On the frequency of Friday the thirteenth

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This article begins with a new look at earlier work on this topic and continues with the presentation of a graphical technique for the determination of the months of a year for which the 13th is a Friday.

It might be thought that the likelihood of the 13th day of a month being a Friday is the same as that of the 13th being any other day, but this is not so. The full day/date repeat cycle is 400 years, this being the interval between century years which are leap years. The outcome of an extraordinary counting exercise some 50 years ago by a 13-year-old Eton schoolboy, S. R. Baxter [1, 2], was to show that, over this period, the 13th of a month will be a Friday at least once more than any other day. Of necessity, Baxter's calculation was intricate and an independent confirmation was undertaken as a matter of interest. The 400-year period beginning 1 March 2014 was considered. The work is described in four stages.

1. The period is envisaged as a succession of columns, one for each year, each containing the days of occurrence of the 13th of each month of that year, beginning with the day for March and ending with that for February. Across the whole 400-year span, there are only seven different columns of days, each characterised by the day for March at the head of the column. Successive column headings move on by one day ordinarily but by two days coming into a leap year. A sample portion of these columns, readily constructed with the help of a recent diary, is shown in Table 1. (Here and throughout the days of the week are denoted by the initials M, Tu, W, Th, F, Sa and Su.)

	2014 -15	2015 -16	2016 -17	2017 -18	2018 -19	2019 -20	2020 -21	2021 -22	2022 -23	2023 -24	2024 -25
Mar	Th	F	Su	M	Tu	W	F	Sa	Su	M	W
Apr	Su	M	W	Th	F	Sa	M	Tu	W	Th	Sa
May	Tu	W	F	Sa	Su	M	W	Th	F	Sa	M
Jun	F	Sa	M	Tu	W	Th	Sa	Su	M	Tu	Th
Jul	Su	M	W	Th	F	Sa	M	Tu	W	Th	Sa
Aug	W	Th	Sa	Su	M	Tu	Th	F	Sa	Su	Tu
Sep	Sa	Su	Tu	W	Th	F	Su	M	Tu	W	F
Oct	M	Tu	Th	F	Sa	Su	Tu	W	Th	F	Su
Nov	Th	F	Su	M	Tu	W	F	Sa	Su	M	W
Dec	Sa	Su	Tu	W	Th	F	Su	M	Tu	W	F
Jan	Tu	W	F	Sa	Su	M	W	Th	F	Sa	M
Feb	F	Sa	M	Tu	W	Th	Sa	Su	M	Tu	Th

TABLE 1

2. Within the 400 columns for the 400-year repeat period there are ‘lesser’ repeat periods of 28 years, interrupted as a result of the omission of the leap day (February 29th) in the years 2100, 2200 and 2300. The eventual

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counting operation is greatly eased by the fact that within each 28-year repeat block each day appears as the 13th of the month the same number of times, i.e. 28 × 12 ÷ 7 = 48 times. The inclusion of the 13ths from these blocks into the totals, therefore, presents no problem.

At this point, it is convenient to tabulate the number of occurrences of each day as the 13th underneath each of the seven different column headings in Table 1 – see Table 2.

		Column heading						
		M	Tu	W	Th	F	Sa	Su
Number	M	2	1	2	1	2	2	2
of days in	Tu	2	2	1	2	1	2	2
column	W	2	2	2	1	2	1	2
with the	Th	2	2	2	2	1	2	1
heading	F	1	2	2	2	2	1	2
indicated	Sa	2	1	2	2	2	2	1
	Su	1	2	1	2	2	2	2

TABLE 2

3. The 400 columns are now sorted (hypothetically) into sequences of the lesser repeat cycle of 28 years and those between one such sequence and the next, as dictated by the occurrence of the non-leap years (2100, 2200, 2300). The governing factor is that a non-leap year cannot be included in a 28-year cycle. The results, showing the starting day for the 1st of March of each particular year, are as follows:

2014 (Sa) – 2097 (F)	3	28-year cycles
2098 (Sa) – 2100 (M)	3	individual columns
2101 (Tu) – 2184 (M)	3	28-year cycles
2185 (Tu) – 2200 (Sa)	16	individual columns
2201 (Su) – 2284 (Sa)	3	28-year cycles
2285 (Su) – 2300 (Th)	16	individual columns
2301 (F) – 2412 (Th)	4	28-year cycles
2413 (F)	1	individual column

These are totals of thirteen 28-year cycles and 36 individual columns.

4. The numbers of the different starting days in the 36 individual columns are noted. Each of these numbers is multiplied by the corresponding figure given in Table 2 above to give the number of times each day is the 13th of the month in the 36 years represented by the individual columns. (The other occurrences, i.e. those in the 28-year cycles, will be the same for each day –

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see later.) The results are given in Table 3.

		Column heading								
		M	Tu	W	Th	F	Sa	Su		
Number of starting days in 36 individual columns		4	6	5	5	6	4	6	Total	Baxter*
Number of days dated 13th of month	M	8	6	10	5	12	8	12	61	13
	Tu	8	12	5	10	6	8	12	61	13
	W	8	12	10	5	12	4	12	63	15
	Th	8	12	10	10	6	8	6	60	12
	F	4	12	10	10	12	4	12	64	16
	Sa	8	6	10	10	12	8	6	60	12
	Su	4	12	5	10	12	8	12	63	15

TABLE 3

* Baxter's figures are numbers in excess of an unstated common basis.

It can be seen that (by the shortest of heads) the 13th of a month falls more frequently on a Friday than on any other day. The derived numbers correspond in ranking precisely with those of Baxter, highlighting the remarkable abilities of this boy.

Absolute figures for the occurrences of the different days as the 13th of a month require consideration of the thirteen 28-year cycles mentioned in stages 3 and 4 above. It is readily shown that each day occurs 48 times as the 13th of a month in a single 28-year cycle; in the thirteen cycles there are therefore $48 \times 13 = 624$ such days and this figure must be added to the 'Total' figures in Table 3 to give absolute values. There are therefore 688 Friday 13ths in the 400-year cycle. (An approximate figure can be obtained as the number of months in 400 years divided by 7 which is 686.)

Following the work outlined above, a means of identifying the months of a year in which the 13th will be a Friday was developed, based on a graphical technique devised earlier [3]. The resulting nomogram is shown in Figure 1.

In constructing the nomogram the year (e.g. 2019) was considered in two parts viz. the century number C (i.e. 20) and the year number Y (i.e. 19). The horizontal axis of the first grid is the year number Y (< 100); this has a 28-year repeat period. The vertical axis of the first grid is the quantity $(5Y/4)$ (i.e. the number of 4's in 5 times Y) less multiples of 7. (The lines linking successive sets of four points in the grid are included for convenience only,)

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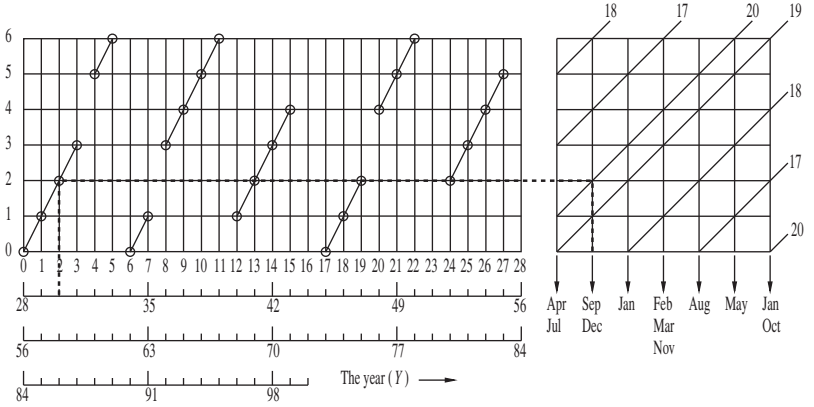


FIGURE 1: The nomogram

The diagonal lines in the second grid are labelled 17, 18, 19, 20. These are the century numbers C; they can be increased indefinitely by multiples of 4 e.g. 21, 22, 23, 24. Each of the vertical lines in this grid extends beyond the grid to a group of one, two or three months.

In order to determine the months in a particular year for which the 13th falls on a Friday, one enters the first grid at the stated year and moves up to the marked point in the grid. From there, one moves horizontally across to the appropriate diagonal in the second grid and thence down to give the required month(s). An example marked with a broken line in the figure shows that in 2030 the 13th will be a Friday in the months of September and December.

The nomogram is less than perfect. Attention is required when the predicted months in a leap year are sought, as follows.

(a) If the predicted months are Feb, Mar, Nov, then delete February.

(b) If the predicted months are Jan, Oct, then delete January.

(c) If the predicted month is Aug, then add February.

(d) If the predicted months are Apr, Jul, then add January.

References

1. S. R. Baxter, Friday the 13th, Letters to the Editor, *The Times* (4 March, 1970.)

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