Idealization and Formalism in Bohr's Approach to Quantum Theory

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I reinterpret Bohr's attitude towards quantum mechanical formalism and its empirical content, based on his understanding of the correspondence principle and its approximate applicability. I suggest that Bohr understood complementarity as a limitation imposed by the commutation relations upon the applicability of the idealizations which had grounded the use of the correspondence principle. By discussing this interpretation against the contemporary background of discussions regarding "naïve realism" about operators (as observables), I suggest that a Bohrian view on the empirical content of quantum mechanical operators may provide a middle ground between complete contextualism and an untenable realism about quantum properties.

1. Introduction. Niels Bohr's philosophy of quantum mechanics has been often misinterpreted and seldom understood. I claim that this problem of interpretation has been largely caused by the general oversight of the importance of his correspondence principle to his philosophical views and the subsequent misunderstanding of the meaning and role of the correspondence principle in his approach to quantum theory. For Bohr, the correspondence principle was not merely a heuristic tool for the development of quantum theory; nor was it *primarily* concerned with the asymptotic agreement between classical and quantum theory. Rather, its main purpose within Bohr's empirical approach was to bridge the epistemological gap between empirical phenomena and the unknown atomic structure, which is responsible for the phenomena, by associating classical properties of spectra with atomic properties. Moreover, I argue that Bohr maintained the need for such a tool in his interpretation of quantum mechanics, and this approach begat a view which combines a fundamental realism about quantum mechanical properties with a subtle understanding of their contextual empirical meaning. I will present this picture of Bohr in the context of an issue concerning the meaning of quantum mechanical

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operators which was raised by Goldstein at the 1996 PSA conference. In the published paper from this talk, Daumer, Dürr, Goldstein, and Zanghì et al. (1996) argue against taking too seriously the standard quantum mechanical formalism and cite Bohr as having recognized some of the central theses of their argument. I will compare my interpretation of Bohr with the Bohmian view presented in that paper in order to help clarify Bohr's views on quantum formalism and classical physics. I consider Bohr in this context because I believe it will help make clear how we today have lost sight of some different Bohrian insights.

2. Naïve Realism. Daumer et al. blame what they call "naïve realism about operators" for the "contortions and perversions" of many interpretations of quantum mechanics. They argue from a Bohmian perspective that we ought not take too seriously the idea of "operator-as-observable" because this leads to our treating operators as straightforward representations of actual properties of the system; they claim that it is only once one accepts the idea that quantum systems have properties corresponding to each operator that one has to explain what happens to those properties when they are not being measured. Daumer et al. argue that rather than take operators to represent independent properties of a quantum system, we ought to understand them only as something like emergent properties of an entire experimental arrangement.[1]

Of course, from the Bohmian perspective, the only real property is position. Other so-called "properties" emerge from the wave equation representing the interaction of the particle with the apparatus. That is, these properties are "*merely* in the wave function" (Daumer et al. 1996, 389), although they caution even against this terminology, for they argue that we cannot *simply* make the move of attributing the property to the wave function instead of the particle, since the wave function is a specific "extremely complicated function" intimately tied to a particular experiment, and different experimental arrangements for what we may tend to think of as the same property may have "entirely different functions" (389). Thus, we ought not even think of this operator as a general property—rather, it is a function appropriate *only* for this particular experiment:

1. They take their inspiration from Bell, who argued partly for these reasons that the word "measurement" should be banished from our vocabulary regarding quantum mechanics. Bell himself refers to Bohr as having taught us that properties must be taken to apply to the whole system-plus-apparatus complex and as such only pertain to a particular experimental arrangement. Daumer et al. extend this critique of the meaning of the quantum mechanical formalism and argue that the idea of operator-as-observable is even more pernicious than the idea of "measurement."

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> The fact that the same operator plays a role in different experiments does not imply that these experiments have much else in common, and certainly not that they involve measurements of the same thing. It is thus with detailed experiments, and not with the associated operators, that random variables might reasonably be expected to be associated. (391)

Now, since we ought not treat operators as representing properties, the noncommutativity of incompatible operators does not inform us at all about the nature of the properties of a quantum system. It can only tell us something about experimental arrangements, as Daumer et al. indicate in this remark about the various no-go theorems:

> The state of affairs described by the theorems nonetheless logically implies the obvious conclusion, namely, that the incompatible joint values refer to different, and incompatible, experimental set-ups, just as Bohr told us all along. This mathematical incompatibility of "joint values" thus seems to attain genuine significance only to the extent that we are seduced by naive realism about operators. (391)

That is, we ought not take this formalism as an indication of the incompatibility of the actual or possible properties of a quantum system.

Now, it is clear that the no-go theorems speak against a particular form of naïve realism. I would like to suggest that a truly Bohrian approach offers a middle ground that does not commit these alleged sins of naïve realism but that retains the idea that operators correspond to real properties. It does so by considering the classically-defined properties we attribute to a system to be *idealized descriptions* of the actual properties of that system while in interaction with a particular experimental apparatus. They are idealizations in the sense that they capture the *essential features* of the system's behavior within this particular context, and they can be taken to represent actual properties to whatever degree any idealization can be in general—although many of the properties dealt with in science are in fact idealizations, this has not in general stopped realists from understanding these idealizations as being at least approximately true. The foundation for this type of view can be seen in Bohr's work on quantum theory associated with the correspondence principle.

3. The Correspondence Principle. Bohr's approach to the development of quantum theory was empirically driven. He attempted to use well-defined phenomena as a secure starting point for the development of theory and searched for the most general features that any atomic theory was to have in the face of the novel introduction of quantum effects. Thus Bohr attempted to determine atomic structure from the properties, spectral ra-

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diation without having any real understanding of the quantum mechanism for the emission of that radiation. Bohr sometimes described conclusions reached through this method as *"empirical deductions from the spectral evidence"* ([1924b] 1976, 224).

This approach assumed a sort of epistemological split between empirical phenomena and the atomic structure responsible for those phenomena and it explicitly required a tool to allow one to bridge the gap between phenomena and theory. The correspondence principle was this tool, which together with Bohr's frequency condition acted as bridge principles that let one make claims about atomic structure based on empirical phenomena. For example, one of the simplest and very first of these arguments was Bohr's determination of the hydrogen energy levels from the established phenomenon of the Balmer series; the energies could be read off the Balmer formula $\nu = (K/n''^2 - K/n'^2)$ once one assumed the frequency condition $\Delta E = h\nu$. The correspondence principle extended this form of reasoning by assuming the general terms of the classical relationship between the wave properties of radiation and the oscillations of electrons in the atom which cause the light waves. This "connection between the spectrum and the atomic model of hydrogen" ([1920] 1976, 26) thus allowed one to reach conclusions about the oscillations of electrons in orbits that were independent of any particular model of the atom or the mechanism of the emission of radiation.

Specifically, the correspondence principle connected the frequencies of radiation in an atomic spectrum with the Fourier components of the motion of an electron in an orbit. While classically the motion of an electron in an orbit would simultaneously emit light in the frequencies of each of the Fourier components of its motion, the correspondence principle held that in a transition between stationary states the light emitted (or absorbed) would correspond in its properties to only one of these harmonics:

> On account of the general correspondence *between the spectrum of an atom and the decomposition of its motions into harmonic components*, we are led to compare the radiation emitted during the transition between two stationary states with the radiation which would be emitted by a harmonically oscillating electron on the basis of the classical electrodynamics. ([1920] 1976, 51; emphasis added)

One thus gains access to the motions of the electrons through the comparison the correspondence principle legitimates between the quantum emission of radiation and the classical account of the emission process. This comparison between classical and quantum theory served as the basis for the true role of the correspondence principle: its epistemologically distinct relata were on the one side the empirical generalizations of spectral

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phenomena and on the other the properties of atomic structure responsible for those phenomena.[2] The true role of the correspondence principle was more interesting philosophically than one might think if it were simply a heuristic for developing possible atomic models; indeed, Bohr understood it as the central tool in his empirical approach. He used it to determine from the presence or absence of certain frequencies of the spectra not only what transitions were allowed, but also what type of orbital motion must be present. Bohr also used the correspondence principle for such purposes as determining the shape of the orbit of the outer electron of a sodium atom when no model of the effects on it from internal electrons was at hand, or for associating the empirical Stark effect with a change in the motion of electron orbits. And despite its limited quantitative applicability (it worked best only for states of high quantum number), Bohr maintained that at the very least a qualitative relation between spectrum and motion obtained, and he held out hope that a more general quantitative relationship could be found.

Of course, when the more general quantitative relationship was found in the form of matrix mechanics, it left behind the actual description of electron orbits. Nevertheless, matrix mechanics was, for Bohr, the culmination of this entire approach:

> In brief, the whole apparatus of the quantum mechanics can be regarded as a precise formulation of the tendencies embodied in the correspondence principle. ([1925] 1984, 852)[3]

Matrix mechanics fulfilled the promise of the correspondence principle not merely in its retention of the forms of classical equations, but more fundamentally in the fact that it used *and solidified* the connection the correspondence principle had provided between empirical properties and the atom:

> [Matrix mechanics] operates with manifolds of quantities, which replace the harmonic oscillating components of the motion and symbolize the possibilities of transitions between stationary states in conformity with the correspondence principle. ([1925] 1984, 852)

2. This idea that the correspondence principle was primarily a connection between spectra and orbital motion (and not between classical and quantum theory in some more abstract sense) has not been entirely missed but has been generally overlooked. Two notable exceptions are Darrigol's (1992) historical treatment of Bohr's correspondence principle, and Tomonaga's (1962) textbook which takes seriously Bohr's ideas about correspondence.

3. Born, Heisenberg, and Jordan made a similar claim in their 1926 paper, calling matrix mechanics "an exact formulation of Bohr's correspondence considerations" (Born, Heisenberg, and Jordan 1926; translated in van der Waerden 1968, 322).

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The matrices were built out of elements which "symbolized" the oscillations that the correspondence principle had connected to the light waves. Thus the Fourier components of the electron motion were retained even though they were no longer assumed to be related to any classical orbital motion. In a certain sense, this development both confirmed Bohr's approach and permanently embedded the correspondence role within quantum mechanics. The question for Bohr was then how to understand this new formalism.

4. Idealizations. Bohr's interpretation of the new formalism explicitly concerned the use of classical theory as necessary idealizations. An indication of how he thought this worked can be found in the following passage:

> All our knowledge concerning the internal properties of atoms is derived from experiments on their radiation or collision reactions, such that the interpretation of experimental facts ultimately depends on the abstractions of radiation in free-space, and free material particles. ([1928] 1985, 586)

Bohr had always believed that the correspondence principle—and subsequently the entire quantum theory which he thought was based on this—had a limited and approximate applicability, just as the use of any idealization is limited to cases when the differences between the idealization and what one believes to be strictly true makes little or no difference for the purpose of capturing the behavior of the particular system. For instance, one of the most important postulates of his quantum theory had been the postulate of stationary states. Yet he explicitly admitted that the use of this postulate was limited to situations where one could justify "the assumption that the motion within the stationary states can be described, to a close approximation, by the laws of ordinary electrodynamics, if only one neglects the reaction connected with the emission of radiation" (1924a, 2; 1976 3: 459). Bohr of course admitted that classical electrodynamics is inconsistent with the concept of stationary states, because the reaction on the electron from the emission of light would lead to a decay of the orbit. However, he argued, we can still use it for those cases when *according to classical physics* the change in motion on the electron from the emission of radiation is small compared to the changes in motion from the forces within the atom—but we can use it *only* in these cases. The applicability of the stationary state concept is limited to those cases where we may in the classical account ignore to a first approximation the forces which correspond to the unknown quantum mechanism of radiation:

> In all processes to which quantum theory has been applied, the ra-

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> diation which, according to the classical theory, is emitted during a period (a revolution of the electron) is but small. Hence, it is a very natural assumption that, in calculating the stationary states, we may neglect the influence of the radiation upon the motion. . . . We emphasize, however, that such a procedure represents only an approximation. ([1922] 1977 4: 352)

The use of the correspondence principle was limited by the same condition, among others. Its use was justified only under certain specifiable conditions, and even then was at best only approximately true. If the interaction force were on the same order as the Coulomb force from the nucleus on the electron, we would expect that the immediate effect of the interaction would not be able to be so neglected. Under such conditions we therefore would not be justified in using the stationary state concept and the correspondence principle, because the neglect of that interaction would not serve as a good approximation in the classical case.

We may thus summarize Bohr's attitude towards quantum properties in the old quantum theory in the following terms: he thought that our ability to attribute properties to the atom or to an electron in the atom depends on the degree to which we can approximately describe them with classical physics, and we in turn determine this degree on the basis of classical reasoning. Only when these conditions are met may we appropriately declare empirical phenomena to be evidence of the otherwise hidden properties of the atom; even then, the claims that we do make are only approximate, as our use of these concepts involves idealizations that do not strictly apply to the atom.

5. Complementarity and the Correspondence Connection. Bohr claimed that the quantum mechanical formalism was *by itself* merely abstract symbolism. Thus the way in which he understood matrix mechanics in terms of the correspondence principle suggests the following reading: the formalism receives its empirical content via a type of correspondence connection between empirical phenomena and quantum properties and can be taken as a representation of actual properties only when the relevant symbols are connected to a grounding empirical setting. It is only through this process of attributing the corresponding classical properties to the quantum system that we can give the abstract symbols empirical meaning. Moreover, the meanings of the symbols are not given once and for all but come via the way in which we describe (in classically defined terms) the interaction between the system and the measuring apparatus:

> We must recognize that a measurement can mean nothing else than the unambiguous comparison of some property of the object under

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> investigation with a corresponding property of another system, serving as a measuring instrument, and for which this property is directly determinable according to its definition in everyday language or in the terminology of classical physics. (Bohr [1939] 1996, 19)

Making a measurement requires making a comparison between some property of the quantum object and some property of the measuring system. One makes this comparison by establishing a correspondence between the quantum mechanical symbol representing some property of the object and a classical quantity with which the measuring instrument, classically understood, interacts. This description of the classical behavior of the apparatus establishes what Bohr calls the "external conditions" of the phenomena.

We need this classical reasoning to fix the external empirical conditions which determine the equations governing the symbols and thus also fix the empirical meaning of an operator in a particular setting. This is how we actually do determine quantum expressions, as we in general start with classical equations and then "quantize" them. The basis for this process begins outside the theory proper—the formalism of nonrelativistic quantum mechanics does not on its own provide the recipe for this process, and so the application of the theory to any particular experimental arrangement depends on conceptual apparatus external to the theory—i.e., it depends on classical physics.

In a particular situation, a quantum symbol receives an essentially classical empirical meaning based on its role in the classical equations used for the description of the experiment. Yet we cannot attribute the whole of that particular classical meaning to this symbol in general. Since the quantum properties of the object are not the same as their classical counterparts—at the very least they are not the same in their relations to one another—this attribution of a classically defined property to a quantum system is at best an approximation or idealization; I argue below that it is because of the commutation rules that we know the limitations of this process. Thus while we may be able to establish a correspondence with a classical idealization in a given empirical setting, this process inherently ignores quantum effects, and so requires that we ignore other aspects of the system not directly involved in the measurement. Given the quantum interaction between a system and the measuring instruments, our ability to attribute properties to the system in the context of an experimental arrangement depends on our ability to neglect this interaction and *independently* describe the "external conditions" of the entire quantum phe-

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nomenon.[4] Bohr does not explain *why* we are able to describe the measurement conditions and results in independent terms. Rather, what he does do is examine one way we may understand this process given that we have this ability.

Likewise, Bohr's argument for complementarity does not really tell us *why* consecutive measurements of different properties cannot always be taken jointly as a measurement of both. Bohr sometimes described this fact in terms of the way our descriptions of experimental results involve a "neglect of quantum effects" where the source of these "incontrollable" effects is the so-called "quantum of action" which we express in the quantum postulate, but does not really define or further describe these key phrases. I claim that despite the appearances of his predominately classical explications of complementarity, Bohr's argument for complementarity cannot be based merely on analyses from outside the quantum formalism. Rather, Bohr claimed that the only way the quantum postulate comes into the quantum mechanical formalism is through the commutation relations. Thus the argument for complementarity fundamentally relies on this formalism, as Bohr states in the following passage:

> In fact, the limited commutability of the symbols by which . . . variables are represented in the quantal formalism corresponds to the mutual exclusion of the experimental arrangements required for their unambiguous definition. (1958, 312)

In fact, Bohr clearly understood the uncertainty relations as "a direct consequence of the commutation rules" ([1939] 1996, 18).

Moreover, Bohr claimed that the statistical nature of quantum theory is "a direct consequence of the fact that the commutation rules prevent us to identify at any instant more than a half of the symbols representing the canonical variables with definite values of the corresponding classical quantities" ([1939] 1996, 14–15). I argue—though the case cannot be made in full here—that Bohr thought that it is *only* from the commutation relations that we learn the limitations of our ability to connect the symbols representing quantum properties to the corresponding classical properties we take our instruments to measure. Complementarity is the *consequence* of this situation, and not something new added to the interpretation from the outside.

6. Naïve Realism Again. On this view, certain aspects of the characterization by Daumer et al. of what Bohr taught us are correct: Bohr's claims

4. I believe that Don Howard's (1990, 1994) take on Bohr's understanding of this interaction in terms of entanglement is illuminating and perhaps the right way to understand Bohr.

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are consistent with their caution against naïve realism, as he takes the content an operator gains in an empirical setting to be dependent on the entire experimental arrangement. However, Bohr's view nevertheless accepts that the quantum operators *represent properties of the system* under measurement. When we attribute a property such as position to a system, it is meant to serve as an idealization which (as is the intent of any idealization) captures the relevant features of a system even though it ignores others. It is *not* classical position in its complete sense, because we know, for example, that a limitation of this idealized description is that the reaction of the system to a momentum measurement will not be what it would be if the system retained *all the features* of the classical position. And whereas in one context the momentum operator may have no empirical meaning for us to extend as a property to the system, in another context we may correlate that operator with a property which an experimental arrangement is independently taken to measure. In this latter case we attribute a momentum to the system. Yet the point of this process is to capture as many as possible of the salient features of the quantum system in the experiment even though we cannot account for whatever it is that limits our ability to maintain this coincidence outside the context of this experiment. Even after we have accepted this limitation, we can nonetheless hold that the idealizations we apply in this context are approximately true.

I would like to present a schematic comparison of the Bohmian caution against naïve realism and what I suggest would be Bohr's view on this topic (Table 1). While Bohr does argue that quantum properties are contextual, he would not go so far as to dismiss the formalism and declare that operators do not represent quantum properties at all. Note, too, the different role classical physics plays in the two approaches: while in Bohmian mechanics it plays a sort of metaphysical role in maintaining determinism and in universally setting position as the principal property, for Bohr the role of classical physics is empirical. Classical physics is used in determining the empirical meaning of the quantum symbols by allowing us to establish a connection between quantum properties and the classically defined properties we take our measuring instruments to measure; moreover, it allows us to determine the validity of the idealizations that are used in this process. Classical physics thus helps us determine which of the quantum properties are being "picked out" in any particular experiment.

Quantum effects are evident only in the relations between incompatible idealizations, and it is only through these commutation relations that we know that these properties are incompatible. The formalism is thus fundamentally important to the Bohrian view in a way it would not be if properties were taken to be merely epiphenomenal or operational. In the

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TABLE 1. Attitudes Regarding Naïve Realism

	Bohmian View	Bohrian View
Operators	Not to be understood as properties	Properties whose empirical content depends on external conditions
Properties	Position only	Many, but idealizations and "approximately" true
Contextuality	At best, other "properties" apply only to entire experimental interaction	Idealizations apply only in context of particular experiment
Mutual exclusivity	Applies only to experimental arrangements, not properties	Expresses limitations to the applicability of idealized properties within the context of an experiment
Role of classical physics	Metaphysical in the retention of position and determinism	Empirical in (1) the idealized description of quantum properties and (2) the determination of the approximate applicability of idealizations
Epistemology	Precise position "hidden"	Access to properties is made possible through approximate correspondence

latter cases, the formalism merely expresses facts about experimental arrangements and nothing interesting about quantum properties themselves. Yet because the Bohrian view allows that (with appropriate reasoning) we can attribute properties *into* to the quantum system, the commutation relations express a fact about both the success and limitations of this attribution of idealized properties to quantum systems.

Complementarity thus reflects how this approximate correspondence between quantum and classical properties in one empirical situation relates to the approximate correspondence one can establish in another. The abstract formalism of quantum mechanics expresses these relationships and indeed demonstrates how different sets of classical quantities are incompatible, but to understand complementarity one needs *both* to understand the way in which noncommutativity and incompatibility is expressed in the formalism and to understand how the formalism is connected to empirical situations.

Thus Bohr offers a possible way in which one might accept the contextuality of quantum properties but still maintain some sort of realism. I do not wish to argue that this Bohrian view offers a novel solution to the measurement problem. I would rather suggest a more modest claim: that this view may help us clarify, justify, or modify some contemporary interpretation which does explicitly deal with the measurement problem. For instance, if one believes with Bub (1997) that the Bohrian view is one type of modal interpretation where the determinate properties change with different experimental settings (rather than stay fixed as in the Bohm-

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ian picture), I would suggest that the foundation, justification, or explication of such a resolution of the measurement problem might be given in terms of the above view. For Bohr, at least part of the task of interpretation is to focus on the way in which the correlation between measuring instruments and the "observed" system is established and to determine the degree to which we can understand the empirically derived "observables" in realistic terms.

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