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R&D SUBSIDIES, INTERNATIONAL KNOWLEDGE DIFFUSION, AND FULLY ENDOGENOUS PRODUCTIVITY GROWTH

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This paper investigates the effects of R&D subsidies on aggregate product variety and endogenous productivity growth without scale effects. In a two-country model with imperfect knowledge diffusion, the larger country has a greater share of firms with higher productivity levels. The concentration of relatively productive firms increases knowledge flows between firms, causing an increase in firm-level employment in innovation. Accordingly, the aggregate growth rate is higher when counties are asymmetric than when they are similar in size. The larger scale of firm-level innovation activity reduces market entry, however, and aggregate product variety falls. In this framework, national R&D subsidies have positive effects on the industry share, relative productivity, and wage rate of the implementing country. If the smaller country introduces an R&D subsidy, aggregate product variety rises and productivity growth falls. If the larger country introduces an R&D subsidy, productivity growth rises, but aggregate product variety may rise or fall.

Keywords: R&D Subsidy, Knowledge Diffusion, Productivity Growth, Scale Effect

1. INTRODUCTION

The discovery of novel ideas in the form of new products and production processes is broadly recognized as an essential driver of economic growth. Thus, policy makers are perpetually looking for opportunities to accelerate growth by stimulating research and development (R&D). In response, a diverse literature investigating the effects of economic policy designed to promote R&D activity has been developed with the objective of providing guidelines for policy makers. In this paper, we

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contribute to this literature with a study of the effects of national R&D subsidies on economic growth in the presence of imperfect knowledge diffusion.

A key theme of the innovation-based endogenous growth literature has been the role of technical knowledge diffusion among firms, and between regions and countries. Indeed, it is the nonrival nature of knowledge as an input in R&D that allows perpetual economic growth when the knowledge created by current innovation activity generates an intertemporal knowledge spillover that raises the labor productivity of all future R&D efforts [Romer (1990)]. Given then the central role of knowledge diffusion in growth theory, a large body of empirical work has set out to verify the existence and measure the extent of knowledge spillovers. Although there is now convincing evidence that knowledge spillovers do occur, their strength appears to diminish significantly with distance [Jaffe et al. (1993); Mancusi (2008); and Coe et al. (2009)].[1] It is this localized aspect of knowledge diffusion that we focus on in our examination of the effects of national R&D subsidies.

Early comparative analysis of the policy implications of global versus local knowledge spillovers can be found in Rivera-Batiz and Romer (1991) and Grossman and Helpman (1991, Chap. 8). In these studies, an improvement in the flow of ideas between countries, as a result of greater economic integration, raises labor productivity in R&D, thereby bolstering the rate of innovation. Within this context, we find that an exogenous reduction in innovation costs induces a reallocation of resources from production to innovation activity, leading us to conclude that R&D subsidies affect economic growth positively, with the strength of this effect increasing in the level of knowledge diffusion.

One potential concern with these results, however, is that they are inherently tied to a scale effect: an increase in population size raises total employment in innovation activity, causing an increase in the rate of economic growth. This scale effect is a common feature of first-generation growth models. There is now a host of empirical evidence, however, that generally rejects the existence of scale effects [Jones (1995b); Dinopoulus and Thompson (1999); Barro and Sala-i-Martin (2004); Laincz and Peretto (2006)], indicating that an appraisal of national R&D policy within a framework that corrects for scale effects and allows for imperfect knowledge diffusion would be a valuable exercise. Along these lines, Minniti and Parello (2011) introduce diminishing technical opportunities into the intertemporal spillover associated with knowledge accumulation in a two-country framework. This setting results in a second-generation growth model [Jones (1995a); Kortum (1997); Segerstrom (1998)], within which the rate of innovation depends solely on the rate of population growth and the rate of diminishing technical opportunities, thus leaving no role for either imperfect knowledge diffusion or R&D policy in the promotion of economic growth. In contrast, we develop a fully endogenous growth framework [Smulders and van de Klundert (1995); Peretto (1996); Aghion and Howitt (1998); Dinopoulus and Thompson (1998); Young (1998); Gil et al. (2013); Sedgley and Elmslie (2013)] that accounts for scale effects while preserving a role for imperfect knowledge spillovers and R&D policy.[2] As there is

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strong empirical evidence supporting the validity of the fully endogenous growth approach over the semiendogenous growth approach [Zachariadis (2003, 2004); Laincz and Peretto (2006); Ha and Howitt (2007); Madsen (2008, 2010); Madsen et al. (2010a, 2010b)], this paper contributes to the economic growth literature with a reexamination of the implications for R&D policy when knowledge diffusion is incomplete, within an empirically robust framework.

In particular, extending the in-house process innovation framework [Smulders and van de Klundert (1995); Peretto (1996)], we develop a two-country model, in which monopolistically competitive firms costlessly differentiate their products, but incur R&D costs each period as they employ researchers in process innovation with the aim of lower production costs and increasing firm value. Each firm's labor productivity in process innovation is determined by the weighted average productivity of the production technologies observable by the researchers that it employs, with domestically observed technologies receiving a greater weight to reflect the imperfect nature of knowledge diffusion between countries. With free market entry and exit, profits are driven to zero, and production scale is linked directly to R&D scale at the firm level. In long-run equilibrium, productivity growth rates equalize across countries and all firms employ the same number of researchers in innovation. Accordingly, the number of firms producing in each country depends on population size. Because innovation costs must equalize across countries, however, a rise in the share of firms producing in a given country is paralleled by an increase in the country's relative productivity. Thus, we find that the country with the larger population attracts a larger share of firms, each with a higher level of productivity than the firms producing in the smaller country, a result that is consistent with empirical evidence [Melo et al. (2009)].

The positive relationship between relative productivity and the share of firms producing in a given country has important implications for overall product variety and the aggregate rate of productivity growth. Specifically, the higher concentration of relatively productive firms in the larger country generates a higher productivity level in flows of technical knowledge between firms, thereby raising R&D productivity and inducing a higher level of employment in process innovation. With both labor productivity and labor employment rising at the firm level, we find that an economy with asymmetric market sizes will have a higher rate of productivity growth than an economy with symmetric market sizes. As the overall level of market entry is determined by R&D employment, however, the lower cost of process innovation associated with symmetric markets induces greater market entry, so that the symmetric equilibrium allows the largest number of firms, and hence the highest level of product variety.

Our framework has interesting implications for the effects of national R&D subsidies targeted at raising economic growth. Consider the effects of an R&D subsidy financed through a lump-sum income tax on domestic households. In first-generation growth models that exhibit scale effects, the R&D subsidy will raise the rate of economic growth by increasing the aggregate scale of innovation employment. In contrast, in our model the relationship between R&D subsidies

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and economic growth is dictated by the manner in which the R&D subsidy affects the firm scale of innovation employment, and thus depends on the relative size and productivity of the implementing country. If the smaller country introduces the R&D subsidy, its share of firms and relative productivity rise. The firm scale of innovation employment decreases for both countries, however, causing a rise in the level of product variety and a fall in productivity growth, as the economy moves toward the symmetric equilibrium. On the other hand, if the larger country implements the R&D subsidy, its share of firms and its relative productivity rise, and the firm scale of innovation employment increases as the economy moves away from the symmetric equilibrium. Thus, productivity growth rises, but the level of product variety may rise or fall. In both cases, the national and aggregate scales of innovation employment increase, even though the firm scale of innovation may rise or fall, highlighting the importance of considering how R&D subsidies influence incentives at the firm level in a growth model that is not biased by scale effects.

Direct comparisons of our results with the empirical literature are difficult at the firm level, as empirical studies have generally focused on whether R&D subsidies stimulate or displace the private R&D expenditures of firms that have received grants or tax incentives, with mixed results [David et al. (2000); Hall and van Reenen (2000); García-Quevedo (2004)], rather than on how they affect the overall R&D expenditure of the average firm in a given industry. A number of the features of our framework, however, can be matched with empirical results. Returning to the effects of national R&D subsidies discussed earlier, the relative productivity increase for firms in the implementing country is supported by Einiö (in press), whereas the increases in the national and aggregate scales of innovation employment are accordant with the growing consensus of the empirical literature [David et al. (2000); Hall and van Reenen (2000); Garcia-Quevedo (2004)]. In addition, our results also match findings that suggest that tax incentives do influence the location of industry [Hines (1999); Devereux et al. (2007)], as R&D subsidies increase the share of firms locating in the implementing country.

Turning to welfare, national R&D subsidies affect the level of product variety and the rate of economic growth and cause adjustments in the terms of trade. Although the complexity of these opposing effects prevents a general analysis of welfare, we are able to extract several conclusions using simple numerical examples. First, when a national R&D subsidy is implemented by the larger country, its level will be too low from the perspective of the smaller country. Second, when countries are symmetric, there is no incentive for either country to promote domestic innovation activity, although both countries would prefer to have the other country provide an R&D subsidy. Third, if the smaller country implements an R&D subsidy, its own welfare level will fall, but the welfare of the larger country may rise or fall, depending on parameter values. In general, therefore, it may be possible to raise the welfare of the average household in the world economy through policy coordination, particularly if the larger country agrees to provide an R&D subsidy that maximizes world welfare. These results

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are, however, closely linked to the level of knowledge spillovers, with convergence in optimal subsidy rates when there is perfect knowledge diffusion.

Our paper is closely related to a number of studies examining R&D subsidies within the endogenous growth literature. For example, Davidson and Sergerstrom (1998) and Cheng and Tao (1999) consider one-country product cycle models and find that R&D subsidies to innovation reduce the growth rate under linear innovation technologies, but raise it for convex innovation technologies. Currie et al. (1999) extend the North–South product cycle framework to consider the case where innovation may occur in both the North and the South, depending on the South's development level. In general, they conclude that R&D subsidies have a positive impact on the pace of growth. Also in North–South product cycle models, Glass and Saggi (2001) show that R&D subsidies to innovation increase the pace of economic growth in the presence of international outsourcing, and Iwaisako et al. (2011) investigate the effects of a northern R&D subsidy that aims to increase the speed of product cycles by inducing the South to implement an optimal patent policy. Overall, the results of these studies suggest a positive relationship between R&D subsidies and the rate of economic growth. All of the frameworks employed, however, generate scale effects. In contrast, in this paper, we show that R&D subsidies may have a positive or negative effect on economic growth, in a framework that is not biased by a scale effect.

The remainder of this paper proceeds as follows. In Section 2 we introduce our model of trade and endogenous productivity growth. Section 3 provides a characterization of long-run equilibrium, and Section 4 investigates the effects of R&D subsidies on the rate of growth and the level of product variety. In Section 5, we discuss the implications of R&D subsidies for national and global welfare levels. Section 6 concludes.

2. BASIC MODEL

This section develops a single-sector model of trade in which firms produce horizontally differentiated product varieties and invest in in-house research with the aim of improving their production technologies. There are two countries, home and foreign, with symmetric household preferences, but asymmetric population sizes, L and L^*, where an asterisk is used to denote variables associated with the foreign country.[3] Investment capital is perfectly mobile between countries, but there is no international labor migration. Home-based firms receive an R&D subsidy, which is paid for with a lump-sum tax on home households. In what follows, we focus on the home setup, but similar conditions can be derived for the foreign setup.

2.1. Households

Each country is populated by dynastic households that choose optimal consumption paths in order to maximize lifetime utility over an infinite time horizon. The

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lifetime utility of a home household is

$$U = \int_0^{\infty} e^{-\rho t} \ln C(t) dt, \tag{1}$$

where $C(t)$ is household consumption at time t and $\rho > 0$ is the rate of time preference. Consumption takes the form of a constant-elasticity-of-substitution quantity index following Dixit and Stiglitz (1977):

$$C(t) = \left(\int_0^{N(t)} c_i(t)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}},$$

where N is the total number of product varieties, c_i is the consumption of a representative variety i, and $\sigma > 1$ is the elasticity of substitution between any two varieties.

At each moment in time, a home household allocates a given level of expenditure E across all available product varieties: $N = n + n^*$, where n and n^* are the respective numbers of product varieties produced in the home and foreign countries. Thus, a home household's demand for a representative product variety m is

$$c_m(t) = \frac{p_m(t)^{-\sigma} E(t)}{\int_0^{n(t)} p_i(t)^{1-\sigma} di + \int_0^{n^*(t)} p_j^*(t)^{1-\sigma} dj}, \quad m \in N, \tag{2}$$

where p_i and p_j^* are the respective prices of home- and foreign-produced varieties. It then follows that the price index associated with aggregate consumption can be defined as

$$P(t) = \left(\int_0^{n(t)} p_i(t)^{1-\sigma} di + \int_0^{n^*(t)} p_j^*(t)^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}}. \tag{3}$$

Thus, household expenditure is the product of the price and consumption indices: $E = PC$.

Households maximize lifetime utility (1) subject to the following intertemporal budget constraint:

$$\int_0^{\infty} e^{-\int_0^t r(s) ds} E(t) dt \leq \int_0^{\infty} e^{-\int_0^t r(s) ds} \left(w(t) - T(t) \right) dt + B(0),$$

where r is the interest rate, w is the wage rate, T is a lump-sum tax, and B is asset holdings in the home country.[4]

Substituting $C = E/P$ into lifetime utility (1), and solving the households' optimization problem, we derive the following Euler equation:

$$\frac{\dot{E}(t)}{E(t)} = r(t) - \rho, \tag{4}$$

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where a dot over a variable denotes time differentiation. Perfect capital mobility ensures a common evolution for home and foreign expenditures, as the interest rate is equalized between countries: $\dot{E}/E = \dot{E}^*/E^* = r - \rho$.

Foreign households face a similar utility maximization problem and therefore have similar demand conditions. Time notation is suppressed where possible in order to economize on notation.

2.2. Production

There is a mass of firms locating in each country, with each firm producing a unique horizontally differentiated variety and competing according to Dixit and Stiglitz (1977) monopolistic competition. Firms costlessly differentiate their products, but each period incur R&D costs that are fixed from the perspective of production (see Section 2.3).

Home and foreign firms employ l_{Xi} and l^*_{Xj} units of labor with the following respective production technologies:

$$x_i = \theta_i l_{Xi}, \qquad x^* = \theta^*_j l^*_{Xj}, \tag{5}$$

where x_i and x^*_j are firm-level outputs, and θ_i and θ^*_j are firm-specific productivity coefficients. We assume that firm-level productivity is symmetric within a country, but may differ across countries.

Given its current production technology, each firm produces to meet the combined demands of both countries. For example, the output of a home firm is $x_i = c_i L + c^*_i L^*$. The large number of firms operating in each market eliminates strategic interaction between firms as they set their optimal production levels. It thus follows that, under monopolistic competition, each firm maximizes operating profit on sales π_i ($\equiv p_i x_i - w l_{Xi}$) by setting price equal to a constant markup over unit cost; that is,

$$p = \frac{\sigma w}{(\sigma - 1)\theta}, \qquad p^* = \frac{\sigma w^*}{(\sigma - 1)\theta^*}, \tag{6}$$

for home and foreign firms, respectively. We suppress the firm indices i and j, because symmetric production technologies ensure symmetric prices within countries. Accordingly, optimal operating profit on sales can be obtained as $\pi = w l_X/(\sigma - 1)$. Substituting the pricing rules (6), the demand conditions (2), and the production functions (5) into $x = c_i L + c^*_i L^*$, and reorganizing the result, we obtain the following expressions for optimal firm levels of employment in production:

$$l_X = \frac{(\sigma - 1)p^{1-\sigma}(EL + E^*L^*)}{\sigma w P^{1-\sigma}}, \qquad l^*_X = \frac{(\sigma - 1)p^{*1-\sigma}(EL + E^*L^*)}{\sigma w^* P^{1-\sigma}}. \tag{7}$$

These conditions determine the optimal production scales of home and foreign firms.

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2.3. Process Innovation

Each firm invests in process innovation with the objective of raising profit through technology improvements that raise the productivity (θ) of labor in production. These investments include both a fixed labor component (l_F) and a variable component (l_R).[5] In particular, given the firm level of variable employment in innovation (l_R), the productivities of home and foreign firms evolve according to

$$\dot{\theta} = K l_R^{\gamma}, \qquad \dot{\theta}^* = K^* l_R^{*\gamma}, \tag{8}$$

where K and K^* are the labor productivities associated with innovation, and $\gamma \in (0, 1)$ is a parameter that ensures a diminishing marginal product of R&D investment.

A key aspect of the innovation process is the application of production processes employed by other incumbent firms. Following the in-house process innovation literature [Smulders and van de Klundert (1995); Peretto (1996)], we assume that technical knowledge regarding production processes accumulates within the firm and can be proxied for using the productivity coefficients θ and θ^*. This allows us to model the labor productivities of home and foreign firms in process innovation as a weighted average of the productivity of currently observable technical knowledge:

$$K = s\theta + \delta s^*\theta^*, \qquad K^* = s^*\theta^* + \delta s\theta, \tag{9}$$

where $s \equiv n/N$ and $s^* = 1 - s \equiv n^*/N$. The parameter $\delta \in (0, 1)$ regulates the level of international knowledge spillovers. As δ approaches unity, perfect knowledge spillovers arise between countries, and as δ approaches zero, knowledge spillovers become completely national in scope.[6] Improvements in production technology raise the level of knowledge, thereby lowering the future cost of process innovation and generating perpetual growth.

A firm's intertemporal optimization problem entails choosing the level of investment in process innovation that maximizes the future flow of profits. Total per-period profit equals operating profit on sales minus the cost of investment in process innovation $l_R + l_F$:

$$\Pi = \pi - (1 - \beta)w(l_R + l_F), \qquad \Pi^* = \pi^* - w^*(l_R^* + l_F), \tag{10}$$

where $\beta \in [0, 1)$ captures the R&D subsidy rate provided to firms in the home country.[7] We assume that fixed innovation costs are symmetric across countries ($l_F = l_F^*$).

Firms choose the optimal level of variable employment in innovation with the objective of maximizing firm value. For example, a home firm sets l_R to maximize $V = \int_0^{\infty} \Pi(\theta, t) e^{-\int_0^t r(\tau)d\tau} dt$ subject to the technology constraint (8). This intertemporal optimization problem can be solved using the following current-value Hamiltonian function: $H = \Pi + \mu K l_R^{\gamma}$, where μ is the current shadow value of an improvement in the productivity of production. Assuming that market shares are small enough so that each firm takes the price index (3) and expenditure levels

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as constant when considering the impact of changes in its price on its profit flow, the first-order conditions are $\partial H/\partial l_R = 0$ and $\partial H/\partial \theta = r\mu - \dot{\mu}$. These can be used to solve for the following no-arbitrage conditions:[8]

$$r = \frac{\gamma l_X l_R^{\gamma-1} K}{(1-\beta)\theta} + \frac{(1-\gamma)\dot{l_R}}{l_R} + \frac{\dot{w}}{w} - \frac{\dot{K}}{K},$$
$$r^* = \frac{\gamma l_X^* l_R^{*\gamma-1} K^*}{\theta^*} + \frac{(1-\gamma)\dot{l_R^*}}{l_R^*} + \frac{\dot{w^*}}{w^*} - \frac{\dot{K^*}}{K^*}, \tag{11}$$

which determine optimal R&D employment levels for home and foreign firms, respectively.

2.4. Government

As discussed earlier, the home government finances the R&D subsidy using a lump-sum tax on households. Aggregating the subsidies provided to home firms, a balanced government budget leads to a lump-sum tax of

$$TL = n\beta w(l_R + l_F), \tag{12}$$

where T is tax per household, and the left-hand side thus denotes aggregate tax revenue. The right-hand side represents the total R&D subsidy payment. As we focus on the effects of R&D subsidies provided to firms in the home country, the R&D subsidy rate is set to zero in the foreign country; that is, $\beta^* = T^* = 0$.

2.5. Market Equilibrium

With zero costs incurred in product development and free market entry, per-period profits are driven to zero in both the home and foreign countries.[9] As a consequence, household expenditures are determined solely as a function of wages earned. To see this, first note that full employment in the labor market requires

$$L = n(l_X + l_R + l_F), \qquad L^* = n^*(l_X^* + l_R^* + l_F). \tag{13}$$

Then, setting per-period profits (10) equal to zero, the free entry conditions for home- and foreign-based firms are

$$l_X = (\sigma - 1)(1-\beta)(l_R + l_F), \qquad l_X^* = (\sigma - 1)(l_R^* + l_F). \tag{14}$$

Next, using (13) and (14), in the home country a balanced government budget leads to a lump-sum tax on each home household of $T = w\beta/[1 + (\sigma - 1)(1-\beta)]$. Once again, R&D subsidies are not provided in the foreign country. Thus, home and foreign expenditures are

$$E = (1 - \eta(\beta))w, \qquad E^* = w^*, \tag{15}$$

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where $\eta(\beta) \equiv \beta/[1 + (1 - \beta)(\sigma - 1)]$ is the effective tax rate, $\eta(0) = 0$, and $\eta'(\beta) > 0$. Thus, from (4), we have $\dot{w}/w = \dot{E}/E = \dot{w}^*/w^* = \dot{E}^*/E^* = r - \rho$ at all moments in time.

3. STEADY-STATE EQUILIBRIUM

This section characterizes the long-run equilibrium of the economy in terms of the relative productivity of home firms and the share of firms based in the home country. We use this characterization of equilibrium to examine the effects of changes in relative labor endowments and the R&D subsidy rate on relative productivity and variable employment in process innovation at the firm level.

We begin by defining the relative productivity of home firms as follows:

$$\tilde{\theta} \equiv \theta/\theta^*. \tag{16}$$

The evolution of relative productivity is determined by the difference between the home and foreign productivity growth rates, as seen from the time derivative of (16):

$$\frac{\dot{\tilde{\theta}}}{\tilde{\theta}} = \frac{\dot{\theta}}{\theta} - \frac{\dot{\theta}^*}{\theta^*} = \frac{K l_R^{\gamma}}{\theta} - \frac{K^* l_R^{*\gamma}}{\theta^*}, \tag{17}$$

where we have used the technology constraints (8).

As we are interested in a steady state with a constant allocation of labor across production and innovation activities, given a constant level of R&D employment, the rate of productivity growth is constant and common across countries in long-run equilibrium. Thus, setting the evolution of relative productivity (17) equal to zero yields

$$\frac{K l_R^{\gamma}}{\theta} = \frac{K^* l_R^{*\gamma}}{\theta^*}. \tag{18}$$

With productivity growth rates equalized across countries, an examination of the time derivatives of K and K^* then reveals that $\dot{\theta}/\theta = \dot{\theta^*}/\theta^* = \dot{K}/K = \dot{K^*}/K^*$ at equilibrium.

Next, we consider the equilibrium investment conditions. Combining $\dot{\theta}/\theta = \dot{K}/K$ and $\dot{\theta^*}/\theta^* = \dot{K^*}/K^*$ with the technology constraints (8) and the free entry conditions (14) yields the steady-state no-arbitrage conditions for home and foreign as follows:

$$\rho = R(l_R) \equiv \left[\gamma(\sigma - 1)\left(1 + \frac{l_F}{l_R}\right) - 1\right]\frac{l_R^{\gamma} K}{\theta}, \tag{19}$$

$$\rho = R^*(l_R^*) \equiv \left[\gamma(\sigma - 1)\left(1 + \frac{l_F}{l_R^*}\right) - 1\right]\frac{l_R^{*\gamma} K^*}{\theta^*}, \tag{20}$$

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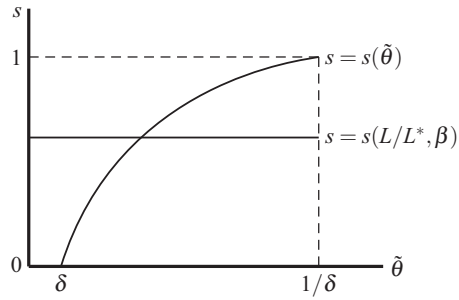


FIGURE 1. Firm shares and relative productivity.

where $R(l_R)$ and $R^*(l_R^*)$ represent the internal rates of return on investment in process innovation. Invoking (18), it is clear from these conditions that $l_R = l_R^*$. Therefore, from the free entry conditions (14), we also have $K/\theta = K^*/\theta^*$ and $l_X/(1-\beta) = l_X^*$ in long-run equilibrium.

We are now ready to solve for relative productivity. The equilibrium conditions enable the derivation of two expressions for the share of firms based in the home country (s). The first is obtained directly, using the technology constraints (9) in $K/\theta = K^*/\theta^*$ to solve for $s = s(\tilde{\theta})$. The second is found using $l_R = l_R^*$ and the free-entry conditions (14) in the labor market conditions (13) to solve for $s = s(L/L^*, \beta)$. Setting these solutions equal yields the following condition for the steady-state value of $\tilde{\theta}$:

$$s = \frac{1-\delta\tilde{\theta}^{-1}}{1-\delta\tilde{\theta}+1-\delta\tilde{\theta}^{-1}} = \frac{L/L^*}{L/L^*+[1+(1-\beta)(\sigma-1)]/\sigma}. \qquad (21)$$

Thus, as illustrated in Figure 1, relative productivity is determined implicitly by the relative labor endowment and the R&D subsidy rate: $\tilde{\theta} = \tilde{\theta}(L/L^*, \beta)$. Proportionate changes in the labor endowments have no effect on relative productivity. Moreover, setting $L/L^* = 1$ and $\beta = 0$ leads to a symmetric equilibrium with $s = \frac{1}{2}$ and $\tilde{\theta}(1,0) = 1$. A change in the R&D subsidy rate has an effect similar to that of a change in the relative labor endowment (L/L^*), and accordingly $s > L/(L+L^*)$ for $\beta > 0$. Note that with perfect knowledge diffusion ($\delta = 1$), the $s(\tilde{\theta})$ curve becomes a vertical line and productivity is symmetric across countries ($\tilde{\theta} = 1$). We summarize the determinants of relative productivity in the following lemma.

LEMMA 1. *Increases in the relative labor endowment (L/L^*) and the R&D subsidy rate (β) raise the relative productivity of home firms.*

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Proof. Total differentiation of (21) gives

$$\frac{d\tilde{\theta}}{d(L/L^*)} = \frac{[1+(1-\beta)(\sigma-1)](\delta-\tilde{\theta})^2}{\sigma\delta[1-\delta^2+(\tilde{\theta}-\delta)^2](L/L^*)^2} > 0,$$

$$\frac{d\tilde{\theta}}{d\beta} = \frac{(\sigma-1)(\delta-\tilde{\theta})^2}{\sigma\delta[1-\delta^2+(\tilde{\theta}-\delta)^2](L/L^*)} > 0.$$

■

Changes in L/L^* and β affect relative productivity through adjustments in the shares of firms locating in each country. For instance, returning to Figure 1, consider a rise in the R&D subsidy rate. The $s = s(L/L^*, \beta)$ line shifts upward, resulting in an increase in the relative productivity of home firms. Intuitively, the increase in the R&D subsidy lowers per-period innovation costs for home firms, leading to an increase in profits and raising the share of firms locating in the home country. This rise in the home country's share of firms coincides with an increase in relative productivity that brings knowledge spillovers back into balance, $K/\theta = K^*/\theta^*$, as the economy returns to steady-state equilibrium.[10]

Next, we consider the relationship between relative productivity and knowledge spillovers, as it will provide an important link in the determination of variable R&D employment at the firm level. Substituting $s = s(\tilde{\theta})$ from (21) into the expression for observable knowledge (9) and reorganizing the result yields the equilibrium level of relative knowledge spillovers as follows:

$$\frac{K}{\theta} = \frac{K^*}{\theta^*} = \frac{1-\delta^2}{2-\delta\tilde{\theta}-\delta\tilde{\theta}^{-1}}. \tag{22}$$

This equilibrium condition provides the following result:

LEMMA 2. *The steady-state level of relative knowledge spillovers is convex in relative productivity with a minimum at* $\tilde{\theta} = 1$.

As shown in Figure 2, an increase in relative productivity lowers relative knowledge spillovers for $\tilde{\theta} < 1$ and raises relative knowledge spillovers for $\tilde{\theta} > 1$. From (21), we can see that $\tilde{\theta} = 1/\delta$ when all firms locate in the home country ($s = 1$), and that $\tilde{\theta} = \delta$ when all firms locate in the foreign country ($s = 0$). Thus, it is clear that an increase in the concentration of industry in one country raises labor productivity in R&D activity. This result is a common feature of new economic geography models that assume imperfect knowledge diffusion between countries [Baldwin and Martin (2004)].

Finally, we investigate the relationship between variable innovation employment at the firm level and the internal rate of return. Figure 3 provides an illustration of the no-arbitrage condition for the home country. At every moment in time, firms choose optimal levels of process innovation, with the aim of maximizing firm value. Thus, when the internal rate of return $R(l_R)$ is greater than the risk-free rate of return $\rho(= r)$, firms increase variable R&D employment, and when

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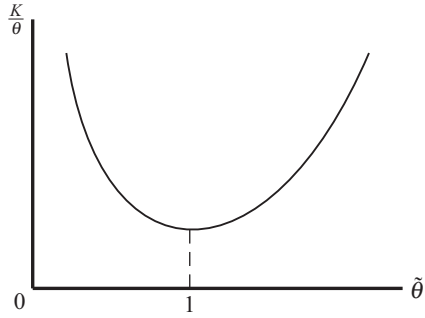


FIGURE 2. Relative knowledge spillovers.

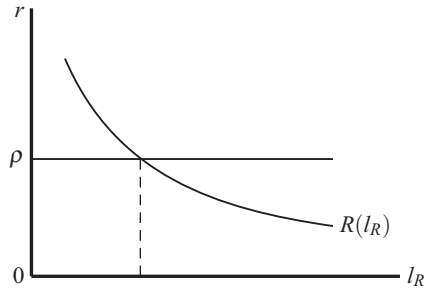


FIGURE 3. In-house R&D employment.

$R(l_R) < \rho$, they reduce R&D employment. This investment behavior implies that the internal rate of return should be decreasing in R&D employment, as depicted by the negative slope of $R(l_R)$ in Figure 3, in order to ensure a positive and finite level of variable employment in process innovation, thereby ensuring a balanced growth path. In the Appendix, we examine the local dynamics of the economy around a symmetric long-run equilibrium for $L/L^* = 1$ and $\beta = 0$. We find that if $2 > (\sigma - 1)(1 - \delta + 2(1 + \delta)\gamma)$, then

$$\frac{\partial R(l_R)}{\partial l_R} = -\left[1 - \gamma(\sigma - 1) + \frac{(1 - \gamma)(\sigma - 1)l_F}{l_R}\right]\frac{\gamma l_R^{\gamma - 1} K}{\theta} < 0, \qquad (\mathbf{23})$$

and the symmetric equilibrium is saddlepoint stable.[11] Henceforth, we characterize the long-run equilibrium and investigate the determinants of productivity growth and product variety under the assumption that $\partial R(l_R)/\partial l_R < 0$.

With relative productivity determined as a function of the relative labor endowment and the R&D subsidy rate, and relative knowledge spillovers determined as a function of relative productivity, it is now possible to examine the effects of changes in L/L^* and β on firm-level employment in process innovation ($l_R + l_F$).

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LEMMA 3. *Increases in the relative labor endowment (L/L^*) and the R&D subsidy rate (β) lower firm-level employment in innovation for $\tilde{\theta}(L/L^*, \beta) < 1$ and raise it for $\tilde{\theta}(L/L^*, \beta) > 1$.*

Proof. Substituting (22) into (19) and taking the derivative with respect to l_R and $\tilde{\theta}$ yields

$$\frac{dl_R}{d\tilde{\theta}} = -\frac{1}{R'(l_R)} \frac{\rho\delta(1-\tilde{\theta}^{-2})(K/\theta)}{1-\delta^2}.$$

This derivative can be signed using (23) and Lemma 1. ■

Changes in L/L^* and β affect l_R indirectly through $\tilde{\theta}$ and subsequently K/θ. For example, consider the effect of an increase in the R&D subsidy rate. When the home country has a greater share of firms and $\tilde{\theta} > 1$, the rise in β increases $\tilde{\theta}$, thereby raising relative knowledge spillovers. Returning to Figure 3, the internal rate of return to process innovation $R(l_R)$ shifts upward and firms increase variable employment in process innovation until the internal rate of return falls back to the risk-free level ρ. On the other hand, when the home country has a smaller share of firms and $\tilde{\theta} < 1$, the increase in β shifts the $R(l_R)$ curve downward and firms reduce innovation employment.

4. PRODUCT VARIETY AND PRODUCTIVITY GROWTH

This section investigates the effects of changes in the home R&D subsidy on equilibrium product variety and the rate of productivity growth.

Combining the labor market conditions (13) with the free-entry conditions (14) and the steady-state condition $l_R = l_R^*$, we can solve for the equilibrium level of product variety as

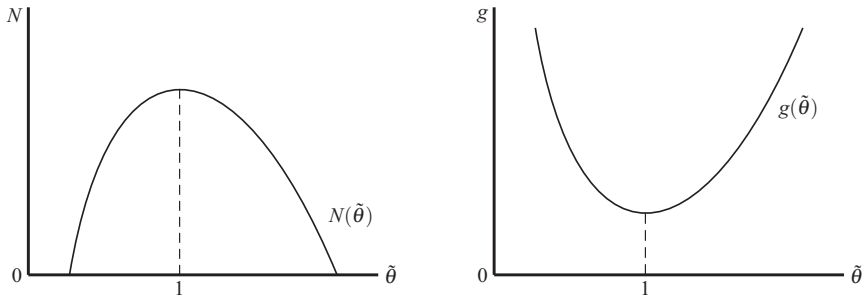
$$N = \frac{1}{l_R + l_F}\left[\frac{L}{1+(1-\beta)(\sigma-1)} + \frac{L^*}{\sigma}\right]. \tag{24}$$

As illustrated in Figure 4a, equilibrium product variety and relative productivity have a concave relationship with a maximum at $\tilde{\theta} = 1$. The mechanics of this relationship can be understood by beginning from a symmetric equilibrium, in which productivity is equalized across countries, and then considering the effects of a change in relative productivity. Either an increase or a decrease in $\tilde{\theta}$ raises relative knowledge spillovers, and at the firm level variable innovation employment rises, leading to an increase in per-period R&D costs. As a result, a portion of firms exit the market and equilibrium product variety falls.

Investigating the effects of the home R&D subsidy on equilibrium product variety, we obtain the following result.

PROPOSITION 1. *For $\tilde{\theta} \leq 1$, an increase in the R&D subsidy rate (β) raises product variety. For $\tilde{\theta} > 1$, however, product variety may rise or fall.*

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(a) Total Number of Firms (b) Rate of Productivity Growth

FIGURE 4. Product variety and productivity growth.

Proof. Taking the total derivative of (24) and applying Lemma 3, we have

$$\frac{dN}{d\beta} = \frac{(\sigma - 1)L}{(l_R + l_F)[1 + (1 - \beta)(\sigma - 1)]^2} - \frac{N}{l_R + l_F}\frac{dl_R}{d\beta}.$$

■

The R&D subsidy has two effects on equilibrium product variety. The first is a positive direct effect that shifts the $N(\tilde{\theta})$ curve upward in Figure 4a. A rise in β directly lowers per-period R&D costs, causing a decrease in firm scale through the free entry conditions (14), and allowing the market to support a larger number of firms for all levels of relative productivity. The second, in contrast, is an indirect effect that moves the economy along the $N(\tilde{\theta})$ curve as firms adjust their optimal investment levels in response to the change in relative knowledge spillovers resulting from the increase in the R&D subsidy rate. Invoking Lemma 3, through changes in l_R, an increase in β will raise N for $\tilde{\theta} < 1$ and lower N for $\tilde{\theta} > 1$. Thus, we conclude that when the home country has the smaller number of firms and $\tilde{\theta} < 1$, an R&D subsidy can be used to raise product variety, but for $\tilde{\theta} > 1$ the overall effect will be ambiguous.

Next, we calculate the steady-state rate of productivity growth. Substituting the no-arbitrage conditions (19) into the innovation function (8) and manipulating the result yields

$$g \equiv \frac{\dot{\theta}}{\theta} = \frac{l_R^{\gamma} K}{\theta} = \frac{\rho}{\gamma(\sigma - 1) - 1 + \gamma(\sigma - 1) l_F / l_R}. \tag{25}$$

An examination of (25) shows that the rate of productivity growth is determined independent of the overall size of the labor force, and is therefore not biased by a scale effect, as $\tilde{\theta} = \tilde{\theta}(L/L^*, \beta)$ is unaffected by proportionate changes in the labor endowments. Intuitively, an increase in the overall labor force is absorbed fully by a rise in the number of incumbent firms (24). Then, although both the national level of innovation employment, $n(l_R + l_F) = L/[1 + (1 - \beta)(\sigma - 1)]$, and the aggregate

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level of innovation employment, $N(l_R + l_F) = L/[1 + (1 - \beta)(\sigma - 1)] + L^*/\sigma$, rise with proportionate increases in L and L^*, firm-level employment in process innovation remains unchanged. As such, our model does not include a scale effect and is consistent with empirical evidence.[12]

Figure 4b depicts the convex relationship between productivity growth and relative productivity, with the rate of productivity growth minimized at $\tilde{\theta} = 1$. Starting again from a symmetric equilibrium, with productivity equalized across countries, either a rise or a fall in relative productivity increases relative knowledge spillovers, leading to increases in both labor employment and labor productivity in process innovation. Consequently, the rate of productivity growth rises. Intuitively, as investment in process innovation rises, per-period R&D costs ($l_R + l_F$) increase and firms are pushed out of the market. With the number of incumbent firms decreasing, the resources required to cover economywide per-period fixed costs Nl_F fall, and the amount of labor available for employment in innovation rises.

We examine the relationship between the home R&D subsidy and productivity growth and obtain the following proposition:

PROPOSITION 2. *An increase in the R&D subsidy (β) lowers productivity growth for $\tilde{\theta} < 1$ and raises productivity growth for $\tilde{\theta} > 1$.*

The R&D subsidy only affects productivity growth indirectly through changes in relative productivity. Returning once again to Lemma 1, $d\tilde{\theta}/d\beta > 0$ and an increase in the R&D subsidy leads to a rightward movement along the $g(\tilde{\theta})$ curve in Figure 4b. Accordingly, the R&D subsidy depresses productivity growth for $\tilde{\theta} < 1$ and accelerates it for $\tilde{\theta} > 1$. Note that the R&D subsidy has no effect on productivity growth for a symmetric equilibrium evaluated at $\beta = 0$.

5. WELFARE ANALYSIS

The results of the preceding section suggest a possible role for national economic policy that uses R&D subsidies as a means of raising the manufacturing share and the relative productivity of the implementing country. The resulting welfare effects are rather complex, however, given the different effects of a national R&D subsidy on the rate of productivity growth, the level of product variety, and the terms of trade. In this section, we provide a simple discussion of welfare and use a numerical example to show the optimal home R&D subsidy from the perspectives of the home, foreign, and world economies.

Although the relative wage drops out of the model in the equilibrium analysis of the preceding two sections, it assumes a role in the welfare effects associated with adjustments in the terms of trade. Substituting the pricing rules (6), the price index (3), and optimal firm-level employment in production (7) into $l_X/(1-\beta) = l_X^*$ and simplifying yields the relative wage rate as $w/w^* = (1 - \beta)^{-1/\sigma}\tilde{\theta}(L/L^*, \beta)^{\sigma-1}$, where $w/w^* = 1$ in the symmetric equilibrium for $L/L^* = 1$ and $\beta = 0$. Hence,

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the terms of trade for the home country are

$$\frac{p}{p^*} = \left[(1-\beta)\tilde{\theta}(L/L^*, \beta)\right]^{\frac{1}{\sigma}}.$$

Thus, although the direct effect of an increase in the R&D subsidy rate is a deterioration of home's terms of trade, a rise in relative productivity will lead to an improvement in p/p^*.

Moving on to national welfare levels, the present values of utility flows to home and foreign households are found by substituting (3), (6), (15), (21), and the terms of trade into lifetime utility (1):

$$\rho U_0 = \ln(1-\eta(\beta))A + \frac{1}{(\sigma-1)}\ln\left\{s(\beta) + (1-s(\beta))\left[(1-\beta)\tilde{\theta}\right]^{-\frac{\sigma-1}{\sigma}}\right\}N + \frac{g}{\rho}, \tag{26}$$

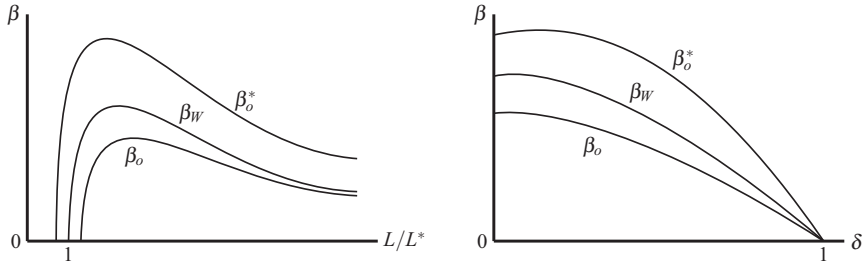
$$\rho U_0^* = \ln A^* + \frac{1}{(\sigma-1)}\ln\left\{s(\beta)\left[(1-\beta)\tilde{\theta}\right]^{\frac{\sigma-1}{\sigma}} + (1-s(\beta))\right\}N + \frac{g}{\rho}, \tag{27}$$

where $A = (\sigma-1)\theta(0)/\sigma$ and $A^* = (\sigma-1)\theta^*(0)/\sigma$ are constants and $s(\beta)$ is defined in the second expression of (21).

Focusing again on the home country, a change in the R&D subsidy affects welfare through four separate channels. First, $\eta(\beta)$ captures the negative income effect of the lump-sum tax placed on home households to pay for the R&D subsidy. Next, the second term in parentheses describes the terms-of-trade effect that results from changes in production shares (s), the relative wage rate (w/w^*), and relative productivity ($\tilde{\theta}$). The third term is the effect of the R&D subsidy on the level of product variety. Finally, the fourth term describes the effect of the R&D subsidy on the productivity growth rate. The opposing effects of the home R&D subsidy on each of these terms makes a general analysis of welfare intractable. As an alternative, we use a simple numerical example to discuss the implications of changes in the R&D subsidy for specific cases.

Figure 5 plots the home R&D subsidy rates that maximize home and and foreign welfare levels, respectively, as β_o and β_o^*. When the home country has the larger population, it hosts a larger share of firms that are relatively more productive ($s > 1/2$ and $\tilde{\theta} > 1$) and from both panels in Figure 5, it is clear that it will set the R&D subsidy rate at a lower level than is optimal for the foreign country, suggesting an opportunity for policy coordination between the home and foreign countries, with the aim of maximizing the welfare of the average household in the world economy, $U^W \equiv [L/(L+L^*)]U_0 + [L^*/(L+L^*)]U_0^*$. The home R&D subsidy rate that maximizes this welfare level is depicted by β_W. As $\beta_W > \beta_o$, average world welfare can be increased by raising the home R&D subsidy rate above the optimal level for home residents.

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(a) Relative Population Size (b) Level of Knowledge Diffusion

FIGURE 5. Optimal home R&D subsidies. For both panels, $\sigma = 1.75$, $\gamma = 0.5$, $l_F = 0.01$, and $\rho = 0.02$. In Panel (a) $\delta = 0.5$, and in Panel (b) $L/L^* = 2$.

Figure 5a illustrates the effects of a change in relative population size on the optimal home R&D subsidy rates. A positive level for β_o only occurs when the home country has the larger population. When population sizes are symmetric, $L/L^* = 1$, the welfare effects of a change in the R&D subsidy can be evaluated as follows:

$$\left.\frac{dU_0}{d\beta}\right|_{\beta=0} = -\frac{(\sigma-1)(1-\delta)}{4\sigma^2\delta\rho} < 0, \qquad \left.\frac{dU_0^*}{d\beta}\right|_{\beta=0} = \frac{(\sigma-1)(1-\delta)}{4\sigma^2\delta\rho} > 0.$$

Thus, in the symmetric case we find that the R&D subsidy lowers home welfare and raises foreign welfare. Hence, although the optimal home R&D subsidy rate is positive for foreign ($\beta_o^* > 0$) and zero for the world ($\beta_W = 0$), the home country would prefer to set the R&D subsidy to zero, or perhaps even to implement an R&D tax ($\beta_o < 0$). Figure 5a shows that these results continue to hold for $L/L^* < 1$, with $\beta_o < 0$, although β_o^* falls to zero and then turns negative as L/L^* approaches zero. Finally, as shown in Figure 5b, an increase in the level of knowledge diffusion leads to policy convergence with the optimal subsidies β_o and β_o^* equalized at $\delta = 1$, given that productivity is symmetric across countries ($\tilde{\theta} = 1$).

6. CONCLUSION

In this paper, we have developed a two-country model of fully endogenous growth in order to investigate the effects of national R&D subsidies on aggregate product variety and economic growth. Monopolistically competitive firms assume a central role in the model as they invest in process innovation with the aim of improving production technologies and lowering unit costs. In a world of perfect capital mobility, but imperfect knowledge diffusion, the country with the larger market maintains a larger share of manufacturing firms, a higher level of productivity, and a higher wage rate. The higher concentration of relatively productive firms in the larger country generates higher average productivity in flows of technical knowledge between firms, thereby raising R&D productivity and inducing a higher

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level of employment in in-house process innovation. As a result, we find that equilibria with asymmetric country sizes produce higher rates of growth than a symmetric equilibrium. As the overall level of market entry is determined by optimal R&D employment, however, the lower cost of process innovation associated with symmetric markets generates greater market entry, so that the symmetric equilibrium produces the largest number of firms and the highest level of product variety.

We use this framework to investigate how national R&D subsidies affect firm-level innovation activity. In particular, we consider the case where one country finances an R&D subsidy to the process innovation of domestic firms through a lump-sum income tax collected from domestic households. The R&D subsidy has positive effects on industry share, relative productivity, and the wage rate in the implementing country. The effects of the R&D subsidy on overall product variety and aggregate productivity growth, however, depend on whether the implementing country initially has the smaller or larger market. If the smaller of the two countries introduces an R&D subsidy, the larger country's share of firms and relative productivity will fall, pushing the economy toward the symmetric equilibrium, causing a rise in product variety, and inducing a fall in productivity growth. In contrast, if the larger country implements the R&D subsidy, its share of firms and relative productivity will rise, thus pushing the economy away from the symmetric equilibrium and raising productivity growth. In this case, however, overall product variety may rise or fall.

Although the large number of opposing effects generated by national R&D subsidies makes a direct study of welfare issues intractable, simple numerical examples lead us to conclude that the optimal level for a national R&D subsidy that is implemented by the larger country will be too low from the perspective of the smaller country. As such, there may be opportunities for policy coordination between the two countries with the aim of raising average world welfare by increasing the R&D subsidy.

Given these preliminary results, an interesting extension of our framework might be an analysis of strategic R&D policy. For example, Kondo (2013) investigates the relationship between R&D subsidies and economic growth in a variety expansion model that allows strategic policy interaction in the presence of imperfect knowledge diffusion, but that is subject to a scale effect. A simplified version of the model presented in this paper might allow an investigation of the relationship between strategic R&D policy and endogenous growth that is not biased by a scale effect. We leave this issue as a topic for future work.

NOTES

1. See Keller (2004) for more detail on the various channels through which technical knowledge diffuses between firms and across regions.

2. See Etro (2009) for an introduction to models of endogenous market structure and endogenous growth.

3. We consider fixed population sizes, but adding population growth would not affect our results.

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4. The value of asset holdings (B) will be driven to zero by free entry in the manufacturing sector. See Section 2.5 for more details. Furthermore, we assume that initially there is no international borrowing or lending in the financial market, as the home and foreign countries have the same rate of time preference ρ.

5. The fixed per-period labor cost is a standard feature of the endogenous market structure and endogenous growth framework and is needed to ensure a finite level of market entry and active investment in process innovation in the long-run equilibrium [Smulders and van de Klundert (1995); Peretto (1996); Etro (2009)].

6. This method of modeling imperfect knowledge diffusion is adapted from Baldwin and Forslid (2000).

7. R&D subsidies are typically provided in the form of grants or corporate tax credits. See Hall and van Reenen (2000) for a survey of tax exemptions offered for private R&D investment in a large number of countries.

8. These no-arbitrage conditions describe the firm's choice between the rate of return on process innovation and the rate of return on a risk-free asset, which is traded in the financial market.

9. With free market entry and exit, firm numbers adjust instantaneously in each country, and $\dot{n} = 0$ and $\dot{n}^* = 0$ at each moment in time [Peretto (1996)].

10. This result is supported by empirical literature that finds that firms are more productive in regions with higher concentrations of economic activity [Melo et al. (2009)].

11. See condition (A.6) in the Appendix.

12. The merit of removing the scale effect is seen here, as the R&D subsidy has a strictly positive effect on aggregate innovation employment, but an ambiguous effect on firm-level innovation employment, following from Lemma 3.

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APPENDIX: STABILITY OF SYMMETRIC EQUILIBRIUM

This Appendix derives a sufficient condition for the stability of the symmetric equilibrium. To examine the stability of the model, we first require a condition for the home country's share of firms outside the steady-state equilibrium. As both countries have the same rate of time preference and there are no flows of investment income, trade flows balance at every moment in time. Therefore, using the product demand conditions (2), the pricing rules (6), and household expenditures (15) in the trade balance condition $npc_i^* L^* = n^* p^* c_j L$, we obtain the home share of firms on the dynamic path as follows:

$$\bar{s} = \frac{(1-\eta(\beta))L/L^*}{(1-\eta(\beta))L/L^* + (w/w^*)^{-\sigma}\tilde{\theta}^{\sigma-1}}. \tag{A.1}$$

Next, conditions (11) and (17) can be combined with $K/\theta = \delta\tilde{\theta}^{-1} + \bar{s}(1-\delta\tilde{\theta}^{-1})$ and $K^*/\theta^* = 1 + \bar{s}(\delta\tilde{\theta} - 1)$ to obtain the following dynamic system:

$$\dot{\tilde{\theta}} = \tilde{\theta}\left[\frac{l_R^{\gamma} K}{\theta} - \frac{l_R^{*\gamma} K^*}{\theta^*}\right], \tag{A.2}$$

$$\dot{l_R} = \frac{l_R}{1-\gamma}\left[\rho - \frac{\gamma(\sigma-1)l_R^{\gamma} K}{\theta}\left(1 + \frac{l_F}{l_R}\right) + \frac{\dot{K}}{K}\right], \tag{A.3}$$

$$\dot{l_R^*} = \frac{l_R^*}{1-\gamma}\left[\rho - \frac{\gamma(\sigma-1)l_R^{*\gamma} K^*}{\theta^*}\left(1 + \frac{l_F}{l_R^*}\right) + \frac{\dot{K}^*}{K^*}\right], \tag{A.4}$$

where

$$\frac{\dot{K}}{K} = \bar{s} l_R^{\gamma} + \frac{(1-\bar{s})\left[1 + (\delta\tilde{\theta} - 1)\bar{s}\right]\delta l_R^{*\gamma}}{\left[\delta + (\tilde{\theta} - \delta)\bar{s}\right]} + \frac{\bar{s}(1-\bar{s})(\sigma-1)(\delta - \tilde{\theta})}{\left[\delta + (\tilde{\theta} - \delta)\bar{s}\right]}\frac{\dot{\tilde{\theta}}}{\tilde{\theta}},$$

$$\frac{\dot{K}^*}{K^*} = (1-\bar{s})l_R^{*\gamma} + \frac{\bar{s}\left[\delta + (\tilde{\theta} - \delta)\bar{s}\right]\delta l_R^{\gamma}}{\left[1 + (\delta\tilde{\theta} - 1)\bar{s}\right]} + \frac{\bar{s}(1-\bar{s})(\sigma-1)(1-\delta\tilde{\theta})}{\left[1 + (\delta\tilde{\theta} - 1)\bar{s}\right]}\frac{\dot{\tilde{\theta}}}{\tilde{\theta}}.$$

In our study of this system, we set the relative wage equal to its steady-state level, $w/w^* = (1-\beta)^{-1/\sigma}\tilde{\theta}(L/L^*, \beta)^{\sigma-1}$, because $(\dot{w/w^*}) = (1-\eta(\beta))^{-1}(\dot{E/E^*}) = 0$. Then (A.1), (A.2), (A.3), and (A.4) provide an autonomous system in $\tilde{\theta}$, l_R, and l_R^*. We investigate the local dynamics around the symmetric equilibrium that arises for $L/L^* = 1$ and $\beta = 0$:

$$\tilde{\theta} = 1, \quad l_R = l_R^* = \bar{l}_R, \quad \frac{w}{w^*} = 1, \quad s = \frac{1}{2}, \quad \frac{K}{\theta} = \frac{K^*}{\theta^*} = \frac{1+\delta}{2}. \tag{A.5}$$

Taking a linear expansion of (A.2), (A.3), and (A.4) and evaluating it around (A.5) yields the following Jacobian matrix:

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$$J = \begin{bmatrix} -\dfrac{[2\delta + (1-\delta)(\sigma-1)]\, l_R^{\gamma}}{2} & \dfrac{\gamma(1+\delta) l_R^{\gamma-1}}{2} & \dfrac{\gamma(1+\delta) l_R^{\gamma-1}}{2} \\ \dfrac{\partial \dot{l_R}}{\partial \tilde{\theta}} & \dfrac{\partial \dot{l_R}}{\partial l_R} & \dfrac{[2\delta + (1-\delta)(\sigma-1)]\, \gamma l_R^{\gamma}}{4(1-\gamma)} \\ \dfrac{\partial \dot{l_R^*}}{\partial \tilde{\theta}} & \dfrac{[2\delta + (1-\delta)(\sigma-1)]\, \gamma l_R^{\gamma}}{4(1-\gamma)} & \dfrac{\partial \dot{l_R^*}}{\partial l_R^*} \end{bmatrix},$$

where

$$\frac{\partial \dot{l_R}}{\partial \tilde{\theta}} = -\frac{\partial \dot{l_R^*}}{\partial \tilde{\theta}} = \frac{[2\delta + (1-\delta)(\sigma-1)][(1-\delta)(\sigma-2) l_R + \gamma(\sigma-1)(1+\delta)(l_R + l_F)]\, l_R^{\gamma}}{4(1-\gamma)(1+\delta)},$$

$$\frac{\partial \dot{l_R}}{\partial l_R} = \frac{\partial \dot{l_R^*}}{\partial l_R^*} = \frac{\{2(1-\gamma)(1+\delta)(\sigma-1) l_F - [2(\sigma-2) + (1-\delta)(\sigma-1)]\, l_R\}\, \gamma l_R^{\gamma-1}}{4(1-\gamma)}.$$

As the system consists of two jump variables (l_R and l_R^*) and one state variable ($\tilde{\theta}$), we require two eigenvalues with positive real parts and one eigenvalue with a negative real part for saddlepoint stability. Although we cannot solve for the eigenvalues directly, saddlepoint stability is established for $\mathrm{tr}(J) > 0 > |J|$, where $\mathrm{tr}(J)$ and $|J|$ denote the trace and determinant of J, respectively. Calculating $\mathrm{tr}(J)$ and $|J|$, we obtain

$$\mathrm{tr}(J) = -\frac{[2\delta + (1-\delta)(\sigma-1)] l_R^{\gamma}}{2(1-\gamma)} - \frac{2 l_R}{(1-\gamma)} \frac{\partial R(l_R)}{\partial l_R},$$

$$|J| = \frac{\gamma(1+\delta)(\sigma-1)[2\delta + (1-\delta)(\sigma-1)] l_R^{2\gamma} l_F}{4(1-\gamma)^2} \frac{\partial R(l_R)}{\partial l_R},$$

where $\partial R(l_R)/\partial l_R = -[1 - \gamma(\sigma-1) + (1-\gamma)(\sigma-1) l_F/l_R]\, \gamma(1+\delta) l_R^{\gamma-1}/2$. Thus, saddlepoint stability is achieved if $\partial R(l_R)/\partial l_R$ is sufficiently negative. Returning to the steady-state no-arbitrage condition (19), we can see that $l_F/l_R > [1 - \gamma(\sigma-1)]/[\gamma(\sigma-1)]$ is required for active process innovation. Then, substituting $l_F/l_R = [1 - \gamma(\sigma-1)]/[\gamma(\sigma-1)]$ with $\partial R(l_R)/\partial l_R$ into $\mathrm{tr}(J)$ and manipulating the result, we obtain

$$(1+\delta)\,[1 - \gamma(\sigma-1)] > \frac{2\delta + (1-\delta)(\sigma-1)}{2} > 0. \tag{A.6}$$

This inequality can be reduced to $2 > (\sigma-1)[1 - \delta + 2(1+\delta)\gamma]$, which represents a sufficient condition for $\partial R(l_R)/\partial l_R < 0$, $\mathrm{tr}(J) > 0$, $|J| < 0$, and the saddlepoint stability of the dynamic path around the symmetric equilibrium.

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