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Temperature relaxation in a parametrically unstable inhomogeneous plasma

V. G. PANCHENKO

Institute for Nuclear Research, Prospect Nauki 47, 252028, Kiev 28, Ukraine

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Abstract. On the basis of kinetic fluctuation theory, the relaxation process between the electron and ion temperatures in a magnetized inhomogeneous plasma is considered when an external pump wave parametrically decays into lower-hybrid and electron drift waves. In the region above the instability threshold, the inverse relaxation time is calculated, and its dependence on the density gradient and the pump-wave intensity and frequency is obtained. It is shown that the presence of the parametric instability leads to a decrease of the relaxation time.

1. Introduction

The temperature relaxation process between electrons and ions in isotropic plasmas were investigated by Spitzer (1962). The problem of temperature relaxation in magnetized plasma was studied by Ichimaru and Rosenbluth (1970). It was found that the relaxation rate between the electron and ion temperatures contains, in addition to the usual Spitzer rate, an anomalous term that diverges logarithmically.

The influence of external electromagnetic radiation on the temperature relaxation process in isotropic plasmas was considered by Puchkov (1975), who found that the presence of parametric instability leads to a decrease in the relaxation time.

Pavlenko and Panchenko (1986) found that a high-frequency electrostatic field with frequency close to the lower-hybrid resonant frequency has a significant influence on the relaxation rate between the electron and ion temperatures in a magnetized uniform plasma. It was shown that the inverse relaxation time contains an additional term due to the pump-wave field under conditions in which a parametric instability develops. This term increases anomalously when the pump-wave amplitude tends to the threshold strength of the electric field. This indicates that linear fluctuation theory is incorrect in this case.

In the present paper on the basis of kinetic fluctuation theory, the relaxation process between the electron and ion temperatures in a magnetized inhomogeneous plasma with density gradient is considered. The case when the external pump wave parametrically excites lower-hybrid and electron drift waves is investigated. In the region above the instability threshold, the inverse relaxation time is calculated and its dependence on the pump-wave intensity and the density gradient is obtained. It is shown that the presence of the parametric instability leads to a decrease in the relaxation time.

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2. The basic equations

Consider an inhomogeneous magnetoactive plasma (with constant magnetic field directed along the z axis) imbedded in an external electric field $\mathbf{E}_0(t) = E_0 \mathbf{y} \cos \omega_0 t$, where we assume the dipole approximation. The pump-wave frequency ω_0 is close to the lower-hybrid frequency region

$$\omega_0 \sim \omega_{lk} \approx \omega_{LH}\left(1+\frac{m_i}{m_e}\cos^2\theta\right)^{1/2}, \qquad \omega_{LH} \approx \omega_{pi}$$

(Porkolab 1974), where $\omega_{p\alpha}$ is the Langmuir frequency of particles of species α, $\alpha = e, i$, and θ is the angle between the direction of wave propagation and the magnetic field. We choose the inhomogeneity to be in the y direction. Then the Maxwellian distribution function corresponding to an exponential density gradient may be written as

$$f_{0\alpha}(p, y) = \frac{n_0}{(2\pi m_\alpha T_\alpha)^{3/2}} \exp\left[-\alpha\left(\frac{p^2}{m_\alpha \Omega_\alpha} - y\right)\right] \exp\left(-\frac{p^2}{2m_\alpha T_\alpha}\right), \tag{1}$$

where

$$p^2 = p_x^2 + p_y^2 + p_z^2,$$

and

$$\alpha = \frac{1}{n_0}\frac{dn_0}{dy}$$

is the plasma inhomogeneity parameter. Here we have assumed the gyro-frequency Ω_α to be homogeneous, and we shall also neglect the temperature gradient.

It is well known (Akhiezer et al. 1978) that the change in the electromagnetic field energy density per unit time (i.e. the power density) for the plasma ion component is defined by the formula

$$W_i = \tfrac{3}{2} n_e \dot{T}_e, \tag{2}$$

where n_α and T_α are the density and the temperature of the particles of species α. On the other hand, we have

$$\dot{T}_e = -\frac{T_e - T_i}{\tau_{ei}} \approx -\frac{T_e}{\tau_{ei}}, \tag{3}$$

where τ_{ei} is the relaxation time between the electron and ion temperatures. Comparing the expressions (2) and (3), we find (Pavlenko et al. 1986)

$$\frac{1}{\tau_{ei}} = -\frac{W_i}{\frac{3}{2} n_e T_e} \tag{4}$$

where the power density is connected with the collision integral by the expression (Silin 1971)

$$W_i = n_i \int \frac{p^2}{2m_i} I_i \, d\mathbf{p} \tag{5}$$

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In (5), I_α is the collision integral, which may be represented in the form

$$I_\alpha = \frac{e_\alpha}{n_\alpha}\,\mathrm{Re}\left\{\int\frac{d\omega}{2\pi}\int\frac{d\mathbf{k}}{(2\pi)^3}\frac{\partial}{\partial\mathbf{p}}\left\langle\,\delta f_\alpha(\omega,\mathbf{k},p)\,\delta\mathbf{E}(\omega,\mathbf{k})\right\rangle\right\}, \tag{6}$$

where $\delta\mathbf{E}$ is the fluctuating electric field, which satisfies the equation

$$\boldsymbol{\nabla}\cdot\delta\mathbf{E} = 4\pi\sum_\alpha e_\alpha\int d\mathbf{p}\,\delta f_\alpha, \tag{7}$$

and δf_α is the fluctuation in the distribution function, which is governed by (Klimontovich 1982)

$$\delta f_\alpha(\mathbf{k},\mathbf{p},t) = -e_\alpha\int d\tau\, e^{i\mathbf{k}\cdot(\mathbf{R}-\mathbf{r})}\,\delta\mathbf{E}(\mathbf{k},t-\tau)\frac{df_{0\alpha}(\mathbf{p})}{d\mathbf{p}}. \tag{8}$$

Using the method of integration along unperturbed orbits for the Fourier component of the distribution-function fluctuation, we have (Pavlenko and Panchenko 1990)

$$\begin{aligned}\delta f_\alpha(\omega,\mathbf{k},\mathbf{p}+\Delta\mathbf{p}) &= \delta f^s_\alpha(\omega,\mathbf{k},\mathbf{p}+\Delta\mathbf{p}) - e_\alpha n_a\sum_{n,m}J_n(a_\alpha)\,J_m(a_\alpha)\\ &\times\int_0^\infty d\tau\exp\left\{i(\omega+n\omega_0-k_\parallel v_\parallel)\,\tau - i\frac{k_\perp v_\perp}{\Omega_\alpha}[\sin(\varphi+\Omega_\alpha\tau)-\sin\varphi]\right\}\\ &\times\left[\frac{k_\parallel}{k}\frac{\partial}{\partial p_\parallel}+\cos(\varphi+\Omega_\alpha\tau)\frac{k_\perp}{k}\frac{\partial}{\partial p_\perp}\right]f_\alpha(p_\perp,p_\parallel)\,\delta\mathbf{E}(\omega+(n-m)\,\omega_0,\mathbf{k}).\end{aligned} \tag{9}$$

Here $\Delta\mathbf{p}$ is the momentum change due to the pump wave, φ is the azimuthal angle in velocity space, J_n is a Bessel function of order n, and $a_\alpha = \mu\Omega_\alpha^2/(\omega_0^2-\Omega_\alpha^2)$, where $\mu = k_\perp E_0 c/\omega_0 B_0 \ll 1$. Substituting (9) into (7), we obtain a system of equations for the fluctuating electric field (Porkolab 1974; Pavlenko et al. 1984):

$$\delta\mathbf{E}(\omega,\mathbf{k}) = \delta\mathbf{E}^s(\omega,\mathbf{k}) + \sum_{m,n}\sum_\alpha I_m(a_\alpha)I_n(a_\alpha)\,\chi^n_\alpha\,\delta\mathbf{E}(\omega+(n-m)\,\omega_0,\mathbf{k}), \tag{10}$$

where $\delta\mathbf{E}^s$ is the fluctuating electric field of the source (i.e. when the interaction between particles is neglected), and χ^n_α is the linear magnetized inhomogeneous plasma susceptibility in the presence of a high-frequency (HF) electric field (Pavlenko and Panchenko 1999):

$$\chi^n_\alpha = \frac{\omega^2_{p\alpha}}{k^2 v^2_{T\alpha}}\left[1-\sum_{r=-\infty}^{\infty}\frac{\omega(1+k_x\alpha T_\alpha/\omega m_\alpha\Omega_\alpha)+n\omega_0}{\omega+n\omega_0-r\Omega_\alpha}A_r(z_\alpha)\,J_+\left(\frac{\omega+n\omega_0-r\Omega_a}{k_\parallel v_{T\alpha}}\right)\right]. \tag{11}$$

Here, $A_r(z_\alpha) = I_r(z_\alpha)\exp(-z_\alpha)$, I_r is the modified Bessel function, $z_\alpha = (k_\perp\rho_\alpha)^2$, and $v_{T\alpha} = (T_\alpha/m_\alpha)^{1/2}$ is the thermal velocity of particles of species α.

We note that, for $n\omega_0\to 0$, the expression (11) determines the linear susceptibility of an inhomogeneous magnetized plasma in the absence of an external electric field, and coincides with the corresponding expression of Ginzburg and Rukhadze (1968).

Using in (6) the expressions (9) and (10), carrying out the statistical

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averaging and the averaging over the period of the HF pump wave ($2\pi/\omega_0$), one can obtain a Fokker–Planck-type collision integral:

$$I_\alpha(p) = \sum_n d\mathbf{k}\,(L_{\alpha n} D_{\alpha n} L_{\alpha n} + L_{\alpha n} A_{\alpha n}) f(p). \tag{12}$$

The quantities $D_{\alpha n}$ and $A_{\alpha n}$ are respectively the diffusion coefficient in velocity space and the dynamic friction coefficient, and the notation $L_{\alpha n}$ means

$$L_{\alpha n} = k_\parallel \frac{\partial}{\partial p_\parallel} + \frac{n\Omega_\alpha}{v}\frac{\partial}{\partial p}$$

3. The influence of parametric instability and plasma inhomogeneity on the temperature relaxation process

It is known that in a plasma subjected to a lower-hybrid pump wave, the excitation of different parametric instabilities is possible (Porkolab 1978). Thus we assume that the pump wave parametrically decays into daughter lower-hybrid and electron drift waves:

$$\omega_0 = \omega_{lk} + \omega_{De}. \tag{13}$$

The frequency and the damping rate of the electron drift wave are (Krall and Trivelpiece, 1973)

$$\omega_{De} = -\frac{k\alpha T_e}{m_e \Omega_e}, \tag{14a}$$

$$\gamma_{De} = -\left(\frac{\pi}{2}\right)^{1/2} \omega_{De}\left[-\frac{\omega_{De} k_x^2 \rho_i^2}{2k_\parallel v_{Te}}\left(1+\frac{T_e}{T_i}\right) + \frac{T_e \omega_{De}}{T_i k_\parallel v_{Ti}}\left(1+\frac{T_i}{T_e}\right)\exp\left(-\frac{\omega_{De}^2}{2k_\parallel^2 v_{Ti}^2}\right)\right], \tag{14b}$$

where ρ_i is the ion Larmor radius.

According to (14b), the interaction between the wave and the electrons is destabilizing, and we have the well-known drift instability, which for a long time was considered to be unavoidable in a finite-size plasma. We note, however, that the ion Landau-damping term will become important in a short device where $k_\parallel$ has to take rather large values. Moreover, in a device with magnetic shear, $k_\parallel$ can take small values only locally, and damping is obtained by convection into regions with larger $k_\parallel$.

In the region above the instability threshold, electric field fluctuations develop, and the plasma becomes turbulent. Under these conditions, super-thermal fluctuations of the field intensity are substantially higher than the level of thermal noise. In view of the anomalous increase in the fluctuations under parametric instability conditions, the velocity-space diffusion coefficient (Pavlenko et al. 1986)

$$D_{\alpha n} = \frac{e_\alpha^2}{16\pi^3 k^2} J_n^2\left(\frac{k_\perp v_\perp}{\Omega_\alpha}\right) \int d\omega \,\langle \delta\mathbf{E}\,\delta\mathbf{E}\rangle_{\omega,\mathbf{k}}\, \delta(\omega - k_\parallel v_\parallel - n\Omega_\alpha) \tag{15}$$

can grow substantially. We therefore assume below that the main contribution to the collision integral (12) for a parametrically unstable plasma is made by the diffusion coefficient (in comparison with the dynamic-friction coefficient).

The correlator $\langle \delta\mathbf{E}\,\delta\mathbf{E}\rangle_{w,\mathbf{k}}$ in (15) has been obtained previously for a spatially

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homogeneous plasma, and is expressed through the correlators of non-interacting particles, which in turn depend on the correlator of source fluctuations, and through the plasma dispersion functions, which depend on the intensity and frequency of the pump wave (Pavlenko and Panchenko 1990). For the inhomogeneous plasma investigated in the present paper the correlator of source fluctuations depends not only on the difference $\mathbf{r}-\mathbf{r}'$ but also on each of the coordinates $\mathbf{r}$ and $\mathbf{r}'$ separately. Since we consider the case of a weakly inhomogeneous plasma, we may assume that the dependence of the correlator of source fluctuations on $\mathbf{r}-\mathbf{r}'$ is stronger than the dependence on the coordinate $\mathbf{r}$, because the former dependence is related to the wavelength of drift oscillations, whereas the latter is related to the characteristic scale length of the plasma inhomogeneity.

Taking into account the dominant role of the diffusion coefficient, we can present the power density in the form (Silin 1971; Pavlenko et al. 1986)

$$W_i = \int \frac{d\mathbf{k}}{(2\pi)^3} \int \frac{d\omega}{2\pi} \frac{\langle \delta\mathbf{E}\, \delta\mathbf{E} \rangle_{\omega,k}}{4\pi} \omega \,\mathrm{Im}\, \chi_i^0, \tag{16}$$

where $\langle \delta\mathbf{E}\, \delta\mathbf{E} \rangle_{\omega,\boldsymbol{k}}$ is the spectral density of the turbulent fluctuations of the electric field in the region above the instability threshold. The correlator $\langle \delta\mathbf{E}\, \delta\mathbf{E} \rangle_{\omega,\boldsymbol{k}}$ near the natural plasma frequencies is obtained from the well-known expression (Pavlenko and Panchenko 1986) in which the plasma eigenmodes are taken to be $\omega_j = \omega_j + i\nu_{\mathrm{eff}}$ ($j = lk, De$ for the lower-hybrid and electron drift waves respectively). Here for the saturation of parametric instability we introduced the effective collision frequency $\nu_{\mathrm{eff}} \approx \gamma_{De} E_0^2 / E_{\mathrm{th}}^2$ (Pavlenko and Panchenko 1999), which defines the additional wave damping due to the scattering of charged particles by turbulent electric field fluctuations; here

$$E_{\mathrm{th}}^2 = \frac{8\omega_0^2 B_0^2}{k_\perp^2 c^2} \frac{\gamma_{De}\gamma_{lk}}{\omega_{De}\omega_{lk}} (k r_{De})^2$$

is the threshold electric field for the decay instability (13).

Substituting the redefined expression for the field-fluctuation spectral density into (16) and integrating with respect to ω and $\mathbf{k}$, we obtain the formula that determines the power density absorbed in the plasma:

$$W_i \approx \frac{1}{16} \frac{e^2 n_e \omega_{lk}(k^*)\, \omega_{De}(k^*)}{m_e \omega_0^4 \gamma_{lk}(k^*)} \frac{E_0^4}{B_0^2} \frac{(k^* c)^2}{(k^* r_{De})^2}, \tag{17}$$

where

$$k^* \approx \frac{(\omega_{lk} - \omega_0)\, m_e \Omega_e}{a T_e}$$

is the wavenumber satisfying the decay condition (13).

Substituting the expression (17) in to (4), we have (Pavlenko and Panchenko 1999):

$$\frac{1}{\tau_{ei}} \approx \frac{1}{24} \frac{e^2 \omega_{lk}^* \omega_{De}^*}{T_e m_e \omega_0^4 \gamma_{lk}^*} \frac{E_0^4}{B_0^2} \frac{(k^* c)^2}{(k^* r_{De})^2}. \tag{18}$$

It can be seen from (18) that the inverse relaxation time grows with increasing density gradient and pump-wave intensity. We note also the sharp dependence of the inverse relaxation time on the pump frequency ($1/\tau_{ei} \sim \omega_0^{-4}$).

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4. Conclusions

Comparing the expression (18) with the value of the inverse relaxation time between the electron and ion temperatures in a magnetized homogeneous plasma in the absence of a pump wave (Ichimaru and Rosenbluth 1970)

$$\frac{1}{\tau_{ei}^{0}} = \frac{2\pi^{1/2} n_e e^4}{m_e m_i}\left(\frac{m_e}{T_e}\right)^{3/2} \ln\left(\frac{\Omega_e^2}{\omega_{pe}^2}\right)\ln\left(\frac{m_i}{m_e}\right), \tag{19}$$

we find

$$\frac{1}{\tau_{ei}}\Big/\frac{1}{\tau_{ei}^{0}} \approx \frac{\pi^{3/2}}{6}\frac{n_e r_{De}^3}{\ln\left(\dfrac{\Omega_e}{\omega_{pe}}\right)\ln\left(\dfrac{m_i}{m_e}\right)}\left(\frac{m_i}{m_e}\right)^{1/2}\frac{\omega_{lk}\,\omega_{De}}{\gamma_{lk}\,\omega_{pi}}\frac{\mu^2}{(kr_{De})^2}\left(\frac{v_E}{v_{Te}}\right)^2, \tag{20}$$

where $v_e = eE_0/m_e\omega_0$ is the electron quiver velocity.

For the characteristic parameters of a tokamak plasma, $n_e r_{De}^3 \approx 10^8$, $\mu \approx 5\times10^{-2}$, $\omega_{lk}/\gamma_{lk} \approx 10^3$, $v_E/v_{Te} \approx 10^{-4}$, $kr_{De} \approx 10^{-1}$, $T_e \approx 1$ keV and $B_0 = 50$ kH, we find that

$$\frac{1}{\tau_{ei}}\Big/\frac{1}{\tau_{ei}^{(0)}} \approx 10^3.$$

Thus the presence of parametric instability in a magnetized inhomogeneous plasma leads to a decrease in the relaxation time, i.e. the rate of HF plasma heating grows.

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