*Macroeconomic Dynamics*, **2**, 1998, 49–71. Printed in the United States of America.

# DECOMPOSITION OF ECONOMIC RELATIONSHIPS BY TIMESCALE USING WAVELETS

## *Money and Income*

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Economists have long known that timescale matters in that the structure of decisions as to the relevant time horizon, degree of time aggregation, strength of relationship, and even the relevant variables differ by timescale. Unfortunately, until recently it was difficult to decompose economic time series into orthogonal timescale components except for the short or long run in which the former is dominated by noise. Wavelets are used to produce an orthogonal decomposition of some economic variables by timescale over six different timescales. The relationship of interest is that between money and income, i.e., velocity. We confirm that timescale decomposition is very important for analyzing economic relationships. The analysis indicates the importance of recognizing variations in phase between variables when investigating the relationships between them and throws considerable light on the conflicting results that have been obtained in the literature using Granger causality tests.

**Keywords:** Wavelets, Timescale, Velocity, Money, Income, Permanent Income Hypothesis, Money-income Causality

## 1. INTRODUCTION

For decades, the idea of time period in economic analysis was enshrined in textbooks as the short run and the long run. Marshall, Edgeworth, Schumpeter, Hicks, and others of the period realized that there were more time periods involved in economic decision making, but the pedagogical advantages of just two periods dominated the relevance of many periods. In any event, it was recognized early in the profession that the time period of analysis, or as we would now term the matter, the timescale of the analysis, is very important for determining those aspects of decision making that are relatively more important and those that are relatively less important. In the physical sciences, the notion of timescale captures the notion of the distinction between slow and fast variables; the former are

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https://doi.org/10.1017/S1365100598006038 Published online by Cambridge University Press

variables whose values are changing slowly relative to those of the latter variables. For example, in the analysis of turbulence, there is a cascade of timescales each of which determines a particular mode of behavior. Similarly, in economics, one can envisage a cascade of timescales within which different levels of decisions are being made, determining different modes of behavior. Some decisions are taken with long horizons, others are taken with short horizons. The choice of timescale determines not only the length of the period over which one requires forecasts of future events, but the very choice of variables that are to be the focus of the decision maker's processing of information.

For example, the Federal Reserve Board can be envisaged as operating on several timescales at the same time. For long-range policy, they may be concerned with very long-run changes in the economy and the corresponding financial instruments will focus on long-term growth with a horizon of interest in decades. Simultaneously, they will be monitoring the performance of the economy and the money supply over business-cycle length periods, as well as ensuring the liquidity of very short-run deposits day by day. The horizons in each case are very different, the focus of policy is different at each horizon, the variables that are of main interest are different, and the degree of time aggregation will be different. Often, there will be different sets of agents involved for the analysis undertaken at each horizon. In these examples, it usually will be the case that the longer-run horizons will set the context for the shorter, but this constraint is not necessary. Even when the variables and degree of time aggregation are the same, the values of the parameters involved may well be different across timescales. Although the qualitative mode of behavior might be the same, the intensity may be different. This result has been demonstrated with respect to the permanent-income hypothesis discussed in Ramsey and Lampart (1997b). The issue involved in this one is reminiscent of Friedman's concept of horizon in his formulation of the permanent income hypothesis (Friedman, 1963). Although Friedman restricted himself to two scales and we contemplate a cascade of scales, the fundamental principle is the same.

We investigate some of these issues empirically by using wavelet analysis that enables one to separate out different timescales of variation in the data. We investigate the role of timescale in economic relationships in terms of the money income relationship; a companion article, Ramsey and Lampart (1997b), investigates similar issues with respect to the permanent-income hypothesis. There is some controversy about the nature of the relationship between money and income and the extent to which the relevant theory, or theories, are supported by the empirical evidence. We attempt to shed light on these issues by separating the empirical analysis of the relationship between variables into that between the variables after separation into timescale components. Instead of considering the net relationship over all timescales as in the conventional analysis, we will consider a set of relationships, one for each timescale.

The discussion to follow is quite condensed; readers wishing more details, tables, and graphs, as well as further elaboration on some points may obtain the original working paper, Ramsey and Lampart (1997a), on request.

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### 1.1. Money and Income

The interaction between monetary aggregates and income has been used to explore a number of different economic questions, from financial development to the money demand function to the operation of monetary policy. The investigation of these various questions differs with respect to the time period over which money and income are assumed to be related. For example, to solve the basic simultaneity problem encountered in estimating the relationships between money demand and supply, it has been customary to pose the money demand equation as a long-run (cointegration) relationship between velocity and interest rates, whereas the short-run relationship between money and income is affected by monetary policy and constitutes the money supply relationship.

In recent years, the long-run money–income relationship has come under increasing scrutiny. In the United States, attempts to model the demand for money has progressed from a general agreement about the existence of a stable velocity relationship to concern about what appeared to be systematic overprediction of money balances during the 1970's. Subsequently, there emerged further concerns that estimates of the money demand function were tending to underpredict balances. Levantakis and Brissinis (1991) provide a thorough survey of these developments.

McMillin (1991) reviewed the evidence for a structural break in M1 velocity widely presumed to have occurred in the early 1980's. He found that the break was due more to changes in the process-generating velocity than to variability in money and income. Conversely, Bomhoff (1990) found that estimates using a stochastic trend in M1 are not significantly different from those for a constant trend.

Boyle (1990) and Dueker (1993, 1995) explore the idea of variable velocity in the theoretical literature. Artis et al. (1992) investigated the stability of money demand functions using spectral techniques. They estimated the evolutionary spectral density using a monthly proxy for M1 velocity by taking the product of industrial production and producer price indices as a substitute for monthly nominal GDP. Stability tests were performed on the evolutionary coefficients for different frequency levels. For the United States they did not find evidence for a structural break in M1 velocity.

The analysis of the stationary parts of the money and income series generally is understood as an examination of the short- to medium-run relationship between the two variables. Analysis of short-run money–income relationships have concentrated on testing the strength of Granger causality between various monetary aggregates and nominal or real income.

The seminal piece in this literature is a paper by Sims (1972) who found, using quarterly data in which output is measured by the industrial production index, that money Granger causes real output. However, when he reestimated this equation in 1980 (Sims, 1980), using the six-month commercial paper rate, his results changed. These findings spawned a literature investigating the strength and direction of Granger causality between measures of monetary activity and output. Most of this

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literature used logarithmic differences with either quarterly or monthly data, to obtain short-run stationary data.

Stock and Watson (1989) used monthly data from 1960 to 1985 and found that output was influenced by changes in the growth of M1. Friedman and Kuttner (1990) extended the same data to 1990 and reported that the strength of the Stock and Watson result is weaker using the extended period. The research continued with many papers that tested for improved methods of trend removal and the possibility of cointegrating structures, checked out-of-sample performance, and examined alternative lag structures and the relative strength of other financial variables. Feldstein and Stock (1993), using quarterly data, attempted to show that the strongest relationship exists between M2 and nominal GDP. Spence (1989) showed that VAR results are very sensitive to trend removal, lag length, and the level of temporal aggregation. Abate and Boldin (1993, p. 36) concluded that money-output specifications suffer from significant heteroskedasticity and uncorrected serial correlation. They concluded that "contrary to the claims of some researchers, the money output relationship does not break down in the 1980s and M2 helps forecast into the early 1990s."

The general conclusion from the literature is that the relationship between money (defined in several ways) and output (also defined in several ways) still is not understood completely. This is true both in the long run and in the short run. Further, there is disputed evidence on the existence of structural breaks and the possibility for nonlinearity in the relationships. The ambiguity of these results may well be due to the fact that there are several timescales involved in the relationships and that a single dichotomy between trend and short-run stationary fluctuations may be inadequate to separate out the timescale-structured relationships between the variables. There is also the possibility that the overly restrictive dichotomy between trend and stationary components has masked shifting dynamical relationships between the variables. The timescale at which the introduction of checkable interest deposits affect M1 demand is most likely different from the one at which a change in Fed operating procedures affects the money output link and is different from the timescale at which disintermediation might affect M2 velocity.

We conclude this review of the literature by stating that the main requirements of an exploratory analysis of the data involve the recognition that nonstationarity in addition to the unit-root problem are relevant issues; that an allowance for effects that are localized in time is needed; that relationships may well vary by timescale of analysis; and that, given the lack of knowledge about the underlying functional forms involved, nonparametric procedures will be required. This combination of conditions indicates that, as an exploratory tool, an analysis using wavelets should prove useful and to this topic we now turn.

## 2. WAVELET ANALYSIS

The discussion to follow is meant to give only an intuitive feel for wavelet analysis and to stress those points that are at the heart of the examination of the statistical properties of the data; a more elaborate statement is contained in Ramsey and

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Lampart (1997a) and references therein. For a thorough review of the basics of wavelet analysis, Chui (1992) is an excellent reference and Daubechies (1992) contains a detailed analysis of the mathematical properties of wavelets. A useful recent article that helps to link wavelet analysis to more conventional time-series analysis is Priestley (1996) and a useful nontechnical review article is Rioul and Vetterli (1991). An important article that explores the use of the wavelet approach to the estimation of complex signals contaminated by noise is Donoho et al. (1995). Further, Brillinger (1994) develops some of the relevant distribution theory.

Although there are many useful properties of wavelets, in this paper we are interested in only three major facets of wavelet analysis: the ability to handle nonstationary data, localization in time, and the resolution of the signal in terms of the timescale of analysis. The nonstationarity that we are concerned about is a broader notion than the existence of a mere unit root process in that some allowance should be made for variation in the processes over time as well as for local effects. The evaluation of the effects of timescale on the relationships between economic variables is central to our purpose.

Wavelet analysis has points of comparison and points of contrast to Fourier analysis. Recognizing both is important for understanding what wavelet analysis can bring to the examination of a data series. Both procedures involve the projection of a signal onto an orthonormal set of components, trigonometric in the case of Fourier-series representations, wavelets that are to be defined below in the case of wavelet analysis. Fourier projections are defined most naturally for functions restricted to $L^2(0, 2\pi)$, because Fourier series have infinite energy, but finite power, when extended to being defined over the entire real line. Intuitively, this is the source of the difficulty that a single disturbance to a signal affects the analysis at all frequencies and that a single disturbance in time is interpreted by Fourier analysis as an event of period $T$, where $T$ is the length of the observed series. Although Fourier analysis allows for the complex superposition of individual harmonics, or waves, the hypothesis is that over any subsegment of the observed time series the precise same frequencies hold at the same amplitudes; the signal is homogeneous over time.

In contrast, the functions that are to be represented by wavelets are naturally defined within $L^2(R)$ and are not necessarily homogeneous over time. This means that the basis functions to be used must have narrow compact support so that the basis functions rapidly converge to zero as the index $t$ approaches plus or minus infinity; such basis functions are known as wavelets. This is in contrast to the trigonometric functions that traditionally have been associated with waves. However, this approach brings its own difficulty in that functions that do not have narrow support have to be approximated by a sequence of functions that do. It is this difficulty that leads to the necessity for defining wavelets with respect to specific locations and then considering a sequence of such functions, each indexed by a particular location. By stringing together a sequence of such wavelet functions, each localized to a particular position on the time axis, complex functions can be approximated.

There is another aspect of wavelets that produces much of their appeal, and that aspect derives from the rescaling capability of wavelets. Instead of considering a

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single sequence of basis functions $g(t, k)$, where $k$ denotes a sequence of positions about which the function $g(\cdot)$ is centered, we consider a double sequence of functions

$$g(t) = \frac{1}{\sqrt{s}} g\left(\frac{t-k}{s}\right), \tag{1}$$

where $s$ is a sequence of scales. The term $1/\sqrt{s}$ maintains the norm of $g(\cdot)$ at 1. The basis function $g(\cdot)$ is centered at $k$ with a scale of $s$; that is, the energy of $g(\cdot)$ is concentrated in a neighborhood of $k$, the size of which is proportional to $s$. Essentially, as $s$ is increased, the length of the support of $g(\cdot)$ in terms of $t$ increases. For example, if the support of $g(\cdot)$ for $s = 1$ is $[-d, d]$ when $k = 0$, then the effect of $s$ is to broaden the support to $[-ds, ds]$. In effect, the rescaling characteristic of wavelets in the time domain is similar to the rescaling of frequencies in frequency domain for Fourier analysis. The process also is known as integral dilation. In Fourier analysis, all frequencies are generated by rescaling, or dilating, a single fundamental frequency. If $\omega_0$ is the fundamental frequency, then the function to be approximated is projected onto a sequence of expressions of the form

$$\{e^{-is\omega_0 t}\}. \tag{2}$$

In short, $2\pi$-periodic functions are representable by integral dilations of the single function $e^{-i\omega_0 t}$.

Dilation in the time domain has advantages in addition to those stemming from frequency dilations. In the latter case, one is considering projecting the entire signal onto ever lower frequencies. In the former case with wavelets, a localized component of the signal is projected onto an ever broader base. The two important aspects here are that the projection is for a *local* component of the signal and that each projection is onto a wavelet whose narrow compact support is a function of $s$. In a sense, the choice of a dilation, or scale, level indicates the size of the packets used to represent the signal. A very important aspect of the wavelet approach is that the sequence of basis functions is orthogonal across scales so that any signal can be decomposed easily in terms of its constituent scales. The view using wavelets is that the total variation of the data centered at any given point in time is obtained by adding the components extracted at each of the admissible scales. A broad support wavelet yields information on signal variations on a large scale; a small support wavelet yields information on signal variations on a small scale. One way to visualize the process is in terms of maps, in that a large-scale map gives the broad picture without details and a small-scale map fills in the details.

An alternative view is that obtained by aggregating the data to various levels using a windowing technique, which in turn can be interpreted as a projection onto the vector $\{1/N, 1/N, \ldots, 1/N\}$, where there are $N$ observations selected to be in each projection, $N = \{2, 3, 4, 5, \ldots\}$. However, by taking this view, one soon recognizes the nonoptimality of such a sequence of projections, in that, at the very least, orthogonality of the projections has been lost, not to mention the fact that one does not have a basis. Consequently, the wavelet approach potentially adds to our

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understanding of the relationships between variables by enabling us to separate by orthogonal decomposition the relevant timescales that may be involved.

There is an understandable temptation to regard timescale and frequency as similar concepts. However, as indicated by Priestley (1996), there is only an intuitive and very indirect connection between frequency and the scale of the analysis. It is only true in the simplest of cases that large-scale wavelets are associated with low frequencies in that the detection of low frequencies requires components with very wide support in the time domain. Correspondingly, the interpretation that the analysis of high frequencies requires a high sampling rate which is provided by components that have narrow support is naive. At each scale, the strength of the projection determines the magnitude of the corresponding coefficient. In essence, one is extracting at the chosen scale those components of the data that resonate most strongly at that scale. The link between scale and frequency is only interpretable when the data are stationary; the wavelet projections are still meaningful even if there are no oscillations in the data.

Consider, for example, a given scale, say $2^3$ or eight months, and that the signal contains components that are at that scale. It still might be true that the power of the projections depends in a cyclic manner on the value of $k$, the index of position; for example, the power of the projections indexed by $k$ might oscillate with a period of 10 years, or indeed any period. More importantly, this intuitive idea of packets of information that arises out of the dilation procedure can be used and interpreted, even when a signal cannot usefully be regarded as a superposition of trigonometric components. Priestley (1996) illustrates this idea in the context of a signal with a fixed frequency, but a time-varying amplitude; Fourier analysis, of course, incorrectly detects power at all frequencies.

We can consider two ways of examining a given signal. At a fixed point in time, say $k$, we can examine how the localized signal projects onto wavelets of varying length of support. In effect, this procedure examines the signal at the given location for variations in the strength of projections onto wavelets of various scales, or dilations. The other view is to examine how the signal projections vary by position as indicated by the value of $k$, but at a given fixed scale, or dilation, $s$.

There are a variety of functions that have been developed for use as the fundamental wavelet that is to be dilated by $s$ and translated by $k$—the Haar, Daublet, Symmlet, etc. Depending on normalization rules, there are two types of wavelet within a given family, such as the Symmlet—father and mother wavelets:

$$\begin{aligned} \Phi_{j,k} &= 2^{-\frac{j}{2}}\Phi\left(\frac{t-2^j k}{2^j}\right), \\ \Psi_{j,k} &= 2^{-\frac{j}{2}}\Psi\left(\frac{t-2^j k}{2^j}\right), \\ \text{Father wavelets: } &\int \Phi(t)\,dt = 1, \\ \text{Mother wavelets: } &\int \Psi(t)\,dt = 0. \end{aligned} \tag{3}$$

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Father wavelets are used for the lowest-frequency smooth components, those requiring wavelets with the widest support; mother wavelets are used for the higher-frequency detail components. Father wavelets are used for the trend components and Mother wavelets are used for all deviations from trend.

Any function $f(t)$ to be represented by a wavelet analysis can be built up as a sequence of projections onto Father and Mother wavelets indexed by both $\{k\}$, $k = \{0, 1, 2, \ldots\}$ and by $\{s\} = 2^j$, $\{j = 1, 2, 3, \ldots\}$. In actual data analysis using discretely sampled data, it is necessary to create a lattice over which the calculations will be made. Mathematically, it is convenient to use a dyadic expansion, as illustrated in equation (3).

The coefficients in the expansion are given by the projections

$$\begin{aligned} s_{J,k} &= \int f(t)\Phi_{J,k}(t)\,dt, \\ d_{j,k} &= \int f(t)\Psi_{j,k}(t)\,dt, \\ j &= 1, 2, \ldots, J, \end{aligned} \tag{4}$$

where $J$ is the maximum scale sustainable by the number of data points. The representation of the signal $f(t)$ now can be given by

$$\begin{aligned} f(t) = &\sum_k s_{J,k}\Phi_{J,k}(t) + \sum_k d_{J,k}\Psi_{J,k}(t) \\ &+ \sum_k d_{J-1,k}\Psi_{J-1,k}(t) + \cdots + \sum_k d_{1,k}\Psi_{1,k}(t). \end{aligned} \tag{5}$$

We can represent the approximation in a more revealing manner for our purposes:

$$\begin{aligned} f(t) &= S_J + D_J + D_{J-1} + \cdots D_j + \cdots D_1 \\ \text{where } S_J &= \sum_k s_{J,k}\Phi_{J,k}(t), \\ D_J &= \sum_k d_{J,k}\Psi_{J,k}(t) \\ \text{and} \quad D_j &= \sum_k d_{j,k}\Psi_{j,k}(t), \;\; j = 1, 2, \ldots, J-1. \end{aligned} \tag{6}$$

The large $J$ refers to the highest level of dilation that is used for the low-frequency, smooth variation of $f(t)$, and the small $j$ refers to the higher-frequency detail coefficients. We can define the multiresolution decomposition of a signal by specifying

- $S_J$ coarsest scale,
- $S_{J-1} = S_J + D_J$,
- $S_{j-1} = S_j + D_j$.

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$\{S_J, S_{J-1}, \ldots, S_1\}$ is a sequence of multiresolution *approximations* of the function $f(t)$ at ever-increasing levels of refinement. The corresponding multiresolution *decomposition* of $f(t)$ is given by

- $\{S_J, D_J, D_{J-1}, \ldots, D_j, \ldots, D_1\}$.

The sequence of terms $S_J(t), D_J(t), D_{J-1}(t), \ldots, D_1(t)$ represents a set of *orthogonal* signal components that provide representations of the signal at resolutions 1 to $J$; each $D_{J-k}$ provides the orthogonal increment to the representation of the function $f(t)$ at the scale, or resolution, $2^{J-k}$.

With each set of macroeconomic data, the idea is to examine in some detail the relationship between the variables when the variation in each variable has been restricted to a specific scale. Instead of looking at the relationship between money and income averaged over all timescales, we examine the relationship at each timescale separately. If $M[S_J]_t$ and $Y[S_J]_t$, and $M[D_j]_t$ and $Y[D_j]_t$ represent money and income at each scale as determined by the wavelet decomposition [recall equation (6)], we can estimate a sequence of regressions using

$$M[S_J]_t = \alpha + \beta Y[S_J]_t + \varepsilon_t$$

$$\text{or} \quad M[D_j]_t = \alpha + \beta Y[D_j]_t + \varepsilon_t, \qquad j = 1, 2, \ldots, J-1.$$

## 3. METHODS AND DATA USED

Seasonally unadjusted monthly data that were obtained from Citibase were used to have a sufficiently high sampling rate to carry out the wavelet analysis, to introduce as little preprocessing of the data as possible, and to allow for the effects of short-term dynamics on the results. The variables used were M1, M2, and nominal personal income (NPI). The last variable listed was used instead of the desired GDP, because GDP does not exist in a monthly series.

We note a difficulty in that monthly measures of nominal GDP are not available. Christiano (1986) and Artis et al. (1992) both used as a proxy the product of the industrial production and producer price indices. We used an alternative proxy for nominal GDP, namely, NPI, which is available in monthly data. Our proxy obviously underestimates nominal GDP because we have left out business income. However, a regression of GDP on our proxy and on the Christiano index produced $R^2$ values of 0.996 and 0.978, respectively, and a regression of log-differenced GDP on log-differenced NPI and on the log-differenced index produced $R^2$ values of 0.123 and 0.020, respectively. Although NPI as a proxy for the monthly values for GDP leaves much to be desired in terms of the detail variation, the proxy seems to be closer to monthly GDP than the index. Further, if the remarks that are to follow concerning results are restricted to our definition of income, little confusion should be occasioned. Our results should be easily interpretable for the definition of income used and should be indicative of the measure of income that we could not use.

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Our main interest is in the reconstructions of the time series by timescale and the relations between the variables at each timescale. The process is exploratory. Our objective is to examine the extent to which an allowance for different effects by scale and for variations in the relationships over time lead to insight into the total variation of the signal over time.

We used the Wavelets package produced by StatSci of MathSoft that was written by Bruce and Gao (1994). We choose the Symmlet as the basic wavelet, designated S12. This wavelet is a compromise between competing requirements. S12 is nearly symmetric, is intermediate in support length with 11 units, has five vanishing moments, and is twice differential. This choice of wavelet is an intermediate choice in that it has reasonably narrow compact support, is fairly smooth, is nearly symmetric, and has a moderate degree of flexibility. We experimented with alternative choices of scaling functions and of wavelets, but the qualitative results were very robust to such changes and the initial choice of wavelet seemed to be the best on balance. We used the boundary condition infinite to reflect the fact that the data contain a trend component.

## 4. MONEY AND INCOME BY TIMESCALE

In this section we use the wavelet decompositions of monetary aggregates and nominal income to investigate whether there are distinct differences in the relationship between these variables at different timescales. We also wish to explore the dynamical relationships between the variables at each scale in more detail.

The remainder of this section is in three subsections: the first provides an overview of the wavelet results and addresses the issue of whether there is any wavelet evidence for a structural break in velocity. The second examines the spectral analysis of the timescale components, and the third section examines the effect of timescale on the evidence for Granger causality.

### 4.1. Review of Wavelet Multiresolution Decompositions

Figures 1 and 2 are time-series plots of the individual components for V1 and V2, where V1 is velocity defined with respect to M1 and V2 is velocity defined with respect to M2. To provide a comparison using the standard analysis, we have included in each set of graphs plots of the log first differences for each variable.

At the highest scale representing trend, we note for V1 that velocity peaks in about 1981 and declines steadily thereafter. Somewhat more interesting is the observation that at the scale S6 for the V2 variable, there is a very definite cycle in the upward growth of velocity with a period of oscillation of 17 years. In interpreting this result, one should recall that our measure of income does not include corporate income.

The next group of scales include the timescales for deviations around the longest timescale and varies from 64 months to 16 months. For V1, the evidence seems to indicate that there has been a long-run buildup in volatility at these scales. There is

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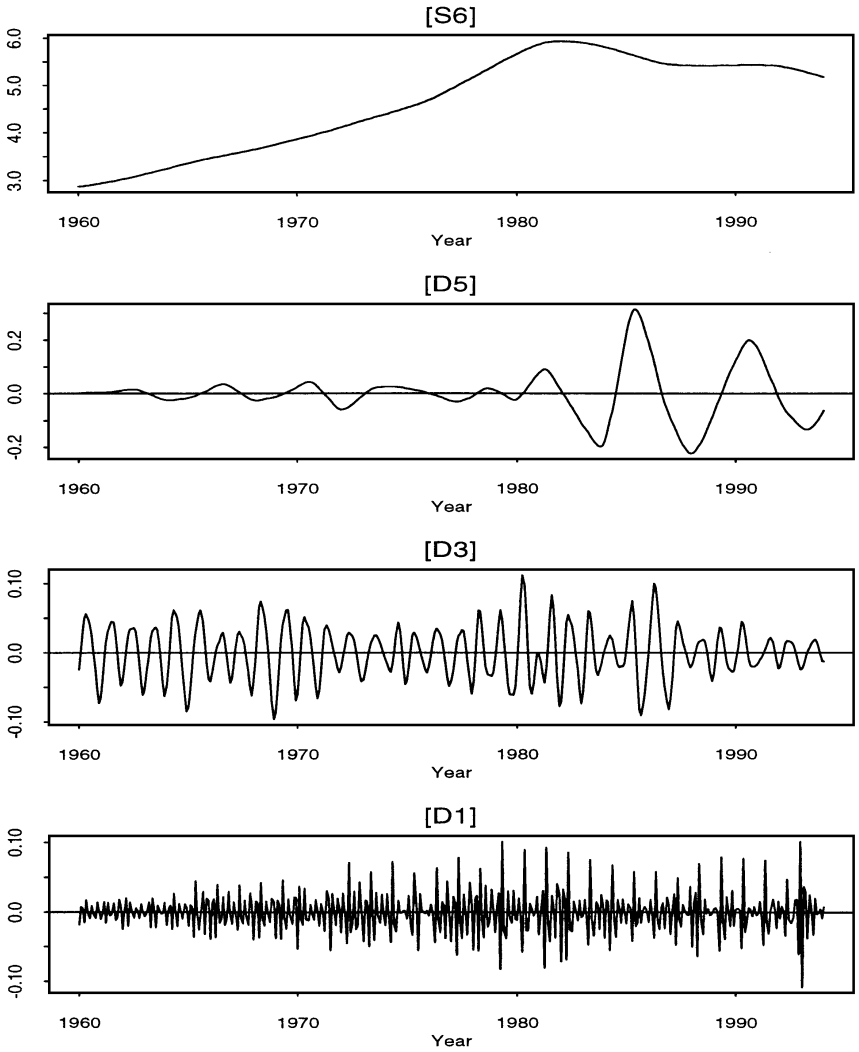



**FIGURE 1.** Time-series plots of individual crystals and log-differenced data for M1 velocity.

no obvious evidence from the distribution of coefficients for a structural break in the data at these timescales. For V2, the evidence is stronger against the postulation of a structural break in the data in 1980. The evidence indicates that events that were in place before 1981, but not yet noticeable, became so after 1981. Attempts to identify a structural break using the Haar wavelet, which is most suitable for such purposes, failed to reveal a break in the data in the vicinity of 1981.

The last group of scales involves variation in the data over periods of eight months or less. In this group of timescales as well, the evidence for the hypothesis that 1980 marked a transition period, rather than a structural break, is even clearer.

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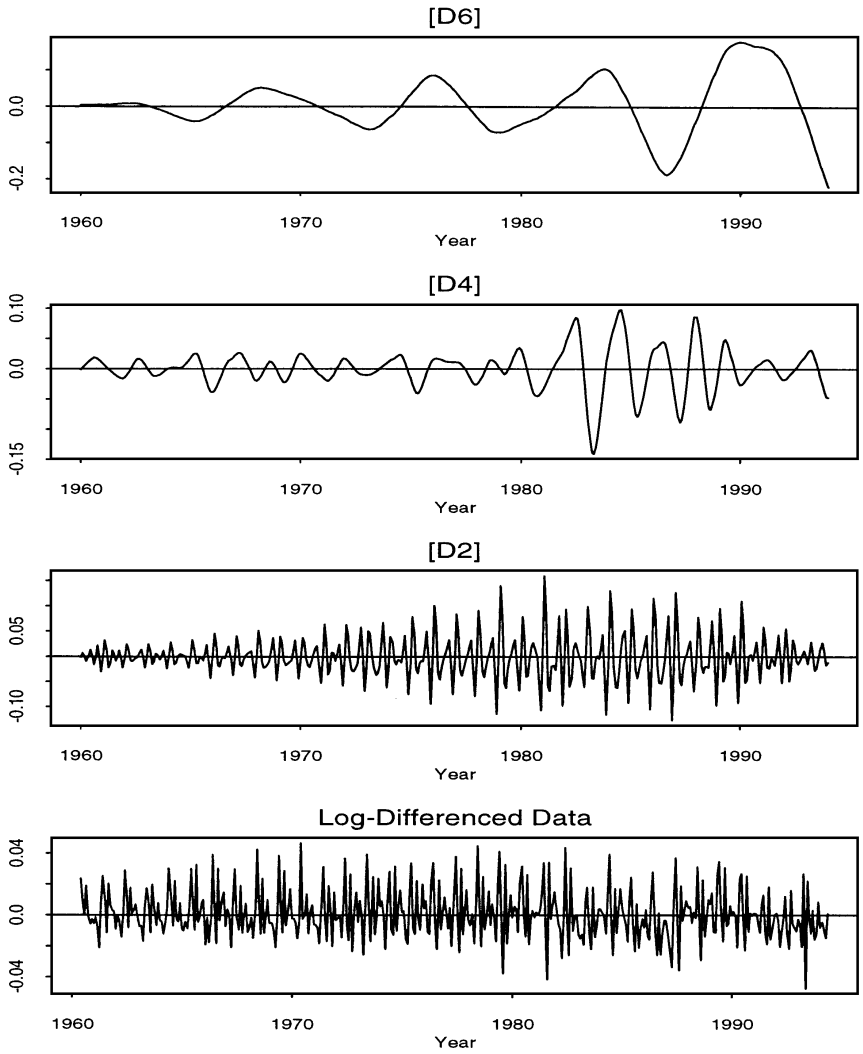



**FIGURE 1.** (Continued.)

One of the most striking features of this group of timescales is the remarkable periodicity in V1 that occurs at the four- and two-month scales, but that is not apparent in the constituent variables. This periodicity does not occur in V2 at these timescales.

### 4.2. Spectral Analysis of the Crystals

We mentioned on two occasions above the presence of well-pronounced oscillations in some of the components. In this section, we explore this observation more

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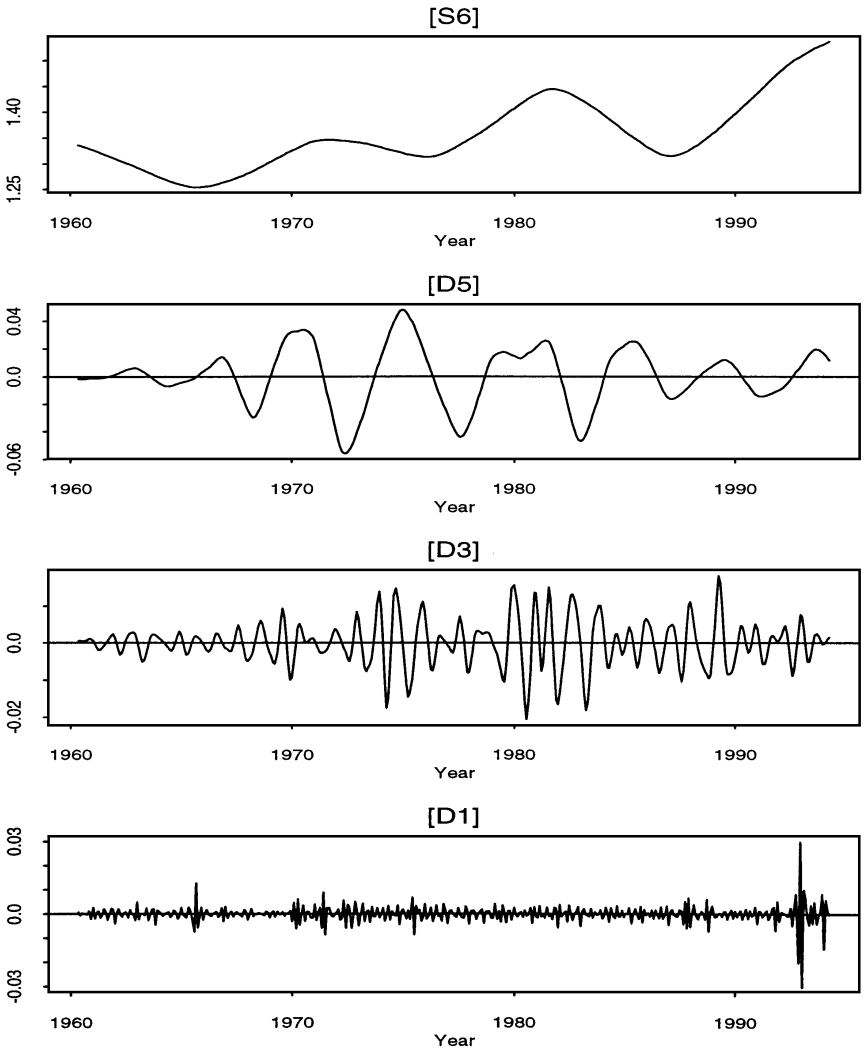



**Figure 2.** Time-series plots of individual crystals and log-differenced data for M2 velocity.

fully and examine some of the dynamical relationships more carefully. M1 and M2 have approximately the same periods and they are in phase. However, M1 and NPI have quite different periods at the D6 and D4 levels, but there seems to be a common period at the D5 level. A more detailed analysis reveals that decomposing the data by timescale reveals oscillations that are not apparent in the raw data. It is only at the very shortest timescales that we observe the same evidence for periodicity in both the differenced data and in the timescale decomposed data.

Another example of interactive dynamics producing interesting results is illustrated by the relationship between M1 and NPI at the timescale of D4. Figure 3

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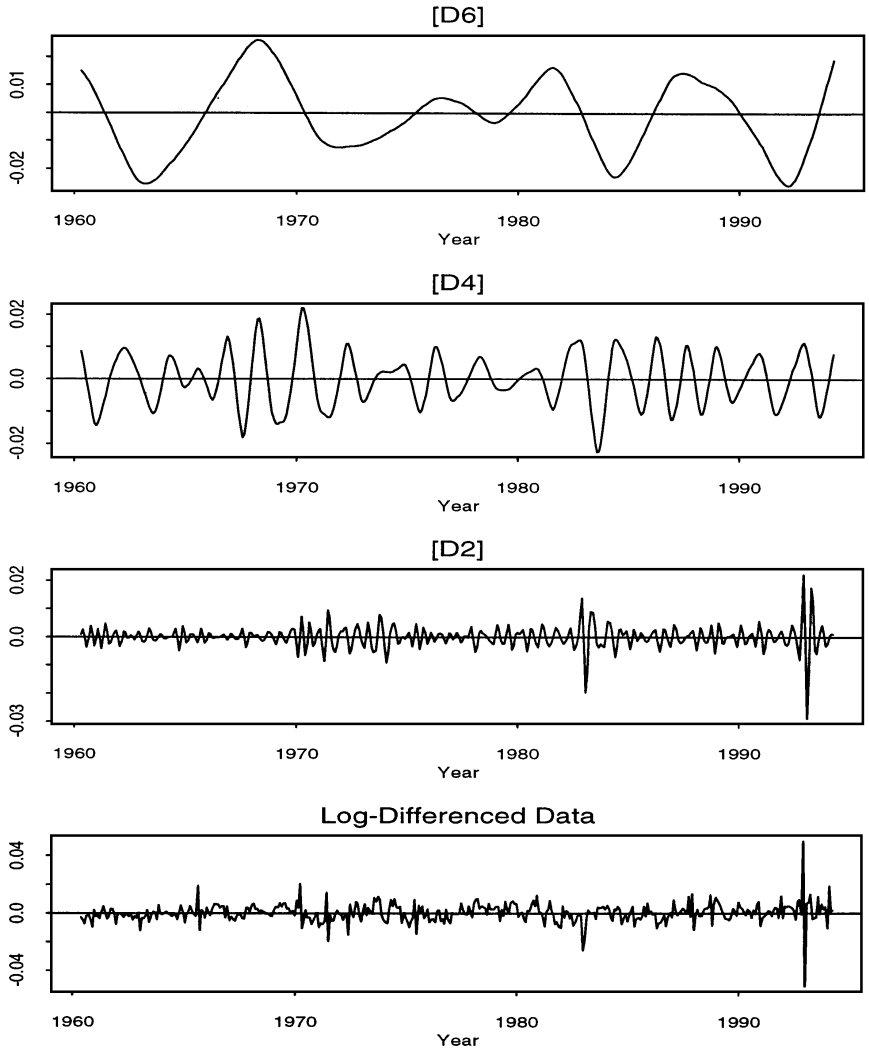



**FIGURE 2.** (Continued.)

has overlaid the plots for the D4 crystals for both M1 and NPI. What is interesting here is that, at this timescale, the two variables M1 and NPI repeatedly move in and out of phase with respect to each other over the sample period. In part, this is due to the fact that at this scale the two series have different spectral peaks; that of NPI is approximately 24 months and that of M1 is approximately 21 months. Consequently, the whole series of observations is made up of stages in which the two series are moving into phase and periods when they are moving out of phase.

If we calculate the coherence and phase relationship between the two variables at this scale, the results indicate very high coherence, almost one, for periods of

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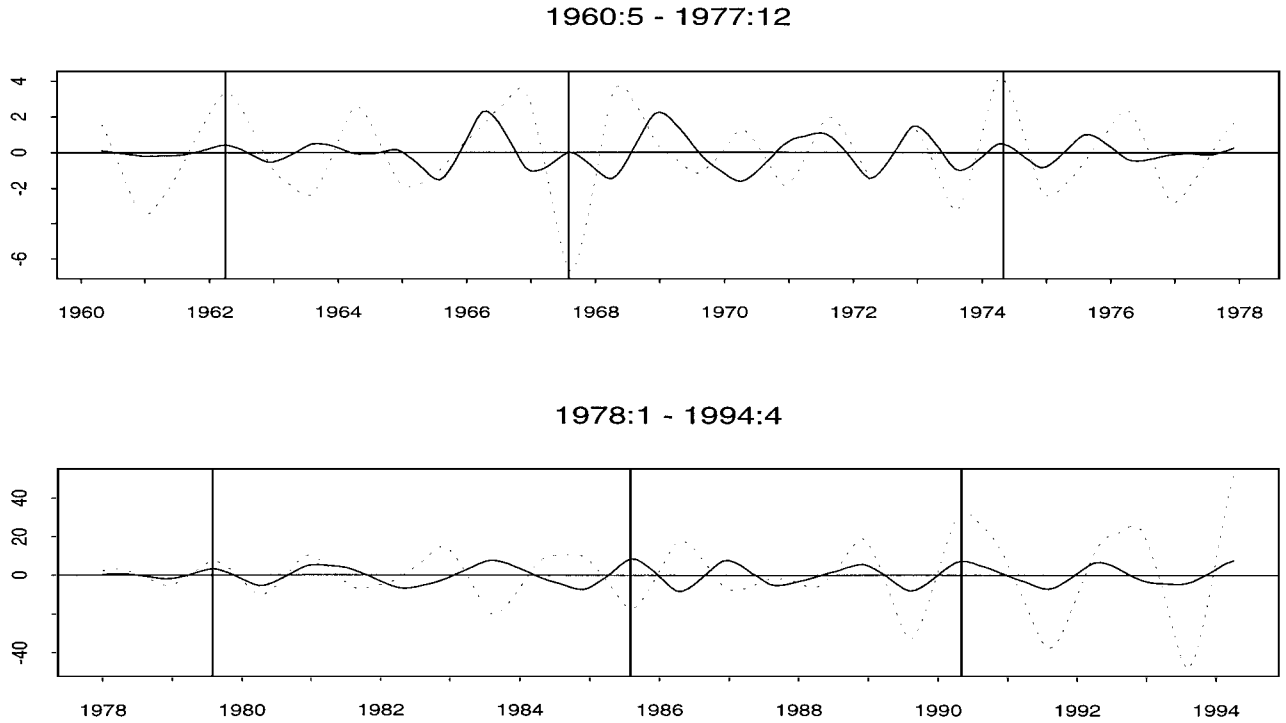



**FIGURE 3.** Time series plots of M1 (solid) and nominal personal income (dots) at crystal [D4].

100 months, and for period ranges of 10 to 7.4 months, 4.5 to 3.8 months, 2.86 to 2.6, and 2.1 to 2.0; these high coherence ranges are demarcated very precisely (see Figure 4). The phase spectrum increases monotonically, but nonlinearly, over the range from zero frequency to $\pi$, rotating about $\pi/4$ radians. We define the group delay, or envelope delay, by

$$d(\omega) = -\left\{ \frac{d\phi_{12}(\omega)}{d\omega} \right\},$$

where $\phi_{12}(\omega)$ is the phase spectrum between the two series. In the simple situation in which one of the variables is linearly related to the other with a lag of $R$ months, the group delay would be a linear function of frequency with a slope coefficient of '$R$'. In the more general situation, as exemplified here, the group delay is no longer a simple constant, but varies with frequency. A group delay of this type can be created by relating two variables with multiple lags, or by allowing the lag structure to vary over time.

Given the results at the D4 level, we can consider the potentiality that the timing of the relationship between money and income may change over time; that is,

$$m_t = \beta y_{t-d(x)} + \varepsilon_t,$$

where $m_t$ and $y_t$ are real logarithms of money and of income, respectively, and $d(x)$ is a variable lag that for suitable values of $x$ could well be a 'lead'. A potential explanation for such a timing function $d(x)$ in the money–income relationship is provided by variations in Federal Reserve policy. For example, the Federal Reserve might change from a reactive to a proactive policy with a consequent change in the timing of the relationship between changes in monetary actions and variations in income. Alternatively the Federal Reserve might have asymmetric strategies between tight and loose monetary policies, so that the lead/lag relationship might well vary over the business cycle. Certainly, these issues are worth exploring more fully and their existence is plausible.

### 4.3. Granger Causality

As we documented above, Granger causality tests have provided over two decades of debate concerning the possible line of causality between money, however defined, and income. Given the discussion above, two aspects of wavelet analysis might well provide some insight into this matter. First, it is likely, given our experience so far, that separating money and income into timescales and analyzing the relationships within timescales will provide considerable insight into the mechanism linking money and income. Second, the research results quoted in the preceding section indicate that another potential complicating factor stems from the variation in phase between the variables at certain timescales. In this section, we investigate each of these aspects.

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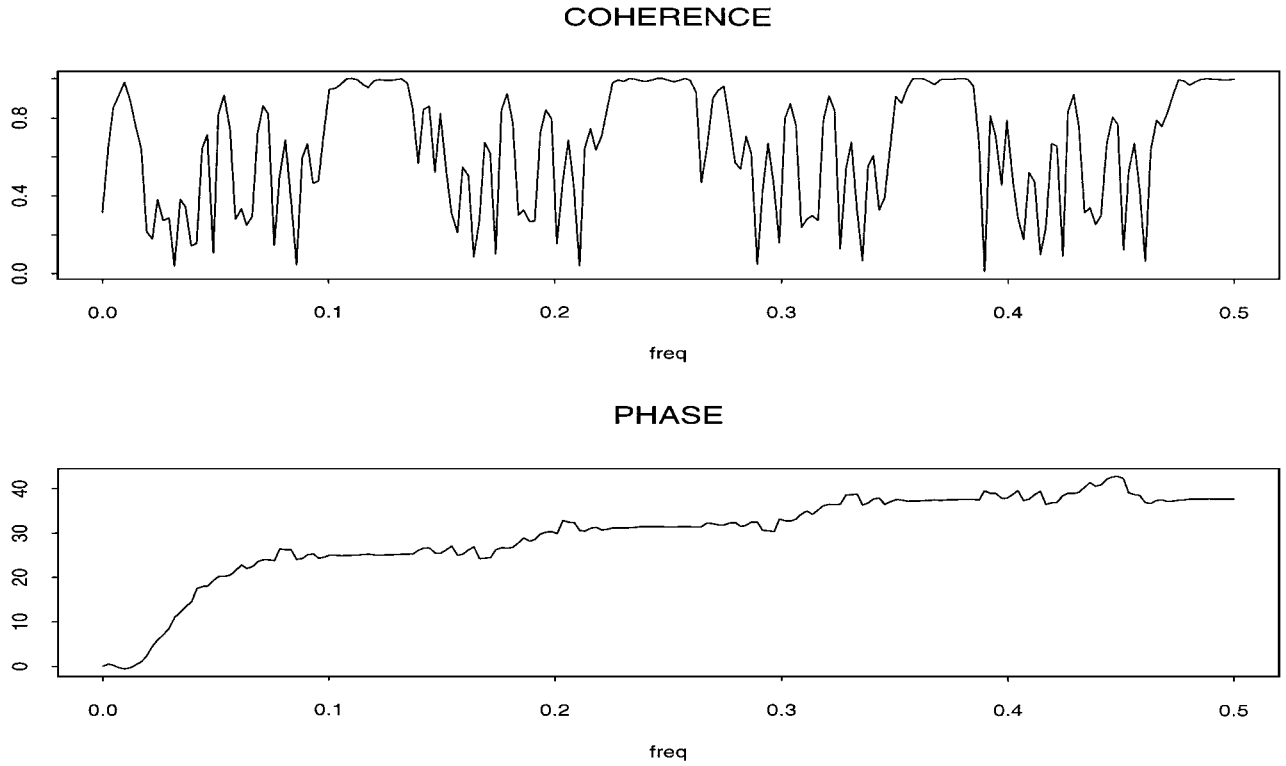



**Figure 4.** Coherency and phase between M1 and nominal personal income at [D4].

**TABLE 1.** Results of Granger causality tests on individual crystals and log-differenced data for M1 and NPI

| | | Null hypotheses[a] | |
|---|---|---|---|
| Timescale (lags) | Results | M1 $\nrightarrow$ NPI | NPI $\nrightarrow$ M1 |
| D6 (5) | Feedback | 6.398 | 6.968 |
| | | (0.000) | (0.000) |
| D5 (20) | Feedback | 4.491 | 5.242 |
| | | (0.000) | (0.000) |
| D4 (19) | M1 $\Rightarrow$ NPI | 3.334 | 0.809 |
| | | (0.000) | (0.695) |
| D3 (17) | M1 $\Rightarrow$ NPI | 1.838 | 1.294 |
| | | (0.023) | (0.193) |
| D2 (23) | M1 $\Rightarrow$ NPI | 5.620 | 1.146 |
| | | (0.000) | (0.293) |
| D1 (14) | M1 $\Leftarrow$ NPI | 1.558 | 5.194 |
| | | (0.089) | (0.000) |
| Log diff. (12) | Inconclusive | 0.534 | 2.063 |
| | | (0.892) | (0.186) |

[a] *P* values in parentheses.

Table 1 summarizes the first set of results. We ran Granger causality tests using the version cited by Granger and Newbold (1986) and restricted attention to M1 and NPI. The joint $F$-tests for the inclusion of lagged value of income in an AR representation of the money equation and for the lagged values of money in the AR representation of the income equation are quoted in the third and fourth columns of Table 1. The null hypothesis for each $F$-test is that the added coefficients are zero and therefore that lagged income does not reduce the variance of money forecasts or that lagged money does not reduce the variance of income forecasts. If neither null hypothesis is rejected, the results are quoted as inconclusive, and if both pairs of $F$-tests reject, the result is labeled as a feedback mechanism. A unique direction of causality is indicated only when one of the pair of $F$-tests rejects and the other accepts the null hypothesis.

The results indicated in Table 1 are very interesting. At the very finest scale, D1, the evidence indicates (weakly) that NPI Granger causes M1, but at the intermediate timescales represented by D2 to D4, we observe with varying degrees of strength that M1 Granger causes income. At the highest levels of timescale, the test results indicate that the mechanism is a feedback one. These results are intuitively plausible. At the finest timescale, it is reasonable to suppose that variations in economic activity initiate corresponding changes in checking-account balances, so that one concludes that NPI Granger causes M1. At the higher timescales, the monetary authorities are trying to control the money supply so that it is plausible that M1 Granger causes NPI. At the very highest scales, it might well be true that M1 and NPI interact in a feedback mechanism. Consequently, we have evidence that not

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**Table 2.** *F*-tests of Granger causality between M1 and NPI at D4 across phase shifts[a]

| Null hypothesis | In-out, 1962:4–1967:8, M1⇒ NPI | Out-in, 1967:8–1974:5, M1 ⇐ NPI | In-in, 1962:4–1974:5, inconclusive |
|---|---|---|---|
| NPI↛ M1 | 6.401 (0.00) | 0.997 (0.434) | 0.643 (0.696) |
| M1↛ NPI | 1.465 (0.208) | 2.714 (0.020) | 0.933 (0.474) |

[a] *P* values in parentheses.

only is it true that the degree of causal relationship varies across scales, but that the *direction* of causality differs by timescale. Examining the log first differences for the historical period under examination, we obtain the unremarkable result that the Granger causality tests are inconclusive. The various outcomes obtained for different timescales provide one explanation of this result in that the differenced data average over the individual scale effects, so that the change in direction across scales can easily lead to an inconclusive outcome for a time aggregate expressed in differenced form.

A second question that arises from this analysis concerns the dynamical links between the variables and the way in which they may vary over time. We may question how sensitive our results are to the choice of time period over which we perform the tests. For example, using the data in log first-difference form, one can show that M1 Granger causes NPI during the period 1970–1979, but that the tests are inconclusive over the period 1980–1994. We also examined in light of this result the Granger *causality* tests for M1 and NPI at the D1 level with the data separated into two periods: 1970–1979 and 1980–1994. For the former period, M1 Granger causes NPI, but for the latter period, the situation is reversed in that NPI Granger causes M1. Given these results, we decided to explore the sensitivity of Granger causality tests to variations in phase between any two variables. Table 2 presents the results of one such investigation.

Refer to Figure 3 in which the D4 crystals are plotted for M1 and NPI. We have mentioned in the preceding section the differences in fundamental periods between the two series: Figure 3 plots out, as a pair of time series, the two variables using the D4 crystals. It is immediately clear that the two series at this timescale are moving into and out of phase with each other. Consequently, depending on whether the series are moving into phase, or out of phase, one is likely to obtain different results from a Granger causality test, depending on which part of the cycle is included. The idea is to go beyond the usual observation that different causality results can be obtained from different segments of the data. Our point is that a key element in obtaining different causality results in the presence of phase-varying series is that different causality results depend on one's selection, implicit, or otherwise, of the sequence of phase relationships. Table 2 illustrates this notion. We observe that, if the series are moving into phase, Granger causality tests indicate that M1 causes

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NPI, whereas when the series are moving out of phase, one obtains with the same tests the opposite result, that NPI causes M1; and if one picks a period in which the series move into and out of phase with each other, Granger causality tests can produce an inconclusive result.

One lesson here is that results that are apparently sensitive to the subset of data that are chosen might well be caused by the effects of the variation over time of the phase relationship between the two variables, even after one has allowed for different effects across timescales. With respect to the different results obtained before and after 1980, we note from Figure 3 that 1970–1979 is a period in which the series moved from out of phase to in phase; whereas the period after 1980 is characterized by a move from in phase to out of phase.

## 5. SUMMARY AND CONCLUSIONS

A key element in any set of economic time series of any length is the potential presence of nonstationarity beyond the notion of a unit root. Further, the theoretically imprecise statement of the functional relationships between economic variables has led many researchers to consider nonparametric procedures. An important class of such procedures is that provided by wavelets. More important, wavelets have some characteristics that make them particularly suitable as a vehicle for analyzing economic data. Because of the translation and scale properties, nonstationarity in the data is not a problem when using wavelets and prefiltering is not needed. Further, because of the flexibility in choice of basis function, that is, choice of wavelet function, and because of the property of narrow compact support, wavelets are particularly well suited to handle complex signals that involve cusps, discontinuities, and rapid changes in modeling regime.

For the objectives of this paper, an even more important property of wavelets involves the separation of timescales of variation into a sequence of scales that can be decomposed orthogonally. A key premise underlying the research in this paper is that the relationships between economic variables may be expressed better in terms of restrictions to given timescales.

A separate concept that can be investigated easily in the context of a wavelet decomposition of economic variables is that of the timing of economic relationships and its variation over the business cycle. For example, the timing of the relationship between money and income may vary with Federal Reserve policy, even to the point where monetary changes sometimes lead and sometimes lag income changes. These variations in timing are recognized as phase drift in terms of the spectral analysis of the data, even after restricting attention to a specific timescale. The importance of phase drift is that if ignored in an empirical study, one may conclude easily that a relationship is a complicated nonlinear function, when in fact the relationship can be quite simple, but subject to phase drift.

The reader will recall that because monthly measures of GDP are not available, we used NPI, which is available monthly, as a proxy; this measure, although clearly underestimating GDP, seems to be closer to the variation in GDP than the index

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measure that has been used in the past. In any event, all comments to follow are easily interpretable for NPI and should be indicative for GDP. In terms of the money–income relationships, we observe little evidence of a structural break in either velocity measure in 1980, although there is a definite, but smooth, change in the rate of increase in velocity using the M1 measure of money at the longest scales. Indeed, there is weak evidence that the changes that generally are presumed to have been induced by the Volker experiment between 1979 and 1981 were to some extent anticipated by subtle changes that were only fully apparent after the end of 1980, or even later. However, there was a rather surprising degree of evidence for periodicities at certain timescales for the M1 velocity and a 17-year cycle for M2 velocity at the very longest timescale. These strongly observed periodicities are not apparent in the constituent series, so that one may well conclude that the observed periodicities are due to the interaction between money and income at specific scales.

Although the distribution of energy, that is, the distribution of variance, across timescales is quite different between M1 and M2 and their corresponding measures of velocity, the two measures of money are in phase throughout the period. For M1 and income at the D4 scale, however, the two series move into and out of phase with respect to each other over the whole observed period. A spectral analysis of these two series indicated that, over very well pronounced frequency bands, M1 and income were very highly coherent and the phase relationship drifted up in a smooth fashion. This latter finding is consistent with a relationship between money and income in which the timing of the relationship is changing over the observed period.

The results of applying Granger causality tests between money and income at various timescales provided very interesting results. First, at the very lowest timescales, we observed that income Granger causes money, but at business-cycle periods, money Granger causes income and, at the longest scales, there is apparently a feedback mechanism at work. However, these conclusions are tentative because of the observed sensitivity of Granger causality tests to phase drift.

At the business-cycle scales, the same scales in which there is phase drift between money and income, the observation of Granger causality depends on whether the two series are moving into phase with respect to each other, or are moving out of phase. More precisely, in one case, one can obtain the result that money Granger causes income and in the other that income Granger causes money, or neither, or both, depending on the combination of phase changes that are observed in the data used for the Granger causality tests. This is a result that requires considerable further research in that its implications for the nature of the relationship between economic variables and how we discover apparent lines of causality is crucial.

Although further work is necessary to pursue these tentative findings, they are of interest for those wishing to model macro data successfully. The immediate requirements for further work involve first the confirmation, possibly using data from other countries, of the qualitative results presented in this paper. The second step is to explore the potential causes of the changes that were observed in the time series at the various scales. The empirical results lead us to rethink our theoretical

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development to include the role of timescale as a characterization of economic data and to recognize that sometimes the loss of simple relationships between variables merely indicates the occurrence of a phase shift in the relationship, that is, a change in the timing of the relationship between variables. Of course, the corresponding challenge is to discover why the phase shift occurs and what factors bring the shift to closure.

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