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Baker domains of meromorphic functions

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Abstract. Let f be a transcendental meromorphic function and U a Baker domain of f. We obtain new estimates for the behaviour of the iterates of f in U and we use these estimates to improve an earlier result relating the geometric properties of U to the proximity of f to the identity function in U. We also apply these estimates to Baker domains of transcendental meromorphic functions f of the form

$$f(z) = az + bz^k e^{-z}(1 + o(1)) \quad \text{as } \Re(z) \to \infty,$$

where $k \in \mathbb{N}$, $a > 1$ and $b > 0$, and show that these Baker domains contain an unbounded set of critical points and an unbounded set of critical values.

1. *Introduction*

Let f be a transcendental meromorphic function and denote the nth iterate of f by f^n, where $n \in \mathbb{N} = \{1, 2, \dots\}$. The *Fatou set* $F(f)$ is defined to be the set of points $z \in \mathbb{C}$ such that $(f^n)_{n\in\mathbb{N}}$ is well defined, meromorphic and forms a normal family in some neighbourhood of z. The complement of $F(f)$ is called the *Julia set* $J(f)$ of f. An introduction to the properties of these sets can be found in [**6**].

The set $F(f)$ is completely invariant so for any component U of $F(f)$ there exists, for each $p \in \mathbb{N}$, a component U_p of $F(f)$ such that $f^p(U) \subset U_p$. If $U_p = U$, then we say that U is a periodic component of *period* p, assuming p to be minimal, and there are then five possible types of periodic components; see [**6**]. In particular, U is called a *Baker domain* if there exists $z_0 \in \partial U$ such that $f^{pn}(z) \to z_0$ as $n \to \infty$, for $z \in U$, but $f^p(z_0)$ is not defined. In this case, there is at least one component U_k, $0 \le k \le p-1$, with the property that $f^{pn}(z) \to \infty$ as $n \to \infty$ for $z \in U_k$.

If U is a Baker domain of a transcendental *entire* function f, then $f^n(z) \to \infty$ as $n \to \infty$ for $z \in U$ and, moreover, U is simply connected [**1**, Theorem 3.1]. This is not true, in general, for transcendental meromorphic functions; see [**10**, Example 1].

Information about the rate at which iterates tend to infinity in a Baker domain can be obtained by using estimates for the hyperbolic metric. For example, it is known that if U is

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an invariant Baker domain of a transcendental meromorphic function and $z_0 \in U$, then for any path $\Gamma = \bigcup_{n=0}^{\infty} f^n(\Gamma_0)$, where Γ_0 joins z_0 to $f(z_0)$ in U and $0 \notin \Gamma$, there is a positive constant C such that

$$|z|^{1/C} \leq |f(z)| \leq |z|^C \quad \text{for } z \in \Gamma;$$

see [**4**, Lemma 4.1]. In this paper we use several fundamental results of Bonfert from [**9**] to show that the above estimates for f can be improved to

$$\frac{1}{C}|z| \leq |f(z)| \leq C|z| \quad \text{for } z \in \Gamma. \tag{1.1}$$

An estimate of this form was previously known to be true only in a Baker domain whose complement includes some unbounded closed connected set, such as a simply connected Baker domain, where G is a domain in $\mathbb{C}$; see [**2**] and [**6**, Lemma 7]. The estimate (1.1) is a special case of one part of Theorem 1, which we state and prove in §2.

In §3, we use Theorem 1 to deduce a result that relates the geometric properties of a Baker domain U of a transcendental meromorphic function f to the proximity of f to the identity function in U. For an invariant Baker domain, this result shows that if $z_0 \in U$ and $\Gamma = \bigcup_{n=0}^{\infty} f^n(\Gamma_0)$, where Γ_0 joins z_0 to $f(z_0)$ in U, then for each $z \in \Gamma$, the distance from z to ∂U is comparable to $|f(z) - z|$ divided by the hyperbolic distance in U from z to $f(z)$. A less precise result of this type was proved in [**12**, Theorem 1] and used there to show the *non-existence* of Baker domains under certain circumstances; see [**12**, Theorem 3]. Also, a version of [**12**, Theorem 1] was used in [**3**, Lemma 3.3] to classify univalent Baker domains of transcendental entire functions.

Finally, in §4, we apply the estimates in Theorem 1 to Baker domains associated with meromorphic functions of the form

$$f(z) = az + bz^k e^{-z}(1 + o(1)) \quad \text{as } \Re(z) \to \infty,$$

where $k \in \mathbb{N}$, $a > 1$ and $b > 0$, and we also show that these Baker domains contain infinitely many critical points and infinitely many critical values of f.

In the rest of the paper we use the following notation:

- $d_G(z) = \inf\{|z - w| : w \in \partial G\}$;
- ρ_G is the density of the hyperbolic metric in G and ρ_G^* is the density of the quasi-hyperbolic metric in G, given by $\rho_G^*(z) = 1/d_G(z)$;
- $[z, w]_G$ is the distance from z to w with respect to the hyperbolic metric in G and $[z, w]_G^*$ is the distance from z to w with respect to the quasi-hyperbolic metric in G;
- $z_n = f^n(z_0)$, for $n = 1, 2, \ldots$, where $z_0 \in \mathbb{C}$ and f are given.

2. *Theorem 1*

In addition to showing that (1.1) holds in a Baker domain, Theorem 1 gives good estimates of the behaviour of an analytic function f along orbits in invariant domains.

THEOREM 1. *Let G be an unbounded domain in $\mathbb{C}$ with at least two finite boundary points, and let f be a function analytic in G such that $f(G) \subset G$, $f^n(z) \to \infty$ as $n \to \infty$, for $z \in G$, and f does not extend analytically to ∞ with $f(\infty) = \infty$.*

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Then, for any compact subset K of G, there exist positive constants C and n_0 such that

$$\frac{|f^n(z') - f^n(z)|}{d_G(f^n(z))} \leq C[f^n(z'), f^n(z)]_G \quad \text{for } z, z' \in K,\ n \in \mathbb{N}, \tag{2.1}$$

and

$$|f^n(z')| \leq C|f^n(z)| \quad \text{for } z, z' \in K,\ n \geq n_0. \tag{2.2}$$

Also, for all $z \in G$,

$$\ln|f^n(z)| = O(n) \quad \text{as } n \to \infty, \tag{2.3}$$

where the constant in $O(n)$ depends on z.

Finally, for any $z_0 \in G$ and any path $\Gamma = \bigcup_{n=0}^{\infty} f^n(\Gamma_0)$, where Γ_0 joins z_0 to $f(z_0)$ in G and $0 \notin \Gamma$, there is a positive constant C such that

$$\frac{1}{C}|z| \leq |f(z)| \leq C|z| \quad \text{for } z \in \Gamma. \tag{2.4}$$

If the function f in Theorem 1 is an iterate of a function meromorphic in $\mathbb{C}$, then $G \subset F(f)$, by Montel's theorem, and G must be a subset of a periodic Baker domain. In particular, Theorem 1 applies if G is itself a p-periodic Baker domain of a transcendental meromorphic function, in which $f^{pn} \to \infty$.

We prove Theorem 1 using several results from [**9**, pp. 49–50], collected together, for convenience, in the following proposition. Note that the assumptions in [**9**] about the domain G are more general than those in the present paper.

PROPOSITION 1. *Let G and f satisfy the hypotheses of Theorem 1.*

(a) *Whether or not $\lim_{n\to\infty}[z_n, z_{n+1}]_G = 0$ is independent of the choice of $z_0 \in G$.*

(b) *If $z_0 \in G$, then there exists a positive constant c_0 depending only on z_0 such that*

$$c_0 \leq \frac{\rho_G(z_n)}{\rho_G^*(z_n)} \leq 1 \quad \text{for } n = 1, 2, \ldots .$$

(c) *For any positive constants c_0 and C_0, there exists a positive constant c_1, depending only on c_0 and C_0, such that if $z, z' \in G$ satisfy*

$$\rho_G(z) \geq c_0\rho_G^*(z) \quad \text{and} \quad [z, z']_G \leq C_0,$$

then

$$\rho_G(z') \geq c_1\rho_G^*(z').$$

Proof of Theorem 1. Let K be a compact subset of G and put

$$C_K = \sup\{[z, w]_G : z, w \in K\}.$$

Then take $z, z' \in K$ and put $z_n = f^n(z)$ and $z_n' = f^n(z')$, for $n = 0, 1, \ldots$. By Proposition 1(b), there exists $c_0 > 0$ such that

$$c_0 \leq \frac{\rho_G(z_n)}{\rho_G^*(z_n)} \leq 1 \quad \text{for } n = 0, 1, \ldots . \tag{2.5}$$

By the contracting property of the hyperbolic metric,

$$C_K \geq [z, z']_G \geq [z_n, z_n']_G = \int_{\gamma_n} \rho_G(\zeta)|d\zeta| \quad \text{for } n = 0, 1, \ldots, \tag{2.6}$$

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where γ_n is a hyperbolic geodesic in G from z_n to z_n'. For $\zeta \in \gamma_n$, we have

$$[\zeta, z_n]_G \leq [z_n', z_n]_G \leq C_K.$$

Hence, by (2.5) and Proposition 1(c), there exists $c_1 > 0$ such that

$$\rho_G(\zeta) \geq c_1 \rho_G^*(\zeta) \quad \text{for } \zeta \in \gamma_n. \tag{2.7}$$

Using (2.6) and (2.7), we obtain

$$\begin{aligned}
[z_n, z_n']_G &= \int_{\gamma_n} \rho_G(\zeta)|d\zeta| \\
&\geq c_1 \int_{\gamma_n} \frac{|d\zeta|}{d_G(\zeta)} \\
&\geq c_1 \int_{\gamma_n} \frac{|d\zeta|}{d_G(z_n) + |z_n - \zeta|} \\
&\geq c_1 \int_0^{|z_n' - z_n|} \frac{dt}{d_G(z_n) + t} \\
&= c_1 \ln\left(1 + \frac{|z_n' - z_n|}{d_G(z_n)}\right) \\
&\geq c_2 \frac{|z_n' - z_n|}{d_G(z_n)},
\end{aligned}$$

where $c_2 > 0$, since $|z_n' - z_n|$ is bounded above by (2.6). Hence, (2.1) follows.

The estimate (2.2) follows from (2.1) and (2.6) by noting that if $w \in J(f)$ and n is so large that $|z_n| \geq |w|$, then

$$\begin{aligned}
|z_n'| &\leq |z_n| + CC_K d_G(z_n) \\
&\leq |z_n| + CC_K(|z_n| + |w|) \\
&\leq (1 + 2CC_K)|z_n|,
\end{aligned}$$

where C is the constant in (2.1), as required.

The estimate (2.3) is obtained by applying (2.2) with $z' = f(z)$ and $K = \{z, z'\}$, and then using induction. Finally, to obtain (2.4) we express z in the form $z = f^n(\zeta)$, where $\zeta \in \Gamma_0$ and $n \geq 0$, and then apply (2.2) to the points $\zeta, f(\zeta)$ with $K = \Gamma_0 \cup f(\Gamma_0)$. □

3. *Theorem 2*

Our next result relates the geometric properties of a Baker domain U of a transcendental meromorphic function f to the proximity of f to the identity function in U. We state the result in the more general context of Theorem 1.

THEOREM 2. *Let G and f satisfy the hypotheses of Theorem 1, and for $z_0 \in G$ let $\Gamma = \bigcup_{n=0}^{\infty} f^n(\Gamma_0)$, where Γ_0 is a path in G joining z_0 to z_1.*

(a) *There exists $C_1 > 0$ such that*

$$d_G(z) \leq C_1 \frac{|f(z) - z|}{[f(z), z]_G} \quad \text{for } z \in \Gamma. \tag{3.1}$$

If we also have $[z_{n+1}, z_n]_G \not\to 0$ as $n \to \infty$, then there exists $C_2 > 0$ such that

$$d_G(z) \leq C_2 |f(z) - z| \quad \text{for } z \in \Gamma. \tag{3.2}$$

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(b) *There exist* $c_1 > 0$ *and* $c_2 > 0$ *such that*

$$d_G(z) \geq c_1 \frac{|f(z) - z|}{[f(z), z]_G} \geq c_2 |f(z) - z| \quad \text{for } z \in \Gamma. \tag{3.3}$$

Theorem 2 improves [**12**, Theorem 1] in several respects. First, it applies if G is any p-periodic Baker domain of a transcendental meromorphic function in which $f^{pn} \to \infty$. Also, part (a) applies to all points $z \in \Gamma$, not just those of the form $z_n = f^n(z_0)$.

Note that the condition $[z_{n+1}, z_n]_G \not\to 0$ as $n \to \infty$ is essential to prove (3.2); this follows from (3.3). Recall from Proposition 1 that whether $\lim_{n\to\infty}[z_n, z_{n+1}]_G = 0$ is independent of the choice of $z_0 \in G$.

Proof of Theorem 2. It follows from the standard estimate $\rho_G(z) \leq 2/d_G(z)$ that

$$d_G(z) \leq |f(z) - z| \left(\frac{2}{[f(z), z]_G} + 1 \right) \quad \text{for } z \in G;$$

for the details, see, for example [**12**, Lemma 2.1(a)]. Thus, to prove (3.1) we need only note that, by the contracting property of the hyperbolic metric, there exists a constant C such that

$$[f(z), z]_G \leq C \quad \text{for } z \in \Gamma. \tag{3.4}$$

To prove (3.2), we need to show that the hypothesis that $[z_{n+1}, z_n]_G \not\to 0$ as $n \to \infty$ implies that there exists $c > 0$ such that

$$[f(z), z]_G \geq c \quad \text{for } z \in \Gamma. \tag{3.5}$$

Suppose that (3.5) is false. Then there exist $w_k \in \Gamma$, $k \in \mathbb{N}$, such that

$$[f(w_k), w_k]_G \to 0 \quad \text{as } k \to \infty. \tag{3.6}$$

Now $w_k = f^{n_k}(\zeta_k)$, where $\zeta_k \in \Gamma_0$ and $n_k \geq 0$, for $k \in \mathbb{N}$. We may assume without loss of generality that

$$\zeta_k \text{ converges to some point } \zeta_0 \in \Gamma_0. \tag{3.7}$$

By Proposition 1(a), and the hypothesis that $[z_{n+1}, z_n]_G \not\to 0$ as $n \to \infty$, there exists $c_0 > 0$ such that, for $k \in \mathbb{N}$,

$$\begin{aligned} c_0 &\leq [f^{n_k}(\zeta_0), f^{n_k+1}(\zeta_0)]_G \\ &\leq [f^{n_k}(\zeta_0), f^{n_k}(\zeta_k)]_G + [f^{n_k}(\zeta_k), f^{n_k+1}(\zeta_k)]_G + [f^{n_k+1}(\zeta_k), f^{n_k+1}(\zeta_0)]_G \\ &\leq 2[\zeta_0, \zeta_k]_G + [w_k, f(w_k)]_G. \end{aligned}$$

For large k, this inequality is contradicted by (3.6) and (3.7). Hence, (3.5) is true. This completes the proof of Theorem 2(a).

To prove part (b), note that if $z \in \Gamma$, then $z = f^n(\zeta)$ for some $\zeta \in \Gamma_0$ and $n \geq 0$. Now we apply inequality (2.1) from Theorem 1 to the points ζ, $f(\zeta)$ with $K = \Gamma_0 \cup f(\Gamma_0)$, and also (3.4), to deduce that

$$|f(z) - z| = |f^{n+1}(\zeta) - f^n(\zeta)| \leq C_1 d_G(z)[f(z), z]_G \leq C_2 d_G(z),$$

for positive constants C_1 and C_2. This completes the proof of Theorem 2. □

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4. *Theorem 3*

For a meromorphic function f we denote by $\text{sing}(f^{-1})$ the set of *inverse function singularities* of f; that is, the set of critical values and asymptotic values of f. If f has an invariant Baker domain U, then $\text{sing}(f^{-1})$ must be an unbounded set; see [**6, 11**]. However, U may or may not contain such singularities. See [**7**] for an example of an entire function f with an invariant Baker domain that does not meet $\text{sing}(f^{-1})$.

In some cases it is possible to prove that Baker domains must contain an unbounded set of inverse function singularities. For example, if f is a transcendental entire function of the form

$$f(z) = az + bz^k e^{-z}(1 + o(1)) \quad \text{as } \Re(z) \to \infty,$$

where $k \in \mathbb{N}$, $a > 1$ and $b > 0$, then f has an invariant Baker domain U such that $U \cap \text{sing}(f^{-1})$ is unbounded; see [**12**, Theorem 2]. The example in [**7**] shows that the case $k = 0$ cannot be included in this result. In [**8**, Theorem 3] the same conclusion was obtained, by a different method, when f is a transcendental entire function of finite order of the form

$$f(z) = z + a + o(1) \quad \text{as } z \to \infty, |\arg z| \leq \eta,$$

where $a > 0$ and $\eta > 0$.

Here we obtain a refinement of the former result, which covers transcendental meromorphic functions of this form. For such functions, $[z_{n+1}, z_n]_U \not\to 0$, for all $z_0 \in U$, and the set of inverse function singularities in U includes an unbounded set of critical values of f.

THEOREM 3. *Let f be a transcendental meromorphic function of the form*

$$f(z) = az + bz^k e^{-z}(1 + o(1)) \quad \text{as } \Re(z) \to \infty, \tag{4.1}$$

where $k \in \mathbb{N}$, $a > 1$ and $b > 0$. Then:

(a) *f has an invariant Baker domain U that, for each $\rho > 0$ and large enough values of $R > 0$, contains an invariant set of the form*

$$D_{\rho,R} = \{z : |z^k e^{-z}| < \rho, |z| > R\};$$

(b) *$[z_{n+1}, z_n]_U \not\to 0$ as $n \to \infty$, for all $z_0 \in U$;*

(c) *both the set of critical points of f in U and the set of critical values of f in U are unbounded.*

Proof. Put $g(z) = z^k e^{-z}$ and note that, for $z = x + iy$ and $\rho > 0$,

$$|g(z)| < \rho \quad \text{if and only if } x > k \ln |z| - \ln \rho. \tag{4.2}$$

It is straightforward to show that for each $\rho > 0$ there exists $R(\rho) > 0$ such that

$$|f(z)| \geq \tfrac{1}{2}(a + 1)|z| \quad \text{for } z \in D_{\rho,R}, R > R(\rho), \tag{4.3}$$

and

$$f(D_{\rho,R}) \subset D_{\rho,R} \subset \{z : \Re z > 0\} \quad \text{for } R > R(\rho); \tag{4.4}$$

see [**12**, Proof of Lemma 3.1]. The existence of a Baker domain U containing $D_{\rho,R}$ then follows from Montel's theorem.

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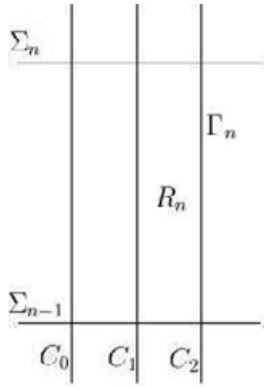


FIGURE 1. The quadrilateral R_n.

To prove part (b), we fix $\rho > 0$ and take $R > R(\rho)$ and $z_0 \in D_{\rho,R}$. Then, by (4.3) and (4.4), we have

$$|z_{n+1}| \geq \tfrac{1}{2}(a+1)|z_n| \quad \text{for } n = 0, 1, \ldots, \tag{4.5}$$

so $|z_n| \to \infty$ as $n \to \infty$. Now choose $w \in J(f)$ such that

$$d_U(z_n) \leq |z_n - w| \leq 2|z_n|,$$

for sufficiently large values of n. It follows from (2.1) in Theorem 1 and (4.5) that, for such n,

$$C[z_{n+1}, z_n]_U \geq \frac{|z_{n+1} - z_n|}{d_U(z_n)} \geq \frac{1}{4}(a-1) > 0,$$

so $[z_{n+1}, z_n] \not\to 0$ as $n \to \infty$. Part (b) now follows from Proposition 1(a).

To prove part (c), we put

$$\theta(z) = k \arg z - y, \tag{4.6}$$

where $\arg z \in (-\pi, \pi]$. For $R > R(\rho)$, we have $D_{\rho,R} \subset \{z : \Re z > 0\}$, by (4.4), so θ is a continuous argument function for g in $D_{\rho,R}$. Then put

$$Q = \{z : \Re(z) > 0, \Im(z) > 0\}, \quad \rho_0 = 8a/b, \quad \rho_1 = 2a/b, \quad \rho_2 = \tfrac{1}{2}a/b,$$
$$C_j = \{z \in Q : |g(z)| = \rho_j\}, \quad j = 0, 1, 2,$$

and

$$\Sigma_n = \{z \in Q : \theta(z) = -(2n+1)\pi\}, \quad n = 1, 2, \ldots .$$

From now on we assume that $R > R(\rho_0)$.

Consider the quadrilateral R_n defined by the four curves C_1, C_2, Σ_n and Σ_{n-1}, and let $\Gamma_n = \partial R_n$, as shown in Figure 1.

By (4.2) and (4.6), the quadrilateral R_n is uniformly close, for large n, to the rectangle

$$\{x + iy : |x - (k\ln(2n\pi) - \ln(a/b))| \leq \ln 2, |y - (2n\pi + k\pi/2)| \leq \pi\},$$

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and R_n lies at a distance from C_0 that is asymptotically equal (as $n \to \infty$) to $\ln 4$. We shall examine the behaviour of f' on Γ_n for large values of R, with n taken so large that $\Gamma_n \subset D_{\rho_0,R}$.

To do this, we define the function h by

$$f(z) = az + bg(z) + h(z).$$

By (4.1), we have $h(z) = bg(z)o(1)$ as $\Re(z) \to \infty$. Hence, by (4.2) and the above observations about the position of R_n for large n, we can take $R > R(\rho_0)$ so large that

$$|h(z)| \leq \tfrac{1}{4}a \quad \text{whenever } \operatorname{dist}(z, \Gamma_n) \leq 1 \text{ and } \Gamma_n \subset D_{\rho_0,R}.$$

Thus, by Cauchy's estimate,

$$|h'(z)| \leq \tfrac{1}{4}a \quad \text{for } z \in \Gamma_n \subset D_{\rho_0,R}. \tag{4.7}$$

Since $g'(z) = -g(z)(1 - k/z)$, we can also take $R > R(\rho_0)$ so large that

$$|g'(z) + g(z)| = |kg(z)/z| < \frac{a}{4b} \quad \text{for } z \in \Gamma_n \subset D_{\rho_0,R}. \tag{4.8}$$

Hence, for such R,

$$|f'(z) + bg(z) - a| \leq b|g'(z) + g(z)| + |h'(z)| < \tfrac{1}{2}a \quad \text{for } z \in \Gamma_n \subset D_{\rho_0,R}, \tag{4.9}$$

by (4.7) and (4.8).

Now, as z traverses Γ_n once anticlockwise starting at the intersection of C_1 and Σ_n, the image $w = bg(z)$ traverses the following curves in turn:

(1) the circle $\{w : |w| = b\rho_1\}$ once anticlockwise starting at $-b\rho_1$;
(2) the interval $[-b\rho_1, -b\rho_2]$ from left to right;
(3) the circle $\{w : |w| = b\rho_2\}$ once clockwise starting at $-b\rho_2$;
(4) the interval $[-b\rho_1, -b\rho_2]$ from right to left.

Therefore, since $b\rho_1 = 2a$ and $b\rho_2 = \frac{1}{2}a$, the image $w = -(bg(z) - a)$ winds exactly once round the closed disc $\{w : |w| \leq \frac{1}{2}a\}$. So if $\Gamma_n \subset D_{\rho_0,R}$ and R is large enough, then the image of Γ_n under f' winds exactly once round 0, by (4.9). Thus, f' has a single (simple) zero inside Γ_n, for all sufficiently large n. This proves that the critical points of f which lie in $D_{\rho_0,R}$, and hence in U, form an unbounded set. It follows from (4.1) that the corresponding critical values of f also form an unbounded set. This completes the proof of Theorem 3. □

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