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Stability of molecular clouds in partially ionized self-gravitating space plasmas

G. JACOBS and P.K. SHUKLA

Institut für Theoretische Physik IV, Fakultät für Physik und Astronomie,
Ruhr–Universität Bochum, D-44780 Bochum, Germany
(ps@tp4.ruhr-uni-bochum.de)

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Abstract. We investigate the gravitational stability of partially ionized astrophysical plasmas, embedded in a large-scale magnetic field. The collisions between the charged particles and neutrals are incorporated into our model, and we study the influence of the ion–neutral collisions on the gravitational stability of a large space plasma, e.g. a large molecular cloud, and on the relevant growth/damping rates. The effects of the ion–neutral friction on the growth rate of a gravitationally unstable space plasma are presented for a range of possible wavenumbers.

1. Introduction

It is widely thought that stars are formed as a result of the gravitational (Jeans) collapse of dense clumps in interstellar clouds. In the Milky Way, stars account for roughly 90% of the visible material, and the remaining 10% is then distributed among stars in the form of interstellar gas and fractional amounts of small interstellar dust particles. Spectroscopic observations have shown that the major part of the interstellar material is concentrated in giant molecular clouds, i.e. clouds with a total mass that exceeds $10^5\ M_\odot$ [1]. These giant molecular clouds are typically over 1000 times denser than the interstellar gases which surround them and can reach up to 300 lightyears in diameter, making them the most massive objects in the Galaxy.

Large molecular clouds contain conglomerates of plasma and neutral gas where the densities are thousands of times higher than the average of the cloud and, in particular in these densest parts of the cloud, substantial self-gravitational forces among the cloud particles are present [2, 3]. Therefore, in dense large molecular clouds the possibility of a gravitational collapse arises, thus providing the preferred scene for active star formation in the Galactic Disk [4–7].

Giant molecular clouds mainly consist of molecular hydrogen and helium but also contain smaller amounts of heavier gases, which may be partially ionized. Furthermore, giant clouds are usually permeated by a magnetic field, the strength of which is typically observed to range from 10^{-10} to 10^{-7} Tesla [8]. The presence of the magnetic field may inhibit the contraction of the cloud [5, 9].

In this paper, we consider the model of a large astrophysical magnetoplasma containing electrons, ions and neutral particles, and investigate the effect of the magnetic field and the charged particle–neutral collisions on the Jeans instability. The presence of the magnetic field affects the dynamics of electrons and ions. On

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the other hand, ions and neutrals are influenced by the self-gravitational force and neutrals interact with the charged particles through drag forces. We derive a new dispersion relation by using the multifluid and Maxwell equations as well as Poisson's equation for the self-gravitating potential. We examine the influence of the magnetic field and collisions between the charged particles and neutrals on the growth/damping rates of the Jeans instability involving coupled Jeans, Alfvén, and sound/acoustic modes. In particular, the influence of the ion–neutral friction on the growth rate of the Jeans instability is investigated over a range of possible wavenumbers that are of relevance to interstellar molecular clouds.

2. Governing equations

We consider a multi-component partially ionized astrophysical plasma composed of electrons, ions, and neutrals in an ambient magnetic field $B_0\hat{\mathbf{z}}$, where B_0 is the strength of the magnetic field and $\hat{\mathbf{z}}$ is the unit vector along the z-axis. Further, we consider $\nabla = \partial/\partial_x\hat{\mathbf{x}}$, as only the perturbations perpendicular to the ambient magnetic field can affect gravitational stability [10].

The equations governing the dynamics of low-frequency modes include the equations of continuity for each species α (with $\alpha = \mathrm{e}, \mathrm{i}, \mathrm{n}$)

$$\partial_{\mathrm{t}} n_\alpha + \nabla\cdot(n_\alpha \mathbf{v}_\alpha) = 0, \tag{2.1}$$

and the equations of motion

$$\rho_{\mathrm{e}}(\partial_{\mathrm{t}} + \mathbf{v}_{\mathrm{e}}\cdot\nabla)\mathbf{v}_{\mathrm{e}} = -en_{\mathrm{e}}(\mathbf{E} + \mathbf{v}_{\mathrm{e}}\times\mathbf{B}) - k_{\mathrm{B}}\nabla(T_{\mathrm{e}}n_{\mathrm{e}}) - \rho_{\mathrm{e}}\nu_{\mathrm{en}}(\mathbf{v}_{\mathrm{e}} - \mathbf{v}_{\mathrm{n}}) - \rho_{\mathrm{e}}\nu_{\mathrm{ei}}(\mathbf{v}_{\mathrm{e}} - \mathbf{v}_{\mathrm{i}}), \tag{2.2}$$

$$\rho_{\mathrm{i}}(\partial_{\mathrm{t}} + \mathbf{v}_{\mathrm{i}}\cdot\nabla)\mathbf{v}_{\mathrm{i}} = en_{\mathrm{i}}(\mathbf{E} + \mathbf{v}_{\mathrm{i}}\times\mathbf{B}) - k_{\mathrm{B}}\nabla(T_{\mathrm{i}}n_{\mathrm{i}}) - \rho_{\mathrm{i}}\nabla\psi - \rho_{\mathrm{i}}\nu_{\mathrm{in}}(\mathbf{v}_{\mathrm{i}} - \mathbf{v}_{\mathrm{n}}) - \rho_{\mathrm{i}}\nu_{\mathrm{ie}}(\mathbf{v}_{\mathrm{i}} - \mathbf{v}_{\mathrm{e}}), \tag{2.3}$$

$$\rho_{\mathrm{n}}(\partial_{\mathrm{t}} + \mathbf{v}_{\mathrm{n}}\cdot\nabla)\mathbf{v}_{\mathrm{n}} = -k_{\mathrm{B}}\nabla(T_{\mathrm{n}}n_{\mathrm{n}}) - \rho_{\mathrm{n}}\nabla\psi - \rho_{\mathrm{n}}\nu_{\mathrm{ni}}(\mathbf{v}_{\mathrm{n}} - \mathbf{v}_{\mathrm{i}}) - \rho_{\mathrm{n}}\nu_{\mathrm{ne}}(\mathbf{v}_{\mathrm{n}} - \mathbf{v}_{\mathrm{e}}), \tag{2.4}$$

where for each species α the quantities $n_\alpha, m_\alpha, v_\alpha, T_\alpha$ stand for the particle number density, mass, velocity and temperature, respectively, and k_{B} is the Boltzmann constant. The particle mass density is denoted by $\rho_\alpha = n_\alpha m_\alpha$. Furthermore, the collision frequencies $\nu_{\alpha\beta}$ and $\nu_{\beta\alpha}$ are interchangeable through the identity $\rho_\alpha\nu_{\alpha\beta} = \rho_\beta\nu_{\beta\alpha}$. The gravitational potential ψ is determined from Poisson's equation

$$\nabla^2\psi = 4\pi G(\rho_{\mathrm{i}} + \rho_{\mathrm{n}}). \tag{2.5}$$

We focus on low phase speed (in comparison with the speed of light) perturbations and neglect the displacement current, so that Maxwell's equations for our purposes are

$$\nabla\times\mathbf{B} \simeq \mu_0\mathbf{J} = \mu_0 e(n_{\mathrm{i}}\mathbf{v}_{\mathrm{i}} - n_{\mathrm{e}}\mathbf{v}_{\mathrm{e}}), \tag{2.6}$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}, \tag{2.7}$$

$$\nabla\cdot\mathbf{E} = \frac{e(n_{\mathrm{i}} - n_{\mathrm{e}})}{\varepsilon_0}, \quad \nabla\cdot\mathbf{B} = 0, \tag{2.8}$$

where ε_0 and μ_0 are the electric permittivity and magnetic permeability of free space, respectively.

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For astrophysical plasmas, we typically have $\nu_{\rm en} \ll \omega_{\rm ce}$ and $\nu_{\rm in} \ll \omega_{\rm ci}$, where $\omega_{\rm ce}$ and $\omega_{\rm ci}$ are the electron and ion gyrofrequencies, respectively. Assuming quasineutrality ($n = n_{\rm i} = n_{\rm e}$), in addition to the electron and ion motion equations, i.e. (2.2) and (2.3), respectively, we have

$$\partial_{\rm t}\mathbf{v}_{\rm i} \simeq \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{\mu_0 n_0 m_{\rm i}} - \frac{C_{\rm s}^2}{n_0}\nabla n - \nabla\psi - \nu_{\rm in}(\mathbf{v}_{\rm i} - \mathbf{v}_{\rm n}), \tag{2.9}$$

where $C_{\rm s}^2 = k_{\rm B}(T_{\rm e} + T_{\rm i})/m_{\rm i}$.

3. Dispersion relation

We now derive the dispersion relation for low-frequency electromagnetic waves in self-gravitating partially ionized plasmas by linearizing the governing equations in Sec. 2. Upon linearization, the x and y components of (2.9) read

$$\partial_{\rm t} v_{{\rm i}x} = -\frac{V_{\rm A}^2}{B_0}\partial_x B_z - \frac{C_{\rm s}^2}{n_0}\partial_x n_1 - \partial_x\psi_1 + \nu_{\rm in}(v_{{\rm n}x} - v_{{\rm i}x}), \tag{3.1}$$

$$\partial_{\rm t} v_{{\rm i}y} = -\nu_{\rm in}(v_{{\rm i}y} - v_{{\rm n}y}), \tag{3.2}$$

where the Alfvén speed is $V_{\rm A} = B_0/\sqrt{\mu_0 n_{\rm i0} m_{\rm i}}$, and we denote the small perturbation n_1 in the unperturbed plasma number density n_0. On the other hand, the x and y components of the equation of motion for the neutrals (2.4) are, respectively,

$$\partial_{\rm t} v_{{\rm n}x} = -\frac{c_{\rm sn}^2}{n_{\rm n0}}\partial_x n_{\rm n1} - \partial_x\psi_1 + \nu_{\rm ni}(v_{{\rm i}x} - v_{{\rm n}x}), \tag{3.3}$$

$$\partial_{\rm t} v_{{\rm n}y} = -\nu_{\rm ni}(v_{{\rm n}y} - v_{{\rm i}y}), \tag{3.4}$$

where $n_{\rm n1}$ is the small density perturbation of the equilibrium neutral density $n_{\rm n0}$, and $c_{\rm sn} = \sqrt{k_{\rm B}T_{\rm n}/m_{\rm n}}$ the speed of sound for the neutral gas. The linearized gravitational Poisson equation can be expressed as

$$\partial_x^2\psi_1 = 4\pi G(n_1 m_{\rm i} + n_{\rm n1} m_{\rm n}), \tag{3.5}$$

and we note that here the 'Jeans swindle' [11] is assumed for the analysis of the gravitational stability. The presence of self-gravitational forces unavoidably brings about a degree of non-uniformity in molecular clouds. Because of the assumption of the Jeans swindle, the perturbations under consideration therefore have much smaller wavelengths than the inherent inhomogeneity lengthscale of the self-gravitating cloud.

Finally, Maxwell's equations yield

$$v_{{\rm e}x} = v_{{\rm i}x}, \tag{3.6}$$

$$-\partial_x B_z = \mu_0 e n_0 (v_{{\rm i}y} - v_{{\rm e}y}), \tag{3.7}$$

$$\partial_x E_y = -\partial_{\rm t} B_z, \tag{3.8}$$

$$\partial_x E_z = \partial_{\rm t} B_y. \tag{3.9}$$

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Hence, the equation of motion for the electrons (2.2) is of the form

$$\partial_t v_{ex} = -\frac{e}{m_e} E_x + \omega_{ce} v_{ey} - \frac{c_{se}^2}{n_0} \partial_x n_1 + \nu_{ei}(v_{ix} - v_{ex}), \tag{3.10}$$

$$\partial_t v_{ey} = -\frac{e}{m_e} E_y - \omega_{ce} v_{ex} + \nu_{ei}(v_{iy} - v_{ey}). \tag{3.11}$$

Finally, the continuity equations per species are

$$\partial_t n_{\alpha 1} + n_{\alpha 0} \partial_x v_{\alpha x} = 0. \tag{3.12}$$

Combination of (3.2) and (3.4) yields $v_{ny} = 0 = v_{iy}$. Furthermore, we consider wave frequencies $\omega \gg \nu_{ei}$. Upon Fourier transformation of all the previous expressions, the substitution of (3.6), (3.7), (3.8) and (3.12) into (3.11) leads to

$$\frac{B_z \varepsilon_0}{e} \left(k^2 c^2 + \omega_{pe}^2\right) = -\omega_{ce} n_1, \tag{3.13}$$

where B_z is the compressional magnetic field perturbation, c is the speed of light in vacuum, and ω_{pe} is the electron plasma frequency. Substituting (3.13), the continuity equations and the gravitational Poisson equation into expression (3.1), we finally obtain

$$\left[\omega(\omega + i\nu_{in}) - \frac{k^2 V_A^2}{1 + k^2 \lambda_c^2} - k^2 C_s^2 + \omega_{Ji}^2\right] \frac{n_1}{n_0} = \left[i\nu_{in}\omega - \omega_{Jn}^2\right] \frac{n_{n1}}{n_{n0}}, \tag{3.14}$$

where $\lambda_c = c/\omega_{pe}$ is the electron skin depth, $\omega_{J\alpha} = (4\pi G n_{\alpha 0} m_\alpha)^{1/2}$ is the Jeans frequency, and $n_{\alpha 0}$ represents the equilibrium densities for species α.

Analogously, (3.3) and the continuity equation for the neutrals yield

$$\left[\omega(\omega + i\nu_{ni}) - k^2 c_{sn}^2 + \omega_{Jn}^2\right] \frac{n_{n1}}{n_{n0}} = \left[i\nu_{ni}\omega - \omega_{Ji}^2\right] \frac{n_1}{n_0}. \tag{3.15}$$

Eliminating the density perturbations in (3.14) and (3.15), we obtain a dispersion relation

$$\left[\omega(\omega + i\nu_{in}) - \frac{k^2 V_A^2}{1 + k^2 \lambda_c^2} - k^2 C_s^2 + \omega_{Ji}^2\right] \left[\omega(\omega + i\nu_{ni}) - k^2 c_{sn}^2 + \omega_{Jn}^2\right]$$

$$= \left[\omega_{Ji}\omega_{Jn} - i\omega\nu_{ni} \frac{\omega_{Jn}}{\omega_{Ji}}\right]^2. \tag{3.16}$$

If one neglects the electron inertia and electron pressure, the dispersion relation can be reduced to those of Pudritz and/or Langer [12, 13]. For our purposes, we rewrite the dispersion relation (3.16) as

$$\omega^4 - A\omega^2 + B + i\tilde{\nu}_{ni}\omega[\omega^2 + C] = 0, \tag{3.17}$$

where $\tilde{\nu}_{ni} = \nu_{ni}(\omega_{Ji}^2 + \omega_{Jn}^2)/\omega_{Ji}^2$ and the other coefficients are

$$A = k^2 \left[C_s^2 + c_{sn}^2 + \frac{V_A^2}{1 + k^2 \lambda_c^2}\right] - \omega_{Ji}^2 - \omega_{Jn}^2,$$

$$B = \left[\frac{k^2 V_A^2}{1 + k^2 \lambda_c^2} + k^2 C_s^2 - \omega_{Ji}^2\right] \left[k^2 c_{sn}^2 - \omega_{Jn}^2\right] - \omega_{Ji}^2 \omega_{Jn}^2, \tag{3.18}$$

$$C = \omega_{Ji}^2 + \omega_{Jn}^2 - k^2 \frac{\left[C_s^2 + V_A^2/\left(1 + k^2 \lambda_c^2\right)\right] \omega_{Ji}^2 + c_{sn}^2 \omega_{Jn}^2}{\omega_{Ji}^2 + \omega_{Jn}^2}.$$

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For our further use, we now introduce the magnetosonic speed $V_{\rm ms} = (V_{\rm A}^2 + C_{\rm s}^2)^{1/2}$. The signs of the coefficients A, B and C are important for the analysis of the dispersion relation (3.17), and therefore it is necessary to determine the positive k^2-values that render the respective coefficients zero, namely

$$\begin{aligned} k_A^2 &\simeq \frac{\omega_{\rm Ji}^2 + \omega_{\rm Jn}^2}{V_{\rm ms}^2 + c_{\rm sn}^2}, \\ k_{\rm J}^2 &\simeq \frac{\omega_{\rm Jn}^2}{c_{\rm sn}^2}, \\ k_C^2 &\simeq \frac{\left(\omega_{\rm Ji}^2 + \omega_{\rm Jn}^2\right)^2}{V_{\rm ms}^2\omega_{\rm Ji}^2 + c_{\rm sn}^2\omega_{\rm Jn}^2}. \end{aligned} \tag{3.19}$$

For the determination of $k_A, k_{\rm J}$ and k_C we have assumed $\omega_{\rm Jn}^4 \ll |\omega_{\rm ce}|\omega_{\rm ci}\omega_{\rm Ji}^2$, $C_{\rm s}$, $c_{\rm sn} \ll V_{\rm A}, V_{\rm ms}$ and also $\omega_{\rm Ji}^2 c_{\rm sn}^2 \ll \omega_{\rm Jn}^2 C_{\rm s}^2$. We note that $k_A < k_C < k_{\rm J}$. We observe that a self-gravitating cloud becomes unstable for wavenumbers $k < k_{\rm J}$. Retaining collisions between ions and neutrals in the model will not change the instability criterion. However, the growth rate of a possible gravitational instability will be reduced when collisions between ions and neutral particles are more frequent.

We also note that our expressions for k_A, $k_{\rm J}$ and k_C are analogous to those obtained by Verheest et al. [14], if we formally replace the dust–acoustic speed there by the magnetosonic speed. They visualized the evolution of the wavemodes for increasing collision frequencies in a qualitative fashion by means of the root-locus method [15]. Here, we numerically analyze our dispersion relation (3.17) for parameters that are relevant to molecular clouds, and present the growth rate with and without collisions.

4. Influence of ion–neutral friction on the growth rate

We have established that the influence of ion–neutral collisions will diminish the growth rate of a gravitationally unstable plasma. Now we more closely investigate the peculiarities of the damping due to the friction between ions and neutrals. For this, we keep the collision frequency constant with a fixed value, $\nu_{\rm ni} = (\rho_{\rm i}\langle\sigma v\rangle)/(m_{\rm i} + m_{\rm n})$, where $\langle\sigma v\rangle$ is the ion–neutral collision rate coefficient [5, 16], which typically has a value $\langle\sigma v\rangle = 2\times10^{-15}$ m^3 s^{-1}. Since neutrals are considerably heavier than ions, the ion Jeans frequency will be considerably smaller than the neutral Jeans frequency, and thus $\tilde{\nu}_{\rm ni} \simeq \nu_{\rm in} \simeq n_{\rm n0}\langle\sigma v\rangle$.

Keeping the collision frequency fixed, we investigate the magnitude of the imaginary part of the roots (in ω) of the dispersion relation (3.17) for different wavenumbers, and make a comparison with the collisionless limit.

We note that for large molecular clouds, the relevant wavenumbers are so small that electron inertia effects are not significant ($k\lambda_{\rm c} \ll 1$) and we can simplify (3.18) to

$$\begin{aligned} A &\simeq k^2V_{\rm A}^2 - \omega_{\rm Jn}^2, \\ B &\simeq k^2V_{\rm A}^2\left(k^2c_{\rm sn}^2 - \omega_{\rm Jn}^2\right), \\ C &\simeq \omega_{\rm Jn}^2 - k^2c_{\rm sn}^2 - k^2\frac{V_{\rm A}^2\omega_{\rm Ji}^2}{\omega_{\rm Jn}^2}. \end{aligned} \tag{4.1}$$

For the neutrals, we assume $m_{\rm n} \simeq 10\, m_{\rm i}$, and for the particle densities, temperatures and the magnitude of the magnetic induction we use the following values:

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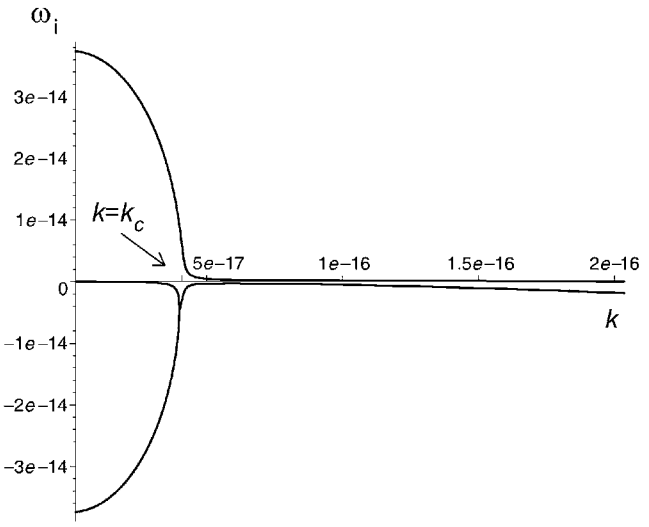


Figure 1. Imaginary parts $\omega_{\rm i}$ of the solutions of the full dispersion relation versus wavenumber.

$n_{\rm e0} = n_{\rm i0} \simeq 4.5 \times 10^4\ {\rm m}^{-3}$, $n_{\rm n0} \simeq 10^8\ {\rm m}^{-3}$, $T_{\rm n} \simeq 30\ {\rm K}$, $T_{\rm e} = T_{\rm i} \simeq 100\ {\rm K}$, $B_0 \simeq 1.3 \times 10^{-9}$ Tesla [17]. Accordingly, the collision frequency $\tilde{\nu}_{\rm ni}$ has a value $\tilde{\nu}_{\rm ni} \simeq 2 \times 10^{-7}$ Hz. For these data, $V_{\rm A} \simeq 5000 C_{\rm s}$, so that our earlier assumption $C_{\rm s} \ll V_{\rm A}, V_{\rm ms}$ is indeed justified, the pressure provided by the magnetic field sways the gravitational instability much more than the particle pressures.

For the above set of parameters, Fig. 1 displays the imaginary part of the frequency ω of the dispersion relation (3.17) versus wavenumber for unstable wavenumbers, i.e. for $k < k_{\rm J}$. The wavenumber range in Fig. 1 is chosen to vary from $k = 2\pi/(100$ parsec) to $k = 2\pi/(1$ parsec), so that very large molecular clouds of 1–100 parsec are kept in mind.

Figure 1 shows that there is an unstable solution for the entire wavenumber range of interest, but the magnitude of the growth rate decreases dramatically around $k = k_C$, and for $k > k_C$ the corresponding curve practically coincides with the k-axis in the figure. This unstable solution has no real frequency and naturally corresponds to a purely growing Jeans mode. As an illustration, we present some numerical estimates: for $k = 10^{-17}\ {\rm m}^{-1}$, the obtained collapse time $\tau = 1/({\rm Im}\,\omega) \simeq 0.9$ million years, for $k = 3 \times 10^{-17}\ {\rm m}^{-1}$ it is $\tau \simeq 1.2$ million years, whereas for $k = 5 \times 10^{-17}\ {\rm m}^{-1}$ we obtain $\tau \simeq 72$ million years, and finally for $k = 10^{-16}\ {\rm m}^{-1}$ the collapse time is $\simeq 212$ million years.

On the other hand, Fig. 1 shows two damped solutions, one of which has a damping rate that also decreases dramatically around $k = k_C$, whereas the other has a maximum damping rate at $k = k_C$. Both damped solutions practically coincide for wavenumbers $k > k_C$. There is another, heavily damped solution of (3.17) for the investigated wavenumber range, namely $\omega = -i\tilde{\nu}_{\rm ni}$, which is not depicted in Fig. 1.

Figure 1 illustrates that the growth rate reduces much more steeply when ion–neutral friction is included in the basic model. The difference in obtained growth rates between a model with and without friction is especially noticeable for the

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wavenumber range $k_C < k < k_J$, as the collisionless model fails to describe the rapid decline of the growth rate of the Jeans instability in the vicinity of k_C.

5. Conclusions

We have examined the Jeans instability in partially ionized self-gravitating astrophysical magnetoplasmas whose constituents are electrons, ions, and neutrals. We have derived a new dispersion relation by employing the hydrodynamic equations for the charged particles and neutrals, supplemented by the Ampère and Faraday laws and Poisson's equation for the self-gravitational potential, which affects the dynamics of both the ions and neutrals. The dispersion relation has been numerically analyzed for the plasma parameters of large molecular clouds in which the plasma β (the ratio of the plasma kinetic to magnetic energy densities) is pretty low. Our results reveal that the magnetic field and ion–neutral collisions are stabilizing factors for the Jeans instability in a partially ionized astrophysical plasma. Specifically, the inclusion of ion–neutral friction into a fluid-like description of the Jeans instability results in a drastic change of the obtained growth rates for wavenumbers corresponding to unstable lengthscales, namely wavenumbers $k < \omega_{Jn}/c_{sn}$. The obtained collapse times of a large cloud are typically much larger than those obtained in a collisionless model, especially in the wavenumber range $k_C < k < k_J$.

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