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KOLMOGOROV STORIES

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Authors of this paper were educated and have spent many years in Lomonosov Moscow State University (MSU), in the Interfaculty Laboratory of Statistical Methods (ILSM). Prof. Yu. K. Belyaev was student of Kolmogorov, and for several years his deputy in the ILSM, created and led by the great A. N. Kolmogorov. Now Prof. Yuri Belyaev is Emeritus Professor at Umea University (Sweden). Asaf Hajiyev was a PhD student in Kolmogorov's ILSM. In this paper some unusual incidents about the legendary Kolmogorov, and people around him are presented. Some stories were taken from the books and the internet (see references) dedicated to the memory of Kolmogorov; some were told by his students, and some happened with authors of this paper.

Men are cruel but Man is kind.
– A. N. Kolmogorov

Keywords: reliability theory

1. INTRODUCTION

A. N. Kolmogorov was born in 1903 and for all his life he was associated with Lomonosov Moscow State University, where he created the well-known and famous mathematical school. After Kolmogorov's demise his student, academician A. N. Shiryaev prepared and published, a three-volume book about Kolmogorov [1], wherein a biography and all of Kolmogorov's papers have been published. Thanks to A. N. Shiryaev, there exists today a Kolmogorov dedicated museum in a house in Komarovka (a village near Moscow), where Kolmogorov lived with a well-known mathematician, his friend, Academician P. S. Aleksandrov since 1935, until last days of his life.

Also published under the editorship of another one of Kolmogorov's students, Professor V. M. Tikhomirov, a very interesting book [2] entitled, "Kolmogorov—Extraordinary Phenomenon," where papers about Kolmogorov have been published by many of his students, a noteworthy one being the one by V. A. Uspensky. Today there are a lot of publications

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(books and articles) about A. N. Kolmogorov, where his illustrious life in science, his social life, and some of his scientific breakthroughs are given [1–10].

The idea to prepare this paper was suggested by Prof. Nozer D. Singpurwalla of The City University of Hong Kong, and The George Washington University in Washington, DC, several years ago, but affirmatively so, after MMR 2019 (the 2019 Conference in Mathematical Methods of Reliability), where Asaf Hajiyev made his latest presentation on "Kolmogorov Stories." Before MMR 2019, its presentation was done in the USA (at The George Washington University), in China (at Chengdu University), in Portugal (at Lisbon University), and in Hong Kong (at the City University of Hong Kong). In this version new stories have been added by the authors. Why is an individual like Kolmogorov so important to the scientific world?

Kolmogorov made essential contributions to science and hence, his name is next to the names of distinguished individuals like—Newton, Euler, Gauss, and Poincare. We would like to start with Kolmogorov's famous words [1]: "I always go with the principle that sincerity is a virtue and our mission is to find it and to cultivate it." We claim that during his life Kolmogorov strongly followed this principle.



A. N. Kolmogorov was born in 25 April 1903 in Tambov. His father Nikolai Kataev, the son of a priest, was an agronomist. Kolmogorov's mother Maria Kolmogorova tragically died during his birth. Her sister, Vera Yakovlevna, a very independent woman, raised Kolmogorov and he always had the deepest affection for his aunt.

Kolmogorov's early childhood was spent in the Yaroslavl region, at Vera's father's estate. His grandfather, Yakov Kolmogorov was a landowner. His grandfather and his aunt published a home-town newspaper, wherein Kolmogorov formulated his first mathematical problem, given below. Kolmogorov was then only 5 years old.

$$1 + 3 = 2^2;$$
$$1 + 3 + 5 = 3^2;$$

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$$1 + 3 + 5 + 7 = 4^2;$$

$$\vdots$$

$$1 + 3 + \cdots + (2n - 1) = n^2$$



The main building Lomonosov Moscow State University

In 1910, Kolmogorov moved to Moscow. He studied at a private gymnasium run by two intelligent and progressive ladies Evgeniya Repman and Vera Fedorova. It was due to his family tradition, and the friendly atmosphere at the gymnasium, that Kolmogorov acquired a general principle, namely, to search for himself the right and serious deeds. He also worked at the train library (which was in Russia) and he stated later, that it was a good opportunity to read a lot of books from different fields.

In 1920, Kolmogorov entered the Lomonosov Moscow State University. At the same time, Kolmogorov also studied at the Metallurgical Department of the Chemical-Technological Institute. Besides mathematics and metallurgy, he also studied Russian history. During that period, the study of machinery was also considered as something serious, and as necessary as pure science.

Together with his aunts, Kolmogorov lived in the village of Tunoshna near Yaroslavl. His aunts organized a small school of around 10 children of different ages. Education by then followed modern pedagogical ideas. His aunt's pedagogical principles of raising him with high social ideals were: responsibility, independence of opinion, intolerance toward idleness, intolerance to poorly performed tasks, desire to understand, and not only to remember. In 1925, Kolmogorov published eight scientific papers, graduated from Lomonosov Moscow State University and became graduate student of well-known professor of mathematics N. N. Luzin.

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2. HISTORY LOST BUT MATHEMATICS WON

Kolmogorov was planning to be a historian. He prepared his first paper in history when he was 16 years old. He wrote a very interesting paper in history and was invited to make its presentation at the seminar series of a well-known historian and professor of Lomonosov Moscow State University, Bakhrushin. After Kolmogorov's talk Bakhrushin said that his theory and propositions were quite interesting but that in the field of history, it is necessary to have several different arguments to validate a theory. To that Kolmogorov responded, "Well, I like history, but I would like to work in a field where it is enough to have only one proof to validate a theory." A consequence is that *history might have lost a great historian but mathematics gained a great mathematician*. Kolmogorov started his research on topics in history when he was 17 years old and finished it when he was 19. After Kolmogorov's demise, among his archives there was found a manuscript on matters historical. In 1989 at a meeting, dedicated to memory of Kolmogorov, a well-known historian, Academician V. Yanin said: "This research has a special place in historical science, and even today history has yet to receive its status" [9].

In 1929, Kolmogorov completed his doctorate. At that time, he had authored 18 publications and was researcher at the Institute of Mathematics and Mechanics of Lomonosov Moscow State University.

During the period June 1930 to March 1931 Kolmogorov made his first trip abroad with the well-known mathematician P. S. Aleksandrov. They visited Germany (Berlin, Göttingen, and Munich) and France (Paris), where Kolmogorov got to meet Hilbert, Caratheodory, Courant, Landau, Levi, Frechet, Lebesque, Borel, Weyl, and some other leading mathematicians.

In 1931, at age 28, Kolmogorov was appointed a Professor at Moscow State University, and in 1935, Kolmogorov received the high academic rank of a Full Doctor in Mathematics (without defence). In 1937, he created and became the first Head of the Probability Theory Department at the Lomonosov Moscow State University.

In 1935, P. S. Aleksandrov and A. N. Kolmogorov bought a house in Komarovka (a village outside Moscow). Many famous mathematicians from various countries visited Komarovka: Hadamard, Frechet, Banach, Hopf, Kuratovski, and others.

In 1938, Kolmogorov was elected a Full member of the Academy of Sciences of USSR. Kolmogorov founded two scientific journals: "Soviet (now Russian) Mathematical Surveys" (Успехи математических наук), and the "Theory of Probability and its Applications" (Теория вероятностей и ее применения). For a long time, he was the Editor in Chief of these journals.

In 1941 (during the Second World War), Kolmogorov was evacuated to Kazan; he returned to Moscow in 1942. There, Kolmogorov essentially worked in ballistics, contributed to it.

Kolmogorov actively worked in many areas of science; an inkling of these follows:

In Applied Sciences: biological issues, geology, meteorology, statistical quality control, and crystallography.

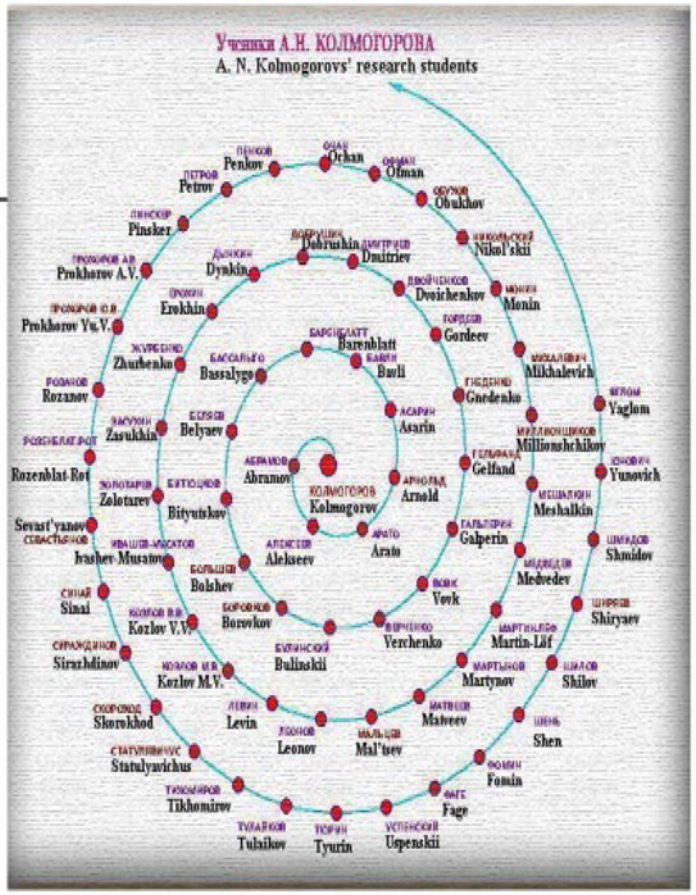
In Mathematics: stationary sequences, interpolation and extrapolation of random processes. The classic work (with B. Gnedenko on)—limit distributions for sums of independent random variables, and foundations of theory of branching processes.

In 1941, he was awarded State Prize of USSR (together with A. Ya. Khinchin) and in 1949 the Chebychev Prize of the USSR Academy of Sciences (together with B. V. Gnedenko). In 1942, Kolmogorov married Anna Egorova. They studied together in the Repman' gymnasium in Moscow, during 1910–1918.

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3. THE KOLMOGOROV SPIRAL

Among Kolmogorov's students, there are many who are famous, and who have worked in the different areas of science. The proverbial Kolmogorov's spiral is giving below.



Achievements of Kolmogorov's Students:

Laureates of the Lenin Prize: V. Arnold, I. Gelfand, A. Maltsev, Yu. Prohorov, and Yu. Rozanov.

Laureates of the State Prize of the USSR: B. Gnedenko, Y. Belyaev, A. Borovkov, A. Khinchin, and S. Nikolskiy.

Academicians: B. Gnedenko, A. Borovkov, A. Gonchar, A. Khinchin, V. Maltzev, V. Mikhalevich, A. Millionshikov, I. Monin, S. Nikolskiy, A. Obukhov, Yu. Rozanov, Ya. Sinai, A. Shiryaev, S. Sirazhdinov, A. Skorohod, A. Vitushkin, and some others.

Professors: V. Alekseyev, T. Arato, S. Barenblatt, L. Bolshev, A. Bulinskiy, E. Dinkin, R. Dobrushin, S. Fomin, V. Kolchin, V. Malishev, M. Malyutov, L. Meshalkin, V. Pisarenko, G. Shilov, V. Tikchomirov, V. Tutubalin, Yu. Tyurin, V. Uspensky, A. Yaglom, I. Zhurbenko, V. Zolotarev, and some others.

Their scientific contributions covered different fields of science:

V. Arnold (Theoretical Physics, Differential Equations, Mechanics, Dynamical Systems), Yu. Belyaev (Theory of Probability and Mathematical Statistics, Reliability Theory,

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Point Processes), A. Borovkov (Theory of Probability and Mathematical Statistics), L. Bolshev (Mathematical Statistics), R. Dobrushin (Theory of Probability, Information Theory and Ergodic theory), Gelfand (Functional Analysis, Differential Equations), B. Gnedenko (Theory of Probability and Mathematical Statistics), A. Gonchar (Complex Analysis), A. Khinchin (Theory of Probability and Mathematical Statistics, Number Theory), V. Maltzev (Algebra, Theory of Functions), V. Mikhalevich (Applied mathematics and Informatics), A. Millionshikov (Differential Equations), S. Nikolsky (Theory of Functions, Algebra), Yu. Prohorov (Theory of Probability and Mathematical Statistics), V. Skorohod (Probability, Stochastic Processes), V. Tickhomirov (Approximation Theory, Optimal Control), A. Shiryaev (Theory of Probability and Mathematical Statistics), Ya. Sinai (Ergodic Theory and Dynamical Systems), V. Sevastyanov (Branching Processes), V. Uspensky (Mathematical Logic), and A. Vitushkin (Differential Equations, Mechanics) to name a few.

Kolmogorov joked that one of his students controls the atmosphere and another, the ocean. His student—academician A. Obuckhov was the Director of the Institute of Atmosphere and—academician A. Monin was the Director of the Institute of Oceanology.

4. THE LEADING MATHEMATICIAN OF THE 20TH CENTURY

Andrey Nikolaevich Kolmogorov was one of the world's leading mathematicians. Some in the mathematical academia claim that if David Hilbert (the great German mathematician) dominated mathematics during the first one-third of 20th century, then A. N. Kolmogorov, without doubt, dominated the rest, i.e., the next two-thirds of the 20th century.

In 1900, at the second International Congress of Mathematicians (in Paris), David Hilbert proposed 23 open mathematical problems, which are known today as Hilbert's problems. It was then considered a unique event in world of mathematics, because even today these problems are very real, and finding a solution to any of these problems is considered, in Mathematics, to be an important event. Not all of Hilbert's problems have yet been solved. In 13th problem Hilbert stated that a continuous function of three variables cannot be represented as a superposition of continuous functions of two variables. Kolmogorov (with Arnold, who was a third year student) proved that was not true. Moreover, Kolmogorov proved that any continuous function can be represented as a superposition of continuous functions of one variable, and unique function of two variables. This result today is a foundation underlying the mathematics of "deep learning."

Kolmogorov's interests in mathematics were so diverse that practically, there is no branch of mathematics where he did not achieve fundamental results. He made very interesting observations, and achieved significant results in various branches of mathematics, such as: theory of probability, mathematical statistics, theory of functions, functional analysis, dynamic systems, ergodic theory, algebra, mathematical logic, theory of approximations, mechanics, and others. Because of this, some scholars in the 50s believed that there is a group of Soviet mathematicians working under the name Kolmogorov and that it is not the work of just one individual (like Nikolas Bourbaki in France).

5. CREATING A THEORY OF PROBABILITY (BY YU. BELYAEV)

A young A. N. Kolmogorov was interested to know in detail different branches of the Mathematical Sciences. In particular, at that time he was interested in the theory of different kinds of sets. As acknowledged by Kolmogorov, the papers of Johann Radon and Otto

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Nikodim in the theory of sets allowed him to extend the concept of conditional probability when the conditioning set had zero probability. This matter played an important role in the construction of a full system of axioms, to the creation of Probability Theory as a branch of the Mathematical Sciences. Within 2 weeks, he developed a full system of axioms of probability, and was the first person to have done so.

This achievement can be considered as the birth and development of a mathematical probability theory. In 1933, Kolmogorov published his famous Grundbegriffe der Wahrscheinlichkeits-rechnung (Foundation of Probability Theory). Given therein was a rigorous definition of conditional expectation. Thus, Probability Theory became a mathematical science. Kolmogorov said that "The theory of probability as a mathematical discipline can, and should be developed, from axioms in exactly the same way as Geometry and Algebra."

6. KOLMOGOROV'S SOLUTION OF A FAMOUS PROBLEM

Kolmogorov's first famous problem was constructing the Fourier series of a cumulative function, which diverged almost everywhere, to which the well-known French mathematician Frechet said (about Kolmogorov): "He is a great young man, who constructed this example." Norbert Weiner wrote (about Kolmogorov): "I stopped my research in functional analysis as soon as I felt that Kolmogorov tread on to my heels." Kolmogorov's comment was: "Competition can be friendly and then it is akin to cooperation. Cooperation between several mathematicians can be very productive, though sometimes, some investigations can be surplus, and then it is reasonably unoffensive to walk away."

Kolmogorov was always saying: "Each person, starting with some age, continues to remain at that age, a characteristic peculiar to this person's perception of the world." When he was asked about his age, his answer was always 14.

7. KOLMOGOROV'S HIGH SCHOOL

Kolmogorov created a special physical–mathematical high school for talented young people. Now it is called Kolmogorov's Boarding School No. 18. From all regions of Soviet Union (and now from Russia) all talented young high school students are invited (on a competitive basis) to continue last 3 years of schooling in this school. Students live and study there. Kolmogorov prepared a special program in mathematics for this School, actively participated in the teaching process, and invited his well-established students also to teach there as well. Several distinguished Soviet and Russian physicists and mathematicians have been through this school.

8. THE FIRST MEETING WITH YURI BELYAEV: CHOOSING AN ADVISOR

It was in 1955. Belyaev was a last year student in the mathematics department of Lomonosov Moscow State University and has prepared his thesis [4]. His scientific adviser was Prof. V. Ya. Kozlov. At that time Belyaev had prepared and published a very interesting paper "Analytical Random Processes." Even scientists from Romania came to Moscow to discuss this paper. When scientists from Romania met Belyaev they expected to meet some old established person with beard, but what they met was a young boy (Belyaev then was 22 years old). It appears that Prof. Kozlov informed Kolmogorov about Belyaev's paper and recommended Belyaev to Kolmogorov. Belyaev afraid to meet with Kolmogorov, because as

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he felt that there was too big distance between them. Kolmogorov, lived in one of the buildings of Lomonosov Moscow State University, and suggested a meeting time with Belyaev near the building, where Kolmogorov liked to take walks. Kolmogorov asked Belyaev about music (Belyaev was a good singer), about sports (Belyaev was a good swimmer), and some other such things, but nothing about mathematics. Afterward Kolmogorov said: "I see that you are very versatile individual and agreed to be his adviser for a PhD."

Comment (by Asaf Hajiyev): Kolmogorov as a great individual, with very deep intuition, felt that Belyaev is a very talented young scholar and was informed about Belyaev's paper; he therefore had agreed to be Belyaev's adviser for a PhD. Yu. Belyaev successfully defended his PhD thesis and later on, prepared his paperwork for a Doctor of Sciences (Dr.Sci.) thesis. Kolmogorov checked this thesis and said that it was interesting work in engineering (applied mathematics not pure mathematics) but that he was ready to support it. Belyaev wanted to prepare a thesis in the pure sciences, and later on got new results, to another Dr.Sci. thesis in pure mathematics which he successfully defended.

9. KOLMOGOROV'S PSYCHOLOGICAL TRICKS (BY YU. BELYAEV)

Kolmogorov was a very reachable person when it came to the formulation of mathematical problems, and always shared them with his students [4]. Once he suggested to me the following problem. Prove that the trajectories of random Gaussian processes are either continuous with probability 1 or are discontinuous at every point. I spent much time but could not solve the problem; once at a seminar Kolmogorov said: "Let us organize a competition so that someone can help Belyaev to solve the problem." Professors Dobrushin and Girsanov (who participated in the seminar) decided to help me to solve the problem. It was an unpleasant situation for me, because now I had competitors. For several nights I could not sleep, and even at nights I was thinking about this problem; at last, I found a solution. It was a very striking problem and I thank Kolmogorov for compelling me to solve it by psychologically drawing others into the arena. For this work, I was cited in a special Encyclopedia.

10. KOMAROVKA AND KOLMOGOROV: WALKS WITH HIS PHD STUDENTS (BY YU. BELYAEV)

Kolmogorov has a style to discuss mathematical problems which he asked his aspiring PhD students to find solutions to. He found that the best way to do this is during the specially organized long walks in nearby forests. These walks took a lot of time, several hours, of the day. As a well-known academician Kolmogorov had the right to order a special auto which could take 7–11 persons. Such big autos were produced in Moscow. Special food (sandwiches) had been prepared for all the participants in these walks. Each participant had two or three appetizing sandwiches during two stops in the walks. Excellent black tee was prepared in several thermoses. Around 7–10 PhD students were invited to Komarovka to be there no later than 9 o'clock in the morning. The auto came just after 9 o'clock. Then all the students with Kolmogorov had places in the auto. Usually, Aleksandrov oversaw all these last minutes details and declared "the crew is ready." The auto had taken all to a starting point about 30 kilometers of Komarovka. Once the walking commenced, Kolmogorov would talk with each aspirant. He did not directly ask them about the assigned mathematical tasks but about their life's problems and achievements. Then Kolmogorov asked about progress in the aspirant's assigned mathematical problems. In case of progress, as an academician

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of the USSR Academy of Sciences, he had the right to recommend publishing the work in "Доклады Академии наук СССР" (*Doklady, Journal of the Soviet Academy of Sciences*).

11. ASAF HAJIYEV'S FIRST MEETING WITH KOLMOGOROV

It was September 1969. I was a first-year student at the mechanical–mathematical faculty of Lomonosov Moscow State University. During the first day of classes, three leading scientists: topologist—academician Pavel Sergeyevich Alexandrov, mechanics—academician Leonid Ivanovich Sedov, and the great Andrey Nikolayevich Kolmogorov delivered lectures as an introduction into mathematics. Kolmogorov finished his speech with the words, "It is very important for mathematicians to get enough sleep. As it is better to stay a little hungry than to be a little sleepy." Following Kolmogorov's advice, only few students showed up to class the next day in the morning. As freshmen, we did not appreciate as to who Kolmogorov was.

I would say that 60 to 70 years of 20th century was renaissance time in mechanical–mathematical faculty of the Lomonosov Moscow State University. In one faculty at that time, worked more than 30 world class mathematicians, and the majority were around Kolmogorov; these were:

> P. Alexandrov, V. Alekseyev, V. Arnold, Yu. Belyaev, E. Dinkin, I. Gelfand, B. Gnedenko, A. Kudryavtsev, A. Nikolsky, B. Sevastyanov, S. Fomin, Yu. Prohorov, Yu. Rozanov, Ya. Sinay, A. Shiryaev, G. Shilov, V. Uspensky, V. Zolotarev, V. Tickhomirov, and others.

Even today it is difficult to imagine any university in the world, where more than 30 leading mathematicians are working at the same time.

12. ASAF HAJIYEV'S SECOND MEETING WITH KOLMOGOROV. CHOOSING A DEPARTMENT. YEAR 1970

At the end of second year at Moscow State University each student had to choose a department for research and a scientific advisor. Each department organizes the so-called "Open Doors Day," where students can come to the department and get information about its scientific activity. The teachers of the department gave information about modern problems of mathematics and presented their own research. There you could choose a field of investigation and an adviser. Prof. E. V. Chepurin recommended to me to be student of Yu. K. Belyaev and told that it is a great opportunity to be in the Laboratory created and led by A. N. Kolmogorov, where Yu. Belyaev worked and was a Deputy of Kolmogorov. Thus, I decided to go to the "Open Doors Day" of the probability department. There were a lot of scientists, who told about their fields of investigation. Suddenly the door opened, and Kolmogorov came in. Immediately all teachers gave way to him and invited Kolmogorov to the blackboard. He took the chalk and wrote the Laplace equation on the blackboard, which we the students were not familiar with. Kolmogorov looked at us and asked, "Did you all study differential equations with partial derivatives?". Of course, the answer was no, because we did not yet study this course. In that time this course was delivered in the third year. Then Kolmogorov told that there is no reason to continue the discussion. He put the chalk down and left the room. We were all shocked. And I started to think how can I go to the probability department if as Kolmogorov said there is no reason to speak to us? After that incident I did not want to go to the probability department and was trying to explain to Y. Chepurin what had happened. He tried to explain me that A. N. Kolmogorov is a special individual and despite his unusual behavior it would be a great opportunity to

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have his advises and to be in this department. Thanks to advice of Y. Chepurin and also my father, they convinced me to enter to probability department and I am happy that I took this advice.

13. ASAF HAJIYEV'S THIRD MEETING WITH KOLMOGOROV. YEAR 1971

I was third year student. Each Tuesday in the student dormitory of Moscow University we had so-called Alexandrov's Tuesdays. Academician P. S. Alexandrov (was an expert in classical music) and every Tuesday he came to the student dormitory with a lot of CD's and we listened to classical music. In the end, the students were told about the history of this music, its composer, and so on. It was an example of richness and the high level of education in Lomonosov Moscow State University. Once Academician P. S. Aleksandrov came with A. N. Kolmogorov and brought, as he said very, interesting music.

Remark. I have a sister and she is a musician, educated from Azerbaijan State Conservatory (now Azerbaijan State Musical Academy). When I was studying in high school, she was preparing to enter an exam for admission to the Conservatory. For this, she had to choose a musical composition (concert, fuge, or something other), and practice it for 8 months before going to the exam. Her choice was J. S. Bach's—Small Prelude and Fugue in Fa Major. Every day in our home she was playing on the piano 6–8 hours and also trying to explain to me how beautiful this music is and also its history of. This was too much for me, and I was waiting for the time she was to go to Conservatory, because it was awful to listen to it every day for 6–8 hours and also its explanation. I knew all the details about it, and frankly was not too thrilled with it as well.

After listening to the music brought by P. S. Aleksandrov and A. N. Kolmogorov, the latter asked: "Who knows this music"? There was only one student who knew it—it was me. The music was of course Bach's "Small Prelude and Fugue in Fa Major."

I explained all the details about it, and Kolmogorov said, it seems you have deep knowledge in classical music. I nothing said. Since this time everybody in the dormitory thought that I am a deep classical music expert. Every Thursday we played soccer in Moscow University and after soccer we wanted to drink beer. At that time, it was not so easy to find beer in Moscow. But one of our friends told us that his parents go every Thursday to the Tchaikovsky Moscow Conservatory for a concert and there is beer in the buffet. After soccer we went to Tchaikovsky Conservatory to drink beer in the buffet. Thus, I came to dormitory very late, because soccer and beer. My friends asked me where you coming? Answer was—from Conservatory. My friends in dormitory were saying, that Kolmogorov mentioned that Asaf is an accomplished expert in classical music.

Once I met A. N. Kolmogorov in the Laboratory and he asked me: Are you still interested in classical music? The reader can guess my answer, because I could not tell a wrong thing to Kolmogorov. He said: "Yes, you are right, for mathematicians, classic music is a very good partner and please continue it."

(Asaf Hajiyev) Year 1978. I was working on my PhD thesis under supervision of Prof. Belyaev and had to show the final version of the thesis to him. He asked me to come to the laboratory, where A. N. Kolmogorov was the Chairman and Belyaev was his deputy. I was waiting for Yu. Belyaev and was checking my thesis and suddenly the door was opened, and A. N. Kolmogorov came in. He was looking for Yu. Belyaev and I told him that I was also waiting for him. A. N. Kolmogorov had a look at my thesis and asked me what was the problem that I had solved. I showed him the problem and he had a look and said: "not an interesting problem" and left the room. I was worried, because I had finished the thesis and without Kolmogorov's support there was no chance to defend it. Soon Belyaev came into

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the room. He looked at me and asked: "What happened to you, is there any problem?" I was trying to explain what had happened when suddenly the door opened, and Kolmogorov came back in. He recommended checking a behavior of the second derivative and finding an optimal function. I answered that I did it and found the optimal function. He looked at my paper and said, "You can prepare paper and publish it" and left the room together with Belyaev. Later on, Belyaev came back and said: "I think you got the answer to your worries. Do not worry anymore and prepare your thesis and the paper for the journal as Kolmogorov said." I was so happy to have the support of Kolmogorov, proud of it, and it is a honor for me to spend more than 15 years in the laboratory, which was created and led by the great man and mathematician Kolmogorov.

Sometimes taking stock of my life, I ask myself: "What is the best achievement in my life, when I was so happy?" My answer is—highest grade, which I got at the entering PhD exam from Kolmogorov.

14. THIS STORY WAS TOLD BY PROF. GENNADY FALIN

He was a PhD student of Gnedenko. He finished his PhD thesis and was going to defend it at the scientific council led by Kolmogorov. Gnedenko told Falin: "As soon as you start to explain your results" Kolmogorov will sleep (he was seriously ill in that time). Usually he slept exactly 13–14 minutes; so, you have to speak for only 11–12 minutes and finish your talk before he wakes up because Kolmogorov can ask complicated questions. Falin considered in his PhD thesis three problems on queueing systems with lost customers. It really was a complicated problem and he solved the problem only partially. But these results were enough for PhD thesis. Falin has spent only 11 minutes and suddenly Kolmogorov woke up. Falin was going to erase everything at the blackboard but Kolmogorov stopped him, had a look at the blackboard and said: "Well, you considered 3 complicated problems and solved them only partially." Afterward Kolmogorov analyzed all the three problems, made several comments, and proposed some new approaches for their solution. In 1 year, Falin completely solved all the three problems, proposed complete solution to the problems and became well a known person in this field.

Later on, Professor A. D. Solovyev mentioned that when a dissertation was in Kolmogorov's interest, he would take it and would solve any problem therein (give the main steps for its solution) in 15–20 minutes, and for the Doctor of Sciences problems 45–50 minutes. The main challenge was to get him interested. But once he was interested students took advantage of his genius to successfully defend their dissertations.

The well-known mathematician, topologist, and Academician P. S. Alexandrov said of Kolmogorov: "Mathematics of the 20th century is so diverse and difficult, I think that there is no one person who can understand all mathematics and their theories, with only one exception. This exception is Kolmogorov."

Kolmogorov's students asserted that, in the formulation of 75% of the dissertations, defended at Moscow State University in mathematics, Kolmogorov saw the solution of the problem before the answer was presented. For the remaining 25% of the dissertations he asked scholars around him to propose solutions. In this way, as quoted by Academician Alexandrov, "Kolmogorov sharpened his expertise in mathematics and was up to date with the most significant developments in the field."

Kolmogorov Created and was First Chair of:

1. The Institute of Mathematics and Mechanics.
2. Department of Theory of Probability in Lomonosov Moscow State University.

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3. Department of Mathematical Statistics in Lomonosov Moscow State University.
4. Department of Mathematical Logic in Lomonosov Moscow State University.
5. Interfaculty Laboratory of Statistical Methods in Lomonosov Moscow State University.
6. Laboratory of Turbulence (now it is The Institute of Oceanology, Russian Academy of Sciences).

Currently, the chairs of all these departments are Kolmogorov's students.

15. KOLMOGOROV'S DELICATE PERSONNA

Kolmogorov was very knowledgeable in classical music. At Komarovka he often organized evenings of classical music for his students. Once while listening to classical music his student Shiryaev (now head of the Probability Department at the Moscow State University) started to tap his fingers on the table, synchronously to the music. Kolmogorov came up to Shiryaev and put a napkin under his fingers, to prevent the noise from disturbing the others but to allow Shiryaev to continue his enjoyment of the music.

16. KOLMOGOROV AS A PSYCHOLOGIST

Kolmogorov together with S. Fomin published a new book: Theory of functions and functional analysis and created a legendary course Analysis—III based on it. It was the course with a very competitive and demanding grading system. During one of the exams, Kolmogorov asked the students if there is anyone who wanted to get the highest grade of an A in it. Everyone was so intimidated and afraid to fail that nobody dared volunteer. Suddenly a girl put up her hand and Kolmogorov invited her to the blackboard. He asked for her grade book (in Soviet Union we had oral exams and grades were recorded in a special grade book) and immediately gave her highest grade of an A. Everybody was shocked and Kolmogorov responded: "I know that you are all afraid of my exams, and if this girl is so sure in her abilities it means that not only she is well prepared for the exam, but that she is also confident in her knowledge. Therefore, I do not need to test her as I am sure that she knows the subject very well."

17. THE CASE OF PROFESSOR Y. SHILOV

Kolmogorov proposed to his student Shilov (later Shilov became a well-known professor of MSU) a mathematical problem as work toward a diploma. It was problem with several parameters that needed a proof for some mathematical fact. Prof. Shilov had spent 6 months to prove this fact for the special case when the parameters take certain fixed values. When he showed his proof, Kolmogorov thought for about 15 minutes and showed how this fact can be proved for any values of the parameters.

18. ASAF HAJIYEV ON KOLMOGOROV'S KINDNESS

Kolmogorov often participated in exams of students. Once he was going to give some student the grade of C. Student asked him to give some additional problems, because if he got a C, then he will lose an opportunity to get a scholarship. In the Soviet Union students with

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high grades were getting a monthly salary (stypendia). This scholarship almost covered all of a student's expenses. Kolmogorov gave him several problems and the student solved them. Kolmogorov asked his grade book and gave him the grade B. Afterward he put inside the grade book 25 rubles and said: "Our mission not only to teach, but also to help." It was half a month's expenses for the student. Kolmogorov always said that students, PhD students, are poor and hence it was necessary to invite them for lunch or dinner once a week. He invited them to Komarovka (the village, where he lived) organized lunch, dinner, classical music evenings, and in winter ski trips. There were several rooms. In one room stayed all scholars interested in functional analysis, in another probability theory, analysis, mechanics, and so on. Students were spending their weekends, discussing scientific problems and got Kolmogorov's advices. Kolmogorov liked to have a long walk (several hours) in the forest with students, where he discussed various scientific problems and gave his advices. After walks students had time to digest Kolmogorov's advices. As a result of this "scientific lambs" were transformed into the "scientific tigers." As an example, we can give the list of the following leading mathematicians who passed through Komarovka and became well-known scholars: Yu. Belyaev, A. Borovkov, A. Bulinsky B. Gnedenko, R. Dobrushin, Yu. Prohorov, Yu. Rozanov, A. Shiryaev, Ya. Sinai, V. Tickhomirov, V. Uspensky, V. Sevastyanov, and others. All of them had been through Komarovka. Once Kolmogorov said: "Do not think that I am interested only in mathematics. I am quite a free person inside, and I am thinking about ideal state." He told that a feature of the ideal state is freedom and education. He always was thinking about education and his high school reforms completely changed education. He has done a lot for university education, preparing even programs for teaching at the mathematical faculties. Moreover, personally he was teaching various courses: mathematical analysis, linear algebra and geometry, differential equations, theory of probability, topology, functional analysis, mathematical logic, information theory, measure theory, and others. It is difficult to imagine how one person can teach these quite different classes. I must say these were nonstandard courses and A. N. Kolmogorov always proposed new approaches.

19. KOLMOGOROV'S HONORS

National Honors: In 1939 he was elected to the Academy of Sciences of the USSR. He received one of the first State Prizes in 1941, the Lenin Prize in 1965, the Order of Lenin on seven separate occasions, and the Lobachevsky Prize in 1987. He had the title of Hero of Socialist Labour.

International Honors: He was also elected to the many other academies and societies including the Romanian Academy of Sciences (1956), the Royal Statistical Society of London (1956), the Leopoldina Academy of Germany (1959), the American Academy of Arts and Sciences (1959), the London Mathematical Society (1959), the American Philosophical Society (1961), The Indian Statistical Institute (1962), the Netherlands Academy of Sciences (1963), the Royal Society of London (1964), the National Academy of the United States (1967), and the French Academy of Sciences (1968).

Kolmogorov received the Balzan International Prize in 1962 and Wolf prize in 1981. Many universities awarded him an honorary degree including Paris, Stockholm, and Warsaw.

20. THE WALK AROUND THE LAKE (AS TOLD BY PROF. A. BULINSKY)

It was in the summer of 1986. A group of scholars were walking with Kolmogorov near Komarovka, in the forest around the lake. This time Kolmogorov was seriously ill, and

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his students visited him to take him out for fresh air. He liked it very much. During a walk Kolmogorov suddenly left the group that was walking in the direction to the lake. When one of his students came to Kolmogorov and asked: "What has happened, Andrey Nikolayevich?" Kolmogorov looked toward the sky and said: "I am proud that I lived a worthy life." The great A. N. Kolmogorov passed away on 20th October 1987.

Everybody, who was lucky enough to be his student, to work with him, or to be beside him, forever experienced a delight of the power of his intelligence, his unpredictable originality of thought, a startling latitude of horizon, and a seductive charm of personality. Over time they became clearer and clearer of his unique place in the world of science, the meaning of his contribution into mathematical education, and the greatness of his activities. Kolmogorov was the person whose mind possessed an insatiable curiosity. Sometimes it is difficult come to terms with the fact that his thoughts and ideas, observations and notes, recommendations and advices remain unrealized and even unrecorded. And as a well-known mathematician, Kolmogorov's student V. M. Tikhomirov said: "Kolmogorov was a Genius who lived among us."

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