Probability in the Engineering and Informational Sciences, **20**, 2006, 183–193. Printed in the U.S.A.

MAINTENANCE OPTIMIZATION OF EQUIPMENT BY LINEAR PROGRAMMING

ARCHANA JAYAKUMAR
Midwest ISO
Carmel, IN 46032

SOHRAB ASGARPOOR
Electrical Engineering Department
University of Nebraska–Lincoln
Lincoln, NE 68588-0511
E-mail: sasgarpoor1@unl.edu

Optimal levels of preventive maintenance performed on any system ensures cost-effective and reliable operation of the system. In this paper a component with deterioration and random failure is modeled using Markov processes while incorporating the concept of minor and major preventive maintenance. The optimal mean times to preventive maintenance (both minor and major) of the component is determined by maximizing its availability with respect to mean time to preventive maintenance. Mathematical optimization programs Maple 7 and Lingo 7 are used to find the optimal solution, which is illustrated using a numerical example. Further, an optimal maintenance policy is obtained using Markov Decision Processes (MDPs). Linear Programming (LP) is utilized to implement the MDP problem.

1. INTRODUCTION

Power system equipment (transformers, circuit breakers, etc.) are subject to deterioration due to factors such as aging, continuous usage, and so forth. As a result, the probability of service interruption due to component failure increases. Maintenance can be perceived as a combination of different actions such as inspection, repair, and/or replacement. The purpose of maintenance is to extend the component lifetime or at least the mean time until the next failure [6].

© 2006 Cambridge University Press 0269-9648/06 $16.00

https://doi.org/10.1017/S0269964806060128 Published online by Cambridge University Press

In traditional maintenance approaches, predefined activities were carried out at regular intervals. This approach was not cost-effective and did not extend component lifetime as much as possible [7]. In the last few years, utilities have employed flexible programs based on an analysis of needs and priorities or on a study of information obtained through predictive maintenance. However, this approach, referred to as reliability-centered maintenance (RCM), is considered heuristic, and its application requires experience and involves time-consuming data collection [3].

Recently, however, several mathematical models are being utilized in maintenance scheduling. In the literature on maintenance optimization models it is stated that "there are a number of case studies published which show that mathematical models are a good means to achieve both effective and efficient maintenance" [5]. In power systems, Markov chains have been extensively applied to the mathematical modeling of reliability and maintenance problems. The problem of replacement or overhaul of equipment, which deteriorates with usage, is one of the standard applications of Markov processes.

In previous studies, preventive maintenance models incorporated transitions from deterioration stage to a single type of maintenance state [4,9]. Based on a two-state transition model, the component model also incorporates deterioration and random failure. From each deterioration stage, there is a transition to the minimal maintenance state, and the minimal preventive maintenance sets back the deterioration process by one step.

However, depending on the level of deterioration, there is a scope for different levels of maintenance [6]. In this paper a component with deterioration and random failures is modeled using Markov processes. Two types of preventive maintenance strategies, namely minor maintenance and major maintenance, are incorporated in the component model. The model is solved for the optimal values of mean times at which to perform maintenance (both minor and major) by maximizing the availability of the component. Maple 7 and Lingo 7 are used to find the optimal solution.

In addition, an optimal maintenance policy is determined using Markov decision processes (MDPs) [8]. Based on expected returns, at each deterioration state the decision is made for the best choice of action (either none or minor or major maintenances). Thus, the optimal maintenance policy describes the action to be taken at each state and yields the maximum expected return. Linear Programming (LP) is utilized to implement the MDP problem and obtain an optimal maintenance policy

The following notation is used in this paper:

K Number of levels of deterioration

D_i Deterioration stage of component, where $i = 1,2,\ldots,k$

f_0 Poisson (random) failure state

f_1 Deterioration failure state

M_i Minor maintenance state of component

https://doi.org/10.1017/S0269964806060128 Published online by Cambridge University Press

MM_i Major maintenance state of component

R Number of stages in the minimal preventative maintenance process before preventive maintenance is started

I Stage of deterioration of the component, $i = 1,2,3,\ldots,k$

J Stage in minimal maintenance process, $j = 1,2,\ldots,r+1$; stage $j = r+1$ indicates that the component is in minimal preventive maintenance

$P(i,j)$ Steady state probability that the component is in state (i,j)

λ_0^{-1} Mean time to random failure

λ_1^{-1} Mean time to deterioration failure

λ_m^{-1} Mean time to minor maintenance

$\lambda_m^{1\,-1}$ Mean time to major maintenance

μ_0^{-1} Mean time to repair random failure

μ_1^{-1} Mean time to repair deterioration failure

μ_m^{-1} Mean duration of minor maintenance

$\mu_m^{1\,-1}$ Mean duration of major maintenance

$A(\lambda_m)$ Availability of component

$1/\lambda_m^*$ Optimal value of mean time to minor maintenance

$1/\lambda_m^{1\,*}$ Optimal value of mean time to major maintenance

P_i Steady state probability that the component is in state i and not undergoing minor or major maintenance, $i = 1,2,\ldots,k, f_0, f_1$

$\sum_x$ Denotes the negative of the sum of all the remaining elements in row x

$\mu_q^{-1} = h_q \mu_m^{-1}; \quad \mu_q^{1\,-1} = h_q^1 \mu_m^{1\,-1}$ h_q and h_q^1 are multiplying factors chosen arbitrarily to vary the mean duration of maintenance (either minor or major) while the component is restored to different improved stages

https://doi.org/10.1017/S0269964806060128 Published online by Cambridge University Press

2. THE PROPOSED METHOD

Based on the two-state model, the component model with deterioration and random failure is developed using Markov processes. Transitions from the deterioration state to maintenance states (both minor and major) are also incorporated. Deterioration (D) of the component is modeled as occurring in k discrete steps. The failure types, namely deterioration failure (F_1) and random failure (F_0), are considered. The state space diagram is shown in Figure 1.

Preventive maintenance leads to an improvement in the deterioration chain; however, allowances are made for the cases where no improvement is achieved. For instance, in the case of minor preventive maintenance being performed, the component improves from stage i to stage $i - 1$ of deterioration or remains in the same deterioration stage i. In the case of major preventive maintenance being performed, the component improves from stage i to stage $i - 2$ of deterioration, to stage $i - 1$ of deterioration, or remains in the same deterioration stage I [1]. If the component is in stage 1 of deterioration, it remains in that stage on completion of either minor or major preventive maintenance.

As a result of preventive maintenance, the probabilities of transitions from minor or major maintenance states to deteriorations states $D_1, D_2, \ldots,$ or D_k are different.

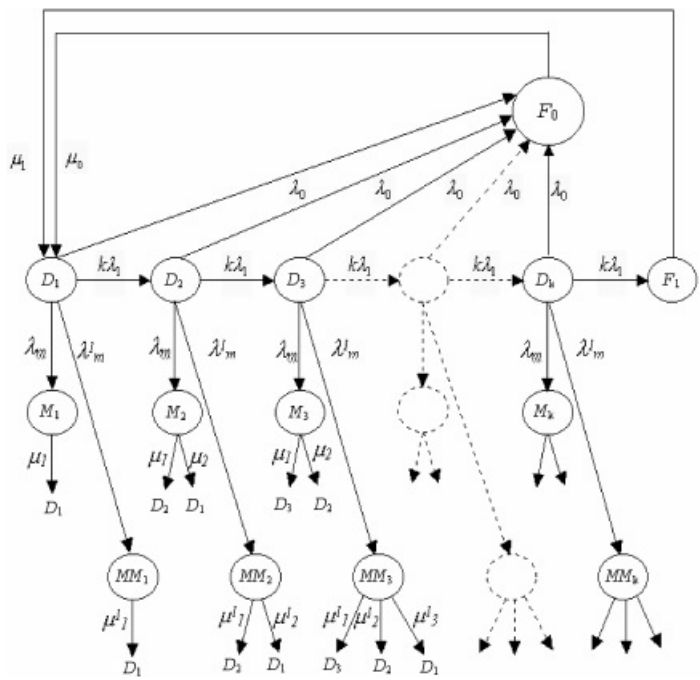


FIGURE 1. State space diagram for the component model.

https://doi.org/10.1017/S0269964806060128 Published online by Cambridge University Press

Transitions from each deterioration stage to both minor and major maintenance states occur at rates λ_m and λ_m^1 correspondingly. For simplicity, consider $\lambda_m = k_\lambda \lambda_m^1$, where k_λ is the proportionality constant. The mean duration to minor maintenance and major maintenance are $1/\mu_m$ and $1/\mu_m^1$, respectively. Repairs following random failure and deterioration failure are completed at rates of μ_0 and μ_1, respectively.

The values of λ_0, λ_1, μ_0, μ_1, μ_m, and μ_m^1 are obtained from the past outage statistics and maintenance practices, and they represent average values. The steady state solution of the component model is obtained by using a recursive approach. The recursive algorithm computes the steady state probabilities of the component [10]. The availability $A(\lambda_m)$, which is defined as the probability that the component is in service, is given by $A(\lambda_m) = P_1 + P_2 + P_3$ where P_1, P_2, and P_3 are the steady state probabilities of the component being in service.

The optimal values of mean time to maintenance (minor and major) are determined by maximizing the availability of the component. Mathematical optimization programs such as Maple 7 and Lingo 7 programs can be used for the optimization of the model. The solution yields the optimal mean time to minor maintenance $(1/\lambda_m^*)$, and from the relation $\lambda_m = k_\lambda \lambda_m^1$, the optimal mean time to major maintenance $(1/\lambda_m^{1\,*})$ is obtained.

3. MARKOV DECISION PROCESSES

Using the state probabilities found in the previous section, the optimal maintenance policy can be determined by setting up the component model as a MDP. At every deterioration stage, a decision process involving a choice from three possible actions, namely do nothing, do minor maintenance, and do major maintenance, is made. The objective is to obtain an optimal policy to select the best action in each state in order to maximize the long-run average return.

In general, an MDP comprises the following:

1. State space, $S = \{s_i \mid, i = 1,2,\ldots,n\}$, where n is assumed to be finite;
2. Action space that defines actions that may be taken over a state space, $A = \{A(s) \mid \forall\ s \in S\}$, where $A(s)$ is a finite set of actions defined on states.
3. Probabilistic state transition function, $P(i,k,j)$, which describes the probability of making a state transition from any one arbitrary state i to a state j_j, when an action k defined on $A(i)$ is taken.
4. Reward function (or cost function) $R(i,k,j)$ over the problem space, which specifies an instant reward that will be received after an action is performed (under a corresponding state transition). These values are predicted from the cost of various maintenance activities and/or their consequences.

A decision rule prescribes a procedure for action selection in each state at a specified decision epoch [8]. A policy is a sequence of decision rules to be used at all decision epochs for a specified time horizon that can be finite or infinite. A stationary policy is a policy where the decision rule is the same in every decision epoch.

https://doi.org/10.1017/S0269964806060128 Published online by Cambridge University Press

If $P(j|i,k)$ represents the transition probability from state i to state j as a result of action k, and $r(i,k,j)$ represents the return that results due to a transition from state i to state j as a result of action k, then the expected return as a result of action k while in state i is given by

$$r(i,k) = \sum_{j\in\{S\}} P(j|i,k)r(i,k,j).$$

Based on the state of a process at any time and the action chosen, an expected reward is earned and the probability distribution for the next state is determined. The objective is to choose a policy that maximizes the expected value of the sum of the rewards earned over a given finite time period. In this paper, Linear Programming is used to determine the optimal maintenance policy.

Linear programming algorithm [8]

Step 1. Apply the simplex method to compute an optimal basic solution (x_{ia}^*) to the linear program.

Minimize

$$\sum_{i\in S}\sum_{a\in A(i)} c_i(a)x_{ia}$$

subject to

$$\sum_{a\in A(i)} x_{ja} - \sum_{i\in S}\sum_{a\in A(i)} p_{ij}(a)x_{ia} = 0, \quad j \in S,$$

$$\sum_{i\in S}\sum_{a\in A(i)} x_{ia} = 1, \qquad x_{ia} \geq 0,\ i \in S,\ a \in A(i).$$

Step 2. Start with the nonempty set,

$$T = \left\{ i \Big| \sum_{a\in A(i)} x_{ia}^* > 0 \right\}$$

and, for any state $i \in T$, set the decision $R_i^* = a$ for some a such that $x_{ia}^* > 0$.

Step 3. If $T = S$, then stop the algorithm with the average cost optimal policy R^*. Otherwise, determine some state $i \notin T$ and action $a \in A(i)$ such that $p_{ij}(a) > 0$ for some $j \in T$ and set $R_i^* = a$ and $T = T \cup \{i\}$. Repeat Step 3.

4. NUMERICAL EXAMPLE

The implementation of the model is illustrated with the following example. Consider a unit for which the average random failure occurs once in 500 days and that the repair lasts for 7 days. If the unit is not maintained, it would fail from deterioration on an average of every 1000 days and the mean repair time is 14 days. Also, the minor maintenance would take an average of half a day and the major maintenance would take an average of 1 day. Assume that all times are exponentially dis-

https://doi.org/10.1017/S0269964806060128 Published online by Cambridge University Press

tributed and the number of stages of deterioration of the unit is 3. Hence, the input parameters are as follows:

$$k = 3 \qquad \mu_1^{-1} = 14 \text{ days}$$
$$\lambda_0^{-1} = 500 \text{ days} \qquad \mu_m^{-1} = 0.5 \text{ day}$$
$$\lambda_1^{-1} = 1000 \text{ days} \qquad \mu_m^{1\,-1} = 1.0 \text{ day}$$
$$\mu_0^{-1} = 7 \text{ days} \qquad k_\lambda = 2$$

The steady state equations and the input parameters are utilized to obtain the optimal solution using both Maple 7 and Lingo 7 programs. The results obtained are as given below.

In order to find the optimal policy, the best action must be chosen in each state. Action I refers to "do nothing," action II refers to "do minor maintenance," and action III refers to "do major maintenance."

State space:

$$S = \{D_1, D_2, D_3, M_1, M_2, M_3, MM_1, MM_2, MM_3, F_0, F_1\},$$

Action space:

$$A = \{\text{Do nothing (I), Do minor maintenance (II), Do major maintenance (III)}\}.$$

Actions I, II, and III can be chosen in states D_1, D_2, and D_3, whereas only Action I is available in states M_1, M_2, M_3, MM_1, MM_2, MM_3, F_0, and F_1. Therefore, the action sets are as follows:

$kD_1 = \{\text{I, II, III}\}$, $kD_2 = \{\text{I, II, III}\}$, $kD_3 = \{\text{I, II, III}\}$

$kM_1 = \{\text{I}\}$, $kM_2 = \{\text{I}\}$, $kM_3 = \{\text{I}\}$, $kMM_1 = \{\text{I}\}$, $kMM_2 = \{\text{I}\}$, $kMM_3 = \{\text{I}\}$, $kF_0 = \{\text{I}\}$, $kF_1 = \{\text{I}\}$

$P(j|i,k)$ = Probability that state i transits to state j as a result of action k

$r(i,k,j)$ = Return due to transition from state i to state j as a result of action k

$r(i,k)$ = Expected return for transition from state i as a result of action $k = \sum_{j\in\{S\}} P(j|i,k)r(i,k,j)$.

The returns are assigned based on the level of deterioration and maintenance state. For instance, the reward for a transition from D_1 to D_2 is chosen to be 900 and that from D_2 to D_3 is 800. The reward decreases because the component is deteriorating and does not work as efficiently. The reward for the component to stay in states M_1, M_2, and M_3 is -100; for states MM_1, MM_2, and MM_3, it is -300; for state F_0, it is -1000; for state F_1, it is -3000 (Case I). The returns in these states are based on the duration of unavailability of the component; the longer the component is out of service either for maintenance or repair of failures, the greater the loss. Also, the return for a transition from any of the maintenance states (minor or major) to state D_1 is greater than the return resulting from a transition to state D_2. This is because there is more improvement in the deterioration chain in state D_1

https://doi.org/10.1017/S0269964806060128 Published online by Cambridge University Press

than in state D_2. Likewise, the return for a transition from any of the maintenance states (minor or major) to state D_2 is greater than the return resulting from a transition to state 3. Also, positive values of returns indicate reward earned and negative values of returns indicate cost incurred. The probabilities are calculated using the transition rates in the component model. Using the transition probabilities and rewards, the cost matrix is obtained as shown in Table 1. The optimal policy algorithms are used to find the optimal maintenance policy for the component model. An APL language software called MDP is utilized for Linear Programming and solving the model [2].

The model was also tested with a different set of values for rewards (Case II). The rewards for the transition from failure state F_0 to D_1 as a result of action {do nothing} is chosen to be 3000 and the reward for transition from failure state F_1 to D_1 as a result of action {do nothing} is also chosen to be 3000. All other values are chosen to be the same as before.

5. RESULTS

Table 2 provides the steady state probabilities (in percent) for different states of the component. The availability of the component is calculated to be 0.9814316. The optimal values of the mean times to minor preventive maintenance and major preventive maintenance for the component model are calculated as 86 days and 172 days, respectively, for a mean time to random failure of 500 days. As expected, the availability decreases as the mean times to preventive maintenance (minor and major) exceed the optimal value.

Figure 2 illustrates the effect of varying mean time to random failure (λ_0^{-1}) for the component model. For a specified value of λ_0^{-1}, the mean times to maintenance corresponding to maximum availability can be determined. For example, for a mean time to random failure (λ_0^{-1}) of 500 days, the optimal values of the mean times to minor maintenance and major maintenance are 86 days and 172 days, respectively, and the maximum availability of the component is 0.98143. The availability decreases as the mean times to maintenance (minor and major) exceed the optimal value, as expected. The optimal values for mean times to maintenance (minor and major) and corresponding maximum availability by varying mean times to random failure (λ_0^{-1}) can also be seen in Figure 2.

By increasing λ_0^{-1}, the optimal values of mean times to maintenance (both minor and major) decrease. It is also observed that the availability of component increases as λ_0^{-1} increases. This is because by increasing λ_0^{-1} (i.e., decrease in random failure rate), more frequent maintenance is needed to bring the component to the new status and, hence, there is higher component availability. By increasing the value of μ_0^{-1}, availability reduces, due to the fact that as more time is spent on repairing random failures (i.e., increasing μ_0^{-1}), the availability of the component reduces. If all parameters remain unchanged, increasing μ_1^{-1} will decrease the value of λ_m^{-1} and will reduce the availability of the component. This is because by increasing the repair time for deterioration failure (i.e., increasing μ_1^{-1}), the availability of the component reduces.

https://doi.org/10.1017/S0269964806060128 Published online by Cambridge University Press

https://doi.org/10.1017/S0269964806060128 Published online by Cambridge University Press

TABLE 1. Transition Probabilities and Returns (Cost Matrix)

State	Action	Transition Probabilities, $P(j\|i,k)$											Returns, $r(i,k,j)$										Expected Return,
i	k	D_1	D_2	D_3	M_1	M_2	M_3	MM_1	MM_2	MM_3	F_0	F_1	D_1	D_2	D_3	M_1	M_2	M_3	MM_1	MM_2	MM_3	F_0	F_1	$r(i,k)$
D_1	I	0	0.6	0	0	0	0	0	0	0	0	0		900								−1000		140
	II	0	0	0	1	0	0	0	0	0	0	0				−100								−100
	III	0	0	0	0	0	0	1	0	0	0	0							−300					−300
D_2	I	0	0	0.6	0	0	0	0	0	0	0	0			800							−1000		80
	II	0	0	0	0	1	0	0	0	0	0	0					−100							−100
	III	0	0	0	0	0	0	0	1	0	0	0								−300				−300
D_3	I	0	0	0	0	0	0	0	0	0	0	1										−1000	−3000	−2200
	II	0	0	0	0	0	1	0	0	0	0	0						−100						−100
	III	0	0	0	0	0	0	0	0	1	0	0									−300			−300
M_1	I	1	0	0	0	0	0	0	0	0	0	0	1000			−100								1000
M_2	I	0.4	0.6	0	0	0	0	0	0	0	0	0	2500	1000			−100							1600
M_3	I	0	0.4	0.6	0	0	0	0	0	0	0	0		900	500			−100						660
MM_1	I	1	0	0	0	0	0	0	0	0	0	0	1000						−300					1000
MM_2	I	0.909	0.091	0	0	0	0	0	0	0	0	0	2000	500						−300				1863.65
MM_3	I	0.769	0.154	0.077	0	0	0	0	0	0	0	0	3000	2000	1000						−300			2692.1
F_0	I	1	0	0	0	0	0	0	0	0	0	0	1000									−1000		1000
F_1	I	1	0	0	0	0	0	0	0	0	0	0	1000										−3000	1000

TABLE 2. Steady State Probabilities (in Percent)

P_1	P_2	P_3	P_{M_1}	P_{M_2}	P_{M_3}	P_{MM_1}	P_{MM_2}	P_{MM_3}	P_{F_0}	P_{F_1}
69.115	22.798	6.23	0.134	0.026	0.0072	0.04	0.011	0.0028	1.374	0.262

Optimal Policy

Using MDPs, the optimal policy obtained in deterioration states D_1, D_2, and D_3 consists of actions {do nothing}, {do minor maintenance}, and {do major maintenance}, respectively, for Case I. The optimal policy yields an average cost per stage equal to 634, as shown in Table 3.

In Case II, it is assumed that the reward for transition from failure states to D_1 as a result of action {do nothing} is 3000, as opposed to 1000 (in Case I). All other values are chosen to be the same as in Case I. The results for Case II consist of actions {do nothing}, {do nothing}, and {do major maintenance} for states D_1, D_2, and D_3, respectively. The gain for this policy is 1009.

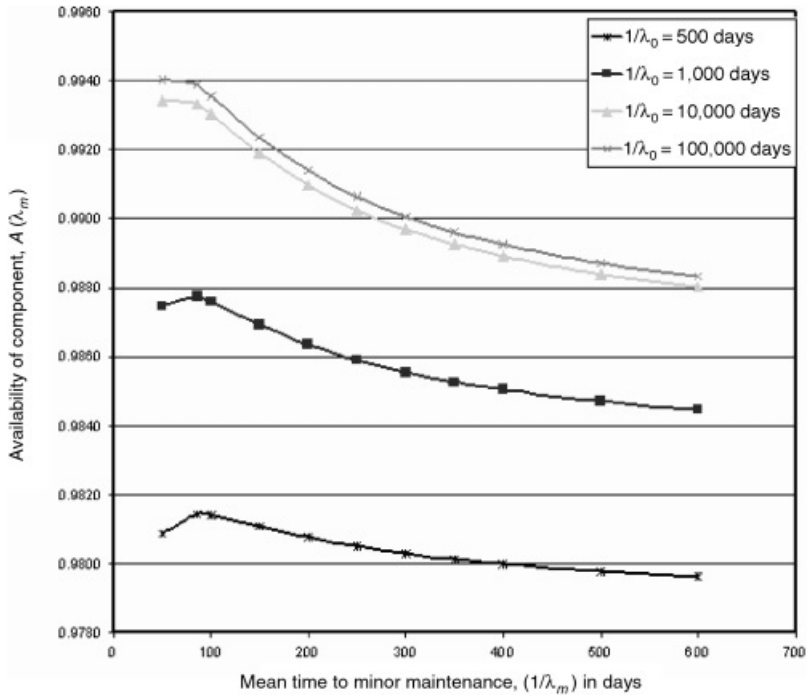


FIGURE 2. Effect of varying mean time to random failure. Note: For $1/\lambda_1 = 1000$ days; $1/\mu_1 = 14$ days.

https://doi.org/10.1017/S0269964806060128 Published online by Cambridge University Press

TABLE 3. MDP Results (Case I)

Start Policy				Revised Policy		
D_1	D_2	D_3	Gain	D_1	D_2	D_3
I	I	I	133.784	I	I	II
I	I	II	592.708	I	II	I
I	II	III	634.091	I	II	III

6. CONCLUSION

For the component model, with three levels of deterioration, the optimal maintenance policy indicates that in the initial level of deterioration, there is no need for preventive maintenance, whereas in the intermediate level of deterioration, minor maintenance is to be carried out, and in the crucial level of deterioration, major maintenance is to be performed. This optimal policy yields maximum long-run average return and contributes to the reliable operation of the component.

A more realistic maintenance model has been developed by incorporating deterioration and both types of failure. The solution for determining optimal mean times to preventive maintenance is fairly simple and can be done through simple nonlinear optimization algorithms.

References

1. Anders, G.J. (1990). *Probability concepts in electric power systems*. New York: Wiley.
2. APL website, http://asrl.ecn.uiowa.edu/dbricker/APL_Software.html.
3. Asgarpoor, S. & Doghman, M. (1999). A maintenance optimization program for utilities' transmission and distribution systems. In *Proceedings of the 1999 North American Power Symposium*, pp. 454–459.
4. Chan, G.K. (1998). Preventive maintenance with Markov processes. M.E. thesis, University of Nebraska–Lincoln, Lincoln NE.
5. Dekker, R. (1996). Applications of maintenance optimization models: A review and analysis. *Reliability Engineering and System Safety* 51: 229–240.
6. Endrenyi, J., Anders, G.J., & Leite da Silva, A.M. (1998). Probabilistic evaluation of the effect of maintenance on reliability: An application. *IEEE Transactions on Power Systems* 13(2): 576–583.
7. Endrenyi, J., et al. (IEEE/PES Task Force on Impact of Maintenance Strategy on Reliability). (2000). The present status of maintenance strategies and the impact of maintenance on reliability. *IEEE Transactions on Power Systems* 16(4): 638–646.
8. Puterman, M.L. (1994). *Markov decision processes: Discrete stochastic dynamic programming*. New York: Wiley.
9. Sim, S.H. & Endrenyi, J. (1988). Optimal preventive maintenance with repair. *IEEE Transactions on Reliability* 37(1): 92–96.
10. Sim, S.H. & Endrenyi, J. (1993). A failure repair model with minimal and major maintenance. *IEEE Transactions on Reliability* 42(1): 134–139.

https://doi.org/10.1017/S0269964806060128 Published online by Cambridge University Press