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Competitive Equilibrium with Debt

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Abstract

This paper studies the interaction among financing, entry, and exit decisions of firms in a competitive industry subject to aggregate uncertainty. In contrast to Fries, Miller, and Perraudin (1997), I do not assume that a firm in default leaves the industry immediately. The implications on the optimal leverage ratios and equilibrium credit spreads are discussed. By incorporating the effect of competition, I show that the model results in significantly higher credit spreads than those predicted by traditional single firm models. Dynamic capital structure strategies in a competitive industry are also examined. The model renders a number of empirical predictions regarding leverage ratios and credit spreads of firms in a competitive industry.

I. Introduction

Understanding the mechanism that drives firms' choices of capital structure has been the focus of many corporate finance papers starting with the seminal work of Modigliani and Miller (1958). Black and Scholes' (1973) discovery of their famous option pricing formula suggested an innovative analytic framework for valuing corporate debt, which drew on the similarity between corporate equity and a financial call option. Since then, this avenue of research has been followed by a number of studies and many interesting results have been obtained. However, the vast majority of contingent claims dynamic models constructed to explain optimal capital structure ignore the effect of a competitive environment on firms' financing decisions. Many of these models, which usually focus on a single firm setting, tend to predict unrealistically high leverage ratios and low credit spreads.

Few attempts have been made to study the relation between firms' investment and financing decisions in the presence of strategic interaction among firms in a continuous-time real-option framework. Lambrecht (2001) examines the exit and entry strategies of levered firms in a duopoly. Zhdanov (2002) develops a model of optimal capital structure in a duopoly setting. Fries, Miller, and Perraudin (1997) study a competitive industry equilibrium under the assumption that a firm in default leaves the industry immediately. This assumption leads them to

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conclude that if equityholders are not cash constrained and are able to raise both debt and equity (for a fair price), then the industry equilibrium implies that all corporate debt is riskless. The intuition is that as long as the credit spreads are strictly positive, firms have an incentive to "undercut" each other in leverage. This incentive disappears when all debt becomes riskless. Miao (2005) examines the evolution of a competitive industry when firms experience idiosyncratic technology shocks. He focuses on a stationary equilibrium with a constant output price, while in this paper the output price follows a stochastic process whose evolution shapes the major predictions of the model.

In this paper, I focus on a perfectly competitive industry similar to Fries et al. (1997). I do not suppose, however, that defaulting firms exit instantly; instead I assume that a firm in default is taken over by its debtholders and continues to operate until it becomes optimal to exit at some future date. This assumption is consistent with the empirical evidence, especially in the U.S. Most large corporations in default do not file under Chapter 7 (liquidation) of the U.S. Bankruptcy Code, but, on the contrary, undergo a reorganization process regulated by Chapter 11. On the other hand, private workouts and renegotiation between claimholders are also common in practice.[1]

The equilibrium constructed in this paper implies strictly positive credit spreads on corporate debt, whether or not the equityholders are cash constrained. Moreover, the predicted credit spreads are nearly twice as high[2] as those resulting from the benchmark single firm model and better fit observable data. I also extend the model to incorporate the possibility of dynamic adjustments to leverage and examine the effect of the recapitalization option on a firm's financing and investment policies.

The model entails a number of empirical predictions regarding various firm and industry characteristics. In particular, credit spreads are predicted to be higher in competitive industries and to negatively depend on the cost of entry. The equilibrium entry threshold is lower and the optimal initial coupon payment is higher if shareholders can adjust their capital structure dynamically. The relation between a firm's leverage and the price in the output market depends on recapitalization costs. When these costs are high, the leverage ratio is likely to be negatively related to the output price, and this relation reverses for low adjustment costs. Finally, I show that the industrial organization result of the optimality of myopic behavior does not hold if firms are allowed to issue debt.

The remainder of the paper is organized as follows. The next section provides a review of the relevant literature. The model is developed in Section III. Section IV reviews numerical results. In Section V, I discuss the implications of debt financing on the optimality of myopic behavior. Section VI examines the optimal investment and financing strategies of a firm in a competitive industry when shareholders can dynamically adjust the level of debt. Section VII concludes. Proofs and technical details can be found in the Appendix.

[1] Wruck (1990) documents that over 97% of firms in financial distress successfully reorganize and emerge.

[2] And can be eight times as high for certain parameter values.

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II. Related Literature

This paper is related to both the industrial organization and the corporate finance literature. On the industrial organization side, some relevant work is performed by Dixit (1989) who studies optimal entry and exit strategies of a firm facing uncertain profits. Caballero (1991) examines industry equilibrium by the shortcut of modeling just one firm's decision and parametrically varying the price elasticity of demand. Leahy (1993) constructs a full-fledged competitive equilibrium with entry and exit and discovers an interesting feature of the equilibrium, namely the optimality of myopic behavior. Baldursson (1998) and Grenadier (2001) examine the case of continuous investment in oligopoly by focusing on symmetric Nash equilibrium strategies. All of these studies incorporate aggregate uncertainty only and assume that firms are financed entirely with equity. Caballero and Pindyck (1996) study the case of combined firm-specific and aggregate uncertainty (but still all-equity financed firms).

The modeling approach of this paper is also related to the growing body of literature analyzing corporate securities in a contingent claims framework. Merton (1974) was the first to apply the valuation insights of Black and Scholes (1973) to the pricing of corporate debt contracts. Leland (1994) examines corporate debt values and capital structure in a unified analytical framework and derives closed-form results for optimal leverage in a single firm scenario. A number of papers add additional flavors to his work, including Anderson and Sundaresan (1996) (a discrete-time model capable of pricing various types of debt contracts), Fan and Sundaresan (2000) (include a model of renegotiation between claimholders), and Francois and Morellec (2004) (a model motivated by the legal liquidation and reorganization procedures in the U.S.), among others. All of these papers focus on a single firm case. Models that link competition in product markets and financing decisions include Lambrecht (2001) and Fries et al. (1997) and are discussed above.

Other industrial organization papers, which explicitly look at optimal capital structure, typically capture the strategic effect of debt only. They are usually based on one- or two-period models and do not accommodate such important aspects of debt as bankruptcy costs and tax benefits. These articles focus primarily on the way in which debt financing may enable firms to commit to particular ex post behavior in imperfectly competitive markets. While they are useful in determining relations between leverage and various industry parameters, they cannot be used to justify observed leverage ratios or to price corporate securities. The first formal model of this type was developed by Brander and Lewis (1986). They incorporate a relation between financing and operating decisions due to the limited liability effect of debt. Maksimovic (1990) investigates the effect of debt financing on the ability of firms to collude in the context of a model of repeated oligopoly. Showalter (1995) extends the analysis of Brander and Lewis (1986) to the case of cost uncertainty and Bertrand competition. Lyandres (2006) extends the literature by applying a model similar to that of Brander and Lewis (1986) to the oligopoly case and provides an empirical study of the model predictions. Interesting empirical studies are also performed by Phillips (1995) and MacKay and Phillips (2005).

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III. The Model

A. A Benchmark Model of All-Equity Firms

I begin by describing a model of all-equity financed firms to demonstrate the underlying mechanism of the industry equilibrium and to establish a useful benchmark. The model is based on Leahy (1993) and Dixit (1993). Some relevant discussions can also be found in Dixit and Pindyck (1994).

I consider an industry with a large number of identical firms endowed with a technology to produce a unit (or a constant number of units) of output per period of time. To start production, an irreversible investment I is required. Once a firm is active, a fixed instantaneous cost c, must be incurred to maintain operations. Entry into the industry is free, i.e., each potential entrant has an option to pay a fixed irreversible cost I and to join the industry at any time. I assume that the inverse demand function for the industry output is given by

$$p_t = x_t D(q_t), \tag{1}$$

where q_t is the number of firms active at time t, and x_t is an exogenously specified stochastic process, which is assumed to follow a geometric Brownian motion,

$$dx_t = \mu x_t dt + \sigma x_t dW_t, \tag{2}$$

where W_t is a standard Brownian motion, and $D(.)$ is a continuously differentiable, monotonically declining function.

The intuition behind the industry equilibrium is as follows. When the price reaches a certain threshold p_{hu}, it becomes attractive to enter the industry. Since there are assumed to be infinitely many potential entrants, at each passage time to the threshold p_{hu} entry into the industry occurs so that the number of active firms increases and the price falls below p_{hu}. In other words, allowing free entry to the industry is equivalent to applying a stochastic regulator M_t^h at the upper price boundary p_{hu}. A similar regulator must be applied at p_{lu}, a lower exit threshold, i.e., a point at which it becomes optimal to exit the industry. When p_{lu} is hit, a mass of firms exits and this generates a floor—a lower reflecting barrier on the price process. This is equivalent to applying a regulator M_t^l at the lower boundary p_{lu}. The regulated stochastic process for the price becomes

$$dp_t = \mu p_t dt + \sigma p_t dW_t - dM_t^h + dM_t^l, \tag{3}$$

where

$$M_t^h = \sup_{0 \le s \le t} \left(\max \left[p_s' - p_h + M_s^l, 0 \right] \right), \tag{4}$$

$$M_t^l = \sup_{0 \le s \le t} \left(\max \left[p_l - p_s' + M_s^h, 0 \right] \right), \tag{5}$$

and $p_s' = x_s D(q_0)$.

The shareholders of each firm have rational expectations about the price process their firm faces, namely, a geometric Brownian motion between the two barriers. The firm's own entry and exit decisions again take the form of upper and

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lower price thresholds. In equilibrium, each firm's individual optimal entry and exit thresholds must coincide with the aggregate ones, i.e., the ones generated by the behavior of all firms in the industry.

It can be shown (see the Appendix for details) that the value of an active firm is given by

$$v(p) = B_1 p^{\beta_1} + B_2 p^{\beta_2} + (1-\tau)\left(\frac{p}{r-\mu} - \frac{c}{r}\right), \tag{6}$$

where β_1 and β_2 are the positive and the negative roots of the quadratic equation,

$$\frac{1}{2}\sigma^2\beta(\beta-1) + \mu\beta - r = 0.$$

The constants B_1 and B_2 together with the equilibrium entry threshold p_{hu} and the optimal exit threshold p_{lu} are given by the solution to the following system of (nonlinear) equations:

$$B_1 p_{hu}^{\beta_1} + B_2 p_{hu}^{\beta_2} + (1-\tau)\left(\frac{p_{hu}}{r-\mu} - \frac{c}{r}\right) = I, \tag{7}$$

$$\beta_1 B_1 p_{hu}^{\beta_1-1} + \beta_2 B_2 p_{hu}^{\beta_2-1} + \frac{1-\tau}{r-\mu} = 0, \tag{8}$$

$$B_1 p_{lu}^{\beta_1} + B_2 p_{lu}^{\beta_2} + (1-\tau)\left(\frac{p_{lu}}{r-\mu} - \frac{c}{r}\right) = 0, \tag{9}$$

$$\beta_1 B_1 p_{lu}^{\beta_1-1} + \beta_2 B_2 p_{lu}^{\beta_2-1} + \frac{1-\tau}{r-\mu} = 0. \tag{10}$$

In (6), the third term represents the value of the perpetual entitlement to the profit of the firm in the case when there is no entry into the industry. The first and the second terms represent the reduction in the value of the firm due to competitive entry into the industry and the increase in the value because of the option to exit.

B. A Model with Debt and Equity Financing

I now move to the focal point of this paper, i.e., the relation between competition in product markets and the optimal investment and financing strategies of firms in the industry. I assume that at the time of entry each entrant makes its financing decisions and selects its capital structure by choosing a fixed instantaneous contractual coupon payment s. At this stage, I rule out any dynamic adjustments to leverage by assuming that such adjustments are prohibitively costly. I extend the analysis to incorporate dynamic capital structure in Section VI. Capital markets are frictionless and firms are not financially constrained and are capable of issuing both debt and equity for a fair price. Firms find debt attractive because of the tax deductibility of interest payments. Costly liquidation limits the amount of debt that firms find optimal to issue. All agents are assumed risk neutral and discount their future payoffs at a constant rate r.

Unlike Fries et al. (1997), I assume that upon default a firm does not leave the industry immediately, but instead the ownership and control rights are transferred

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to the debtholders, a sunk reorganization cost is incurred, and the firm continues to operate as an all-equity entity until it becomes optimal to exit the industry at some future date.

As in the benchmark model of all-equity firms, the firms are price takers and the instantaneous profit of each firm is given by the inverse demand function $x_t D(q_t)$, net of the contractual coupon payment s and the fixed cost of production c:

$$\pi_t \quad = \quad x_t D(q_t) - c - s.$$

A particular form of $D(q)$ does not affect the solution as long as $D(q)$ is monotonically declining and continuously differentiable. The latter condition is imposed in order for the number of entering and exiting firms to be well defined.

Similar to Fries et al. (1997), I focus on aggregate uncertainty and assume that all firms are identical and at each point in time face the same shock to their profits (the case of both aggregate and idiosyncratic uncertainty remains an interesting extension). Before proceeding to the formal analysis of the model, it is useful to outline the basic intuition of the game structure. As in the benchmark all-equity model, at a certain upper threshold p_h a mass of new firms enters the industry (each of them has to incur a fixed investment cost I). As before, new entry is equivalent to applying a stochastic regulator to the price process that reflects the price off the entry threshold p_h. Upon entry, each firm selects a strictly positive coupon payment (and therefore a strictly positive leverage ratio) because of the tax benefits of debt. Positive debt levels lead to the possibility of default. Each firm defaults at the first passage time of the price process to an endogenously determined threshold p_d at which the ownership is transferred to the debtholders of the firm.[3] A crucial assumption here is that debtholders continue to employ the assets of the firm in their current use, so default does not alter the industry's competitive structure.

If the price reaches an even lower exit threshold p_l, a mass of firms exits the industry, which leads to the corresponding adjustment to the price process. When the upper threshold p_h is reached again, I observe the new entry of a mass of *levered* firms. Obviously, the following inequality must hold: $p_h > p_d > p_l$.

The thresholds p_h, p_d, and p_l must satisfy the equilibrium condition, i.e., provided that all other firms find it optimal to enter at p_h and exit at p_l, and the same must hold for any particular firm. Also, assuming free entry into the industry, it must be the case that at p_h any potential entrant is indifferent between entering and staying aside, i.e., its expected gain from entering at p_h must be exactly offset by the investment cost I. A similar condition must hold for p_l: at this price, all active firms must be indifferent between staying or leaving the industry. If it were not the case, then either nobody would want to leave the industry or a "catastrophic" exit would occur.

[3] I also assume that debtholders, having taking the control of the assets, do not lever the firm up. Therefore, once defaulted, the firm remains unlevered until it exits the industry at some future date. A possible alternative assumption is that debtholders optimally relever the firm either immediately or at some future date. However, following this assumption would complicate the model without providing much additional insight.

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I now discuss the formal solution of the model. The derivation of the equilibrium proceeds in three steps. First, I need to determine the value of a firm in default, which I call "the post-default" value of the firm. This is the value captured by the debtholders after the sunk reorganization cost has been paid. Second, the optimal coupon payment of each firm at the time of entry must be found. Finally, I must ensure that the equilibrium condition of free entry holds at the entry threshold p_h.

The post-default value of a firm is equal to the value of an unlevered firm when the price fluctuates between the entry and exit thresholds, p_h and p_l. The post-default value together with the optimal exit threshold (as a function of the entry threshold yet to be determined) is given in the following proposition.

Proposition 1. The post-default value of a firm is given by

$$A(p) = C_1 p^{\beta_1} + C_2 p^{\beta_2} + (1-\tau)\left(\frac{p}{r-\mu} - \frac{c}{r}\right), \tag{11}$$

where

$$C_1 = \frac{\beta_2(1-\tau)\left(p_l^{\beta_2-1} - p_h^{\beta_2-1}\right)}{\beta_1\beta_2(r-\mu)\left(p_l^{\beta_1-1}p_h^{\beta_2-1} - p_l^{\beta_2-1}p_h^{\beta_1-1}\right)}, \tag{12}$$

$$C_2 = \frac{\beta_1(1-\tau)\left(p_h^{\beta_1-1} - p_l^{\beta_1-1}\right)}{\beta_1\beta_2(r-\mu)\left(p_l^{\beta_1-1}p_h^{\beta_2-1} - p_l^{\beta_2-1}p_h^{\beta_1-1}\right)}, \tag{13}$$

where p_h is the equilibrium entry threshold, and p_l is the equilibrium exit threshold, given by the positive root of

$$\begin{aligned}(\beta_2-\beta_1)p_l^{\beta_1+\beta_2-1} + \beta_1 p_l^{\beta_2} p_h^{\beta_1-1} - \beta_2 p_h^{\beta_2-1} p_l^{\beta_1} \\ + \beta_1\beta_2\left[p_l - \frac{c(r-\mu)}{r}\right]\left(p_l^{\beta_1-1}p_h^{\beta_2-1} - p_l^{\beta_2-1}p_h^{\beta_1-1}\right) = 0.\end{aligned} \tag{14}$$

Proof. See the Appendix. □

In (11), the last term represents the value of the perpetual entitlement to the profit of the firm operating in a monopoly environment. The first and the second terms represent the reduction in the value of the firm due to free entry into the industry and the increase in the value due to the option to exit.

The shareholders of each firm select their capital structure by optimally choosing the contractual coupon payment s at the time of entry. The coupon payment, in turn, determines the optimal default threshold. A firm's default threshold as a function of p_l and p_h together with the values of the firm's equity and debt is determined in Proposition 2.

Proposition 2. The value of a firm's equity is

$$e(p) = C_3 p^{\beta_1} + C_4 p^{\beta_2} + (1-\tau)\left(\frac{p}{r-\mu} - \frac{c+s}{r}\right),$$

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where

$$C_3 = \frac{\beta_2(1-\tau)\left(p_d^{\beta_2-1} - p_h^{\beta_2-1}\right)}{\beta_1\beta_2(r-\mu)\left(p_d^{\beta_1-1}p_h^{\beta_2-1} - p_d^{\beta_2-1}p_h^{\beta_1-1}\right)},$$

$$C_4 = \frac{\beta_1(1-\tau)\left(p_h^{\beta_1-1} - p_d^{\beta_1-1}\right)}{\beta_1\beta_2(r-\mu)\left(p_d^{\beta_1-1}p_h^{\beta_2-1} - p_d^{\beta_2-1}p_h^{\beta_1-1}\right)},$$

and the optimal default threshold is the positive root of

$$(\beta_2-\beta_1)p_d^{\beta_1+\beta_2-1} + \beta_1 p_d^{\beta_2}p_h^{\beta_1-1} - \beta_2 p_h^{\beta_2-1}p_d^{\beta_1} + \beta_1\beta_2\left[p_d - \frac{(s+c)(r-\mu)}{r}\right]\left(p_d^{\beta_1-1}p_h^{\beta_2-1} - p_d^{\beta_2-1}p_h^{\beta_1-1}\right) = 0. \quad (15)$$

The value of the firm's debt is

$$d(p) = C_5 p^{\beta_1} + C_6 p^{\beta_2} + \frac{s}{r},$$

where

$$C_5 = \frac{\beta_2 p_h^{\beta_2-1}\left((1-\alpha)A(p_d) - \frac{s}{r}\right)}{\beta_2 p_h^{\beta_2-1}p_d^{\beta_1} - \beta_1 p_h^{\beta_1-1}p_d^{\beta_2}}, \quad (16)$$

$$C_6 = -\frac{\beta_1 p_h^{\beta_1-1}\left((1-\alpha)A(p_d) - \frac{s}{r}\right)}{\beta_2 p_h^{\beta_2-1}p_d^{\beta_1} - \beta_1 p_h^{\beta_1-1}p_d^{\beta_2}}, \quad (17)$$

where $A(p_d)$ is the post-default value of the firm derived in Proposition 1.

Proof. See the Appendix. □

The objective of the equityholders is to maximize the value of their firm ex ante, i.e., the total value of the firm at the time of debt issuance (immediately upon entry). The formal maximization problem of the shareholders can be stated as:

$$V^*(p_h) = \max_s (e(p_h, s) + d(p_h, s)),$$

subject to (14) and (15). In other words, at the time of entry the equityholders of each firm choose their contractual coupon payment s to maximize the total ex ante value of their firm's securities. When making their decision, they optimally trade off tax benefits of debt against expected bankruptcy costs, while taking into account the investment and financing strategies of the other firms in the industry.

The final condition that determines equilibrium is the free entry restriction imposed on the value of an entering firm,

$$V^*(p_h, s^*) = I,$$

which stipulates that at the entry threshold each potential entrant must be exactly indifferent between entering the industry and staying aside. If this condition does not hold, then there either would be no entry at all or, in the opposite case, a "catastrophic" entry would happen, and the entry threshold would be driven down by a mass of entering firms.

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IV. Numerical Analysis

I start by comparing the optimal equilibrium strategies of all-equity financed firms to the case when firms can optimally choose their capital structures. Figure 1 presents the equilibrium entry thresholds of both an unlevered and an optimally levered firm in a competitive industry as functions of the volatility of cash flows σ and the investment cost I. The optimal investment thresholds are increasing in both volatility and investment cost. This is a standard real-option result—both high volatility and high investment costs increase the value of the option to wait. Also, as follows from Figure 1, if firms are allowed to issue debt, the equilibrium entry threshold slightly decreases. This is because the option to issue debt is valuable to the potential entrants into the industry because of the tax shields on interest payments. This option increases the total value of an entering firm. For the equilibrium zero profit condition to hold, firms must enter at a lower price threshold so that the decrease in value caused by lower expected profits is exactly offset by the additional value of the option to issue debt. The results presented in Figure 1 are based on the following set of parameter values: $I = 5$, $c = 0.3$, $\tau = 0.15$, $r = 0.05$, $\mu = 0.01$, $\sigma = 0.2$, and $\alpha = 0.2$.

FIGURE 1

Equilibrium Entry Thresholds

Figure 1 presents the equilibrium entry thresholds of an unlevered firm (dashed line) and an optimally levered firm (solid line) in a competitive industry as functions of the investment cost I and the volatility of the stochastic shock σ. The input parameter values are set as follows: $I = 5$, $c = 0.3$, $\tau = 0.15$, $r = 0.05$, $\mu = 0.01$, $\sigma = 0.2$, and $\alpha = 0.2$.

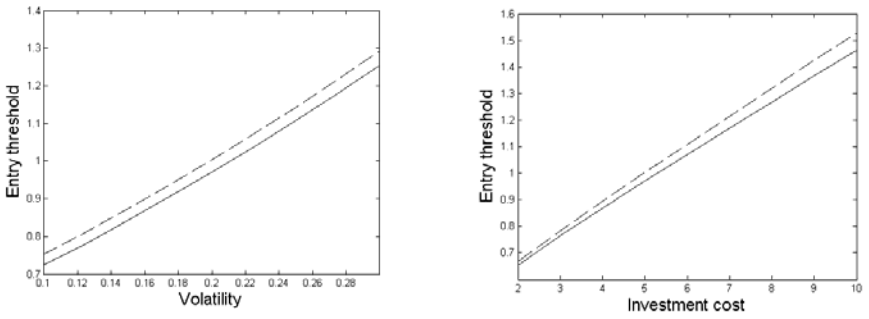


Figure 2 presents the optimal market leverage ratios (the market value of debt divided by the total value of the firm) of a firm in a perfectly competitive industry and of a monopolist. The monopolist does not experience any threat of subsequent entry and therefore does not expect the price process to be "regulated" by the entry of a mass of firms as described by (3)–(5). Furthermore, the zero profit condition does not hold for a monopolist who extracts rents from the market. Therefore, the option to enter the industry is valuable for a monopolist, while in a perfectly competitive industry the value of the option to enter net of the discounted investment cost is zero. A formal discussion of the monopolist's optimization problem is presented in the Appendix. The results displayed in Figure 2 reveal that the equilibrium leverage of a firm in a perfectly competitive environment is generally comparable to the optimal leverage of a monopolist. The

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optimal leverage of a competitive firm is also more sensitive to the cost of investment. Reducing the investment cost (while keeping the fixed cost of production c constant) leads to higher operating leverage and decreases the optimal financial leverage. When the cost of investment goes up from $I = 2$ to $I = 10$, the optimal leverage in a perfectly competitive industry grows by about 4%.

FIGURE 2
Optimal Leverage Ratios

Figure 2 displays the optimal leverage ratios of a monopolist (dashed line) and a firm in a perfectly competitive environment (solid line) as functions of the investment cost I and the reorganization cost α for the base set of parameter values.

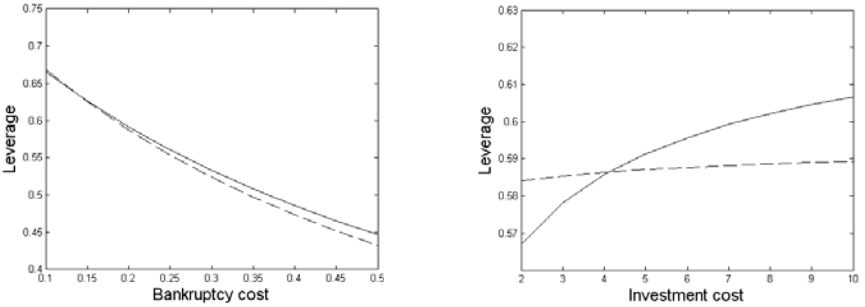


Figure 3 provides some important results by comparing the equilibrium credit spreads in a competitive industry with the credit spread on debt issued by a monopolist. Credit spreads are calculated as the difference between the current yield and the riskless rate r for firms entering the industry:

$$cs = \frac{s^*}{d(p_h)} - r. \tag{18}$$

While the optimal leverage ratios are just slightly affected by competition in product markets, credit spreads are significantly higher in a perfectly competitive environment. For example, for the base parameter values $I = 5$, $c = 0.3$, $\tau = 0.15$, $r = 0.05$, $\mu = 0.01$, $\sigma = 0.2$, and $\alpha = 0.2$ the credit spread of a monopolist is 158 basis points, while in a perfectly competitive industry the equilibrium credit spread is 216 basis points, or about 40% higher. For some values of the input parameters (e.g., low investment cost I) the credit spreads in a competitive industry can be nearly twice as high as the credit spreads in the single firm case.

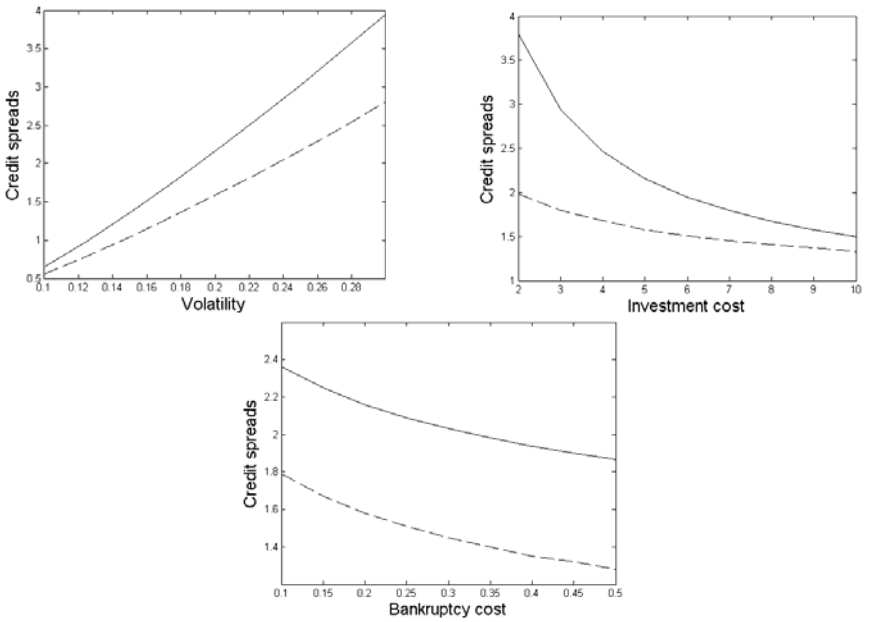
One important remark is in order. The credit spreads in Figure 3 are measured at the *equilibrium* leverage ratios implied by the model, i.e., about 60%. Leverage ratios of this magnitude correspond to firms issuing high-yield junk bonds. To meaningfully compare the credit spreads generated by the model with the empirically observed values, one would have to account for the difference in leverage ratios. That is, to use the model for the pricing of corporate debt issued by a typical firm, one needs to use the appropriate input parameters, i.e., the typical leverage ratio. I therefore also measure the credit spreads implied by the model for lower leverage ratios. Figure 4 presents the equilibrium credit spreads both in a competitive industry and in a monopoly environment by requiring that

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FIGURE 3

Equilibrium Credit Spreads

Figure 3 presents the equilibrium credit spreads (in percentage points) of a monopolist (dashed line) and a firm in a perfectly competitive industry (solid line) as functions of the volatility of the stochastic shock σ, the investment cost I, and the reorganization cost α.

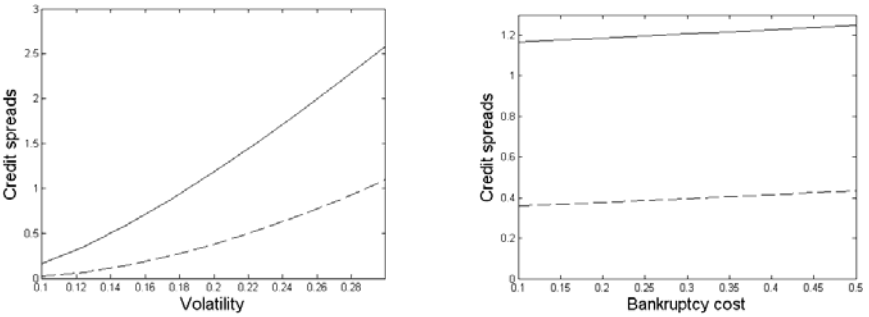


the market leverage ratio of an entering firm be 22% (which is about the average market leverage of industrial firms in the U.S.).

FIGURE 4

Equilibrium Credit Spreads for the Base Set of Input Parameters

Figure 4 presents the equilibrium credit spreads (in percentage points) of a monopolist (dashed line) and a firm in a perfectly competitive industry (solid line) as functions of the volatility of the stochastic shock σ, the investment cost I, and the reorganization cost α for the base set of input parameters. The leverage ratio of a firm upon entry is set to 22%.



The results in Figure 4 reveal that the credit spreads in a competitive industry are again considerably higher than the credit spreads on debt issued by a monopolist. This result is particularly important because of the general tendency

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of traditional contingent claims capital structure models, which mostly focus on the single firm case, to predict unrealistically low credit spreads.[4] The present analysis offers a potential explanation for this discrepancy—competition in product markets may significantly influence the equilibrium credit spreads. The major factor causing credit spreads to rise is the effect of the strategic interaction among firms on the price process. As argued above, the free entry condition effectively places an upper reflecting barrier on the price process, and the price is regulated down at each passage time to the barrier. Two major implications of this cause the equilibrium credit spreads to increase. First, the probability of default in any given period of time increases since it becomes more likely that the default threshold will be hit. Second, the post-default value of the firm (assumed to be equal to the value of the unlevered firm evaluated at the default threshold net of the proportional reorganization cost) goes down as some of the "good" states of the demand shock are ruled out by the presence of the reflecting barrier. Debtholders are rational and take into account the implications of competition in product markets on the value of their claims and therefore charge higher interest.

Credit spreads in a competitive industry produced by the model for a representative firm with a 22% market leverage seem to agree well with the data. For example, Delianedis and Geske (2002) report mean credit spreads of about 0.5%, 0.7%, and 1.1% for AA, A, and BBB rated bonds, respectively. Duffee (1998) reports credit spreads between 0.67% and 1.84% on a sample of 2,814 non-callable bonds. These numbers are consistent with the competitive industry model of this paper, but exceed the spreads predicted by single firm models. As displayed in Figure 4, for the base set of input parameters the competitive industry model results in a credit spread of 1.18%, while the single firm model predicts the credit spread of 0.37%. When the volatility of the stochastic shock σ goes down to 15%, the predicted credit spreads are 0.60% and 0.14% for the competitive and the monopoly models. Moreover, when volatility is further reduced to 10%, the single firm model results in a credit spread of 0.02%, an unrealistically low value, while the prediction of the competitive industry model is eight times as high, about 0.16%.

Credit spreads are positively related to the volatility of cash flows σ because high volatility makes debt riskier and increases the probability of costly default. The relation between equilibrium credit spreads and the bankruptcy cost is negative in Figure 3 (equilibrium leverage) and positive in Figure 4 (fixed leverage of 22%). While high bankruptcy costs apparently reduce the value of debt and force debtholders to increase the interest rate, they also lead to lower optimal leverage ratios. Lower leverage makes debt less risky and therefore reduces the equilibrium credit spreads for firms both in a competitive industry and in a monopoly environment. This leads to the negative relation between credit spreads and bankruptcy costs in Figure 3. On the other hand, keeping the leverage constant, higher bankruptcy costs increase the risk of a particular bond issue and result in higher yield spreads (as in Figure 4). The credit spreads are also negatively related to the cost of investment, I. A low investment cost implies earlier

[4]For example, the model in Leland (1994) predicts credit spreads below five basis points for firms with 30% leverage (Figure 5 in Leland's paper).

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entry and (keeping the annual cost of production c constant), results in higher operating leverage, making debt contracts riskier.

V. On the Optimality of Myopic Behavior

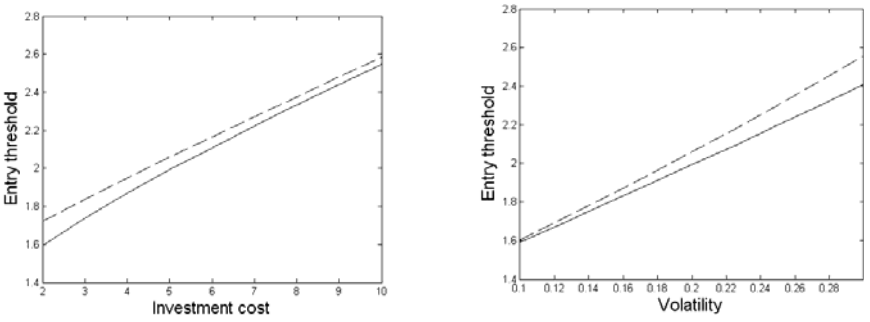
Leahy (1993) established a remarkable property of the competitive equilibrium among all-equity financed firms. He shows that by behaving myopically and disregarding the effect of the subsequent entry of other firms on the price process, each firm will still follow the optimal strategy. In other words, each firm can make its entry decision by finding the expected present value of its profits as if it were the last firm that would enter this industry, and then making the calculation of the entry option value. While the firm should have rational expectations about the stochastic demand shock x, it can totally disregard the other firms' entry decisions. The intuition of this interesting result is as follows. When a firm pretends that it is the last to enter the industry, it is ignoring two things. First, it does not incorporate the reduction in the upside potential of its profit caused by the subsequent entry that occurs if x reaches a certain threshold. Second, it ignores the fact that the prospects of future entry reduce the value of its option to wait. The two effects exactly offset each other.

In this section, I show how the option to issue debt invalidates the optimality of myopic behavior. Figure 5 plots the optimal myopic entry threshold and the competitive one as functions of the volatility of cash flows and of the investment cost I for the following case: $I = 5$, $c = 1$, $\tau = 0.15$, $r = 0.05$, $\mu = 0.02$, $\sigma = 0.2$, and $\alpha = 0.2$. Both the myopic and the competitive thresholds are obtained for an optimally levered firm.

FIGURE 5

Optimal Investment Thresholds

Figure 5 presents the optimal myopic (dashed line) and competitive (solid line) entry thresholds as functions of the volatility of the stochastic shock σ and of the investment cost I for the following set of input parameters: $I = 5$, $c = 1$, $\tau = 0.15$, $r = 0.05$, $\mu = 0.01$, $\sigma = 0.2$, and $\alpha = 0.2$.



As the results in Figure 5 indicate, the competitive equilibrium can no longer be achieved as a result of myopic behavior on the part of the firms. For a firm in a perfectly competitive environment, the option to issue debt has no value ex ante (as long as all competitors are endowed with the same option). For the equilibrium free entry condition to hold, the increase in value due to this option must be fully

offset by the lower entry thresholds and therefore lower expected profits. All additional value arising from the tax benefits of debt is fully eroded by earlier entry. On the other hand, a monopolist has the advantage of exercising its options optimally without having to look at the actions of other firms. The option to issue debt will lead to an increase in the total value of its securities, and result in an earlier entry. However, there is no free entry condition that must be satisfied and the monopolist does not have to ensure that its value upon entry is the same as in the all-equity case. Therefore, its entry threshold will go down but not by as much as the entry threshold in the competitive industry. As Figure 5 shows, the myopic (monopolistic) threshold always exceeds the perfectly competitive one.

As can be inferred from Figure 5, the difference between the myopic and competitive equilibrium entry thresholds is particularly noticeable for lower investment costs and higher volatility. Higher volatility increases the value of the option to wait, while low investment costs imply a higher relative effect of the option to issue debt on the value of the firm (since in a perfectly competitive industry the value of an active firm upon entry exactly equals the incurred investment cost). For example, for $I = 1$ and $\sigma = 0.4$, the myopic entry threshold exceeds the competitive one by 23%.

VI. Dynamic Capital Structure

In this section, I relax the assumption that a firm's leverage is fixed up to the moment of bankruptcy. This assumption is critical in the preceding sections. However, it is somewhat unrealistic. A growing literature on target leverage suggests that firms tend to adjust their capital structures in response to changes in the economic environment (see, for example, Leary and Roberts (2005), Flannery and Rangan (2006), and Strebulaev (2007)). At times of economic expansions when profits are high, shareholders may decide to issue additional debt to keep the leverage ratio close to the target. On the contrary, in a recession they may optimally choose to retire some debt to avoid costly default.

The problem of dynamic capital structure in a single firm case is studied by, among others, Kane, Markus, and McDonald (1984), (1985), Fischer, Heinkel, and Zechner (1989), Fries et al. (1997), Goldstein, Ju, and Leland (2001), and Mauer and Triantis (1994).

Fries et al. (1997) consider a case of costless upward adjustments of leverage and assume that these adjustments occur so as to maximize the total value of the firm's securities, i.e., its debt and equity. They propose a solution technique to find the values of the firm's debt and equity together with the optimal releverage policy.

Goldstein et al. (2001) focus on a firm with zero production costs and assume that a fixed restructuring cost must be incurred when the leverage ratio is adjusted. Similar to Fries et al. (1997), they allow the firm to increase but not decrease its leverage. The scaling property of a geometric Brownian motion keeps their model tractable and allows for closed-form solutions for the values of debt and equity.

Titman and Tsyplakov (2007) construct a model for a firm that can dynamically adjust both its capital structure and its investment policy. They use numeri-

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cally based finite difference schemes to solve the corresponding stochastic control problem and obtain the value of the firm's equity.

My solution approach is also based on using a finite difference scheme to solve for the values of firms' securities. Unlike Titman and Tsyplakov (2007), I do not allow firms to continuously adjust their production capacities. However, I incorporate the effect of competitive entry into my analysis.

Unlike the majority of related studies that consider only positive changes in leverage, I allow equityholders to adjust leverage both upward and downward. They will do so in response to prevailing economic conditions and anticipated changes in the competitive environment. In economic expansions, when demand is high, shareholders have an incentive to increase the level of debt and obtain greater tax benefits since bankruptcy becomes less likely. On the contrary, in economic downturns it is optimal to reduce leverage and avoid costly default.

As before, I examine a firm in a perfectly competitive environment that is financed by both equity and perpetual debt with a constant instantaneous coupon payment s. The stochastic shock to price follows a geometric Brownian motion,

$$\frac{dx_t}{x_t} = \mu dt + \sigma dW_t,$$

and the firm is endowed with a technology to produce one unit of output per period of time. For the sake of computational tractability, I assume zero production costs $c = 0$,[5] so the instantaneous profit is given by $\pi_t = p_t - s$, where $p_t = x_t D(q_t)$ is the price in the output market.

At any time, the equityholders are able to adjust leverage by changing the contractual coupon payment s. To do so, they renegotiate their current debt contract with the debtholders. I assume that renegotiation is costly, and at any restructuring date a sunk restructuring cost $q\Delta s$ must be borne, where Δs is the change in the coupon payment and q is a constant of proportionality. Any change in leverage can be viewed as follows: the equityholders repurchase all outstanding debt for its fair market price and then issue new debt. It is also equivalent to assuming that equityholders initiate a side payment to debtholders to account for the difference between the values of debtholders' claims in the firm before and after restructuring. The value of equity after the existing debt has been repaid is the sum of the values of the new issue of debt and the ex post equity value (i.e., equity value immediately after the issuance of new debt). Therefore, when selecting a new coupon, equityholders maximize the gain in the total value of the firm's securities net of the restructuring costs. The optimization program of the equityholders can be formalized as

$$\begin{aligned} s_1^*(p) &= \arg\max_{s_1}(E(s_1,p) - E(s,p) + D(s_1,p) - D(s,p) - q|s_1 - s|) \\ &= \arg\max_{s_1}(V(s_1,p) - V(s,p) - q|s_1 - s|), \end{aligned} \tag{19}$$

[5] Assuming non-zero production costs will not lead to any additional insights while complicating the model by making it necessary to numerically compute not only the entry and the default thresholds, but also the exit one.

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where s is the current coupon payment, s_1^* is the new optimal coupon, and $V(.)$ is the total value of the firm's securities. If the shareholders follow the optimal restructuring policy, then the value of equity becomes equal to

$$E(s,x) = E\left(s_1^*(x),x\right) - q|s_1^*(x) - s| + D\left(s_1^*(x),x\right) - D(s,x). \tag{20}$$

The problem of the shareholders is an optimal stochastic control one, and I use a finite difference method to solve for the values of debt and equity and to determine the optimal restructuring policy. This method is explained in detail in the Appendix.

Figure 6 plots the optimal investment threshold as a function of the costs of adjusting the debt level, q. When the adjustment cost is low, the total value of the firm's securities goes up. In this case, the equityholders can easily relever their firm in response to a change in the economic conditions by issuing new debt or retiring a fraction of the existing debt. This allows them to more fully utilize the tax benefits of debt and avoid costly foreclosure, leading to an increase in the total value of the firm. Because the industry is perfectly competitive and entry is free (anybody can pay the fixed investment cost I and enter), a higher value upon entry leads to an earlier entry, when profits are lower. A reduction in the present value of the stream of profits is exactly offset by the increase in the present value of the tax shields so the total value of the firm's securities upon entry is still equal to the cost of investment, I, and the equilibrium free entry condition is satisfied.

FIGURE 6

Equilibrium Entry Threshold as a Function of the Adjustment Cost

Figure 6 plots the equilibrium entry threshold in the dynamic capital structure case as a function of the adjustment cost, q, for the following set of input parameters: $I = 5$, $c = 0$, $\tau = 0.15$, $r = 0.05$, $\mu = 0.01$, $\sigma = 0.2$, and $\alpha = 0.2$.

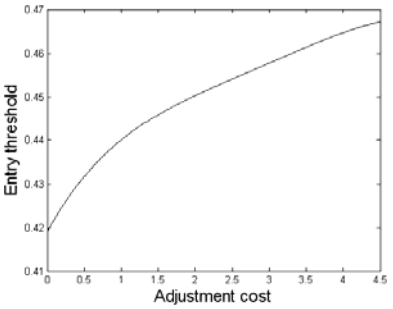


Figure 7 provides the optimal contractual coupon payment that the equityholders choose upon entry into the industry as a function of the cost of adjusting capital structure, q. When the adjustment cost q is low, it is relatively easy to reduce the level of debt if a low realization of the stochastic shock to profits occurs. Therefore, initially at the time of entry, the shareholders choose a higher coupon payment that allows them to obtain greater tax benefits. On the contrary, when the cost of adjusting leverage is high, a high initial coupon payment can easily lead to a costly default, as the benefit of changing the level of debt may not be able to outweigh the corresponding adjustment costs. In this case, it is optimal to resort to a lower initial coupon. As the results in Figure 6 suggest, for the base set of

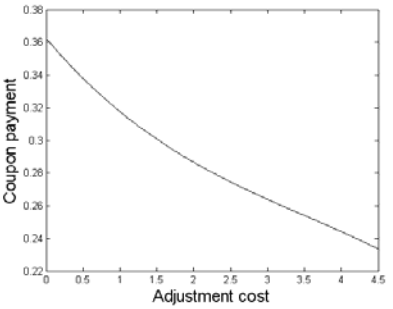
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input parameters, the optimal initial contractual coupon payment s goes down by over 30% from 0.36 to 0.24 when the adjustment cost q goes up from 0 to 4.[6]

FIGURE 7

Optimal Coupon a Function of the Adjustment Cost

Figure 7 plots the optimal contractual coupon payment upon entry in the dynamic capital structure case as a function of the adjustment cost, q, for the following set of input parameters: $I = 5$; $c = 0$; $\tau = 0.15$; $r = 0.05$; $\mu = 0.01$; $\sigma = 0.2$, and $\alpha = 0.2$.



A remark is in order. The model implies that in the dynamic capital structure-setting firms issue debt more aggressively. A majority of existing models (see, for example, Goldstein et al. (2001)) reach the opposite conclusion.[7] The reason for this discrepancy is twofold. First, the majority of papers (including Goldstein et al. (2001)) focus on the single firm case. In the single firm setting, due to the positive drift of the underlying asset process the firm's value is expected to grow and thus firms issue debt less aggressively to preserve the option to rebalance. On the other hand, in the competitive industry setting, the output price is capped above at p_h, reducing the option value of rebalancing. Second, Goldstein et al. (2001) focus on *upward* changes in leverage only, while I allow for both *upward* and *downward* adjustments. Precluding downward adjustments in the competitive industry setting would make the dynamic capital structure case trivial as the firms would never adjust their leverage ratios since in the model they choose their capital structure upon entry when the price process is at its maximum feasible value p_h.

Figure 8 provides the simulated leverage ratios of a firm in a competitive industry when dynamic adjustments of leverage are feasible. Figure 8, Graph A presents the average leverage ratio as a function of price when the adjustment cost is low, $q = 0.01$, while Graph B plots the same function for the case with a high adjustment cost, $q = 2.5$. Interestingly, the relation between the leverage ratio and the output price depends on the adjustment cost q. This relation is positive when the adjustment cost is low, but becomes negative for high adjustment costs. Indeed, when the adjustment cost is high, it might be optimal for the equityholders to keep their coupon payment fixed even when the profit is low. In this case, the expected bankruptcy cost is lower than the cost that would have to be incurred to reduce the level of debt and avoid bankruptcy. Since the value of equity is more

[6] $q = 4$ implies that to change the coupon payment by \$1 a cost of \$4 must be incurred.

[7] I am grateful to the referee for pointing this out.

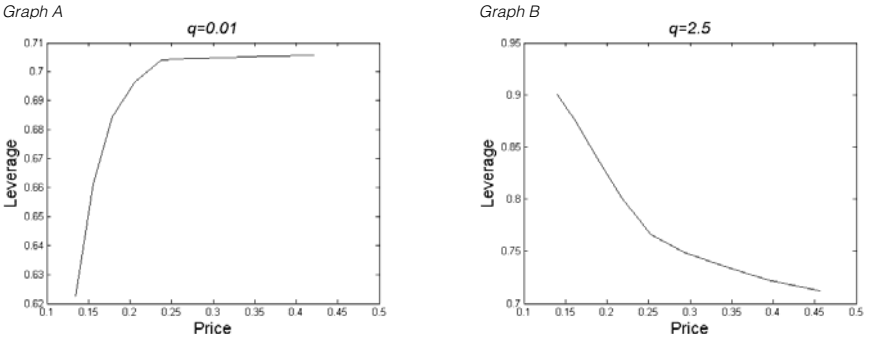
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sensitive to the changes in the stochastic profit than the value of debt, the market leverage ratio increases as the output price decreases. On the other hand, when the adjustment cost q is low, it is optimal to retire some debt if the state of the stochastic shock is low, and reduce the amount of outstanding debt. Therefore, empirically I expect to see a negative relation between the leverage ratio and the state of the economy for firms with high costs of adjusting capital structure. Such firms are likely to have higher leverage ratios in economic downturns than in economic expansions and are more likely to file for bankruptcy. On the contrary, firms with relatively low adjustment costs will retire some debt in recessions and are probably able to avoid foreclosure.

FIGURE 8

Leverage Ratios in the Industry

Figure 8 plots the average simulated leverage ratios in a competitive industry as functions of the price in the output market. Graph A refers to the case with low costs of adjusting leverage ($q = 0.01$), while Graph B refers to the case with high adjustment costs ($q = 2.5$).



VII. Conclusions

This paper constructs a competitive industry equilibrium with debt under the assumption that a firm in default does not leave the industry immediately, but is taken over by its debtholders who continue to employ the assets of the firm. Unlike in Fries et al. (1997), the equilibrium does not imply that all corporate debt is riskless unless equityholders are financially constrained. On the contrary, the model results in strictly positive credit spreads, which are significantly higher than the credit spreads predicted by traditional models.

This result can partially explain the inability of most contingent claims capital structure models to predict realistic credit spreads values. Most of those models focus on a single firm case and ignore competition among firms in product markets. By introducing competition, the model in this paper results in credit spreads that agree better with the data. I also extend the model to examine possible dynamic adjustments of capital structure and to study the dynamics of leverage.

The model entails a number of empirical predictions regarding various firm and industry characteristics. In particular, credit spreads are predicted to be higher in competitive industries and to negatively depend on the investment cost. The

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equilibrium entry threshold is expected to be lower and the optimal initial coupon payment is excepted to be higher if shareholders can adjust their capital structure dynamically. Also, the relation between a firm's leverage and the price in the output market is expected to depend on the magnitude of the costs of adjusting leverage. When adjustment costs are high, the leverage ratio is likely to be negatively related to the output price, and this relation may reverse for low adjustment costs. Finally, I show that the industrial organization result of the optimality of myopic behavior does not hold if firms are allowed to issue debt.

The model can be extended in a variety of ways. One possible extension would be to introduce production flexibility and allow firms to change their production capacities over time. Another way to extend the model would be to introduce idiosyncratic uncertainty among firms in addition to the aggregate shock to their profits. In this case, even under the assumption of immediate exit from the industry upon default, there might be a sustainable equilibrium with risky debt.

VIII. Appendix

A. A Benchmark Model of All-Equity Firms

Here I elaborate on the optimal strategies of all-equity firms defined by the set of equations (7)–(10). While the price is between the lower and the upper boundaries p_l and p_h and there is no entry to or exit from the industry, the value of an active firm $v(p)$ must satisfy the following ODE:

$$\frac{1}{2}p^2\sigma^2 v_{pp}(p) + \mu p v_p(p) - rv(p) + (1-\tau)(p-c) = 0, \tag{21}$$

where $v_p(p)$ and $v_{pp}(p)$ are the first and the second derivatives of the value function with respect to price. The solution of (21) is given by

$$v(p) = B_1 p^{\beta_1} + B_2 p^{\beta_2} + (1-\tau)\left(\frac{p}{r-\mu} - \frac{c}{r}\right), \tag{22}$$

where β_1 and β_2 are the positive and the negative roots of the quadratic equation,

$$\frac{1}{2}\sigma^2\beta(\beta-1) + \mu\beta - r = 0. \tag{23}$$

Equation (21) must be solved subject to four relevant boundary conditions. A value-matching condition (7) requires that the value of an active firm upon entry be exactly equal to the investment cost I, $v(p_u) = I$. A similar condition must hold at the equilibrium exit threshold: $v(p_l) = 0$. The remaining boundary conditions are the smooth-pasting condition (10) and the no-arbitrage condition (8). They ensure the optimality of the exit threshold p_l and preclude arbitrage in financial markets. For a detailed exposition and discussions see, for example, Dixit and Pindyck (1994).

B. Proofs of Propositions 1 and 2

Proof of Proposition 1. Under the assumption that upon default the firm is taken over by its debtholders and continues to maintain its regular operational activities, the post-default value of the firm equals the corresponding value of the unlevered firm. While the price is between the lower and the upper boundaries p_l and p_h, and there is no entry to or exit from the industry, this value must satisfy the following ODE:

$$\frac{1}{2}p^2\sigma^2 A_{pp}(p) + \mu p A_p(p) - rA(p) + (1-\tau)(p-c) = 0, \tag{24}$$

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whose solution is given by

$$A(p) = C_1 p^{\beta_1} + C_2 p^{\beta_2} + (1-\tau)\left(\frac{p}{r-\mu} - \frac{c}{r}\right). \tag{25}$$

To preclude arbitrage opportunities, it must be that $A_p(p_h) = 0$. The optimality of the exit threshold also requires that $A_p(p_l) = 0$. Therefore,

$$C_1\beta_1 p_l^{\beta_1-1} + C_2\beta_2 p_l^{\beta_2-1} + \frac{1-\tau}{r-\mu} = 0 \tag{26}$$

and

$$C_1\beta_1 p_h^{\beta_1-1} + C_2\beta_2 p_h^{\beta_2-1} + \frac{1-\tau}{r-\mu} = 0. \tag{27}$$

Solving (26) and (27) together yields

$$C_1 = \frac{\beta_2(1-\tau)\left(p_l^{\beta_2-1} - p_h^{\beta_2-1}\right)}{\beta_1\beta_2(r-\mu)\left(p_l^{\beta_1-1}p_h^{\beta_2-1} - p_l^{\beta_2-1}p_h^{\beta_1-1}\right)}, \tag{28}$$

$$C_2 = \frac{\beta_1(1-\tau)\left(p_h^{\beta_1-1} - p_l^{\beta_1-1}\right)}{\beta_1\beta_2(r-\mu)\left(p_l^{\beta_1-1}p_h^{\beta_2-1} - p_l^{\beta_2-1}p_h^{\beta_1-1}\right)}. \tag{29}$$

The equilibrium optimal exit threshold must be consistent with that of any particular firm. In other words, every firm must be indifferent between staying in the industry and exiting at p_l. Therefore, $A(p_l) = 0$ must hold, which implies that p_l must be the positive root of the following equation:

$$\begin{aligned}&(\beta_2-\beta_1)p_l^{\beta_1+\beta_2-1} + \beta_1 p_l^{\beta_2} p_h^{\beta_1-1} - \beta_2 p_h^{\beta_2-1} p_l^{\beta_1} \\ &+ \beta_1\beta_2\left[p_l - \frac{c(r-\mu)}{r}\right]\left(p_l^{\beta_1-1}p_h^{\beta_2-1} - p_l^{\beta_2-1}p_h^{\beta_1-1}\right) = 0. \qquad \square\end{aligned}$$

Proof of Proposition 2. An argument similar to that used in the proof of Proposition 1 yields that the value of equity is given by

$$e(p) = C_3 p^{\beta_1} + C_4 p^{\beta_2} + (1-\tau)\left(\frac{p}{r-\mu} - \frac{c+s}{r}\right). \tag{30}$$

The constants C_3 and C_4 together with the optimal default threshold are determined through the following set of boundary conditions:

$$e(p_d) = C_3 p_d^{\beta_1} + C_4 p_d^{\beta_2} + (1-\tau)\left(\frac{p_d}{r-\mu} - \frac{c+s}{r}\right) = 0, \tag{31}$$

$$e_p(p_d) = \beta_1 C_3 p_d^{\beta_1-1} + \beta_2 C_4 p_d^{\beta_2-1} + \frac{1-\tau}{r-\mu} = 0, \tag{32}$$

and

$$e_p(p_h) = \beta_1 C_3 p_h^{\beta_1-1} + \beta_2 C_4 p_h^{\beta_2-1} + \frac{1-\tau}{r-\mu} = 0. \tag{33}$$

The first two are the value-matching and the smooth-pasting conditions for the equityholders' maximization problem; the third is a no-arbitrage condition similar to (27).

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Solving (31) through (33) yields

$$C_3 = \frac{\beta_2(1-\tau)\left(p_d^{\beta_2-1}-p_h^{\beta_2-1}\right)}{\beta_1\beta_2(r-\mu)\left(p_d^{\beta_1-1}p_h^{\beta_2-1}-p_d^{\beta_2-1}p_h^{\beta_1-1}\right)}, \tag{34}$$

$$C_4 = \frac{\beta_1(1-\tau)\left(p_h^{\beta_1-1}-p_d^{\beta_1-1}\right)}{\beta_1\beta_2(r-\mu)\left(p_d^{\beta_1-1}p_h^{\beta_2-1}-p_d^{\beta_2-1}p_h^{\beta_1-1}\right)}, \tag{35}$$

and

$$\begin{aligned}(\beta_2-\beta_1)p_d^{\beta_1+\beta_2-1}+\beta_1 p_d^{\beta_2}p_h^{\beta_1-1}-\beta_2 p_h^{\beta_2-1}p_d^{\beta_1}\\ +\beta_1\beta_2\left[p_d-\frac{(s+c)(r-\mu)}{r}\right]\left(p_d^{\beta_1-1}p_h^{\beta_2-1}-p_d^{\beta_2-1}p_h^{\beta_1-1}\right) = 0.\end{aligned} \tag{36}$$

The value of the firm's debt is given by

$$d(p) = C_5 p^{\beta_1}+C_6 p^{\beta_2}+\frac{s}{r}, \tag{37}$$

and must satisfy the following boundary conditions,

$$d(p_d) = (1-\alpha)A(p_d) \tag{38}$$

and

$$d_p(p_h) = \beta_1 C_5 p_h^{\beta_1-1}+\beta_2 C_6 p_h^{\beta_2-1} = 0. \tag{39}$$

Therefore, constants C_5 and C_6 are given by

$$C_5 = \frac{\beta_2 p_h^{\beta_2-1}\left((1-\alpha)A(p_d)-\frac{s}{r}\right)}{\beta_2 p_h^{\beta_2-1}p_d^{\beta_1}-\beta_1 p_h^{\beta_1-1}p_d^{\beta_2}}, \tag{40}$$

$$C_6 = -\frac{\beta_1 p_h^{\beta_1-1}\left((1-\alpha)A(p_d)-\frac{s}{r}\right)}{\beta_2 p_h^{\beta_2-1}p_d^{\beta_1}-\beta_1 p_h^{\beta_1-1}p_d^{\beta_2}}. \tag{41}$$

□

C. The Optimal Strategy of a Monopolist

Here I derive the optimal leverage together with the optimal entry, default, and exit thresholds of a monopolist, which are used throughout the paper for comparison reasons. I start by assuming that both entry and financing decisions have already been made and derive the optimal default and exit thresholds. Then I focus on the optimal entry and coupon payment chosen by the monopolist upon entry. The monopolist does not worry about the threat of subsequent entry, and the price process it faces is an (unregulated) geometric Brownian motion,

$$\frac{dp_t}{p_t} = \mu dt+\sigma dW_t. \tag{42}$$

The following proposition provides the values of the monopolist's debt and equity together with the optimal default threshold.

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Proposition 3. The monopolist's debt and equity values are given by

$$d_m(p) = \frac{s_m}{r} - \left[\frac{s_m}{r} - (1-\alpha)A(p_d))\right]\left(\frac{p}{p_d}\right)^{\beta_2} \tag{43}$$

and

$$e_m(p) = (1-\tau)\left[\frac{p}{r-\mu} - \frac{c+s_m}{r} - \left(\frac{p}{p_d}\right)^{\beta_2}\left(\frac{p_d}{r-\mu} - \frac{c+s_m}{r}\right)\right], \tag{44}$$

where p is the output price, s_m is the coupon payment, and $A(p_d)$ is the post-default value of the firm. The optimal default threshold is given by

$$p_d = \frac{\beta_2}{\beta_2 - 1}\frac{(c+s_m)(r-\mu)}{r}. \tag{45}$$

Proof. Similar arguments to those used in the proofs of Propositions 1 and 2 lead to the following specification for the value of equity:

$$e_m(p) = Ap^{\beta_1} + Bp^{\beta_2} + (1-\tau)\left[\frac{p}{r-\mu} - \frac{c+s_m}{r}\right], \tag{46}$$

subject to the following set of boundary conditions,

$$\lim_{p\to\infty} e_m(p) = (1-\tau)\left[\frac{p}{r-\mu} - \frac{c+s_m}{r}\right], \quad e_m(p_d) = 0, \quad \frac{de_m(p_d)}{dp} = 0. \tag{47}$$

Solving (46) and (47) together yields the desired result. The debt value $d_m(p)$ is found in a similar vein. □

The post-default value $A(p_d)$ is simply the value of the unlevered firm (since it is assumed that at the time of default the debtholders take over the firm and continue to stay in business). This value can be calculated along the lines of Proposition 3 and is given by

$$A(p_d) = (1-\tau)\left[\frac{p_d}{r-\mu} - \frac{c}{r} - \left(\frac{p_d}{p_e}\right)^{\beta_2}\left(\frac{p_e}{r-\mu} - \frac{c}{r}\right)\right], \tag{48}$$

where

$$p_e = \frac{\beta_2}{\beta_2 - 1}\frac{c(r-\mu)}{r} \tag{49}$$

is the optimal exit threshold. The monopolist stops its production activity at the first passage time to p_e.

The optimal entry threshold and the optimal coupon payment are chosen by the monopolist so as to maximize the total ex ante value of the firm's securities. This value is given by $V_m(p) = (p_0/p)^{\beta_1}(d_m(p, s_m) + e_m(p, s_m) - I)$, where I is the cost of investment.

D. Numerical Algorithm

The original problem is an infinite horizon optimization problem and therefore a numerical solution requires reformulating the model to a finite horizon one. This is done in a way similar to the one followed by Titman and Tsyplakov (2007) by adding the partial

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derivatives with respect to time to the right-hand side of the equation determining the value of equity.[8] The modified equation is:

$$\begin{aligned}(50)\qquad &\frac{1}{2}\sigma^2 p^2 E_{pp}(p,s,t) + \mu p E_p(p,s,t) - rE(p,s,t) \\ &+ (1-\tau)\left[\frac{p}{r-\mu} - \frac{s}{r}\right] + E_t(p,s,t) \quad = \quad 0.\end{aligned}$$

By making a change in variables $y = \log p$, (50) is transformed to

$$\begin{aligned}(51)\qquad &\frac{1}{2}\sigma^2 E_{yy}(y,s,t) + \left(\mu - \frac{1}{2}\sigma^2\right) E_y(y,s,t) - rE(y,s,t) \\ &+ (1-\tau)\left[\frac{\exp y}{r-\mu} - \frac{s}{r}\right] + E_t(y,s,t) \quad = \quad 0.\end{aligned}$$

The partial derivatives are approximated according to the Euler method:

$$\begin{aligned}(52)\qquad E_y(y,s,t) &= \frac{E(y+dy,s,t) - E(y-dy,s,t)}{2dy}, \\ E_{yy}(y,s,t) &= \frac{E(y+dy,s,t) - 2E(y,s,t) + E(y-dy,s,t)}{dp^2}, \\ E_t(y,s,t) &= \frac{E(y,s,t) - E(y,s,t-dt)}{dt}.\end{aligned}$$

The steps in y and t, are chosen so as to ensure stability of the algorithm and subject to the following restriction,

$$dy \quad = \quad \sigma\sqrt{dt}.$$

At each node of the grid, I check if there are any admissible profitable changes in leverage, a profitable releveraging occurring if the following condition is satisfied for some u,

$$(53)\qquad L(u) \quad = \quad E(y,u,t) + D(y,u,t) - q\,|u-s| - [E(y,s,t) + D(y,s,t)] \quad > \quad 0,$$

subject to $E(y,u,t) > 0$ (i.e., that a change in leverage does not move the firm into the default region). The optimal adjustment is then chosen so as to maximize $L(u)$. Here $D()$ is the value of debt in the static coupon case.

At the terminal boundary, the values of debt and equity are approximated by the corresponding values for a firm with a static coupon. Those values are given by

$$\begin{aligned}E_T(s,p) &= C_1(s)p^{\beta_1} + C_2(s)p^{\beta_2} + (1-\tau)\left(\frac{p}{r-\mu} - \frac{s}{r}\right), \\ D_T(s,p) &= C_3(s)p^{\beta_1} + C_4(s)p^{\beta_2} + \frac{s}{r},\end{aligned}$$

[8] Unlike Titman and Tsyplakov (2007), I do not assume that debt is repaid at the face value corresponding to the case when all debt is riskless. On the contrary, I assume that new debt has exactly the same price as debt in the static capital structure case considered before. At any restructuring point, the net cash flow from equityholders to debtholders is determined by the difference between the corresponding debt values in the case when dynamic adjustments are prohibited. Thus, the value of debt at any time is equal to the present value of the future payments to debtholders both in the static and dynamic capital structure cases.

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where the functions $C_1(s) - C_4(s)$ together with the optimal default threshold p_d are determined by the following system of nonlinear equations:

$$\beta_1 C_1(s)(p^*)^{\beta_1 - 1} + \beta_2 C_2(s)(p^*)^{\beta_2 - 1} + \frac{1-\tau}{r-\mu} = 0, \tag{54}$$

$$\beta_1 C_3(s)(p^*)^{\beta_1 - 1} + \beta_2 C_4(s)(p^*)^{\beta_2 - 1} = 0, \tag{55}$$

$$C_1(s)p_d^{\beta_1} + C_2(s)p_d^{\beta_2} + (1-\tau)\left(\frac{p_d}{r-\mu} - \frac{s}{r}\right) = 0, \tag{56}$$

$$\beta_1 C_1(s)p_d^{\beta_1 - 1} + \beta_2 C_2(s)p_d^{\beta_2 - 1} + \frac{1-\tau}{r-\mu} = 0, \tag{57}$$

$$C_3(s)p_d^{\beta_1} + C_4(s)p_d^{\beta_2} + \frac{s}{r} = Y(p_d). \tag{58}$$

Boundary conditions (54) and (55) are the no-arbitrage conditions for the values of equity and debt, respectively. These conditions ensure that when the price in the output market reaches the free entry threshold p^*, there are no-arbitrage opportunities arising from shorting debt or equity securities. Condition (56) establishes that at the default threshold p_d the value of equity becomes equal to zero, while (57) is the smooth-pasting condition that ensures the optimality of the default threshold. Finally, the value-matching condition (58) requires that the value of debt of a bankrupt firm $Y(p_d)$ be equal to its post-default value net of the corresponding reorganization cost,

$$Y(p_d) = \frac{(1-\tau)(1-\alpha)}{r-\mu} p_d \left[1 - \frac{p_d^{\beta_1 - 1}}{\beta_1 (p^*)^{\beta_1 - 1}}\right], \tag{59}$$

where a is the proportional bankruptcy cost. In (59), the term $((1-\tau)(1-a)/(r-\mu))p_d$ is the post-default value of the firm in a monopoly environment, while the second term represents the reduction in value due to free entry. The initial value of the entry threshold p^* is determined by the equilibrium condition in the static leverage case,

$$\max_s (E_T(s, p^*) + D_T(s, p^*)) = I,$$

where I is the investment cost.

The stochastic optimization problem specified by (50) and (53) is then solved for the terminal conditions described above. The equilibrium entry threshold p^e must be found iteratively as the solution to

$$\max_s (E(s, p^e) + D(s, p^e)) = I,$$

where $E(s, p^e)$ solves (50) with the corresponding boundary conditions.

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