Macroeconomic Dynamics, **26**, 2022, 1021–1034. Printed in the United States of America.
doi:10.1017/S1365100520000462

MONEY AND CREDIT REMIX

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Money and credit are ubiquitous in actual economies, but there is an active theoretical debate on whether they are both necessary if they can both be used in all transactions. Recently, Gu et al. (2016) have shown that money and credit cannot be simultaneously essential and debt limits do not matter for the determination of real allocations in a class of monetary economies. In this paper, we revisit their irrelevance result in a monetary economy based on Lagos and Wright (2005), which exhibits a misallocation of liquidity that is common in search models of money. We show that monetary loans, which naturally require the use of both money and credit, implement Pareto superior allocations in which the size of debt limits matters.

Keywords: Money, Credit, Essentiality, Monetary Policy

1. INTRODUCTION

Money and credit are both commonly used in everyday transactions. However, from a theoretical standpoint, it is challenging to come up with environments in which both instruments can be used in all transactions and, at the same time, are essential as means of payment. The intuition is that money has mostly been conceived as a substitute for missing monitoring and commitment technologies, which are necessary background conditions for the enforcement of credit transactions.[1] Recently, Gu et al. (2016) have taken this point of view to its logical conclusion showing that if traders can perfectly substitute between a spot monetary payment and a delayed credit payment in the purchase of goods, then money and credit are not simultaneously essential. If enforcement works sufficiently well, money is redundant; otherwise, credit is redundant. Moreover, even if money and credit are used together, an increase in debt limits leads to a proportional decrease in the demand and in the value of money, so that the total liquidity available to buyers remains constant in equilibrium.[2]

In this paper, we argue that, even in environments in which money and credit can potentially be used as means of payment in all transactions, it is possible

This paper has been presented at the 2018 Wisconsin Midwest Macro meetings and at the 2018 St. Louis Fed Summer Workshop on Money, Banking, Payments, and Finance. Comments and suggestions by seminar participants are gratefully acknowledged. Remaining errors are ours alone. Address correspondence to: Leo Ferraris, Universita' di Roma, Tor Vergata, via Columbia 2, 00133, Rome, Italy. e-mail: leo.ferraris@uniroma2.it.

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https://doi.org/10.1017/S1365100520000462 Published online by Cambridge University Press

to conceive them, at least for some transactions, as complementary rather than substitute instruments, making them both essential. In fact, we show that this can happen in the same environment considered by Gu et al. (2016), provided one drops their restriction that traders' types are permanent. Precisely, the model we work with is a version of Gu et al. (2016) with random traders' types, as in Lagos and Wright (2005).[3]

This is convenient, since the Lagos and Wright (2005) environment features, as is typically the case in search models of money, some traders who end up holding idle cash, brought in to be used for potential purchasing opportunities that do not always materialize. In such a situation, if there is a technology that allows to enforce limited credit transactions, it can profitably be used to lend idle cash at an interest. We show that bilateral loans of cash lead to an allocation that Pareto dominates the allocation achieved by any feasible alternative arrangement. In particular, the value of money in equilibrium does not decrease proportionally with an increase in debt limits. In fact, the total liquidity available to the buyers turns out to increase with debt limits, thus escaping the irrelevance result of Gu et al. (2016). The assumption of permanent types in Gu et al. (2016) plays a crucial role as it implies that no trader ever holds idle cash during trading sessions, thus preventing monetary loans from playing any useful role.[4] On the other hand, when some traders end up holding some idle cash during a trading session, as it happens when types are random, monetary loans have a potentially beneficial role to play.[5]

These bilateral monetary loans play a similar role as bank credit transactions in Berentsen et al. (2007) and collateralized credit market transactions in Ferraris and Watanabe (2008). However, in these papers, the superiority of monetary credit arrangements relative to alternative credit arrangements, in particular real credit transactions, is not established. Here, we show that the monetary loans, in fact, outperform the feasible alternative arrangements, including those considered by Gu et al. (2016).

Various papers in the literature have carved up a useful role for both money and credit exploiting the presence of extensive margin effects, which basically boils down to assuming that money can be used in all meetings but credit can only be used in some meetings.[6] In contrast, in our case, consistently with the challenge posed by Gu et al. (2016), only intensive margin effects are at work.[7] Hence, extensive margin effects, important as they might be for a better understanding of the interplay between money and credit, are not necessary for this interplay to take place.

Ultimately, our interest in models in which money and credit both have a useful role to play is driven by the aim of understanding the interplay of monetary and credit conditions and the impact of monetary policy. Should one of the two instruments be irrelevant, we could, to a first approximation, simply ignore it and limit attention to the effects of the other or just capture the irrelevant instrument in a reduced form manner as it is often done with money in the New-Keynesian literature. To highlight that the interplay of monetary and credit conditions matters for the determination of allocations, and the effectiveness of policy intervention, in

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the last part of the paper, we briefly consider the implications of conventional and unconventional policies when the debt limit binds at a monetary equilibrium. Interestingly, the Government may be able to help the economy relax its credit constraint both issuing public debt and purchasing private debt. This is reminiscent of the role played by public debt in the credit-constrained economies of Woodford (1990) and Holmstrom and Tirole (1998), and the unconventional credit policies adopted by the Fed in the wake of the 2008 financial crisis.[8]

The paper proceeds as follows. Section 2 presents the model. Section 3 contains the results. Section 4 discusses policy. Section 5 concludes. All proofs are in the Appendix.

2. MODEL

The model is the same as Gu et al. (2016) (henceforth, GMW) but with random buyer and seller types. Time is discrete and continues forever. Each period is divided into two sub-periods, during which two different perishable goods are produced, traded, and consumed by a unit mass of agents in two separate markets, a decentralized (DM) and a centralized (CM) one, opening sequentially. Every period, with probability $\alpha \leq \frac{1}{2}$, the agents become buyers and sellers of the DM good q, traded in bilateral random meetings, or remain inactive with the complementary probability. We describe how transactions occur in the DM in the next section. Consumption of q gives rise to utility $u(q)$ with $u' > 0$, $u'' < 0$ and $u(0) = 0$, while production of q gives rise to cost $c(q)$, with $c' > 0$, $c'' \geq 0$, and $c(0) = 0$. Standard Inada conditions are also assumed. We let q^* denote the unconstrained efficient quantity, that is, $u'(q^*) = c'(q^*)$. In the CM, all the agents consume good x, which is produced using labor, with a linear production function that gives ω units of x per unit of labor. The agents have one unit of time that can be divided between labor and leisure l. Trade of the CM good, which is the numeraire of the economy, is competitive and gives rise to a payoff $U(x, l)$, with $U_x > 0$, $U_l > 0$, $U_{xx} \leq 0$, and $U_{ll} \leq 0$, plus the generalized quasi-linearity assumed by GMW, $\left|U_{xx}U_{ll} - U_{xl}^2\right| = 0$.[9] The agents discount the future at a rate $\beta < 1$ across periods. There is a divisible and durable fiat object, money, m, whose value in numeraire units is ϕ. Its nominal stock, M, is controlled by the government that can alter its net growth rate $\pi > \beta - 1$.

3. MONEY AND CREDIT

First, we consider the same class of trading arrangements analyzed by GMW and show that their irrelevance result extends to the case of random types. Then, we show that the irrelevance result does not hold if one considers a richer class of feasible trading arrangements. Throughout, we limit attention to stationary symmetric equilibria. Moreover, we study equilibrium for given levels of debt and policy.

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3.1. Pure Money

Consider a monetary trading arrangement whereby, within each DM meeting between a buyer and a seller, the seller provides q in exchange for a spot monetary payment whose value in units of numeraire is given by a differentiable, increasing function of the quantity, $v(q)$, with $v(0) = 0$.[10] In the DM, the buyers are subject to the cash constraint $v(q) \leq \phi m$, being their payment limited by the real value of the cash held before entering the meeting, given by m. The budget constraint of an agent during the CM is given by $x + \phi m_{+1} = \omega(1 - l) + \phi \widetilde{m} + T$, where m_{+1} is the amount of money the agents carry into the next period, $\widetilde{m}$ is the amount of money brought into the CM, and T is a lump-sum tax by the Government. The distribution of money holdings at the end of the CM is degenerated, due to the separability of the choice of future holdings from current ones induced by the generalized quasi-linearity in the CM payoff.[11] Let $\zeta \geq 0$ be the multiplier of the cash constraint and $\Lambda > 0$ be the marginal utility of wealth in the CM as in GMW. The first-order condition for q gives $\zeta = \left[w(q) - 1\right] \Lambda$. The envelope condition is $V'(m) = \phi[\alpha(\zeta + \Lambda) + \alpha\Lambda + (1 - 2\alpha)\,\Lambda]$. Inserting ζ into this condition, delaying it one period, and combining it with the optimality condition for m_{+1}, $\phi\Lambda = \beta V'(m_{+1})$, we obtain $\phi = \beta\phi_{+1}\left[\alpha w(q_{+1}) + 1 - \alpha\right]$. Since $M_{+1} = (1 + \pi)\,M$, at a stationary equilibrium, $\frac{\phi_{+1}}{\phi} = \frac{1}{1+\pi}$. Hence, we obtain the Euler equation for the accumulation of money holdings,

$$\frac{1 + \pi}{\beta} = \alpha w(q) + 1 - \alpha, \tag{1}$$

where $w(q) \equiv \frac{u'(q)}{\Lambda v'(q)}$. Henceforth, we assume that $v(q)$ is such that $w'(q) < 0$.[12] The LHS of (1) corresponds to the cost of bringing one unit of real balances into the DM, while the RHS of (1) captures its benefit. Note that buyers use the money to obtain a marginal benefit $w(q)$, while all the other agents hold it barren till the CM. By (1), with $1 + \pi > \beta$, $w(q) > 1$, which implies $\zeta > 0$, that is, that the cash constraint is binding. Hence, by money market clearing, $v(q) = \phi M$. A stationary monetary equilibrium is a time-invariant function q that satisfies (1). Existence of a stationary monetary equilibrium is established under standard Inada conditions on the payoffs; uniqueness is guaranteed by the monotonicity of $w(q)$. The binding cash constraint, together with money market clearing, determines the value of money, once DM output has been determined, $\phi M = v(q)$.

3.2. Pure Credit

Consider, next, a pure credit arrangement, whereby the sellers provide q in the DM in exchange for a promise of delayed payment during the CM, $v(q)$, up to an exogenous debt limit D.[13] The cash constraint is replaced by the repayment constraint $v(q) \leq D$, and the equilibrium must satisfy a complementary slackness condition for such constraint,

$$\left[w(q) - 1\right]\left[D - v(q)\right] = 0. \tag{2}$$

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The first term in square brackets is the multiplier of the repayment constraint and the second the constraint itself, both terms being non-negative. There is no intertemporal optimality condition, since money is not used and all debt is settled in the same period in which it is created. A stationary credit equilibrium is a time-invariant function q that satisfies (2). If $D \geq v(w^{-1}(1))$, the debt limit is sufficiently large and the unique equilibrium is pinned down by $w(q) = 1$. If instead, $D < v(w^{-1}(1))$, the repayment constraint binds and the unique equilibrium is pinned down by $v(q) = D$. Proposition 1 identifies the region of debt limits where the monetary system dominates credit. An arrangement dominates another if the equilibrium day-time allocation, q, induced by the former is strictly larger than the allocation induced by the latter.

PROPOSITION 1. *There exists a positive $\widehat{D}$, such that money dominates credit iff $D < \widehat{D}$.*

Hence, if the debt limit is sufficiently low, money dominates credit, otherwise credit dominates money.

3.3. Credit Irrelevance: Real Credit

Consider now a mixed monetary and credit arrangement whereby the agents can acquire the good partly with money and partly on credit, as in GMW. In this arrangement, within each meeting between a buyer and a seller in the DM, the seller provides q in exchange for a spot payment with cash and a promise of delayed payment during the CM up to the limit D. The cash constraint is replaced by the payment constraint $v(q) \leq D + \phi m$, and the equilibrium is given by an Euler equation for the accumulation of money holdings, identical to (1). The derivation of the equilibrium conditions is identical to the case with pure money, except for $v(q) = \phi M$ which is replaced by $v(q) = D + \phi M$. A stationary real credit equilibrium is a time-invariant function q that satisfies (1). Once the equilibrium output has been determined, the binding payment constraint determines the value of money for any given debt limit. Existence of a real credit equilibrium requires $D < \widehat{D}$, to guarantee that $v(q) - D = \phi M > 0$. Proposition 2 shows that the arrangement with only money and the real credit arrangement that mixes money and credit are equivalent, in the sense that they induce the same day-time allocation.

PROPOSITION 2. *The allocation achieved under real credit is the same as the allocation achieved under money. Moreover, this allocation is independent of the debt limit D.*

Proposition 2 implies that if $D < \widehat{D}$ and the arrangement with a mix of money and credit is an equilibrium, it can always be replaced by the arrangement that uses only money with no impact on the allocation. Since D does not appear in (1), changes in the debt limit simply alter the value of money without any effect on

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the allocation. This is the sense in which the type of credit considered by GMW is irrelevant in monetary economies.

3.4. Credit Relevance: Monetary Credit

We now consider a trading arrangement whereby, within each meeting between a buyer and a seller in the DM, the seller provides an amount d of cash and q of the DM good, in exchange for an immediate cash payment $v(q)$ and a repayment of $r(\phi d)$ in units of numeraire in the CM, where $r(\phi d)$ is a differentiable, increasing function of ϕd, with $r(0) = 0$, which is determined at equilibrium. The enforcement technology is the same as before, with late payment being guaranteed only up to the exogenous debt limit, D. Hence, late payment cannot exceed the debt limit, $r(\phi d) \leq D$. Spot payment, on the other hand, cannot exceed the cash balances held by the buyer at that point, $v(q) \leq \phi d + \phi m$. This arrangement can be interpreted as a monetary credit transaction with a spot commodity transaction paid in cash. The difference with GMW is that the agents lend each other cash rather than the good.[14] Let $\delta \geq 0$ and $\lambda \geq 0$ be the multipliers of the late and spot payment constraints, respectively. The first-order condition for q gives $\lambda = \left[w(q) - 1\right] \Lambda$, and the one for d gives $\delta = \left[w(q) - r'(\phi d)\right] \frac{\Lambda}{r'(\phi d)}$. We consider a scenario in which the sellers have the incentive to lend all their money to buyers and the buyers have the incentive to spend all their money on day-time consumption. With $r'(v(q)/2) > 1$, we have $\lambda > \delta \geq 0$. Hence, the spot payment constraint binds. As a result, since the sellers lend out all their money holdings and the money market clears, we have $\phi d = \phi m = \phi M = v(q)/2$. Substituting these into the late payment constraint, together with δ, we obtain the complementary slackness condition for the late payment constraint,

$$\left[w(q) - r'(v(q)/2)\right]\left[D - r(v(q)/2)\right] = 0. \tag{3}$$

where the first term in square brackets in equation (3) is the shadow value of the constraint and the second term in brackets is the constraint itself, both terms being non-negative. The envelope condition for m is

$$V'(m) = \phi\left[\alpha(\lambda + \Lambda) + \alpha r'(\phi d)\,\Lambda + (1 - 2\alpha)\,\Lambda\right].$$

Inserting λ into it, delaying it one period, and combining it with the optimality condition for the accumulation of cash, we obtain

$$\phi = \beta\phi_{+1}\left[\alpha w(q_{+1}) + \alpha r'(v(q_{+1})/2) + 1 - 2\alpha\right].$$

Since $M_{+1} = (1 + \pi)\,M$, at a stationary equilibrium, $\frac{\phi_{+1}}{\phi} = \frac{1}{1+\pi}$. Hence, we obtain the Euler equation for money holdings,

$$\frac{1 + \pi}{\beta} = \alpha w(q) + \alpha r'(v(q)/2) + 1 - 2\alpha, \tag{4}$$

where, in contrast to (1), the RHS of equation (4) also includes the marginal return that the sellers earn on the cash they lend to the buyers. As pointed out above, we

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focus on situations in which the repayment satisfies $r'(v(q)/2) > 1$, so that the sellers have a strict incentive to lend money and the buyers have a strict incentive to spend it right away. Use (4), to obtain $r'(v(q)/2) = \frac{1+\pi-(1-2\alpha)\beta}{\alpha\beta} - w(q)$, and integrating it with the initial condition $r(0) = 0$, obtain the equilibrium repayment function $r(\cdot)$. We can now rewrite (3) as

$$\left[2\alpha\beta w(q) - 1 - \pi + \beta - 2\alpha\beta\right]\left[D - r(v(q)/2)\right] = 0. \tag{5}$$

A stationary monetary credit equilibrium is a time-invariant q that satisfies (5). Proposition 3 proves existence and uniqueness of a stationary monetary credit equilibrium.

PROPOSITION 3. *There exists $\underline{D}$ such that, if $D > \underline{D}$, a stationary monetary credit equilibrium exists and is unique.*

The bound on the debt limit guarantees that there is the incentive to lend money even when the repayment constraint is binding. Proposition 4 characterizes the monetary credit equilibrium. A monetary credit equilibrium is unconstrained if the repayment constraint is slack, and it is constrained otherwise.

PROPOSITION 4. *There exists a value $\widetilde{D} > \underline{D}$, such that the stationary monetary credit equilibrium is unconstrained iff $D \geq \widetilde{D}$. If the stationary monetary credit equilibrium is unconstrained, q is insensitive to D; if it is constrained, q is increasing in D.*

The equilibrium can be interpreted as follows. The buyers borrow cash from the sellers at an interest, use the cash to acquire the DM good and, then, repay their debt in the CM. If the debt limit is relatively large, it does not bind. In this case, the real interest rate on monetary loans and the allocation do not depend on the debt limit. In contrast, if the debt limit is relatively small, it binds and changes in debt limits affect both the real interest rate on monetary loans and the equilibrium allocation, as an increase in D relaxes the credit constraint, leading to an increase in the quantity produced in a trade meeting, which reduces the liquidity premium of money, and an increase in the real interest rate on monetary loans, which increases its "fundamental" return.

Dominance. Proposition 5 establishes the main result of the paper, that is, that the arrangement with monetary credit dominates all the alternative arrangements examined above, namely pure money, pure credit, and real credit, and that the equilibrium allocation depends on the debt limit.

PROPOSITION 5. *In a non-empty region of parameters, monetary credit dominates all the alternatives: pure money, pure credit, and real credit. Moreover, if $D \in \left(\underline{D}, \widetilde{D}\right)$, the stationary monetary credit equilibrium allocation depends on D.*

Hence, monetary credit can dominate strictly the arrangements with either money or credit alone, as well as the arrangement with real credit. Monetary credit

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helps in two ways. First, relative to pure credit, a buyer obtains extra cash expanding his transaction possibilities. This can be seen by comparing the second term of (3) and (2). This benefit is not specific to monetary credit alone but obtains also under real credit. Second, and more interesting, relative to pure money and real credit, a seller obtains an extra benefit lending out cash that would otherwise remain barren. This can be seen by comparing (4), with $r'(v(q)/2) > 1$, and (1). Finally, the monetary credit equilibrium allocation depends on credit market conditions in an open interval of debt limits, when the credit constraint is binding.

An Example. Consider the following simple example. Payoffs in the DM are $u(q) = \ln q$ and $c(q) = q$, while the payoff in the CM is linear in both x and l. Assume $\alpha = 1/2$ and $\omega = 1$. For simplicity, consider linear payment schedules.[15] Suppose $D < 1$, otherwise pure credit would work too well, giving rise to $q^* = 1$. The purely monetary equilibrium and the real credit equilibrium both give $\widehat{q} = \frac{\beta}{2(1+\pi)-\beta}$, while the pure credit equilibrium gives $\overline{q} = D$. The real credit equilibrium exists if $D < \frac{\beta}{2(1+\pi)-\beta}$, which coincides with the case in which pure money dominates pure credit. The monetary credit equilibrium gives $\widetilde{q} = \frac{\beta}{1+\pi}$ if $D \geq \frac{1}{2}$ and the debt limit does not bind, and $\widetilde{q} = \frac{1+2D}{2}\frac{\beta}{1+\pi}$ if $D < 1/2$ and the debt limit binds. The interest rate on monetary loans in the case where the debt limit does not bind is $\frac{1+\pi-\beta}{\beta}$, and is equal to $\frac{1+\pi-\beta}{\beta} - \frac{1-2D}{1+2D}\frac{1+\pi}{\beta}$ if the debt limit binds. The monetary credit equilibrium only exists if the latter interest rate is non-negative, which requires $D \geq \frac{1}{2}\frac{\beta}{2(1+\pi)-\beta}$. Finally, the value of money is given by $\widehat{\phi}M = \frac{\beta}{2(1+\pi)-\beta} - D$ in the real credit equilibrium, by $\widetilde{\phi}M = \frac{\beta}{2(1+\pi)-\beta}$ in the unconstrained monetary equilibrium, and by $\widetilde{\phi}M = \frac{1+2D}{4}\frac{\beta}{1+\pi}$ in the constrained monetary credit equilibrium. Hence, the value of money is strictly higher in the monetary credit equilibrium. Moreover, while the value of money decreases one-for-one with the debt limit under real credit, it does not vary with the debt limit in the unconstrained monetary credit equilibrium, and it increases with the debt limit in the constrained monetary credit equilibrium.

4. POLICY INTERVENTION

The model generates a region of parameters in which the debt limit matters for the allocation. Next, we ask whether, in such a region, monetary policy may be able to ease tight credit market conditions. We consider three types of intervention: a conventional monetary expansion, an unconventional private debt purchase, and a public bond issuance.

4.1. Monetary Expansion

Expanding the money stock through lump-sum transfers always reduces the amount traded of the DM good at the monetary credit equilibrium, since it erodes the value of money which is the trading instrument in the DM. This may change

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in case of an endogenous debt limit if the punishment for non-repayment is more limited and allows the agents to keep trading with money after having been excluded from credit markets. In this case, lowering the value of money may have a positive role since it would make the off-equilibrium punishment tougher, which in turn would increase the amount traded on-equilibrium.[16] Except for such off-the-equilibrium-path effects, increasing π is not beneficial. This can be easily seen in the example above, where both in the unconstrained and constrained regions, equilibrium output is decreasing in π.

4.2. Unconventional Intervention

Unconventional intervention when credit markets are tight may, instead, be beneficial. To see this point, consider the example presented above. Suppose the debt limit is such that the economy is constrained, $D < \frac{1}{2}$. Suppose also that the Government uses the extra stock of money created every period, $M_{+1} - M = \pi M$, to purchase private debt from the agents during the CM. The repayment constraint is $r(\phi d) \leq D + \pi\phi M$. The intervention makes the repayment constraint slack if $D > \frac{1}{2}\frac{1+\pi(1-\beta)}{1+\pi}$. For values of $D \in \left(\frac{1-\beta}{2}, \frac{1}{2}\right)$ and $\pi \in \left(0, \frac{1-2D}{1+2D}\right)$, the equilibrium DM output is larger with unconventional intervention than with no intervention and conventional intervention. Hence, a moderate purchase of private debt may help the economy relax its credit constraints when debt limits are tight with a positive effect on output, which may be particularly relevant during financial crises.

4.3. Public Bonds

The Government may also be able to help the economy issuing public bonds that cannot be traded in exchange for goods but can be used as collateral to obtain loans. The bonds need to be illiquid in the DM market for the purpose of acquiring the good in order to make the collateral credible, otherwise the guarantee of repayment could be voided. Liquid bonds would simply be another way to increase liquidity, which cannot improve the allocation.[17] Consider the same example above. Suppose the Government issues a stock B of one period bonds that are sold in the CM at a price ρ in units of money and promise repayment of cash during the following CM one-for-one, which can be used as collateral to guarantee the repayment of loans. If a borrower fails to repay, the creditor has the right to seize the posted collateral. In this case, the buyers face a laxer constraint, $r(\phi d) \leq D + \phi b$, since the repayment is guaranteed also by the value of their bond holdings, b. Let the bonds to money ratio be $z \equiv B/M$. The equilibrium with bonds cannot improve upon the equilibrium without bonds if the economy is unconstrained, since the only role of bonds is to relax the borrowing constraint. Suppose $z > \frac{1}{2\beta-1}$ so as to make the repayment constraint slack. The multiplier of the constraint gives $q = \frac{1}{1+r}$. By arbitrage, the price of the bond is

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the inverse of the gross interest rate on the bond, $\rho = \frac{1}{1+r}$. From the Euler condition, $q = \frac{\beta}{1+z(1-\beta)} = \rho$ and $r = \frac{(1-\beta)(1+z)}{\beta}$. At equilibrium, an increase in the bonds to money ratio decreases the price of the bond and increases the interest rate. This is reminiscent of the liquidity effect of open market operations found by Lucas (1990) and others. The allocation with bonds is larger than the monetary credit-constrained allocation. Hence, if the Government has committed to a certain rate of inflation, and the economy is credit constrained, it can help boost output issuing public bonds that can be used as collateral for loans.

5. CONCLUSION

GMW have argued that money and credit cannot matter at the same time: if credit is easy, money is irrelevant; if credit is tight, money is sufficient. The rationale is that if money and debt are perfect substitutes as means of payment, an increase in debt limits leads to an endogenous response of real balances, in such a way that total liquidity remains constant. We consider a small variation of their environment in which money and debt are still potentially perfect substitutes as means of payment, but the existence of idle balances in the decentralized market allows agents to exchange monetary loans, using the available enforcement technology. We obtain that money and credit matter at the same time, and total liquidity are increasing in the debt limit. The reason is that, although money and debt may be perfect substitutes as means of payment, they can complement each other as means of correcting a misallocation of liquidity relative to best use, which is typical of monetary search economies.

NOTES

1. This is the idea that money is a substitute for missing memory, formalized by Kochelakota (1998).

2. Gu et al. (2016) belong to the so-called New-Monetarist literature, but the challenge arises in the New-Keynesian literature as well, although in reverse, since it is money this time that is seen as a dominated asset which is bound to disappear as a result of the historical process of improvements in credit and financial transactions: "the only natural limit to this process is an ideal state of frictionless financial markets, in which there is no positive demand for the monetary base at all," Woodford (1990), p. 174. See Lagos and Zhang (2019) for a rebuttal of this argument.

3. In footnote 5 and 6 of Gu et al. (2016), it is claimed that the credit irrelevance result extends from the Rocheteau and Wright (2005) model with permanent buyer and seller types to the Lagos and Wright (2005) model with random types. The present paper shows that this is not the case.

4. Notice that, in Gu et al. (2016), there are some randomly selected agents that remain inactive every period, who end up holding idle cash. However, their cash balances cannot be put to any profitable use without violating the assumption that these agents remain inactive throughout the trading session.

5. The presence of random types is not the only way to capture the complementarity between money and credit. The key element is the existence of idle liquidity and the possibility of reshuffling it using the monitoring technology that makes debt feasible.

6. A variant of this assumption goes back to Lucas and Stokey (1987), which imposes that only credit can be used in a subset meetings and only money can be used in the complementary subset.

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Examples of papers within this tradition are Gomis-Porqueras and Sanches (2013), Lotz and Zhang (2016), Araujo and Hu (2018), and Lagos and Zhang (2019).

7. Gu et al. (2016) discuss some extensions of their framework in which the credit irrelevance fails in the presence of sufficient agents' heterogeneity. Since in our model the agents are ex-ante identical, our result does not depend on heterogeneity.

8. see Bernanke (2009).

9. Assumption 1 in GMW. Under this assumption, all agents take the same amount of real balances out of the CM, irrespective of whether their wealth and payoffs in the CM are linear in wealth.

10. In more detail, terms of trade in the DM are given by the quantity q produced by the seller and the payment p of the buyer. GMW consider trading arrangements in which the seller produces q^* and the buyer pays $p^* = v(q^*)$ as long as $p^* \leq L$, where L is the total liquidity of the buyer; otherwise, the buyer uses all the liquidity and gets $q = v^{-1}(L) < q^*$. GMW show (Lemma 3 therein) that this trading arrangement includes a broad class of trading mechanisms, including perfect competition, and standard bargaining solutions.

11. Lemmas 1 and 2 in GMW.

12. This assumption is trivially satisfied under perfect competition and standard bargaining solutions.

13. As in the first part of the GMW paper, we assume an exogenous debt limit, which can be endogenized, as in the second part of the GMW paper, assuming that an agent who fails to repay is detected with some probability and excluded from any future possibility of trade. In the Appendix, we work out a simple example along these lines.

14. To stay as close as possible to the GMW setting, we have chosen to formulate the credit transaction in the match between the borrower and the lender. There are other ways of modeling monetary loans. For example, Berentsen et al. (2007) introduce banks that open after the CM and after agents have known their types. In their setting, banks reallocate money from sellers to lenders, and sellers receive an interest on their deposits. The important feature is that agents anticipate that cash will not remain idle, that is, it will be used either to purchase goods or to make loans.

15. This corresponds to a competitive case, in which buyers and sellers meet bilaterally but can be rematched to alternative trading partners at no cost in the same trading session.

16. This disciplining effect of inflation is analyzed by Berentsen et al. (2007), Gomis-Porqueras and Sanches (2013), and Lotz and Zhang (2017), among others.

17. As in Kochelakota (2003).

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A: APPENDIX

Proof of Proposition 1. Define $\Pi \equiv \frac{1+\pi-\beta}{\alpha\beta} > 0$. Let $D < v\left(w^{-1}(1)\right)$. The equilibrium allocation under pure money, given by $\widehat{q} = w^{-1}(1+\Pi)$, is larger than the equilibrium allocation under pure credit, given by $\overline{q} = v^{-1}(D)$, iff $D < v\left(w^{-1}(1+\Pi)\right) \equiv \widehat{D}$. ■

Proof of Proposition 2. With real credit, $q = w^{-1}(1+\Pi) = \widehat{q}$, which is independent of D. ■

Proof of Proposition 3. Let $\Phi(q,\pi,D) \equiv D - r(v(q)/2)$ and $\Gamma(q,\pi) \equiv w(q) - 1 - \Pi/2$, and rewrite (5) as $\Gamma(q,\pi)\,\Phi(q,\pi,D) = 0$. When $\Gamma(q,\pi) = 0$, existence of a monetary credit equilibrium is established under standard Inada conditions; uniqueness is guaranteed by the monotonicity of $w(\cdot)$. When $\Phi(q,\pi,D) = 0$, existence of a monetary credit equilibrium is established for $v(\infty)$ sufficiently large; uniqueness is guaranteed by the monotonicity of $w(\cdot)$. Let $\widetilde{q}$ be the value that satisfies $\Gamma(\widetilde{q},\pi)\,\Phi(\widetilde{q},\pi,D) = 0$. Next, we check $r'(v(q)/2) > 1$. When $\Gamma(\widetilde{q},\pi) = 0$, $r'(v(q)/2) > 1$ for any $\pi > \beta - 1$. When $\Phi(\widetilde{q},\pi,D) = 0$, define $\Psi(D) \equiv 1 + \Pi - w(\widetilde{q}(D))$, a continuous function in D. By definition, $r'(v(q)/2) > 1 \Leftrightarrow \Psi(D) > 0$. We have $\Psi(0) < 0$ and $\Psi(\infty) > 0$ and $\Psi'(D) > 0$. Hence, there exists a unique value $\underline{D}$ s.t. $\Psi\left(\underline{D}\right) = 0$, by the intermediate value theorem. If $D > \underline{D}$, $\Psi(D) > 0$. Therefore, $\widetilde{q}$ is the monetary credit equilibrium. ■

Proof of Proposition 4. Suppose $\Gamma(\widetilde{q},\pi) = 0 \Leftrightarrow \widetilde{q} = w^{-1}(1+\Pi/2)$, which we can substitute into $\Phi(q,\pi,D)$, obtaining $\Phi(\widetilde{q},\pi,D) \geq 0$ iff $D \geq r(v(\widetilde{q})/2) \equiv \widetilde{D}$. Suppose $D \geq \widetilde{D}$. If $\widetilde{D} \geq r(v(q)/2)$, then $\Phi(\widetilde{q},\pi,D) \geq 0$. The inequality in the premise is verified, since $\Gamma(q,\pi) \geq 0 \Leftrightarrow q \leq w^{-1}(1+\Pi/2)$ at any monetary credit equilibrium, and $v(\cdot)$ and $r(\cdot)$ are increasing. By $r'(v(q)/2) > 1$, $\widetilde{D} > \underline{D}$. When $\Gamma(\widetilde{q},\pi) = 0$, we have $\widetilde{q} = w^{-1}(1+\Pi/2)$, which is independent of D. When $\Phi(\widetilde{q},\pi,D) = 0$, we have $D = r(v(\widetilde{q})/2,\pi)$, hence $\widetilde{q}$ depends on D, with $d\widetilde{q}/dD > 0$, since both $r(\cdot)$ and $v(\cdot)$ are increasing and independent of D. ■

Proof of Proposition 5. Suppose $\Gamma(\widetilde{q},\pi) = 0$. The allocation achieved by pure money, $\widehat{q} = w^{-1}(1+\Pi) < \widetilde{q} = w^{-1}(1+\Pi/2)$, since $w(\cdot)$ is decreasing. The allocation achieved by pure credit $\overline{q} = v^{-1}(D) < \widetilde{q} = w^{-1}(1+\Pi/2)$ which holds for $\pi < \overline{\pi} \equiv \beta(1+2\alpha) - 1$, which is positive for $\beta > (1+2\alpha)^{-1}$. Suppose $\Phi(\widetilde{q},\pi,D) = 0$. Define $\Theta(D) \equiv D - v(\widetilde{q}(D))$, continuous and monotonically increasing in D. The allocation under the monetary credit equilibrium dominates the allocation under pure credit strictly iff $\Theta(D) < 0$. We have

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$\Theta(0) < 0$ and $\Theta(\infty) > 0$. Hence, there exists a unique value $\overline{D}$ s.t. $\Theta(\overline{D}) = 0$, by the intermediate value theorem. If $D < \overline{D}$, the monetary credit equilibrium dominates pure credit. By the properties of $\Phi(q, \pi, D)$, we have $D < \underline{D} \Leftrightarrow \Phi(\widehat{q}, \pi, D) > 0$, which implies $\widehat{q} < \widetilde{q}$. The two cutoff values for D are compatible since $\Psi(D) > \Theta(D)$ for all D and both functions are monotonically increasing in D. Hence, monetary credit dominates both pure money and pure credit if $\pi < \overline{\pi}$ and $D \in (\underline{D}, \overline{D})$. By Proposition 4, $d\widetilde{q}/dD > 0$, for $D \in \left(\underline{D}, \widetilde{D}\right)$. ■

Endogenizing the Debt Limit. Consider the situation with monetary loans in the text. Following GMW, suppose there is a monitoring technology that can detect whether an agent honors his debts with probability μ. For simplicity, assume that if an agent fails to repay his debts and is not discovered, nothing happens, while if discovered, the amount of resources corresponding to the unpaid debt is taken away immediately and the agents are excluded from any future trade possibilities. In this case, the incentive constraint for the repayment of debt is $\mu u(q) \geq r(v(q)/2)$, with a multiplier equal to $\frac{w(q) - r'(v(q)/2)}{r'(v(q)/2) - \mu w(q)}$, with $r'(v(q)/2) > \max\{\mu w(q), 1\}$. The Euler condition remains the same as before. The equilibrium system has two equations, (4) in the text and $\left[w(q) - r'(v(q)/2)\right]\left[\mu u(q) - r(v(q)/2)\right] = 0$. Using (4), we can reduce the equilibrium system to just one complementary slackness condition, as before. For the sake of concreteness, we work within the context of a simple example. Payoffs in the DM are $u(q) = 2\sqrt{q}$ and $c(q) = q$, while the payoff in the CM is linear in both x and l. The other parameters are $\alpha = 1/2$ and $\omega = 1$. As before, consider linear payment schedules. For $\mu \in \left(\frac{1}{4}\frac{\beta}{2(1+\pi)-\beta}, \frac{1}{4}\right)$, equilibrium DM output is $q = \left(\frac{1+4\mu}{2}\frac{\beta}{1+\pi}\right)^2$ and the gross interest rate on monetary loans is $\frac{4\mu}{1+4\mu}\frac{2(1+\pi)}{\beta}$, which is larger than both 1 and $\mu w(q) = \frac{\mu}{1+4\mu}\frac{2(1+\pi)}{\beta}$. If $\mu < \frac{1}{2}\frac{\beta}{2(1+\pi)-\beta}$, this equilibrium allocation dominates the allocations achieved with only money and real credit, which are the same as before and independent of μ, and the allocation with pure credit, which is equal to $4\mu^2$, provided $\mu < \frac{1}{2}$. Since DM output depends on the enforcement parameter μ at equilibrium, we conclude that credit conditions matter for the determination of the allocations. Hence, there is a non-empty interval of $\mu \in \left(\frac{1}{4}\frac{\beta}{2(1+\pi)-\beta}, \frac{1}{2}\frac{\beta}{2(1+\pi)-\beta}\right)$, in which the arrangement with monetary credit dominates the alternatives and credit conditions matter.

Alternative Arrangements. In the text, we have shown one feasible arrangement in which credit is not irrelevant. There may exist other feasible arrangements, also using both money and credit, that are not irrelevant. Consider a general trading arrangement, with a (possibly vector valued) payment schedule $p(q, \phi d)$, combining early and late payments, and a general constraint set $F(D, m)$. The monetary credit arrangement is subsumed under this formulation. Any system that gives rise to a constraint of the form $v(q) + r(\phi d) - \phi d \leq D + \phi m$ leads to the equilibrium condition (1), hence to the same allocation as pure money, which, in turn, is equivalent to real credit. A system that gives rise to a constraint of the form $f(q, d) - \phi d \leq D + \phi m$ leads to the binding constraint $f(q, \phi M) = D + 2\phi M$ that relates implicitly $\phi M = g(q, D)$, together with the Euler condition $u'(q) = (1 + \Pi)\Lambda f_1(q, g(q; D))$, where $f_1(q, g(q, D))$ is the first derivative of the function $f(\cdot, \cdot)$ wrt the first argument. Provided the cross derivative $f_{12}(\cdot, \cdot) \neq 0$, conditions can be found on $f(\cdot, \cdot)$ so that such a system dominates the alternative cases, whose equilibrium condition is $u'(q) = (1 + \Pi)\Lambda v'(q)$. Moreover, the equilibrium allocation depends on D, through the function $g(q, D)$. Hence, any system in which: *i.* the cross effects between q

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and ϕd in $p(q, \phi d)$ are absent and *ii.* the debt limit and money holdings are substituted one-for-one so that $F(D, \phi m) = D + \phi m$ attains the same allocation as the pure money or real credit arrangement above. Systems that violate either *i.* or *ii.* have the potential to escape the irrelevance result of GMW. The monetary credit arrangement considered in the text does so violating the latter, but even a system that violates only the former may equally escape the irrelevance result if the cross effects between q and ϕd are present. In sum, the question boils down to whether money and credit are conceived as complements or substitutes.

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