a_{25} + a_4 a_{22} a_{28} + a^2 a_4 a_{22} a_{27} ,$$

$$f_4 = -a_2 a_{13} a_{21} a_{24} + a_{14} a_{21} a_{28} + \gamma^2 a_{14} a_{19} a_{26} - a^2 a_2 a_4 a_{13} a_{19}$$
$$+ \gamma^2 a_{22} a_{24} a_{26} - a^2 a_4 a_{22} a_{28} ,$$

$$a_i^* = \frac{a_{30} k_i^6 + a_{31} k_i^4 + a_{32} k_i^2 + a_{33}}{a_{10} k_i^6 + a_{34} k_i^4 + a_{35} k_i^2 + a_{36}} ,$$

$$b_i^* = \frac{-\gamma^2 k_i^6 + a_{37} k_i^4 + a_{38} k_i^2 + a_{39}}{a_{10} k_i^6 + a_{40} k_i^4 + a_{41} k_i^2 + a_{42}} ,$$

$$c_i^* = (1 + a^2 \eta_1) b_i^* - \eta_1 b_i^* k_i^2 , \; d_i^* = \frac{a_{12}}{(k_i^2 - a_{13})} ,$$

$$L_i = (\delta_1 + \delta_3)(k_i^2 - a^2) + 2\delta_2 k_i^2 - 2\iota a \delta_2 d_i^* k_i + b_1 a_i^* - \beta T_0 c_i^* ,$$

$$M_i = 2\iota a \delta_2 k_i + (a^2 + k_i^2) \delta_2 d_i^* , \; Q_i = b_i^* ,$$
$$N_i = k_i a_i^* ,$$

$$\Delta = (L_1 M_2 - L_2 M_1)(Q_3 N_4 - Q_4 N_3)$$
$$+ (L_3 M_1 - L_1 M_3)(Q_2 N_4 - Q_4 N_2)$$
$$+ (L_1 M_4 - L_4 M_1)(Q_2 N_3 - Q_3 N_2)$$
$$+ (L_2 M_3 - L_3 M_2)(Q_1 N_4 - Q_4 N_1)$$
$$+ (L_4 M_2 - L_2 M_4)(Q_1 N_3 - Q_3 N_1)$$
$$+ (L_3 M_4 - L_4 M_3)(Q_1 N_2 - Q_2 N_1) ,$$

$$\Delta_1 = -p_0 \left(M_2 (Q_3 N_4 - Q_4 N_3) - M_3 (Q_2 N_4 - Q_4 N_2) \right.$$
$$\left. + M_4 (Q_2 N_3 - Q_3 N_2) \right) ,$$

$$\Delta_2 = p_0 \left(M_1 (Q_3 N_4 - Q_4 N_3) - M_3 (Q_1 N_4 - Q_4 N_1) \right.$$
$$\left. + M_4 (Q_1 N_3 - Q_3 N_1) \right) ,$$

$$\Delta_3 = -p_0 \left(M_1 (Q_2 N_4 - Q_4 N_2) - M_2 (Q_1 N_4 - Q_4 N_1) \right.$$
$$\left. + M_4 (Q_1 N_2 - Q_2 N_1) \right) ,$$

$$\Delta_4 = p_0 \left(M_1 (Q_2 N_3 - Q_3 N_2) - M_2 (Q_1 N_3 - Q_3 N_1) \right.$$
$$\left. + M_3 (Q_1 N_2 - Q_2 N_1) \right) .$$

APPENDIX B

$$c_1 = \frac{l_2}{l_1}, \; c_2 = \frac{l_3}{l_1}, \; c_3 = \frac{l_4}{l_1} , \; l_1 = \gamma^2 a_{18} + a_{10} a_{20} ,$$

$$l_2 = -(\gamma^2 a_{19} + \gamma^2 a_{18} a_{25} + a_{10} a_{21} + a_{20} a_{27}) ,$$

$$l_3 = \gamma^2 (a_{18} a_{26} + \gamma^2 a_{19} a_{25} + a_{20} a_{28} + a_{21} a_{27}) ,$$

$$l_4 = -(\gamma^2 a_{19} a_{26} + a_{21} a_{28}) ,$$

$$\Delta^{**} = L_5 (M_6 Q_7 - M_7 Q_6) - L_6 (M_5 Q_7 - M_7 Q_5)$$
$$+ L_7 (M_5 Q_6 - M_6 Q_5) ,$$

$$\Delta_5^{**} = -p_0 (M_6 Q_7 - M_7 Q_6) ,$$

$$\Delta_6^{**} = p_0 (M_5 Q_7 - M_7 Q_5) ,$$

$$\Delta_7^{**} = -p_0 (M_5 Q_6 - M_6 Q_5) ,$$

$$L_j = (\delta_1 + \delta_3)(k_j^2 - a^2) + 2\delta_2 k_j^2 - 2\iota a \delta_2 e_j^* k_j - \beta T_0 g_j^* ,$$

$$M_j = 2\iota a \delta_2 k_j + (a^2 + k_j^2) \delta_2 e_j^* ,$$

$$Q_j = f_j^* , \; e_j^* = \frac{a_{12}}{(k_j^2 - a_{13})} ,$$
$$f_j^* = \frac{a_{20} k_j^2 - a_{21}}{a_{18} k_j^2 - a_{19}} , \; g_j^* = (1 + a^2 \eta_1) f_j^* - \eta_1 f_j^* k_j^2 .$$

NOMENCLATURE

t_{ij}	Components of stress tensor
λ, μ	Lame's constants
β	$(3\lambda + 2\mu)\alpha_t$
α_t	Coefficient of linear thermal expansion
τ_0	Thermal relaxation time
T	Absolute temperature
T_0	Reference temperature
θ	$T - T_0$, Thermodynamic temperature assumed to satisfy $\lvert \theta / T_0 \rvert \ll 1$
φ	$\varphi - T_0$, Conductive temperature
a^*	Two-temperature parameter assumed to satisfy $a^* > 0$
u_i	Components of displacement vector
μ_0	Magnetic permeability
ε_0	Electric permittivity
ρ	Density of the medium
g	Acceleration due to gravity
e_{ij}	Components of strain tensor
e_{kk}	e, Cubical dilatation
k	Thermal conductivity

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c_e	Specific heat at constant strain
ϕ	Change in volume fraction field
δ_{ij}	Kronecker delta function
h_i	Components of equilibrated stress vector
q_i	Components of heat flux vector
α, b, ξ_1	Void material parameters
m	Thermo-void coefficient
χ	Equilibrated inertia
η	Entropy per unit mass
s	Intrinsic equilibrated body force
a'	Thermal constant
t	Time variable

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