Weak Discernibility and Relations between Quanta

Joshua Norton*†

Some authors (Muller and Saunders, Huggett and Norton) have attempted to defend Leibniz's Identity of Indiscernibles through weak discernibility. The idea is that if there is a symmetric, nonreflexive physical relation that holds between two particles, then those particles cannot be identical. In this article I focus only on Muller and Saunders's account and argue that the means by which they achieve weak discernibility is not through a quantum mechanical observable but an alternate mathematical construction that is both unorthodox and incomplete.

1. Introduction. Muller and Saunders (2008) argue that identical particles are weakly discerned by having opposite spin. This is in response to the longstanding concern that identical particles, like fermions, violate Leibniz's Principle of the Identity of Indiscernibles (PII). However, if "identical" particles are discernible by their spins, then they do not differ merely numerically and thus do not violate the PII. If Muller and Saunders are correct, then they will have successfully demonstrated that one form of the PII is immune from challenges posed by identical particles. The first half of this article lays out the relevant issues as well as Muller and Saunders's position. In the second half of this article, I argue that Muller and Saunders's account fails since they do not make use of quantum observables, and what they do make use of we are not justified in interpreting as a physical relation.[1] Without a physically pertinent relation, one cannot even begin the process of weak discernibility.

*To contact the author, please write to: University of Illinois at Chicago, Department of Philosophy, 1423 University Hall (MC 267), 601 South Morgan Street, Chicago, IL 60607-7109; e-mail: joshua.d.norton@gmail.com.

†I would like to thank Nick Huggett, Christian Wüthrich, Jeremy Butterfield, Adam Caulton, Fred Muller, Simon Saunders, as well as the varied participants at the 2014 Philosophy of Science Association conference for their comments on this article or its earlier instantiations.

1. Both French and Redhead (1988) and Huggett and Norton (2014) make a similar mistake in how they build physical relations. Huggett and Norton make the same assumptions

Philosophy of Science, 82 (December 2015) pp. 1188–1199. 0031-8248/2015/8205-0038$10.00
Copyright 2015 by the Philosophy of Science Association. All rights reserved.

https://doi.org/10.1086/683647 Published online by Cambridge University Press

2. The Challenge from Identical Particles and Weak Discernibility. Assuming some familiarity with both the PII and with the challenge raised by identical particles, I will be brief in my retelling of this story. Leibniz posited seven principles that he largely took as being self-evident, the PII being one of them. This principle states that if two objects are indiscernible then they are identical or, a in slightly more interesting formulation, "that it is not true that two substances may be exactly alike and differ merely numerically, *solo numero*" (Leibniz 1686/1974, 417). I say "more interesting" because this quote makes clear that if there are two numerically distinct things then they must be discernible; however, the quote also claims that if two things are indiscernible then they must not be even numerically distinct: there must be only one thing. The second conditional will play an important role at end of this article. Leibniz's principle can be written in the following logically equivalent ways. The first most faithfully captures the words of Leibniz, but the third is the most common way of thinking of the principle:

$$\neg[\exists\, x\, \exists\, y[(x \neq y) \cdot \forall F \in \{F\} \mid (F(x) \equiv F(y))]], \tag{1}$$

$$\forall\, x\, \forall y[(x \neq y) \supset \exists F \in \{F\} \mid \neg(F(x) \equiv F(y))], \tag{2}$$

$$\forall\, x\, \forall\, y[\,\forall\, F \in \{F\} \mid (F(x) \equiv F(y)) \supset (x = y)], \tag{3}$$

where $\{F\}$ represents the set of monadic properties for the objects in our domain.[2] According to this formulation, we define that two objects, a and b, are strongly discerned if and only if (iff)

as Muller and Saunders. French and Redhead consider relations built out of conditional probabilities of single-particle operators and provide no argument that these relations represent physical relations of the particles themselves. See Huggett and Norton (2014) for remarks on French and Redhead's use of nonsymmetrized observables.

2. Some authors include both monadic and relational properties in the set F; however, Leibniz's view on relations suggests that we keep these cases separate. Leibniz scholars debate whether Leibniz was simply an antirealist regarding relations, in which case relations would certainly not be included in his PII, or whether relations, for Leibniz, are unique in the sense that no two objects could partake of the same relation as some other distinct pair of objects. For instance, the relation of brotherhood shared between two men would not the same as the relation of brotherhood shared between any other distinct pair of men. Consequently, if a relation is satisfied by a pair of distinct objects, the relation fails to be reflexive, and as a consequence weak discernibility is easy to satisfy. Under this second interpretation it again seems unlikely that relations were originally included in the PII. For a discussion on Leibniz's views, see Mugnai (1992).

https://doi.org/10.1086/683647 Published online by Cambridge University Press

$$\exists F \in \{F\} \mid \{[F(a) \wedge \neg F(b)] \vee [\neg F(a) \wedge F(b)]\}. \tag{4}$$

If strong discernibility (4) is met, then the inference in (3) cannot go through. In such a case, one could then use Leibniz's Principle of the Indiscernibility of Identicals to infer that the "substances" must not be identical. However, in the quantum case, it seems that (4) fails, and so the inference in (3) does go through.

The literature recognizes other means of discerning objects including that used by Muller and Saunders (2008): two objects, a and b, are defined to be weakly discerned iff there is a physically relevant dyadic predicate that is symmetric and nonreflexive when applied to the two objects.[3]

$$\exists F(a,b) \mid [F(a,b) \wedge F(b,a) \wedge \neg F(a,a) \wedge \neg F(b,b)]. \tag{5}$$

The idea behind weak discernibility is that if two objects are identical then $F(a, b)$ is equivalent to $F(a, a)$ and the above condition fails; thus, if it does hold then a and b cannot be identical. Although the inclusion of weak discernibility shifts the debate away from the exact form of Leibniz's principle, the spirit of the debate remains very much the same.

Quantum mechanics challenges the truth of Leibniz's principle in that identical particles in states like the singlet state,

$$\Psi = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow), \tag{6}$$

are thought to be distinct particles and yet have no property to discern them. The term 'identical particles' is unfortunate given the current context. To be clear, in the context of quantum mechanics, all that is entailed by two or more particles being identical is that each has the same value for their nondynamical properties such as their mass and charge. Identical particles are not necessarily identical in a logical or ontological sense. For the sake of sticking closely to the terms of Leibniz's project, I will assume that we have two objects that do differ at least numerically, but there is an important sense in

3. Quine (1976) as well as Muller and Saunders (2008) formulate the principle of weak discernibility in terms of irreflexive relations and not nonreflexive relations. However, I see no reason to require the logically stronger relation in the context of discerning identical particles. Equation (5) is from Muller and Saunders (2008, 528–29). The authors also note that weak discernibility can be satisfied by a relation that is reflexive and symmetric and that does not hold between all nonidentical objects. Examples of such relations are 'identical to' or perhaps 'same haecceity' (depending on whether haecceities are properties). Such relations will not be of much use to us.

https://doi.org/10.1086/683647 Published online by Cambridge University Press

which entangled states may deny this and thereby trivialize the question of discernibility.[4]

The singlet state in (6) describes an entangled two-particle system expressed in terms of $\hat{z}$-spin, while the particles' location has been suppressed. In order to be relevant for the PII, we insist that the implicit portion of the wave function be identical and that the particles follow the same path through space (this is why it has been suppressed). This state is physically realizable as two entangled electrons in the same orbital of some atom. In such a situation, for all the properties that are specified by quantum mechanics, the particles share said properties. In other words, there is no property held by one of the particles and not by the other, yet we do not think that these particles are ontologically one—as would be required by the PII. Consequently, many have taken the existence of such particles as being counterexamples to the PII.

Although identical particles fail to be strongly discernible, Muller and Saunders (2008) claim that they are weakly discerned by the relation 'has opposite spin'. This relation is symmetric and irreflexive, and yet in order to justify its application to the singlet state we need an 'opposite spin' observable that when applied to the singlet state yields an appropriate eigenvalue. A common assumption, although far from obvious, is that there is a one-to-one mapping from physical properties to quantum mechanical Hermitian operators (observables), such that if a state Ψ is the ith eigenvector of the observable with eigenvalue a_i then the state is said to have the property associated with this eigenvalue. This association between properties and eigenvalues is often referred to as the "eigenvector, eigenvalue link" (EE-link) and serves to coordinate Hermitian operators with properties of a system. In order for us to have a mathematical framework complete enough to describe quantum phenomena, we require that our mathematical machinery be able to represent every physical property that might be possessed by our system. Thus, if we want to play the game of modeling quantum mechanics at all, we must assume that the aforementioned mapping is at least injective.[5]

4. Given the holistic nature of entangled states, one might question whether it even makes sense to speak of having two particles when the state is entangled. It is plausible that particles lose their identity through entanglement and become only a new unity. If this is correct, then the PII cannot be challenged by entanglement since there are no longer two things that differ *solo numero*. I elaborate on this point toward the end of the article.

5. I am leaving open the question of surjectivity; in particular, if there are superselection rules then the mapping will not be surjective. Relatedly, one might worry that injectivity also fails. It seems that there are quantum facts that we do not represent with Hermitian operators. Such facts might include selection rules, superselection rules, the Stone–von Neumann theorem, or that all systems are represented as vectors in a Hilbert space. Two comments on

https://doi.org/10.1086/683647 Published online by Cambridge University Press

Muller and Saunders (2008, 533) use the following generalized operator to build the relation of having or failing to have opposite spin:

$$\sum_{i,j=1}^{d} P_{ij}^{(a)} P_{ij}^{(b)}. \tag{7}$$

Without getting into too many of the details, note that the superscripts on the single-particle operators $P_{ij}^{(a)}$ pick out which slot of the tensor product the operators act on, and the subscripts refer to vectors of a given eigenbasis. For example, $P_{\uparrow\downarrow}^{(2)} = \mathbb{I} \otimes (P_{\uparrow} - P_{\downarrow}) \otimes \mathbb{I} \otimes \ldots$, where the $P_{\uparrow}$ projects onto the $\uparrow \hat{z}$-spin eigenvector and $P_{\downarrow}$ onto the $\downarrow$. Muller and Saunders (2008, 533–35) define the following two relations and claim that both are able to weakly discern the identical particles:

$$Z_{-2}(a,b) \quad \text{iff} \quad \sum_{i,j=1}^{d} P_{ij}^{(a)} P_{ij}^{(b)} \Psi = -2\Psi. \tag{8}$$

$$Z_{2}(a,\ b) \quad \text{iff} \quad \sum_{i,j=1}^{d} P_{ij}^{(a)} P_{ij}^{(b)} \Psi = 2\Psi. \tag{9}$$

In the case considered by Muller and Saunders, the $P_{ij}^{(\cdot)}$ operators each reduce to the Pauli spin matrix σ_z, and we get the following refinement:

$$Z_{-2}(a,b) \quad \text{iff} \quad 2\sigma_z^a \sigma_z^b \Psi = -2\Psi. \tag{10}$$

$$Z_{2}(a,\ b) \quad \text{iff} \quad 2\sigma_z^a \sigma_z^b \Psi = 2\Psi. \tag{11}$$

For $a = 1$, $b = 2$ $(a \leftrightarrow b)$ the relation holds:

$$Z_{-2}(1,2) \equiv 2\sigma_z \otimes \sigma_z \Psi = -2\Psi. \tag{12}$$

For $a = b = 1$ the relation does not hold:

$$Z_{-2}(1,\ 1) \equiv 2(\sigma_z)^2 \otimes \mathbb{I}\Psi \neq -2\Psi. \tag{13}$$

For $a = b = 2$ the relation does not hold:

this: (1) these properties are not properties of physical systems, at best they are relations that hold between the physical world and some language(s) that we use to model the world. (2) The ongoing assumption in literature is that meeting the EE-link is the gold standard for predicating quantum systems and that projects that diverge from it ought to be held as suspect. Setting these concerns aside, I am more than happy to consider alternate mathematical devices for modeling properties of our quantum systems; however, I will not do so here since the EE-link is something that Muller and Saunders are going to want to abide by since they aim to present an orthodox defense of the PII.

https://doi.org/10.1086/683647 Published online by Cambridge University Press

$$Z_{-2}(2,\ 2) \equiv 2\mathbb{I} \otimes (\sigma_z)^2\Psi \neq -2\Psi. \tag{14}$$

In other words Z_{-2} holds iff $a \neq b$. The relation Z_2 holds under the exact opposite conditions. Thus, the particles are weakly discerned by the relation Z_{-2}, and the PII is immune from challenges posed by identical particles. For a more detailed account, see Muller and Saunders (2008).

Although Muller and Saunders's account is straightforward, it is far from unproblematic. Equation (10) captures three different equations, each with its own observable. As we select particular values for a and b, we construct new mathematical objects (see eq. [15]). Consequently, the formal relation Z_{-2}, used to weakly discern, is not a Hermitian operator on the system, and we cannot apply the EE-link to it. Since the EE-link is not applicable to Z_{-2}, we have no reason for thinking that it represents a physical relation. In the following section I develop a series of criticisms that stem from this fact.

3. Challenges. Muller and Saunders are aware that traditionally the EE-link (roughly what they refer to as StrPP) is used to pick out monadic predicates.[6] They nonetheless insist that it can also pick out polyadic predicates:

> Finally, a note on relations. When physical system S is (taken as) a composite system, built up of other physical systems, some of the properties of S determine and are determined by relations of its constituents. . . . Consequently, both WkPP and StrPP [EE-link], although giving rise to properties of S and of its subsystems (expressed by monadic predicates), equally provide conditions for the ascription of relations among constituents of S; the magnitude A may itself be relational (as in relative distance), and likewise the operator A corresponding to it. (This is why one does not need to introduce relation postulates in addition to property postulates.) Typically, however, as we shall see in the next section, where authors have made use of WkPP or StrPP [EE-link], they have used them only to consider monadic properties—that is to say, from our point of view, they have not made use of either property postulate to ascribe relations among constituents of S which is the key step that we shall be taking in this paper. (Muller and Saunders 2008, 515)

According to Muller and Saunders, the conditions for ascribing a physical relation to a formal relation as in (10) are as follows: "When the projectors under

6. According to Muller and Saunders, "We represent a quantitative physical property mathematically by the ordered pair $\langle A, a\rangle$ where A is the operator which corresponds to physical magnitude $\mathcal{A}$ and $a \in \mathbb{C}$ is its value. According to the Strong Property Postulate (StrPP), a physical system S having state operator $W \in \mathcal{S}(\mathcal{H})$ possesses quantitative physical property $\langle A, a\rangle \in \mathcal{MS}(\mathcal{H}) \times \mathbf{C}$ iff W is an eigenstate of A that belongs to eigenvalue a" (2008, 513).

https://doi.org/10.1086/683647 Published online by Cambridge University Press

consideration belong to the spectral family of magnitude-operator A, assumed to be physically meaningful, they are themselves physically meaningful; by the WkPP, when the system is in the state W, so is relation Rt, which is defined in terms of them (Req1)" (Muller and Saunders 2008, 532), where the condition (Req1) is "all properties and relations should be transparently defined in terms of physical states and operators that correspond to physical magnitudes, as in WkPP, in order for the properties and relations to be physically meaningful" (527). Here, Muller and Saunders use the WkPP, which is simply a weaker version of the StrPP (514) and will make no difference to my argument. In summary, according to Muller and Saunders, what justifies assigning a physical relation, 'opposite spin', to Z_{-2} is merely that such a relation gains physical transparency by piggybacking off the physical transparency of the observables that show up in (10). Before examining this claim, I want to make clear that I am in no way questioning whether the observables that show up in (10) have physical meaning and am only questioning how physical content is forced onto a formal relation built out of them.

First, Z_{-2} is not an observable itself but is map from $\{1,2\} \oplus \{1,2\}$ to the following set of observables (acting on Ψ):[7]

$$\{2\sigma_z \otimes \sigma_z \Psi,\ 2(\sigma_z)^2 \otimes \mathbb{I}\Psi,\ 2\mathbb{I} \otimes (\sigma_z)^2 \Psi\}, \tag{15}$$

where Z_{-2} is associated with three different observables, each of which seems to be "manifestly" physical and yet each of which could bear a distinct physical meaning. What then is the relationship between this set of observables and Z_{-2} whereby the map becomes associated with a single physical relation? There are three questions: (1) why must the map represent any physical property of the system or of the particles that make up the system, (2) why is this map associated with a dyadic property (opposite spin) and not a property of some different arity, and (3) even if the mapping is associated with a dyadic property, why must we think that this relation is symmetric and irreflexive? Surely the formal relation $Z_{-2}(a,b)$ is symmetric and irreflexive on slot indices a and b, but why must the observables in (10) bind together to produce a physical relation that is symmetric and irreflexive on the particles? The criterion we are in search of needs to be more than that the objects mapped onto are physically meaningful or are individually a relation of a certain kind since in general it is false that maps have the same properties as the objects in their image.

What Muller and Saunders need to supply is an EE-link for relations that identifies when n-valued functions onto sets of observables pick out m-ary

7. As discussed by Huggett and Norton (2014), we should be cautious of these observables since they are multiples of the identity on the space of fermions and are not symmetrized.

https://doi.org/10.1086/683647 Published online by Cambridge University Press

properties. Such a link would unpack what it means for a relation to be "transparently defined" by selecting which formally defined relations represent physical properties and what the relevant physical property is. However, an EE-link for relations is not provided for by Muller and Saunders, nor is it part of standard quantum mechanics. As things stand, we cannot do much with Muller and Saunders's project until we are given more reason for thinking that Z_{-2} represents something physical. However, for the sake of argument let us assume that Z_{-2} is physically meaningful and consider questions 2 and 3. If Z_{-2} does represent something physical, are there reasons for associating it with the relation 'opposite spin'? Not clearly. Analyzing (12)–(14) and how the operators act on the singlet state does not leave one with any assurance that any weakly discerning relation is being described by them. For instance, there is nothing ruling out the possibility that equation (12) tells us that the state has total spin equal to zero, while (13) and (14) tell us that the total spin fails to be either greater or less than zero. If these are the proper interpretations suggested by the individual equations (12), (13), and (14), then it would be hard to argue that Z_{-2} represents a relation since the individual observables are monadic properties of the system. Similarly, in order for Z_{-2} to represent a weakly discerning relation on the particles, equations (12)–(14) must not bind together to suggest some three-place relation between the particles and their environment and so on. Somehow we must use the information in (12)–(14) to rule out all contrary interpretations and leave only an interpretation attached to some weakly discerning relation. Without this relation, Muller and Saunders (2008) have only a physically salient map (since we have granted them this) and no means to weakly discern. That equations (12)–(14) do select such a relation is not argued for by Muller and Saunders, nor is it even clear what such an argument would look like.

Before exploring other motivations for why we ought to think that 'opposite spin' is true of the singlet state, I want to highlight one further worry about Z_{-2}. If Muller and Saunders are correct in their claim that $Z_{-2}(x, y)$ represents a physical relation since it is built out of physically salient Hermitian operators, then it must also be true that $Z(x) \equiv Z_{-2}(x, a)$ represents a physically relevant monadic property since it too is built out of physically salient Hermitian operators.[8] Moreover, since it is not the case that both $x = 1$ and $x = 2$ satisfy $Z(x)$, the particles are strongly discerned by it. However, we had already concluded that the particles are not strongly discerned.

We are able to bypass our previous conclusion by using non-Hermitian operators to represent monadic properties of the particles. If we think that we have made a mistake in getting to this conclusion (and we have), then I suggest that we have also made a mistake in the case of weak discernibility. Perhaps in filling out or adding to Muller and Saunders's (2008) conditions

8. Where a is either 1 or 2.

https://doi.org/10.1086/683647 Published online by Cambridge University Press

for why $Z_{-2}(x, y)$ represents a physical relation we will discover conditions that fail when applied to $Z_{-2}(x, a)$. However, until these conditions are given, we should have no confidence that $Z_{-2}(x, y)$ is any more physical than $Z_{-2}(x, a)$.

Thus far we have not found anything in the formal structure of Z_{-2} acting on $\boldsymbol{\Psi}$ that suggests 'opposite spin' or any other symmetric and irreflexive physical relation. Why then think that it does? With respect to their claim that $Z_{-2}(a, b)$ represents 'opposite spin', Muller and Saunders claim "Relation Z_{-2} is the one in footnote 5 of (Saunders [2003a], p. 294): 'has opposite direction of each component of spin to'" (2008, 535). Or in other words, the physical property represented by Z_{-2} is that provided by Saunders in 2003: Z_{-2} represents the property 'has opposite direction of each component of spin to'. However, the referred-to passage (in Saunders 2003) does nothing to elucidate the justification for interpreting Z_{-2} in this way. Sauders (2003) argues merely that "the most general anitsymmetrized two-particle state is $\boldsymbol{\Psi} = \frac{1}{\sqrt{2}}(\boldsymbol{\phi} \otimes \boldsymbol{\psi} - \boldsymbol{\psi} \otimes \boldsymbol{\phi})$ where $\boldsymbol{\phi}$ and $\boldsymbol{\psi}$ are orthogonal. Analogues of operators for components of spin can be defined as $S = P_{\boldsymbol{\phi}} - P_{\boldsymbol{\psi}}$, where $P_{\boldsymbol{\phi}}$, $P_{\boldsymbol{\psi}}$ are projections on the states $\boldsymbol{\phi}$, $\boldsymbol{\psi}$. Each of the two particles in the state $\boldsymbol{\Psi}$, has opposite value of $\mathbf{S}$. But no particle can have opposite value of $\mathbf{S}$" to itself (Saunders 2003, 294 n. 5).[9]

What does the heavy lifting in predicating opposite $\mathbf{S}$-value (opposite spin) to the particles is apparently Saunders's assertion that each of the two particles in the state $\boldsymbol{\Psi}$ happen to have opposite values of spin. This is surely troubling; if we are not going to reason in a circle, we must have some way of verifying that the particles do in fact give opposite values of $\mathbf{S}$. Yet given that the state is entangled, we have no way of isolating any single particle to determine its spin. Saunders claims that the observable $\mathbf{S}$ does pick out opposite eigenvalues when applied to the particles in the singlet state; however, in order to make this claim one would need to first identify which part of the singlet state represents some single particle. Although we know that such talk is nonsense, perhaps if we were to look at the actual terms of the $\hat{z}$-spin expansion of the singlet state, we might be able to find some motivation for

9. There are formal problems with Saunders's $\mathbf{S}$: it is a single-particle observable and cannot be applied to the singlet state. This is corrected in Muller and Saunders (2008), although their observable is not symmetrized. In this same footnote (Saunders 2003, 294 n. 5), presumably in defense of his position, Saunders quotes Mermin (1998). In this article, Mermin gives a nonstandard interpretation of quantum mechanics under which all there is are relations and no relata. According to this interpretation, there are no particles to weakly discern since there are no particles. The role that Mermin's interpretation is supposed to play in Saunders's account is far from clear. Surely such an interpretation cannot help decide the fate of the PII, which assumes that there are relata in the world.

https://doi.org/10.1086/683647 Published online by Cambridge University Press

thinking that 'opposite spin' has something to do with the singlet state. It sure seems like the particles in (16) have opposite spin.

$$\Psi = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow). \tag{16}$$

How does one dissect this state to apprehend one particle at a time and thus come to believe that each particle has the opposite **S**-value from the other? If we were to assume that the slots of each term of the singlet state represent the individual particles, then we might conclude that the particles have opposite spin. For instance, let

$$\text{Particle 1 } (\uparrow \otimes \cdot) \text{ as well as } (\downarrow \otimes \cdot), \tag{17}$$

$$\text{Particle 2 } (\cdot \otimes \downarrow) \text{ as well as } (\cdot \otimes \uparrow). \tag{18}$$

Then according to our assumption, the first slot in $\uparrow\downarrow$ represents the first particle and its properties and the second the second particle and its properties, and similarly for $\downarrow\uparrow$. Since the first particle in $\uparrow\downarrow$ has the opposite value of spin as the second particle, we conclude that the particles have opposite spin. Not forgetting that there are two terms, we check the second term $\downarrow\uparrow$ and note that the particles represented here also have opposite spin. Therefore by the linearity of (16), it is true of the singlet state that the particles have opposite spin.

The mistake of course is that $\uparrow\downarrow$ does not represent anything physical: no identical particles are represented by $\uparrow\downarrow$. The eigenvector $\uparrow\downarrow$ is no more essential to the physical description of the state than the minus sign connecting the other nonphysical pieces of syntax. As a whole, the state has physical content, but none of its parts do. To treat the terms $\uparrow\downarrow$ or $\downarrow\uparrow$ as individually giving us physical information of the state is to treat the singlet state as akin to a statistical mixture rather than a genuine entanglement.[10] This line of reasoning corrects an assumption we may have had regarding the EE-link: we may have assumed that since observables are linear in the wave function the properties associated with them are too. This of course is false. Physical properties only hold on physical states. The EE-link associates properties with physical states and not with nonphysical terms of a state's decomposition. Moreover, implicit in my treatment is a denial that

10. Let us lay this objection aside and assume that it makes sense to read physical properties off nonphysical syntax; we still need a story as to why 'opposite spin' is selected and not 'definite and opposite spin'. Surely it is true of both $\uparrow\downarrow$ and $\downarrow\uparrow$ that the particles have definite and opposite spin, and thus should we not also say of the singlet state that its particles have definite and opposite spin? Yet this is false: the particles described by the singlet state do not have definite spin and may or may not have opposite spin (this is the contentious claim we are exploring). This example highlights the danger inherent in inferring state properties from nonphysical states.

https://doi.org/10.1086/683647 Published online by Cambridge University Press

slot indices always represent single particles. Rather, only those slots of factorized states represent single particles: slots on nonphysical syntax cannot represent physical particles.

In summary of these arguments: the singlet state is Ψ, and focusing too much on the expansion $(\uparrow\downarrow - \downarrow\uparrow)$ may tempt us into thinking that its pattern of arrows is revealing hidden relations between the single particles. We must conclude that there is nothing in the singlet state's description that warrants attributing opposite spin to its particles. Moreover, since, as we have seen, there is nothing in the formal structure of Z_{-2} that warrants our association of it to the relation 'opposite spin', quantum mechanics simply does not provide us with a description of the particles' spins.[11] If quantum mechanics does not provide a description, then why do we think there is one?

In conclusion: I have provided three challenges to predicating 'opposite spin' to states like the singlet state. The first challenge stems from Muller and Saunders's (2008) emphasis that their formal relation Z_{-2} should be identified with a physical relation. I have argued that there is nothing in the formal structure of Z_{-2} for thinking that it represents any physical property of the state, or that it is a physical relation, or that the physical relation is suitably structured to weakly discern. I close this article with the following consideration that others have also noted: Why must we think of the singlet as representing two things to begin with? As I have argued throughout, it is exceedingly difficult to justify applying relational properties to the singlet state since an entangled state's description "hides" what is true of any single particle. If we are to extract information about single-particle properties, it is not clear how we are to do so. Given the extent to which entangled states hide the supposed particles that they represent, why not interpret such states as representing only emergent unities?

Whether or not we think the PII is a logical principle (as Leibniz did), for most of us the principle remains highly intuitive; moreover, since the principle comes under attack only to the extent that entangled states represent multiple particles, what is keeping us from flipping the argument around and using the PII to deny the composite nature of entangled states? I will not attempt to address this issue and only note that if we did deny the composite nature of entangled states, we would simultaneously block the PII from attack as well as make it unreasonable to apply relational properties to them. For if there are not two "numerically distinct" things to challenge the PII, then there are not two things that have opposite spin (for instance).

11. It might be the case that Z_{-2} ought to be associated with 'opposite spin'; however, given our current understanding of quantum mechanics, we are not warranted in making this association. In order to be warranted, we require a more robust EE-link than the one we currently have.

https://doi.org/10.1086/683647 Published online by Cambridge University Press

REFERENCES

French, S., and M. Redhead. 1988. "Quantum Physics and the Identity of Indiscernibles." *British Journal for the Philosophy of Science* 39 (2): 233–46.

Huggett, N., and J. Norton. 2014. "Weak Discernibility for Quanta, the Right Way." *British Journal for the Philosophy of Science* 65 (1): 39–58.

Leibniz, G. 1686/1974. *The Rationalists: Descartes: Discourse on Method and Meditations; Spinoza: Ethics; Leibniz: Monadology and Discourse on Metaphysics*. New York: Anchor.

Mermin, D. 1998. "What Is Quantum Mechanics Trying to Tell Us?" *American Journal of Physics* 66:753–67.

Mugnai, M. 1992. *Leibniz' Theory of Relations*. Stuttgart: Steiner.

Muller, F., and S. Saunders. 2008. "Discerning Fermions." *British Journal for the Philosophy of Science* 59 (3): 499–548.

Quine, W. 1976. "Grades of Discriminability." *Journal of Philosophy* 73 (5): 113–16.

Saunders, S. 2003. "Physics and Leibniz's Principles." In *Symmetries in Physics: Philosophical Reflections*, 289–307. Cambridge: Cambridge University Press.

https://doi.org/10.1086/683647 Published online by Cambridge University Press