Paradox as a Guide to Ground

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Abstract
I will use paradox as a guide to metaphysical grounding, a kind of non-causal explanation that has recently shown itself to play a pivotal role in philosophical inquiry. Specifically, I will analyze the grounding structure of the Predestination paradox, the regresses of Carroll and Bradley, Russell's paradox and the Liar, Yablo's paradox, Zeno's paradoxes, and a novel omega plus one variant of Yablo's paradox, and thus find reason for the following: We should continue to characterize grounding as asymmetrical and irreflexive. We should change our understanding of the transitivity of grounding in a certain sense. We should require foundationality in a new, generalized sense, that has well-foundedness as its limit case. Meta-grounding is important. The polarity of grounding can be crucial. Thus we will learn a lot about structural properties of grounding from considering the various paradoxes. On the way, grounding will also turn out to be relevant to the diagnosis (if not the solution) of paradox. All the paradoxes under consideration will turn out to be breaches of some standard requirement on grounding, which makes uniform solutions of large groups of these paradoxes more desirable. In sum, bringing together paradox and grounding will be shown to be of considerable value to philosophy.[1]

Paradox, although it has often led through harsh terrain, has all in all been a good guide in the analysis of important concepts. Our current concepts of *set* and *elementhood*, which are all but undisputed, have been shaped by the need to evade the paradoxes of Russell, Cantor, and Burali-Forti. Our standard model-theoretic way of approaching the concept of *truth* (cautiously, from a meta-level)

[1] I have many people to thank because the history of this paper goes far back. In my talks 'Foundation and Paradox' in May 2015 (*Logica 2015*, Hejnice) and 'Metaphysical Foundationalism and the Paradoxes of Non-Well-Foundedness' in September 2015 (*Congress on Analytical Philosophy*, Osnabrück), I presented the results about the semantic and set theoretical paradoxes as puzzles of grounding that are now recounted in sections 4 and 5. In January 2016, I completed a first draft of the full paper, which I then circulated among a small number of people and later used as the basis of my talk 'Paradox and the Structure of Explanation' in May 2019 (*ExLog2019*, Louvain-la-Neuve). I am grateful to all these audiences for many helpful comments, as well as to the following individuals: Daniel Boyd, Scott Dixon, Kit Fine, Stephan Krämer, Tobias Martin, Peter Milne, Chris Scambler, Niko Strobach, Graham Priest, and Yale Weiss.

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has one important source in the need to evade the Liar paradox.[2] Since Kripke's seminal paper in 1975, the search for an adequate theory of truth that can express more semantic facts than allowed by the Tarskian hierarchical restrictions is an ongoing and thriving project. The Liar paradox is again the main motor of the process. Insofar as the search involves logical revision, paradox can even have an impact on the concept of *consequence*. Finally, Zeno's paradoxes, although they have not been the main motivation behind the modern mathematical theory of the continuum, provide valuable test cases even in our times for the philosophical question of what exactly this mathematical theory tells us about the metaphysics of *space*, *time*, and *movement*.[3]

So, attention to certain paradoxes has furthered our understanding of the concepts of set, truth, consequence, space, time, and movement. Can attention to these and other paradoxes be similarly helpful for our understanding of *explanation*? What I mean here is specifically the non-causal kind of explanation that is of crucial importance in philosophy (and beyond) and has recently been explicated as *metaphysical grounding*. Can paradox be a good guide to ground? That is the question I want to explore in the present paper.

This amounts to developing an approach in the study of grounding that is still new. Of course, grounding already has its own puzzles,[4] and they provide a welcome starting point. But as of now, these

[2] Another important source of our model-theoretic approach to truth is Gödel's first incompleteness theorem. Although not itself paradoxical, it is still close enough to the Liar paradox. For if – *per impossibile* – arithmetic were complete (in a semantic sense), then provability would be equivalent to truth, and a Gödel sentence would not only claim its own unprovability in an indirect way, but also claim its own untruth, albeit in an even more mediated way.

[3] Cf., e.g.: Wesley C. Salmon: *Zeno's Paradoxes* (Indianapolis: Hackett, 2001).

[4] Kit Fine: 'Some Puzzles of Ground', *Notre Dame Journal of Formal Logic* 51 (2010), 97–118; Stephan Krämer: 'A Simpler Puzzle of Ground', *Thought* 2 (2013), 85–59; Johannes Korbmacher: 'Yet Another Puzzle of Ground', *Kriterion – Journal of Philosophy* 29.2 (2015), 1–10; see Stephan Krämer: 'Logical Puzzles of Ground', to appear in: Mike Raven (ed.): *Routledge Handbook of Metaphysical Grounding* [forthcoming]. Johannes Korbmacher studies a paradox of self-reference that does employ the grounding predicate. However, it makes crucial use of the factive aspect of the notion of grounding in question (as has been observed by Stephan Krämer), and should therefore be seen as a variant of the Liar paradox, not a genuine paradox of grounding.

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puzzles and other paradoxes have played only a minor role in work on grounding.[5] Let us change that! I propose to put paradox center stage in the study of grounding, especially in the study of its structural features. Fortunately, to use paradox as a guide in the analysis of grounding we need not press ourselves and try to invent new paradoxes that concern grounding and no other notions. The reason is that it will turn out that we need only look at the grounding side of the well-known paradoxes of sets, truth, space, and so on to discover a whole range of new puzzles of grounding. These will in turn provide us with new arguments concerning several important properties of grounding, in particular concerning the question that has recently been debated of whether grounding is foundational, and if so, then precisely in what sense.[6] Adopting this novel methodology for the study of grounding will thus be rewarding. As a spin-off, there will also be some ground-theoretic insights relevant to the understanding of the paradoxes of sets, truth, and space, as well as about the very notion of a paradox.

[5] For laudable exceptions, cf. Jack Woods: 'Emptying a Paradox of Ground', *Journal of Philosophical Logic* 47.4 (2018), 631–648 and (in a way) Jon Erving Litland: 'Grounding, Explanation, and the Limit of Internality', *Philosophical Review* 124.4 (2015), 481–532. In a number of presentations under the title 'Ground and Paradox', Boris Kment has reported work that is probably closest to the project of the present paper. As far as I know (from one of these presentations), the main overlap concerns the diagnosis that the semantic and set theoretic paradoxes give rise to further puzzles of ground (cf. section 5). From this shared observation, however, Kment appears to draw quite different conclusions. Cf. my methodological considerations in section 11.

[6] Ross P. Cameron: 'Turtles All the Way Down: Regress, Priority and Fundamentality', *The Philosophical Quarterly* 58 (2008), 1–14; Jonathan Schaffer: 'Monism: The Priority of the Whole', *Philosophical Review* 119.1 (2010), 37; Alexander Paseau: 'Defining Ultimate Ontological Basis and the Fundamental Layer', *The Philosophical Quarterly* 60 (2010), 169–175; Ricki Bliss: 'Viciousness and the Structure of Reality', *Philosophical Studies* 166.2 (2013), 399–418; Tuomas Tahko: 'Boring Infinite Descent', *Metaphilosophy* 45.2 (2014), 257–269; Gabriel Oak Rabin / Brian Rabern: 'Well Founding Grounding Grounding', *Journal of Philosophical Logic* (2015), 1–31; Jon Erving Litland: 'An Infinitely Descending Chain of Ground Without a Lower Bound', *Philosophical Studies* (2015), 1–9; and T. Scott Dixon: 'What Is the Well-Foundedness of Grounding?', *Mind*, doi:10.1093/ mind/fzv112 (first published online: February 28, 2016).

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1 Explanation as Metaphysical Grounding

Before we look at the more technical sides of grounding to prepare our investigation of the paradoxes, a more general word on why it makes good sense to conduct this study in terms of metaphysical grounding. Ultimately, we are interested in explanation of a certain sort, which is of particular importance for philosophical inquiry and argument. To characterize it in the roughest strokes, we can say that it is non-causal, and we also have some initial intuitions about its structure. Thus we often assume that nothing explains itself, that no explanation is circular, and (maybe more controversially) that all explanations ultimately come to an end. It is such structural features of explanation – rather than particular instances of *this* explaining *that* – that we are interested in.

But these matters are far from clear, and more rigor would be desirable. Fortunately, this intuitive notion of explanation has in recent years been explicated in more precise ways as *metaphysical grounding*. For the purposes of this paper, I assume that this explication is adequate – or more cautiously, that we can learn so much about explanation by investigating metaphysical grounding that we might as well focus on the latter.

Let us introduce grounding in the words of some main proponents; Kit Fine:

> 'A number of philosophers have recently become receptive to the idea that, in addition to scientific or causal explanation, there may be a distinctive kind of metaphysical explanation, in which explanans and explanandum are connected, not through some sort of causal mechanism, but through some constitutive form of determination. I myself have long been sympathetic with this idea of constitutive determination or "ontological ground".'[7]

And Fabrice Correia and Benjamin Schneider write:

> 'Some of the most important questions in philosophy, we believe, concern matters of priority. […] a particular sort of non-causal priority which we would like to call *grounding*.'[8]

[7] Kit Fine: 'Guide to Ground', in: Fabrice Correia / Benjamin Schneider (eds.): *Metaphysical Grounding: Understanding the Structure of Reality* (Cambridge: Cambridge University Press, 2012), 37–80.

[8] Fabrice Correia / Benjamin Schneider: 'Grounding: An Opinionated Introduction', in: Fabrice Correia / Benjamin Schneider (eds.): *Metaphysical Grounding: Understanding the Structure of Reality* (Cambridge: Cambridge University Press, 2012), 1.

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Grounding is a notion of metaphysical explanation; in natural language it is often expressed by locutions involving phrases like 'because', 'due to', or 'in virtue of' and in philosophical parlance by talk of (systematic) priority. Here are some typical examples: 'There is a labor strike *due to the fact* that the truck workers are refusing to work and instead picketing outside their workplace',[9] 'If something is loved by the gods, it is loved by them *because* it is pious', and 'A substance is *prior* to its tropes or modes'.[10] These examples already illustrate that the notion of explanation explicated by grounding is of great importance to philosophy. As for the explicans – aside from a sizeable debate centering on the notion of grounding itself, the last couple of years have seen its use spreading into various subfields, like the philosophy of science, philosophy of mind, philosophy of time, etc.

Furthermore, metaphysical grounding is by now well-studied, allowing us to import principles about its interaction with other notions and to work more rigorously than with an intuitive notion of explanation alone. In particular, the technical tools developed in recent work on the logic of ground (by Fine and others) become available, which we will use to characterize its structural features in a precise way.

2 Grounding: Preliminaries, Questions, and Puzzles

Let us start out on our study by having a closer look at the tools.

Grounding claims come in two flavors.[11] *Operational* grounding is expressed by a twoplace sentential operator. We will use '$<$', where '$p < q$' reads 'p (operationally) grounds that q', which can be seen as a regimentation of 'q because of p'.[12] *Predicational* grounding is expressed by a two-place predicate. We will use '$\lhd$', where '$x \lhd y$' reads 'x (predicationally) grounds y'.[13]

[9] Ricki Bliss / Kelly Trogdon: 'Metaphysical Grounding', in: Edward N. Zalta (ed.): *The Stanford Encyclopedia of Philosophy (Winter 2014 Edition)*, URL = http://plato.stanford.edu/archives/win2014/entries/grounding/.

[10] Op. cit. note 8, 3; my emphasis.

[11] Op. cit. note 8, 10–12.

[12] '$<$' is a sentential operator *of the object language*, in contrast to the logic of ground of Fine's 'Guide to Ground' (op. cit. note 7), where it is a meta language symbol, in the same way as the turnstile '$\vDash$'.

[13] One might be tempted to see 'x predicationally grounds y' just as a paraphrase of the converse 'y depends ontologically on x'. But the relation

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On the metaphysical level, whereas predicational grounding is *a dyadic relation*, operational grounding could be called *a dyadic modality* (in analogy to the monadic modalities of necessity and possibility).[14] A standard example for operational grounding is that a sentence's being true is grounded in what it says being the case, e.g., Snow is white < 'Snow is white' is true.[15] A standard example for predicational grounding is that a set is grounded in each of its elements, e.g., Socrates ◁ {Socrates}.[16] Here these two grounding claims serve only to illustrate the notion of grounding. But each one of them will later also play an important role in the grounding analysis of the set theoretic and semantic paradoxes.[17]

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between grounding and ontological dependence is more complicated than that; cf., e.g., Benjamin Schneider: 'Grounding and Dependence', *Synthese*, doi: 10.1007/s11229-017-1378-z (first published online: May 18, 2017).

[14] Here my terminology is not standard. Others talk of an 'operation' (e.g., Litland in op. cit note 5) or 'operationalism' (T. Scott Dixon: 'Upward Grounding', *Philosophy and Phenomenological Research* (2017), doi: 10.1111/phpr.12366). But this might be misleading because what the sentential operator of grounding expresses has nothing to do with either the set theoretical notion of an operation as a certain kind of mapping nor the everyday notion of an operation as a certain kind of action.

[15] Cf. Schaffer op. cit. note 6, 35; going back to Aristotle's *Categories*, i.e., Aristotle: 'Categories', in: Jonathan Barnes (ed.): *The Complete Works of Aristotle. Vol. 1* (Princeton: Princeton University Press, 1984), 22.

[16] Schaffer op. cit. note 6, 35; Paseau op. cit. note 6, 169; and Kit Fine: 'Essence and Modality', *Philosophical Perspectives* 8 (1994), 4–5.

[17] In order to do justice to everyone I should add that with regard to predicational grounding, there is no consensus about which sorts of objects it can relate. While Jonathan Schaffer does not appear to accept any restriction (crucially, he discusses whether a physical object is grounded in its parts or *vice versa*; Schaffer op. cit. note 6), others hold that the relata of a claim of predicational grounding must be *facts* or *propositions*, so that the only substantial difference to the corresponding claim of operational grounding would be grammatical. In some places I will also talk as if I made no restrictions for the possible relata of predicational grounding. But this way of speaking can be made acceptable to everyone, because whoever is unhappy with understanding this as a primitive claim is welcome to analyze it (away) in terms of operational grounding and a (non-trivial) existence predicate, according to the following definition of a suitably wide notion of predicational grounding: $x \triangleleft y \Leftrightarrow_{\text{def}} E!(x) < E!(y)$. I would like to thank Stephan Krämer for convincing me of both the importance of this problem and the viability of a solution in terms of existence.

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There are further important distinctions to be made among varieties of grounding.[18]

Most people understand grounding claims as *factive*: If $p <_f q$, then p, and q. But the factive notion can be defined in terms of the non-factive: $p <_f q \Leftrightarrow_{\text{def}} p < q \wedge p \wedge q$. Therefore, and because we will want to deal with hypothetical scenarios when we analyze the paradoxes, let us work with *non-factive grounding*.

There is a *full* and a *partial* notion of ground. E.g., p is a full ground of $p \vee q$ but only a partial ground of $p \wedge q$. Full ground seems to be the fundamental notion. But as we often will know a partial ground of something without knowing (or caring for) its full ground, let us work with *partial grounding*.

Finally, there is a *weak* and a *strict* notion of ground. On the weak notion, everything is self-grounding, $p \leq p$. But as irreflexivity will be crucial in the analysis of the paradoxes, let us work with *strict grounding*.

Note that our resolution to work with non-factive, partial, strict grounding does not amount to a substantive claim about grounding, but concerns only the methodological question of which variety of grounding to focus on in our search for substantive claims.

So much for preliminaries; let us now turn to interesting questions.

What are the structural properties of grounding?

Grounding is usually taken to be asymmetric, hence irreflexive, and transitive. In view of the two flavors, we need to characterize each one of these structural properties twice:

	operational grounding:	*predicational grounding*:
(Asymmetry)	$p < q \vDash \neg(q < p)$	$x \lhd y \vDash \neg\, y \lhd x$
(Irreflexivity)	$\vDash \neg(p < p)$	$\vDash \neg\, x \lhd x$
(Transitivity)	$p < q, q < r \vDash p < r$	$x \lhd y, y \lhd z \vDash x \lhd z$

Further, grounding is often required to be *foundational*. This is most naturally spelled out according to the mathematical notion of well-foundedness, so that foundationality means that each collection of objects[19] has a grounding-minimal element. In the presence of the axiom of choice we might as well require that every descending chain

[18] In the next three paragraphs I use the distinctions of Fine op. cit. note 7, 46–54 to set out a spectrum of options, but I follow my own preferences in picking from them.

[19] From here on, I will speak of grounding only in terms of a *relation* that holds among *objects* – taking the above table to establish that whatever structural properties we require of predicational grounding can be translated into corresponding requirements on operational grounding.

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of grounding terminates, or that there are no infinitely-descending grounding-chains. However, we will later encounter an alternative notion of foundationality that is somewhat weaker.

Sometimes, it is allowed (or even required) that grounding claims are themselves grounded, leading to questions of *meta-grounding*.

Although the asymmetry, irreflexivity, and transitivity of grounding are felt to be very intuitive by most people in the field, even they have been questioned.[20] Matters are more contentious with regard to foundationality – the debate about whether it holds and in what sense has already been mentioned. Concerning meta-grounding, there seems to be the least consensus but some recent interest.[21] When we turn to the paradoxes, we will find something interesting concerning each one of these notions.

Besides these structural questions, there are what can be called *material questions* about grounding – specific instances of the question: What grounds what? Here, questions about what can be called the *polarity* of grounding will be most interesting to us. These occur when there is a candidate for an asymmetrical relation that may well go along with grounding, but it is not yet clear in which direction. We will be concerned with the following two polarity questions: Does *a* instantiate *F*-ness because *a* is *F*, or *vice versa*? Is the part grounded in the whole, or *vice versa*?

Kit Fine has shown that the notion of grounding leads to puzzles,[22] e.g., that the fact that everything exists is (partly) grounded in itself, although grounding intuitively is irreflexive.[23] He argues thus:

> 'Here is an especially simple version of the puzzle. Surely, everything exists. Let f_0 be the fact that everything exists. Then everything exists partly in virtue of f_0's existing. But f_0 exists partly in virtue of everything existing, since f_0 is the fact that everything exists; and so everything exists partly in virtue of everything existing. Which is an absurdity, given that nothing can hold in virtue of, or partly in virtue of, itself.'[24]

[20] Carrie S. Jenkins: 'Is Metaphysical Dependence Irreflexive?', *The Monist* 94.2 (2011), 267–276; Gonzalo Rodriguez-Pereyra: 'Grounding is Not a Strict Order', *Journal of the American Philosophical Association* 1.3 (2015), 517–534; Woods op. cit. note 5.

[21] E.g., cf. again Rabin / Rabern op. cit. note 6.

[22] See Krämer op. cit. note 4 for an up-to-date overview.

[23] Fine op. cit. note 4.

[24] Fine op. cit. note 4, 98.

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Using '[…]' as an operator that takes a sentence and forms a term that refers to the corresponding fact, we can formulate the conclusion of Fine's puzzle thus:

[Everything exists] exists < [everything exists] exists.

Or more concisely:

Everything exists < everything exists.

We will come upon similarly puzzling results in our investigation of the paradoxes. Fine himself, however, observes 'some resemblance to the paradoxes of self-reference',[25] but he thinks that his puzzles of ground are distinct from the paradoxes.

3 The Asymmetry of Grounding: The Predestination Paradox

Let us start on a lighter note, with some paradoxes of backward time travel. In an instance of the *Grandfather paradox*, the time-traveler does something in the past that precludes this very action, e.g., by killing one of his or her grandparents. In an instance of the *Predestination paradox*, the time-traveler does something in the past which brings about this very action, e.g., by bringing information about how to build a time machine and giving directions of what to do with it. The paradoxes of time travel are less dangerous than others for the simple reason that they are *fictional*. The Grandfather paradox thus relates to the Liar paradox as Professor Moriarty relates to Napoleon. But even in the absence of real danger, we can learn something both for the notion of grounding and for the notion of paradox by thinking about them. Specifically, let us think about the Predestination paradox.

The mainstream view in the philosophical debate about time travel is that backward time travel would be logically possible if consistent.[26] This would mean to reject the Grandfather paradox but accept the

[25] Fine op. cit. note 4, 97.

[26] Cf. the seminal paper by David Lewis, with its praise of consistent time travel stories by Robert Heinlein. David Lewis: 'The Paradoxes of Time Travel', *American Philosophical Quarterly* 13.2 (1976), 145–152; Robert A. Heinlein: 'By His Bootstraps', in: Robert A. Heinlein: *The Menace from Earth* (New York: Signet, 1959[1941]), 39–88; and Robert A. Heinlein: '"– All You Zombies –"', in: Robert A. Heinlein: *The Unpleasant Profession of Jonathan Hoag* (New York: Ace, 1959), 138–151. Today, we might want to add the Doctor Who episode 'Blink' as another example for a well-crafted Predestination paradox.

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Predestination paradox, and along with it all other *hypodoxes*, as they have been called.[27] But consistency – logical coherence – is not all. What about physical, or metaphysical, coherence? In the Predestination paradox, A explains B and B explains A; typically, as a cause explains its effect. And this breach of asymmetry can certainly *feel* incoherent. Now, it is of course important to keep apart grounding, or metaphysical explanation, from causality, or physical explanation. But they might turn out to be species of the same genus, and likely have a similar structure.[28] Thus we can take our feeling of incoherence as further evidence that *any* kind of explanation needs to be asymmetrical.

It is worth noting that in some instances of the Predestination paradox there arguably is a *direct* breach of irreflexivity and hence of the asymmetry of grounding. In Robert Heinlein's short story '"– All You Zombies –"', the tropes of intersexuality and sex change are exploited to create a narrative where a human being is their own mother and father.[29] Under the plausible assumption that mammals are (partly!) grounded in their parents, this protagonist would be self-grounding. Although consistent, this feels decidedly paradoxical.

From this, we can learn a more general lesson concerning the notion of a paradox. Today, a paradox is standardly defined as an apparently valid argument with apparently true premises and an apparently unacceptable conclusion. In paradigmatic cases, from the Liar to the Grandfather paradox, the conclusion appears to be unacceptable because it is contradictory; in other cases, like Zeno's paradoxes, because it is false. What we can learn from thinking about the Predestination paradox is that a conclusion can also be unacceptable because, according to the argument in question, it is *grounded in the wrong way*. This gives a new perspective on the paradoxes of material implication – and on instances of Curry's paradox with a true conclusion. E.g., when reasoning about the sentence 'If this sentence is true then snow is white' allows us to infer that snow is white, we want to protest, 'sure, snow is white – but not *solely in virtue of* some facts that concern sentences!'[30] The general lesson is that all hypodoxes should be classified as paradoxical. This is particularly interesting

[27] Peter Eldridge-Smith: 'Paradoxes and Hypodoxes of Time Travel', in: J. Lloyd Jones / P. Campbell / P. Wylie (eds.): *Art and Time* (Melbourne: Australian Scholarly Publishing, 2007), 172–189.

[28] Cf., e.g.: Jonathan Schaffer: 'Grounding in the Image of Causation', *Philosophical Studies* 173.1 (2016), 49–100.

[29] Heinlein op. cit. note 26.

[30] For a fuller version of this argument for the claim that *every* instance of Curry's paradox is paradoxical, cf. Martin Pleitz: 'Curry's Paradox and the Inclosure Schema', in: Pavel Arazim / Michal Dančák (eds.): *The Logica Yearbook 2014* (London: College Publications, 2015), section 4.

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with regard to the hypodoxical cousin of the Liar, the Truth-teller ('This sentence is true'), and certain infinite chains of sentences. Sure, the Truth-teller can consistently have either truth-value. But then it would be true *because* it is true, or false *because* it is false. As far as its grounding structure is concerned, it thus fares no better than the Predestination paradox.

4 The Polarity of Grounding: Of Tortoises and Turtles

One way of understanding what happens in Lewis Carroll's dialogue between Achilles and the Tortoise[31] is that Achilles states that some premises logically entail a certain conclusion, and the Tortoise again and again asks 'Why?', like a young child, but with infinite patience (or malice). A and B already logically entail Z, but Achilles is careless enough to let the Tortoise goad him into adding 'If A and B are true, then Z must be true' as a further premise C. The Tortoise now accepts A, B, and C, but is not convinced that they entail Z, so Achilles adds the appropriate D. And on it goes.

Difficulties of justifying rules of deduction aside, what we can learn here is that the polarity of grounding can be crucial. For it is because A and B entail Z that C is true – not *vice versa*! The whole infinite regress can be stopped – or turned from its head onto its feet and thus transformed into a benign one – by realizing that the ground is in the opposite direction than had been presumed by the philosopher (or reptile) who posed the puzzle originally.

Bradley's regress is another case in point. It can be put like this: With a view to a being F, Bradley asks 'Why?', and answers, like Achilles goaded by the Tortoise, 'Because a intantiates F-ness.' – 'But why?' – 'Because a and *F-ness* instantiate (in a second-level way) instantiation.' – 'But why?' – And on it goes.

Perhaps there is a deep difficulty here. But note how, again, the whole problem dissolves when we allow ourselves to change the polarity of grounding. If we follow David Armstrong and reject the initial presumption that a's being F is in need of explanation in the first place,[32] we arrive at a much less troubling picture:[33]

[31] Lewis Carroll: 'What the Tortoise Said to Achilles', *Mind* 4.14 (1895), 278–280.

[32] David M. Armstrong: *A World of States of Affairs* (Cambridge: Cambridge University Press, 1997), 157–158; cf. Cameron op. cit. note 6, 1–3.

[33] Here we skid over technical details regarding the different *adicity* and (perhaps) different *level* of the various notions of instantiation involved.

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a is $F < a$ instantiates F-ness $<$ a and F-ness instantiate instantiation $< \ldots$

Scott Dixon has recently argued in a very similar way that we should prefer (in his terms) 'upward grounding' to 'downward grounding', but with much more attention to detail and historical predecessors.[34] The lesson about grounding is that while an infinitely-descending grounding chain is bad, its mirror image, an infinitely-ascending grounding chain is unproblematic.

This brings us to what might be called the *Gravitation metaphor*: The words 'ground', 'grounding', 'well-foundedness', 'foundationality', 'fundament' all suggest the same imagery, of building a house or some other static structure in a homogenous gravitation field of perhaps infinite extension. Here, the intuition of foundationality is that although a skyscraper *can* have infinitely many storeys with no end skywards, its basement cannot extend infinitely far down. Or, to use the traditional picture, it makes a big difference whether it is 'turtles all the way down' or 'turtles all the way up'![35] From a grounding point of view, the latter is harmless.[36]

5 The Irreflexivity of Grounding: The Paradoxes of Self-Reference

We come now to the heart of the matter. While the paradoxes discussed so far might be shrugged off as oddities, Russell's paradox has wreaked havoc in its day and the Liar paradox remains dangerous even now, despite all attempts to solve it. What I want to show here is that on top of the contradictions they engender, each one of them is a puzzle of ground.[37]

Each of Fine's puzzles is an argument from plausible assumptions about grounding to the unacceptable conclusion that something is self-grounding. To show that the set theoretic and semantic paradoxes are similar, we need to make explicit some grounding facts, concerning both operational and predicational grounding.

[34] Dixon op. cit. note 14.

[35] Note the similar use of 'upward' and 'downward' in Dixon op. cit. note 14 – despite the slightly different view about foundationality in Dixon op. cit. note 6.

[36] The turtle at ground zero might dissent, and complain (with more justification than Carroll's Tortoise) that it feels *flattened* by the weight on its back.

[37] As mentioned earlier, some of the work reported here has been done independently by Boris Kment.

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For operational grounding, the Truth Schema, $\textsc{True}(\underline{p}) \equiv p$, that plays a crucial role in the Liar paradox, should be split up into two conditionals, each of which needs to be supplemented with a grounding claim. One of these was already mentioned above as a paradigmatic example of operational grounding:[38]

(Truth Schema)	(Grounding and Truth):	
$p \supset \textsc{True}(\underline{p})$	$p < \textsc{True}(\underline{p})$	(GT^{+})
$\neg p \supset \neg\textsc{True}(\underline{p})$	$\neg p < \neg\textsc{True}(\underline{p})$	(GT^{-})

Also for operational grounding, the schema of Naïve Abstraction, $a \in \{x \mid Fx\} \equiv Fa$, that plays a role in Russell's paradox, should be supplemented in a similar way:

(Abstraction)	(Grounding and Abstraction):	
$Fa \supset a \in \{x \mid Fx\}$	$Fa < a \in \{x \mid Fx\}$	(GX^{+})
$\neg Fa \supset \neg a \in \{x \mid Fx\}$	$\neg Fa < \neg a \in \{x \mid Fx\}$	(GX^{-})

In addition, we will need a principle from the (applied) logic of ground:[39]

(Grounding and $\neg\neg$-intro) $\quad \phi < \neg\neg\phi \quad (\mathrm{G}\neg\neg)$

Now, let $\lambda = \underline{\neg\textsc{True}(\underline{\lambda})}$ be our Liar sentence.[40] Then we can reason thus:

(1)	$\neg\textsc{True}(\underline{\lambda}) <$	$\textsc{True}(\underline{\lambda})$	by (GT^{+})
(2)	$\neg\neg\textsc{True}(\underline{\lambda}) <$	$\neg\textsc{True}(\underline{\lambda})$	by (GT^{-})
(3)	$\textsc{True}(\underline{\lambda}) <$	$\neg\neg\textsc{True}(\underline{\lambda})$	by $(\mathrm{G}\neg\neg)$
(4)	$\textsc{True}(\underline{\lambda}) <$	$\neg\textsc{True}(\underline{\lambda})$	(3), (2), (Transitivity)
(5)	$\textsc{True}(\underline{\lambda}) <$	$\textsc{True}(\underline{\lambda})$	(4), (1), (Transitivity)

[38] Recall that we work with a non-factive notion of grounding.

[39] The principle (Grounding and $\neg\neg$-intro) is endorsed in Fine op. cit. note 7, 63. It may well be contested by those who think so classically about negation that they believe that in a logical context, $\neg\neg\phi$ *just is* ϕ. Thanks go to Peter Milne for raising this point. However, as the phrase 'just is' indicates, in that case a stronger principle than $(\mathrm{G}\neg\neg)$ would hold, stating some kind of *grounding equivalence* between $\neg\neg\phi$ and ϕ, thus licensing the intersubstitutability of $\neg\neg\phi$ and ϕ in grounding contexts and providing an alternative way of deriving line (4) of the reasoning above.

[40] We use underlining as an operator that forms a term that refers to the underlined expression. For our reasoning we assume that this sort of term is *robust* in the sense that we can substitute one such term for another such term if they are co-extensional. In particular, to derive line (1) and (2) from (GT^{+}) and (GT^{-}), respectively, we assume that the fact that $\lambda = \underline{\neg\textsc{True}(\underline{\lambda})}$ licenses that we substitute '$\underline{\lambda}$' for '$\underline{\neg\textsc{True}(\underline{\lambda})}$'. As the grounding operator '$<$' creates a hyperintensional context, this is a substantial assumption. I would like to thank Stephan Krämer for alerting me to this.

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The conclusion that $\textsc{True}(\underline{\lambda}) < \textsc{True}(\underline{\lambda})$ contradicts the irreflexivity of operational grounding. With regard to the Russell set $\rho = \{x | x \notin x\}$, an entirely similar proof shows that $\rho \in \rho < \rho \in \rho$, again contradicting irreflexivity.

For predicational grounding, I argue that any meaningful sentence is grounded in the objects it is about. As a Liar sentence refers to itself, this entails that $\lambda \lhd \lambda$, so that the Liar sentence λ is a self-grounding object. With regard to sets, the obvious fact of predicational grounding was already mentioned above; it is that a set is grounded in its elements:[41]

(Grounding and Elementhood) If $x \in y$, then $x \lhd y$.

Note that we can *not* show that $\rho \lhd \rho$ in the same direct way as we have shown that $\lambda \lhd \lambda$, because in the initial stages of the reasoning of Russell's paradox we do not know whether $\rho \in \rho$. But in the presence of (Grounding and Elementhood), requiring the irreflexivity of grounding will entail the irreflexivity of elementhood and thus lead to a set theory where the Russell set does not exist.[42]

We have established in two ways – both for operational and for predicational grounding – that the Liar and Russell's paradox are further puzzles of grounding.

6 The Well-Foundedness of Grounding: Yablo's Paradox

We are not done yet with the set theoretic and semantic paradoxes. These are often called 'paradoxes of self-reference' by those who see a common structure, because each of them involves some circularity, be it of the relation of elementhood or of reference. But the addition of Yabloesque variants of the Liar and Russell's paradox[43] has shown that not the circularity but the more general phenomenon of the non-well-foundedness of the respective relation is the culprit. This will of course reflect in our grounding analysis.

[41] Paseau op. cit. note 6, 169; cf. Fine op. cit. note 16, 4–5.

[42] Because of the irreflexivity of elementhood, every set would satisfy the condition that $x \notin x$. Thus if the Russell set existed, it would be the Universal set. But in this scenario, there cannot be a Universal set, again because of the requirement of irreflexivity.

[43] Stephen Yablo: 'Paradox without Self-Reference', *Analysis* 53 (1993), 251–252; Laurence Goldstein: 'A Yabloesque Paradox in Set Theory', *Analysis* 54.4 (1994), 223–227; cf. Roy T. Cook: *The Yablo Paradox. An Essay on Circularity* (Oxford: Oxford University Press, 2014).

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Let us start by having a look at Yablo's paradox for sentences. Consider the following sequence of sentences, where the quantifier in each sentence ranges over the set of the natural numbers, $\mathbb{N}$ – also known as ω, the first transfinite ordinal:[44]

Y_1 = 'For every $n > 1$, Y_n is false.'
Y_2 = 'For every $n > 2$, Y_n is false.'
Y_3 = 'For every $n > 3$, Y_n is false.'
… …

This sequence is paradoxical, in so far as there is no consistent assignment of truth-values to all the sentences.

In terms of grounding, it constitutes an infinitely descending tree, compatible with the irreflexivity of grounding, but incompatible with its well-foundedness. This is easy to see for predicational grounding. As reference entails grounding, we will have $Y_1 \triangleright Y_2 \triangleright Y_{3\ldots}$,[45] and infinitely many other similar infinitely-descending chains. It is also the case for operational grounding. Although here, we will strictly speaking have to distinguish infinitely many different hypothetical scenarios,[46] it is clear that the principles (GT^+) and (GT^-) will entail that one of the sentences having a certain truth-value will be grounded in all following sentence having certain truth-values. A similar grounding analysis can be given of Yablo's paradox for sets.

In sum, these considerations show that the semantic and set theoretic paradoxes are (further) problems of grounding. While the Liar and Russell's paradox contradict the irreflexivity of grounding, Yablo's paradox contradicts only the stronger requirement of the well-foundedness of grounding. This is also contradicted by Fine's puzzles. The adoption of foundationalism (in the sense of requiring grounding to be well-founded) with regard to the ontologies of meaningful sentences, sets, and facts would therefore solve the semantic and set theoretic paradoxes and Fine's puzzles of ground in a uniform way: It would preclude the existence of the problematic sentences, sets, and facts. Uniformity is desirable in its own right.[47] What is more, this

[44] Here the symbol '>' has it usual meaning 'is greater than', and has nothing to do with grounding.

[45] The symbol '$\triangleright$' is of course the converse of '$\triangleleft$', and can be read as '… is grounded in …'.

[46] Recall that we work with a non-factive notion of grounding.

[47] Graham Priest: 'The Structure of the Paradoxes of Self Reference', *Mind* 103(409) (1994), 25–34.

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would provide us with a single metaphysical motivation for both well-foundedness in set theory, which solves Russell's paradox and other set theoretical paradoxes, and the requirement of semantic (!) groundedness, which solves the Liar paradox[48] and other semantic paradoxes. Thus attention to grounding appears to shed new light on old solutions to paradox.

7 Grounding the Continuum: Zeno's Paradoxes

However, matters will turn out to be more complex with regard to foundationalism. This will become clear after we have given a grounding analysis of some of Zeno's paradoxes.

Many aspects of Zeno's paradoxes have of course been cleared up by the modern mathematical treatment of infinity. However, a few questions would seem to remain. Mathematics models the continuous line as a set of real numbers, each one of which represents a point. But can a continuous line really be made up of unextended points – even if there are strictly more of them than natural numbers? Or do we need to depart from the standard mathematical treatment and think of the continuum as *gunky*, in the sense that none of its parts is a mereological atom? In any case, there is a tension between the set theoretical explication and the venerable Aristotelian intuition that in the case of the continuum, the whole is prior to the parts.

Attention to grounding will help to clear this up. This becomes especially clear with regard to a modern variant of one of Zeno's original paradoxes, the Reverse Dichotomy. Here, Achilles wants to run up to a resting tortoise. But before he can reach his goal, he must reach half way; before that, he must reach a quarter; before that, he must reach an eighth; and so on – so he cannot even start running! If 'before that' is meant here in a temporal way only, the mathematical treatment solves this problem. But there also seems to be a connotation of grounding: How can an interval exist, given that it is partly grounded in its first half, which is partly grounded in the first quarter, and so on? As in the regresses discussed above (those of Carroll and of Bradley), the solution is to realize that the polarity of grounding is different from what we initially thought, guided by our set theoretical explication.

[48] Saul Kripke: 'Outline of a Theory of Truth', *Journal of Philosophy* 72 (1975), 690–716; Tim Maudlin: *Truth and Paradox: Solving the Riddles* (Oxford: Oxford University Press, 2004).

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Consider the following mini-theory about grounding and the continuum.

For $a, b \in \mathbb{R}$, let $[a, b]$ be the set $\{x \in \mathbb{R} \mid a \leq x \leq b\}$. Then the ontology of the continuum is $C = \{[a, b] \mid a \leq b\}$.[49] Its elements represent intervals of the continuous line (for $a < b$) and points (for $a = b$). Now require the following for the polarity of grounding:

(Holism) $\quad (a \leq b \leq c < d) \vee (a < b \leq c \leq d) \Leftrightarrow [a, d] \lhd [b, c]$

Thus a point or interval will be grounded in every interval that it is a proper part of – the parts will be grounded in the whole. This should dispel whatever strangeness still remains in the Reverse Dichotomy. In particular, we need not go gunky: All parts of the continuum exist, including the points, but the points need not bear the weight of explaining the line, because they are explained by it. Progress in the formal treatment of grounding thus makes it possible to explicate the Aristotelian intuition that the continuum is prior to its parts not as a rival alternative to, but as a supplementation of the mathematical model of the continuum.

8 Foundationality Without Well-Foundedness?

So, attention to grounding helps dispel the last remnants of Zenonian paradoxicality, and deepens our understanding of the continuum. However, here we come upon a problem that concerns the notion of foundationality. We have taken Yablo's paradox as establishing that grounding need not only be irreflexive, but well-founded. But according to our theory (Holism), there will be infinitely descending chains of grounding! E.g., $[0.1, 0.9] \rhd [0.01, 0.99] \rhd [0.001, 0.999] \rhd \ldots$, asymptotically approaching but never reaching $[0, 1]$. The problem is more general; and it is independent of the polarity of grounding: If the whole were grounded in its parts, the infinitely-descending chains would simply go in the other direction (with reference to parthood). A similar problem is likely to beset Schaffer's Monism, because he explicitly requires grounding to be well-founded.[50]

So maybe the requirement of well-foundedness is too strong to capture the intuition that grounding is foundational, and we need to search for another condition? This has already been suggested in recent work.[51] Specifically, we may feel some pressure here to

[49] Strengthening this condition to '$a < b$' would lead to a gunky model.

[50] Schaffer op. cit. note 6.

[51] Rabin / Rabern op. cit. note 6; Litland op. cit. note 6; Dixon op. cit. note 6.

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understand foundationality not as well-foundedness or the termination of *all* grounding chains but as the termination of all *downward maximal* grounding chains. For this is the case if grounding goes along with parthood in the continuum – every non-terminating chain can be extended to a downward maximal chain that does terminate (either in the whole, or in a point, depending on the polarity of grounding).[52]

9 An Omega Plus One Paradox

We seem to have found a reason that compels us to weaken the requirement of foundationality to the condition that all downward maximal grounding chains terminate. But this cannot be true. To see this, let us turn back to Yablo's paradox – or rather, to a transfinite variation thereupon.

The basic idea is to add another sentence below all the sentences of Yablo's ω-sequence, thus turning it into an $\omega + 1$-sequence. If the additional sentence is true, every sentence before it must be false, and *can* be false in a consistent way. This might seem to add to the impression that our notion of foundationality as well-foundedness was too strong. However, adding a *false* sentence restores paradoxicality.

Consider the following sequence of sentences, where the quantifier in each sentence ranges over the set $\omega + 1 = \mathbb{N} \cup \{\omega\}$, which is the second transfinite ordinal:[53]

Z_1 = 'For every $n > 1$, Z_n is false.'
Z_2 = 'For every $n > 2$, Z_n is false.'
Z_3 = 'For every $n > 3$, Z_n is false.'
… = …
Z_ω = '$0 = 1$'

This is paradoxical.

Suppose that every Z_i with $i \in \mathbb{N}$ is false. In this situation, Z_1 would be false and, as Z_ω is false, Z_2 and all following sentences would be false, too. But because of what Z_1 says and the Tarskian truth schema, Z_1 would then also be true, a contradiction.

So for some $i \in \mathbb{N}$, Z_i must be true.

Let Z_j be one of these true sentences. Then because of what Z_j says and the Tarskian truth schema, Z_{j+1} and all following sentences are

[52] Given a grounding structure $(O, \lhd)$, a subset C of O is a *chain* if and only if for all $x, y \in C$ either $x \lhd y$ or $x = y$ or $y \lhd x$. A chain C is *downward maximal* if and only if there is no $z \in O$ with $z \notin C$ such that $z \lhd x$ for every $x \in C$ (cf. Dixon op. cit. note 6, 15).

[53] Here, again, the symbol '>' has it usual meaning 'is greater than'.

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false. (This is consistent with the fact that Z_ω is false.) In this situation, Z_{j+1} is false and Z_{j+2} and all following sentences are false, too. But because of what Z_{j+1} says and the Tarskian truth schema, Z_{j+1} must then also be true, a contradiction.[54]

So the $\omega + 1$-sequence of the Z-sentences is paradoxical. And yet, every downward maximal chain terminates – in Z_ω. But the $\omega + 1$-sequence also contains chains that do not terminate, e.g., the ω-sequence of all Z_i with $i \in \mathbb{N}$. Thus the $\omega + 1$-sequence of all the Z-sentences is non-well-founded. This suggests that the requirement of well-foundedness was not too strong, after all.

10 Meta-Grounding: Two Kinds of Transitivity

We are pulled into opposing directions: Considerations about the grounding structure of the continuum suggest that grounding is foundational in the sense that all downward maximal chains terminate, while the Omega Plus One paradox suggests that grounding is foundational in the sense of well-foundedness. How can this apparent incompatibility be resolved?

My guiding intuition is that while terminating but non-well-founded chains are harmless in the case of the continuum, they are dangerous in the case of the Omega Plus One paradox and other Yabloesque structures. So, what is the crucial difference? To learn this, let us compare the following four grounding scenarios:

Scenario 1:

> Snow is white < 'Snow is white' is true < ''Snow is white' is true' is true.
> By (Transitivity): Snow is white < ''Snow is white' is true' is true.

Scenario 2:

> Socrates $\lhd$ {Socrates} $\lhd$ {{Socrates}}.
> By (Transitivity): Socrates $\lhd$ {{Socrates}}.

Scenario 3:[55]

> Object *a* weighs 1kg < *a* weighs between 0.75kg and 1.25kg < Object *a* weighs between 0.5kg and 1.5kg.

[54] I give the proof of the paradoxicality of this $\omega + 1$-sequence, because as far as I know it has not yet been considered.

[55] Adapted from Dixon op. cit. note 6, 11.

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By (Transitivity): Object *a* weighs 1kg < Object *a* weighs between 0.5kg and 1.5kg.

Scenario 4 (from our mini-theory of grounding and the continuum):

[0, 1] ◁ [0.2, 0.8] ◁ [0.4, 0.6].
By (Transitivity): [0, 1] ◁ [0.4, 0.6].

Only in the arguments by (Transitivity) that concern scenarios 1 and 2 something real seems to be happening. In the arguments about scenarios 3 and 4, by contrast, we do not actually need the intermediate step to explain the conclusion. While (Transitivity) holds in all four cases, it does so in a *vacuous* way[56] in scenarios 3 and 4.

As this difference concerns *explanation*, it can be spelled out in terms of grounding – of meta-grounding, to be exact. Let us say that an operational grounding chain $p < q < r$ *collapses* if and only if it is not the case that $(p < q \wedge q < r) < (p < r)$. And let us say that a predicational grounding chain $x \lhd y \lhd r$ *collapses* if and only if it is not the case that $(x \lhd y \wedge y \lhd z) < (x \lhd z)$.[57] Then the grounding chains in scenarios 3 and 4 collapse, but those in scenarios 1 and 2 do not collapse. And only a grounding chain that does not collapse does any significant explanatory work.

Now, the important thing to note is that, with regard to the paradoxes considered so far, it is only where grounding collapses that non-well-founded but terminating chains feel harmless. This suggests a novel, generalized account of foundationality. The basic idea is to require well-foundedness only where grounding does not collapse. To spell this out (in a not yet very formal way), we need an additional notion: For a grounding structure *G*, let the *non-collapsible reduction* of *G* be the structure where all collapsible chains of *G* have been collapsed, but that contains all non-collapsible chains of *G*.[58] Now we can define:

(Foundationality) A grounding structure is *foundational* if and only if its non-collapsible reduction is well-founded.

Thus well-foundedness regains the leading role. Note that in all cases where grounding does *not* collapse, this notion of

[56] Here I re-use a term from Woods op. cit. note 5 with a somewhat different sense.

[57] Friends of predicational grounding can phrase this condition entirely without operational grounding if they have a fact-term-forming operator '[…]' and allow for conjunctive facts, in the following way: '$[x \lhd y \wedge y \lhd z] \lhd [x \lhd z]$'.

[58] This can be made more precise in graph theoretical terms.

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(Foundationality) has our original notion of foundationality as well-foundedness as its limit case. Foundationalism in this sense – given that there is independent warrant for it – would therefore solve all the set theoretic and semantic paradoxes, from the Liar to the Omega Plus One paradox,[59] but is permissive enough to allow for (Holism) about the continuum,[60] as well as for Schaffer's Monism.

11 Ground as a Guide to Paradox?

Let us get back from the more subtle dialectic of the last three sections to our main argument. What we have learned is that all the paradoxes we considered can be construed as puzzles of ground – breaches of some of the intuitive structural requirements on grounding like irreflexivity, asymmetry, and foundationality, which for the most part are uncontroversially endorsed in other contexts of inquiry. What to make of this result? The main moral that we shall draw is that this observation serves to reinforce the view that grounding has these structural features. But before we come to that (in the next, concluding section), we should have a look at whether something might be gained in the converse direction, too: Does this result *also*[61] warrant taking ground as a guide to paradox?

The answer is, in short, that we can indeed learn something substantial from our result for the *diagnosis* of paradox, but will get no clear indication for its *therapy*.

Even before that (and independent of our main result about the structure of grounding), we have learned something about the *anamnesis*, so to say, of the paradoxes. For we have seen (at the end of section 3) how grounding can explain why certain arguments strike us as paradoxical even though they have a true conclusion. This gives us reason to classify more puzzles as paradoxes, and group the Truth-teller and its ilk with the Liar.

What we learn for the *diagnosis* of paradoxes concerns how we should group them together, which will be of some indirect import for how they might be solved. For one clear result is that, contrary

[59] Cf., however, the methodological reflection in section 11.

[60] It will also allow for the converse of (Holism), according to which the whole is grounded in its parts.

[61] To those who ask how there *could* be results in both directions, here is a short answer: Reflective equilibrium. Methodologically, we need not dig deeper on this occasion, because we shall be quite cautious about how much we can learn from ground for paradox.

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to the current orthodoxy, the semantic and set theoretic paradoxes are of the same kind. This pre-theoretically plausible impression has few proponents among current experts, because most of them endorse different kinds of solution for the two groups of paradoxes – most would deny the existence of the paradoxical objects on the set theoretical side, but accept them on the semantic side, trying to rein in the paradoxical consequences by revising truth and logic. But the contrary impression of similarity gets strong support from our observation that these paradoxes give rise to the same grounding puzzles. If we add Graham Priest's plausible principle of uniform solution – 'same kind of paradox, same kind of solution'[62] –, we see that it is indeed desirable to treat the semantic and set theoretic paradoxes in a uniform way.

Beyond that, matters are more murky. For when we turn from diagnosis to therapy, very diverse reactions to our result are possible. The *overly pessimistic* will conclude: Not only naïve truth and sethood, but also our 'naïve' notion of grounding is shown to be incoherent by the paradoxes. Either the notion of grounding is incoherent *tout court* (a lesson Alfred Tarski and Charles Chihara drew from the Liar paradox with regard to truth, at least for natural language), or it must be revised in some gerrymandered fashion (where what we took to be general features of grounding have exceptions in situations that would otherwise be paradoxical). Both variants of the pessimistic reaction strike me as giving up too much of what is intuitive. At the other extreme, the *overly optimistic* may exult: Our intuitions (and maybe some independent reasons) show that grounding is well-founded. Thus the paradoxes are solved! For the paradoxical scenarios simply cannot obtain, on pain of contradicting the foundationality of grounding.[63]

But this cannot be right. It would be no less ad hoc than attempting to solve the Liar paradox by saying: 'Look, if there were Liar sentences, then truth would be contradictory. Hence there cannot be any Liar sentences! Problem solved.' In the vicinity of paradox, *reductio ad absurdum* loses much of its reliability.[64]

[62] Priest op. cit. note 47.

[63] I will admit that I have felt the allure of the overly optimistic reaction myself at one point. And from how I understand it, the proposal that Boris Kment is developing on the basis of the observation that the paradoxes are puzzles of ground is in the same vicinity.

[64] For a fuller version of this consideration about *reductio* and paradox, cf. Martin Pleitz: *Logic, Language, and the Liar Paradox* (Münster: Mentis, 2018), 242–244.

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And yet, we get *some* indication for how to solve the paradoxes, so that a very cautiously optimistic reaction is justified. Although we get no recipe for a specific solution, we are being pointed in a certain direction. For (to anticipate the next section) considering the paradoxes does reinforce our intuition that grounding needs to be foundational in the appropriate sense. Thus attention to grounding not only tells us that a solution to the paradoxes should be uniform, but also that it should be a uniform solution that respects foundationalism.[65]

I should be clear that all this, including the morals drawn from paradox about the structure of grounding, rests on an *assumption of trust* about solving the paradoxes. Maybe contrary to the current mainstream in the field, I expect that eventually every paradox can have a solution worth its name. We may not have found it yet, and it may be incredibly difficult to find, but somewhere out there there must be a solution for every paradox and puzzle![66] Whereas the uninitiated might take this as a truism, and maybe analytic of the notion of a paradox, it is far from clear how many of the current experts would agree. They might rather hold, in contrast, that we have to settle for some kind of trade-off between conflicting desiderata, because a neater resolution is just not to be had.[67]

In order to see why the assumption of trust is crucial, just imagine someone who is thoroughly fatalistic about solving the puzzles and paradoxes of ground. They would take an apparent scenario where some fact grounds itself or some grounding chain never terminates not as an indication that something has gone wrong in our understanding of the situation, but as *data* – data supporting the idea that grounding might be circular or otherwise non-well-founded. In other words, they would view paradoxical reasoning as sound argument. But if this move was legitimate, then most of the intuitive

[65] One promising way to go would be too argue for a uniform non-existence solution, according to which the purported paradoxical objects (hypersets, self-referential expressions, Yablo sequences . . .) do not exist. For this, however, we would need independent justification. With regard to the semantic paradoxes, I have made a detailed proposal in my *Logic, Language, and the Liar Paradox*; op. cit. note 64, 255–629.

[66] If we were inclined to increase the rhetoric, we might vary Hilbert's famous slogan about knowledge in mathematics and proclaim about paradox: 'We must solve! We will solve!'

[67] Cf., e.g., Hannes Leitgeb: 'What Theories of Truth Should be Like (but Cannot be)', *Philosophy Compass* 2.2 (2007), 276–290, with its telling title.

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basis for the characterization of our basic concepts would be called into question.[68]

12 Paradox and Philosophy

What have we learned about the value of paradox to philosophy? Our focus has been on a kind of explanation that is of crucial importance to philosophy, metaphysical grounding. Hence what paradox has taught us about grounding will carry over as something we can learn from paradox for all philosophical inquiry and argument. So let me collect the results of my study of grounding and paradox, and then conclude with a remark about their relevance for philosophy.

We have used paradox as a guide to grounding, and learned a lot. We have considered the grounding structure of the Predestination paradox, the regresses of Carroll and Bradley, Russell's paradox and the Liar, Yablo's paradox, Zeno's paradoxes, and a novel Omega Plus One variant of Yablo's paradox, and we have found reason to endorse these theses:

> We should continue to require that grounding is asymmetrical and irreflexive.
>
> We should change our understanding of the transitivity of grounding, in terms of collapsability.
>
> We should require foundationality in a new, generalized sense, that has well-foundedness (in the mathematical sense) as its limit case.
>
> Meta-grounding is important; and more particularly, we should expect the notion of meta-grounding to be non-trivial in the sense that only some grounding facts are themselves grounded.
>
> We should also be aware that the polarity of grounding matters in cases like those of instantiation and parthood.

[68] In a similar vein, I would see the examples given in Dixon op. cit. note 6 and Litland op. cit. note 5 and op. cit. note 6 for circular or infinitely descending grounding chains not as casting doubt on intuitive structural claims about grounding, but as posing further puzzles that call for a solution. In the case of the infinite disjunctions discussed by Dixon, this would likely concern which infinitary extensions of our languages are legitimate, and for the examples discussed by Litland, which make essential use of truth ascriptions, it would likely need to be in line with the correct solution to the Liar paradox – whatever that might turn out to be.

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Because of its intimate connection to grounding, we thus have learned a lot about the specific non-causal kind of explanation that is ubiquitous in philosophy. Attention to paradox has proven its worth! (In some cases even without looking at any specific proposal for solving the paradox in question.) This value is not diminished by the fact that what we have learned is abstract and at a very general level, concerning not specific questions of what explains what, but the very form that any legitimate explanation can take. Getting back to the gravitational metaphor, it will not be too much to say that we have found important guidelines for anyone who wants to conduct their philosophical work on firm ground.

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