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NOTE

ON RANDOM MATCHING, MONETARY EQUILIBRIA, AND SUNSPOTS

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We study comparative statics results for the steady-state monetary equilibria of a simple random matching model of money with endogenous prices and no extrinsic uncertainty. Some of the results appear counterintuitive (both when take-it-or-leave-it offer or when Nash–Rubinstein bargaining is used in the model). Consistency of the equilibrium expectations causes the partial equilibrium intuitions to be reversed. We then proceed to apply the new insights to the analysis of sunspot equilibria in these type of models of bilateral trade with money.

Keywords: Random Matching, Monetary Equilibria, Multiplicity, Comparative Statics, Sunspot Equilibrium

1. INTRODUCTION

A typical result for monetary models based on decentralized exchange is the multiplicity of equilibria. In the first generation of these models [see, e.g., Kiyotaki and Wright (1993)], prices are exogenous and there is a nonmonetary equilibrium and two monetary equilibria, one with pure and one with mixed strategies. In these models, money has value only because people expect it to.

More recently, price determination through a bargaining game played by the parties in the pairwise meetings was included in the models [see Shi (1995) and Trejos and Wright (1995)]. In these models, there is also multiplicity of steady-state (SS) equilibria. In general, if barter is allowed, there is one nonmonetary equilibrium, two pure-strategy SS monetary equilibria, and no mixed-strategy equilibria. Although the existence of the two pure-strategy SS monetary equilibria, one with low price level and the other with high price level, is well understood, the properties of these steady states have not yet been fully studied.

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In this note, I provide the steady-state comparative statics (i.e., how SS prices change) of the monetary equilibria in the second generation of models—the "endogenous price" models. I first present the underlying ideas using a simple bargaining rule based on take-it-or-leave-it offers by the buyers. I find that the SS equilibrium quantities respond in interesting and non-obvious ways to changes in certain parameters of the model.

Assuming, however, take-it-or-leave-it offers is somewhat simplistic (although it has been widely use in the literature, making its study not only helpful but also interesting in itself). Hence, as a second stage in the analysis, I inquire into the same matters using a more sophisticated rule of bargaining, the Nash–Rubinstein solution. I show that the same ideas apply to this case; in particular, most of the "surprising" results carry over to this setup.

Finally, as an application of these results, I show that the seemingly unintuitive properties of the SS equilibria are very important for analyzing the influence of *extrinsic* uncertainty in random-matching monetary economies.

2. MODEL AND RESULTS

Throughout the paper, I will use the simplest possible model of decentralized exchange with bargaining that allows me to make my points. The environment considered is completely standard in the literature.[1] Time is continuous. There is a continuum of infinite-lived agents with names in the unit interval. There is specialization in production and consumption, and bilateral matches occur according to a Poisson process with arrival rate β. No agent consumes what he/she produces. There is a probability x of a single coincidence of wants and xy is the probability of a double coincidence of wants.[2] By symmetry, these are the same for any pair of agents matched. I normalize $x\beta = 1$ (with no loss in generality). Agents derive utility $u(Q)$ from consumption and have a disutility from producing, given by $c(Q)$. After the initial time, an agent must consume before being able to produce again. Money is indivisible and agents will have either 1 or 0 units of money. Goods are nonstorable and divisible. At the initial time, a fraction $M \in [0, 1]$ of the agents are endowed with one unit of money each; the rest are endowed with a production opportunity. It follows from these assumptions that the distribution of money holdings is invariant from date zero on.

When two agents form a mutually beneficial match, they decide how much to produce through a bargaining game. There are two possible types of matches that would originate trade. On one hand, if an agent holding one unit of money (*buyer*) meets an agent with no money (*seller*) and the latter is able to produce any of the goods that the former wants, then there is a possibility of trade. For this case—the "monetary" match—I will consider two alternative bargaining games that have been widely used in the literature: (1) the take-it-or-leave-it offer by the buyer and (2) the Nash–Rubinstein game.[3] I will show that although the specifics of the results depend on which of the two assumptions is used, the ideas behind them are the same. On the other hand, if two sellers meet and there is a double coincidence

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of wants (the "barter" match), trade can take place and I assume that the symmetric Nash solution results from the bargaining process.

Assume that $u(0) = 0$, $u'(Q) > 0$, and $u''(Q) < 0$ for all $Q \geq 0$ and that $c(0) = 0$, $c'(Q) > 0$, and $c''(Q) \geq 0$ for $Q \geq 0$ [without loss of generality, I assume that $c(Q) = Q$ when convenient]. Also assume that there is a $\hat{Q} > 0$ such that $u(\hat{Q}) = c(\hat{Q})$. Finally, let v_0 and v_1 be the flow return of being a seller and a buyer, respectively. Assume that $v_0 \geq v_1$.

As noted by Trejos and Wright (1995), there are no mixed-strategy monetary equilibria in this model. Therefore, any equilibrium is either monetary or nonmonetary. Also, it is easy to see that there always exists a nonmonetary equilibrium. However, in what follows, I concentrate exclusively on monetary equilibria.

Let V_0 be the value of being a producer (prior to a match) and V_1 the value of holding money (also prior to a match). In any SS monetary equilibrium with $Q \leq \hat{Q}$, (V_0, V_1) satisfy

$$r V_0 = v_0 + \Omega + M[V_1 - V_0 - c(Q)]$$

and

$$r V_1 = v_1 + (1 - M)[u(Q) + V_0 - V_1],$$

where $\Omega = (1 - M)y[u(q^*) - c(q^*)]$, $u'(q^*) = c'(q^*)$ and r is the time discount rate, common to all agents.

Before going into the analysis of equilibrium and its comparative statics, I introduce a basic welfare concept that will be essential for the interpretation of results.

DEFINITION 1.[4] *Let q_0^* and q_1^* be the quantities that maximize V_0 and V_1, respectively. Any $Q \in [q_0^*, q_1^*]$ is ex post Pareto efficient.*[5]

I first present the simpler case of take-it-or-leave-it offers and then discuss how the same ideas also apply to the more complicated Nash–Rubinstein bargaining environment. Finally, I apply my comparative statics results and insights to the analysis of sunspot equilibria of the decentralized-exchange monetary economy.

2.1. Take-It-or-Leave-It Offers

In this case, the basic setup follows closely that of Li and Wright (1998) in their model without government. For simplicity, in what follows, $c(Q) = Q$. Note that assuming take-it-or-leave-it offers by the buyers (when a buyer and a seller meet) implies that the buyer appropriates all the trade surplus from the match. As a consequence, the money price of goods p has to satisfy $p = 1/Q = 1/(V_1 - V_0) > 0$ in any monetary equilibrium. Also, the assumptions on $u(\cdot)$ and $(v_j)_{j=0,1}$ imply that $Q \leq \hat{Q}$ in equilibrium, and so, the incentive compatibility for the buyers is always satisfied $[u(Q) + V_0 - V_1 > 0]$.

Then, using the fact that, in equilibrium, $\Delta = V_1 - V_0 = c(Q) = Q$, we have the SS monetary equilibria being given by the fixed points of the function

$$D(Q) = \frac{(1 - M)}{1 + r - M} u(Q) - \frac{[\Omega + (v_0 - v_1)]}{1 + r - M}. \tag{1}$$

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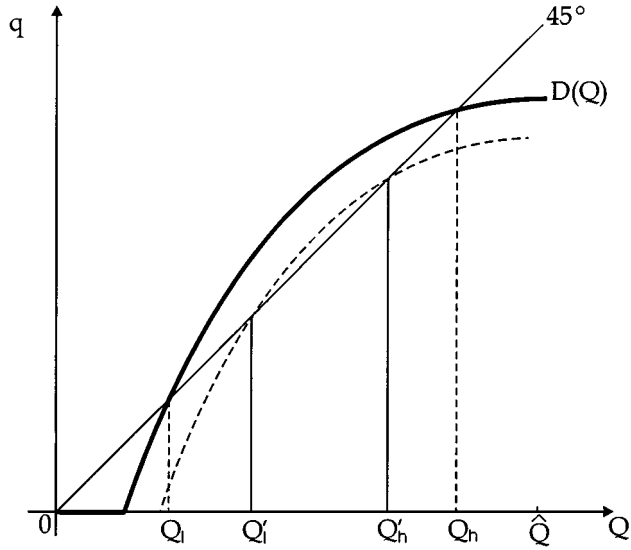


FIGURE 1. Multiplicity of equilibria with take-it-or-leave-it offers.

Under certain conditions on parameter values, there exist two SS monetary equilibria in this model (see Figure 1).[6] The comparative statics properties of these two SS equilibria are interesting and not obvious. Suppose we have a given economy with two SS monetary equilibria. Let us compare it, for example, with another economy, also with two SS equilibria, but where the instantaneous cost of being a producer is lower (i.e., a higher value of v_0). This will result in there being a uniformly[7] higher exogenous part of the payoff from being a seller in the second economy. Accordingly, intuition might tell us that when a buyer and a seller of the new economy meet in a mutually beneficial match, the quantity that results from bargaining would be lower: Given the market quantities, the *net* payoff from becoming a buyer ($V_1 - V_0$) is now lower, and the only terms acceptable to the seller will be to produce a lower quantity for the exchange. When we compare the two economies in the low-price equilibrium (Q_h and Q'_h), this intuition is correct.

However, the equilibrium market quantities have also changed. In the "high-price" situation, the results are reversed (see Figure 1). Even though the seller apparently has a better position in the negotiations, she agrees to produce more to get the unit of money at stake in the match. The logic of this result follows from the action of equilibrium beliefs and the properties of the utility and cost functions. In the high-price SS equilibrium, we have

$$\frac{(1-M)}{1+r-M}u'(Q_l) > 1 = c'(Q); \tag{2}$$

that is, Q_l is ex post Pareto inefficient (see note 5). Suppose a seller is already in a match with an appropriate buyer. If she (the seller) believes that the equilibrium

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price in the economy will be $1/Q_l'$, she will accept an agreement to produce Q_l'.[8] Obtaining the unit of money in this match allows her to enter the next match with a unit of money and, after a successful agreement, to consume Q_l'. From (2), it is clear that we have

$$\frac{(1-M)}{1+r-M}u(Q_l') - Q_l' > \frac{(1-M)}{1+r-M}u(Q_l) - Q_l;$$

that is, the increased expected utility associated with becoming a buyer, a direct consequence of the higher equilibrium quantities, counterbalances the additional production costs and the lower instantaneous cost of being a seller. Now, if every other seller is to believe that $1/Q_l'$ is the price level in the economy, the beliefs will be self-fulfilled.

Another way of seeing this is to note that the seller gets zero surplus from the match in equilibrium. When the economy is in an equilibrium with $Q < q_0^*$, increases in Q imply a higher value of becoming a buyer and this can counterbalance increases in $(\Omega + v_0)$ to leave the seller again indifferent toward a change of state (from seller to buyer). Because of the assumed concavity of $u(\cdot)$ (and the convex cost function), this is no longer possible when the equilibrium quantities are greater than q_0^*.

An interesting situation results when we compare economies with different values of v_1. Some authors [e.g., Li and Wright (1998)] have proposed interpreting $v_1 < 0$ as some kind of inflation tax. If we accept this, then for economies in the "high-price" equilibrium, higher inflation rates (i.e., lower values of v_1) imply a "stronger" currency (lower money prices of goods).[9]

It is important to mention that when one considers nonstationary equilibria in this model, the "high-price" steady state turns out to be stable [see Li and Wright (1998)]. However, the nonstationarity is produced exclusively by the assumed smoothness in the path of expectations (all variables in the model could, in principle, adjust instantaneously). Then, there is no a priori reason why one cannot restrict attention to SS situations. If agents expect stationary prices, then the steady states will be the only equilibria. Partly for simplicity, in the following discussions, I restrict the analysis to steady states.[10]

2.2. Nash–Rubinstein Bargaining

When one considers the Nash–Rubinstein bargaining solution used by Trejos and Wright (1995) and Shi (1995), the ideas developed earlier become more complicated. In this case, the surplus in a "monetary" match, in general, will be divided between the buyer and the seller.

In this version of the model and for values of r, v_1, and v_0 small enough, there are also two SS monetary equilibria. One is a high-price "*constrained*" monetary SS equilibrium that satisfies $c(Q_l) = V_1 - V_0$. The other is an "*unconstrained*" monetary SS equilibrium with a lower price level, $1/Q_h$.

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It is easy to see that inequality (2) holds for the low-quantity equilibrium, Q_l, and then the comparative statics of the high-price (constrained) equilibrium are "unexpected" and similar to that of the high-price equilibrium in the take-it-or-leave-it-offer version.

Finding an unconstrained equilibrium in this economy reduces to finding a feasible zero for the following expresion:

$$\hat{T}(Q, Q) = T(Q, Q) + [(1 - M)u'(Q) - (1 + r - M)c'(Q)](v_0 + \Omega) + [(r + M)u'(Q) - Mc'(Q)]v_1, \quad (3)$$

where $T(Q, Q) = [(r + M)(1 - M)u(Q) - \Phi c(Q)]u'(Q) - [\Phi u(Q) - M(r + 1 - M)c(Q)]c'(Q)$ and $\Phi = r(1 + r) + M(1 - M)$. When $v_1 = 0$, we also have $Q_h < q_0^*$ [see Ennis (1998) for details]. Therefore, for sufficiently small v_1, Q_h is also ex post Pareto inefficient. The comparative statics in this case are slightly more complicated though. From expresion (3), if $Q_h < q_0^*$, an increase in $(v_0 + \Omega)$ will increase Q_h. The direct effect of this change would be that the payoff from consuming and becoming a seller tends to be higher. From this, one would expect the buyer to be willing to accept lower quantities in exchange for her unit of money. However, as was just stated, the equilibrium quantities move in the opposite (counterintuitive) direction.

The complication occurs because, in this equilibrium, the seller and the buyer are dividing the surplus from the match. As a consequence, we can no longer treat changes in $(v_0 + \Omega)$ and changes in v_1 as equivalent. Now, an increase in v_1 increases Q_h [see expresion (3)] and the direct intuition goes through: This change tends to increase the value of becoming a buyer and hence higher quantities result from bargaining to even again the relative payoffs. Finally, note that, for sufficiently large $v_1 > 0$, it will be the case that $Q_h > q_0^*$ and the equilibrium will be ex post Pareto efficient. In this case, the comparative statics are "well behaved" as in the low-price equilibrium with take-it-or-leave-it offers.

2.3. Application to Sunspot Equilibrium

In his 1995 *JET* paper, Shouyong Shi proved, using arguments of continuity, the existence of sunspot equilibria in an economy similar to the one presented in the preceding section. Although the principles in the construction of sunspot equilibrium were similar to the ones used by Wright (1994), the prices were endogenously determined.[11] I show next how those sunspot equilibria can become a good example of the broad applicability of the points made in the preceding sections.

The modeling of sunspots follows that of Wright (1994). Consider the same economy presented in the preceding section but now allowing for the possibility of *extrinsic* uncertainty. Assume the sample space of the *extrinsic* random variable is $S = \{s, s', s''\}$. Let s be associated with the low-price state of the economy, s' with the high price, and s'' with the nonmonetary equilibrium. Again, I consider only stationary outcomes. Assume the sunspot evolves according to a Poisson process.[12]

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When the economy is in state s, it switches to state s' at rate $\lambda_{ss'}$. Similarly, the economy switches from state s' to state s at rate $\lambda_{s's}$. Assume for the moment that the other transition rates are all zero; that is, $\lambda_{ss''} = \lambda_{s''s} = \lambda_{s's''} = \lambda_{s''s'} = 0$. This will simplify the exposition. I come back to the nonsemidegenerate case later.

As it is stated so far, one can show that this economy has a nonmonetary, nonsunspot equilibrium (note the $\lambda_{s''s} = \lambda_{s''s'} = 0$), two SS monetary equilibria in which agents merely ignore the sunspots (these are the same equilibria studied before),[13] and one sunspot equilibrium (for sufficiently small λ's). In the sunspot equilibrium the economy jumps from state s with low prices to state s' with high prices, back and forth, according to the Poisson processes with rates $\lambda_{ss'}$ and $\lambda_{s's}$. In what follows, I concentrate exclusively on this last equilibrium, the proper sunspot equilibrium.

The inclusion of a sunspot variable implies that, in principle, the value of being a seller or of being a buyer will depend on the current state of the economy. Hence, one can show that the value functions are given by[14]

$$r V_0^k = v_0 + \Omega + M\left[V_1^k - V_0^k - c(Q^k)\right] + \lambda_{kh}\left(V_0^h - V_0^k\right),$$

$$r V_1^k = v_1 + (1 - M)\left[u(Q^k) + V_0^k - V_1^k\right] + \lambda_{kh}\left(V_1^h - V_1^k\right),$$

for $k, h = s, s'$, $k \neq h$.

I will proceed with the analysis of the simpler case of take-it-or-leave-it offers within "monetary" matches. Extending the results to the Nash–Rubinstein case presents some additional complications but it does not contribute to my main point, which is to show the relevance of the comparative static results derived above in the nonsunspot economy for analyzing the sunspot equilibria of the model.[15]

First, note that the value function for the seller will satisfy

$$r V_0 = v_0 + \Omega$$

in both states. This, of course, is a consequence of the assumed bargaining solutions (the "take-it-or-leave-it-offer" rule). The value of being a seller only depends on the chances of finding an agent for barter because monetary exchange gives no return to the seller. However, barter is independent of the state of the economy and, as a consequence, so is V_0.

Rearranging terms, we obtain $\boldsymbol{\Delta} = (\Delta^s, \Delta^{s'}) = \boldsymbol{D}(Q^s, Q^{s'})$ (i.e., $\boldsymbol{D} : \mathbf{R}^2 \to \mathbf{R}^2$) is given by

$$\Delta^k = V_1^k - V_0 = \frac{(1 - M)u(Q^k) + \lambda_{kh}[\Delta^h - \Delta^k] - K}{1 + r - M}, \tag{4}$$

for $k, h = s, s'$, $k \neq h$ and where $K = \Omega + (v_0 - v_1)$.

Again, in equilibrium, using $\bar{Q}^k = \bar{\Delta}^k = V_1^k - V_0 = D^k(\bar{Q}^k, \bar{Q}^h)$; $k, h = s, s'$, $k \neq h$, we have a monetary SS equilibrium given by a fixed point of the map $\boldsymbol{\Delta}$. Note that when agents ignore the sunspot variable, we have $\Delta^s = \Delta^{s'} = D(Q)$ and the two monetary equilibrium of Section 2.1 apply [see equation (1)].

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Let (Q_l, Q_h) be the nonsunspot equilibrium quantities. Consider the situation in which sunspots matter. Then we have the following result.

Result. Consider an economy with two SS monetary nonsunspot equilibria, Q_h and Q_l. Let $(\lambda_{ss'}, \lambda_{s's})$ be small enough that there is a sunspot equilibrium with $(\bar{Q}^s, \bar{Q}^{s'})$. Without loss of generality, let $\bar{Q}^s > \bar{Q}^{s'}$ hold. Then, we have $\bar{Q}^s < Q_h$ and $\bar{Q}^{s'} < Q_l$.

Proof. Define $f(Q)$ as

$$f(Q) = \frac{(1-M)}{1+r-M}u(Q) - Q.$$

Clearly, $f(Q_h) = f(Q_l) = K/(1+r-M)$. Note that $\Delta^{s'} - \Delta^s = \bar{Q}^{s'} - \bar{Q}^s \equiv -d < 0$, because we have assumed $\bar{Q}^s > \bar{Q}^{s'}$. Then, by (4), we have $f(\bar{Q}^s) = (K + \lambda_{ss'}d)/(1+r-M)$ and $f(\bar{Q}^{s'}) = (K - \lambda_{s's}d)/(1+r-M)$. However, since we are assuming that the inequality in note 6 holds, we know that $f'(Q_h) < 0$ and $f'(Q_l) > 0$, which implies that $\bar{Q}^s < Q_h$ and $\bar{Q}^{s'} < Q_l$. ■

Recall that the pure-strategy sunspot equilibria in Wright (1994) are merely randomizations over nonsunspot equilibria because prices are exogenous in that model. The above Result shows that, in the random-matchings model of money with endogenous prices, the sunspot equilibria are not pure randomizations over nonsunspot equilibria. Starting with the seminal work by Cass and Shell (1983, Appendix), the literature has paid special attention to the possibility of this kind of example.[16]

The preceding Result also implies that, as a consequence of *extrinsic* uncertainty, *both* possibly observed equilibrium prices will be higher in the sunspot equilibrium. In other words, sunspots reduce the equilibrium value of money uniformly across states. As explained later, this directly follows from the comparative statics analysis of the nonsunspot economy.

The logic in the Result is clear. The appearance of sunspots influences the value of becoming a buyer and hence the resulting quantities from bargaining. In particular, consider an agent with a production opportunity (a seller) that is already in a beneficial match. When the economy is in the low-price state and sunspots matter, she faces a lower expected value of becoming a buyer: With some probability the economy switches to the high-price equilibrium in the following "period" and, after having acquired a unit of money, she would suffer a capital loss. This potential stream of events tends to reduce the quantities produced in the current match (the seller is less willing to incur production costs in order to obtain a unit of money).

On the other hand, when the economy is in the high-price state, the expected value of becoming a buyer would seem to be enhanced by sunspots. There is a chance that, after an agent has acquired a unit of money, the economy will switch to the low-price state and improve the agent's position as a money holder. Accordingly, one would expect equilibrium quantities in the high-price state to be greater in the presence of sunspots. However, in equilibrium, we have $\bar{Q}^{s'} < Q_l$. This is

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closely related to the comparative statics analysis of the nonsunspot economy. The effect that equilibrium beliefs have on the bargaining outcome plays an important role. If the agents believe that a unit of money will trade for smaller quantities of goods in the future (i.e., $\bar{Q}^{s'} < Q_1$), then becoming a buyer is not as attractive. This will allow the lower quantities to be consistent bargaining solutions that confirm these beliefs. Note that lower quantities traded imply lower immediate production costs for the seller, but they also imply a lower value of becoming a buyer and, in fact, in a proportion large enough to compensate for the lower costs *and* the sunspot effects. The fact that this situation is an equilibrium depends on the properties of the utility and cost functions as is clearly seen in the proof of the Result (and was suggested by our previous comparative statics analysis).

Another possible exercise is to consider economies where not all of $\lambda_{ss''}$, $\lambda_{s''s}$, $\lambda_{s's''}$, $\lambda_{s''s'}$ are zero. For example, when $\lambda_{s's''}$ and $\lambda_{s''s'}$ are not equal to zero, if the economy is in the high-price equilibrium, with some probability it will switch to a state in which money has no value (the nonmonetary SS equilibrium). The pattern of equilibria in this case resembles the sunspots of Wright (1994).[17] Once again, some peculiar results appear in this case. Suppose that, for some reason, the rate at which the economy switches from the state s' to s'' increases. This implies a greater chance for a currency collapse. However, the price level in state s', where the economy currently is, decreases. This result seems, at first sight, to be counterintuitive and surprising, but it is a direct consequence of the SS equilibrium properties described earlier.[18]

3. CONCLUSION

I present a standard random matching model of money, in which prices are determined through bargaining, and I proceed to perform comparative statics for the steady-state equilibria. I then show how these results turn out to be useful in analyzing the effects of sunspots in the model.

In an economy with a large number of agents, specialized in consumption and production and meeting pairwise at random, I did comparative-statics analysis for the quantities traded in steady state. It is well known that these economies have two steady-state monetary equilibria. I show that, for the equilibrium with a higher price level, the "response" of the economy to changes in certain important parameters is not simple to interpret. In particular, changes in the exogenous parts of the value of becoming a *seller* and/or a *buyer* have unexpected consequences for the equilibrium price level. This is true under either of the two most widely used assumptions about price determination through bargaining in monetary-exchange models.

As an application of the previous ideas, I analyzed the effect of introducing publicly observed ("aggregate") *extrinsic uncertainty* in the model. I considered a two-state sunspot variable (with high-price and low-price states). Since sunspots change the potential payoff of the agents engaged in bargaining, they affect the final quantities traded in monetary exchanges. The way this happens is not obvious. The unusual properties of some of the steady-state monetary equilibria that were

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studied in the first part of this note explain why, for example, prices may increase in the two possible states (high price and low price) when sunspots are introduced.

NOTES

1. I will not comment here on general assumptions. For a good discussion of some of these, see, for example, Wallace (1997).

2. One can think that there is a continuum of differentiated goods with total measure equal to one. Each agent can produce one good and x is the proportion of commodities that she can consume. Likewise, y is the conditional probability that an agent produces what the other consumes, given that the former consumes what the latter produces. Its value depends, for example, on the sampling technology available (and y, but not x, is the important parameter determining trade).

3. See Li and Wright (1998) and Aiyagari et al. (1996) for the take-it-or-leave-it offer, and Shi (1995) and Trejos and Wright (1995) for the Nash–Rubinstein bargaining.

4. See Trejos and Wright (1995, p. 127).

5. Note that

$$(1-M)u'(q_0^*) - (1+r-M)c'(q_0^*) = 0$$

and

$$(r+M)u'(q_1^*) - Mc'(q_1^*) = 0.$$

Then, it is easily seen that, for any $Q < q_0^*$, both $V_0(Q)$ and $V_1(Q)$ are increasing in Q and, therefore, any small increase in the equilibrium quantities would benefit all buyers and sellers.

6. Note that q_0^* satisfies

$$u'(q_0^*) = \frac{1+r-M}{1-M}. \tag{5}$$

Let $\gamma = [(1-M)/(1+r-M)]u(q_0^*) - q_0^*$; then, as long as $[\Omega + (v_0 - v_1)]/(1+r-M) < \gamma$ holds, there in fact exist two SS monetary equilibria, one with low price and the other with high price.

7. That is, for both SS monetary equilibria.

8. However, note that the buyer in this model is the one making the take-it-or-leave-it offer. So, the buyer has to know that the seller believes that Q_l' will be the quantity in the market in the future. Then, the strong implications of the use of the perfect-foresight assumption in the determination of the equilibrium become apparent. This is especially true since the model deals with decentralized pairwise exchanges.

9. The ideas presented here should be reminiscent of the literature on inflationary financing of government deficits. This is so because expectations play a key role in the determination of SS equilibrium values. However, the foundations of these effects are remarkably different.

10. In the sunspot equilibrium, expectations will change *discretely* between high and low prices. Note that, in general, nonstationarity would complicate the solution of the bargaining problem [see Coles and Wright (1998) and Ennis (1999)].

11. In Wright (1994), the price level is exogenously given and sunspot equilibria are obvious randomizations over the monetary and nonmonetary equilibria. Although establishing that sunspots can matter in these models was very important, the characteristics of those sunspot equilibria were not very surprising.

12. As in Wright (1994), I consider the Poisson process for the sunspot variable to be independent of the Poisson process followed by the pairwise matchings in the economy. Therefore, for the same reason that a Poisson process rules out simultaneous arrivals, a match and a switch of state will not happen simultaneously. This is important when writing down the value functions of the different agents in the sunspot equilibrium.

13. Assume that the restrictions over parameters that allow these to exist also hold in this environment [see note (6)].

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14. Assuming parameter values are such that $Q^s \leq \hat{Q}$.

15. For a thorough analysis of the Nash–Rubinstein case, see Ennis (1998).

16. The early papers by Farmer and Woodford (1997) and Azariadis and Guesnerie (1986) are good examples of this possibility. See also Bhattacharya et al. (1998) and the references therein.

17. An important issue needs to be considered in this case. Specifically, when the economy switches to the state in which money has no value, it is possible that money holders would prefer to dispose of their unit of money and try to barter. In consequence, the economy would change to a permanent barter equilibrium. Following Wright (1994), it is assumed here that an agent needs to consume in order to be able to produce. This rules out that equilibrium. People holding money have just produced and entered a successful monetary trade. They need to consume to be able to produce again. The only way that can happen is if they enter a new monetary match and trade. However, to avoid this equilibrium in which the economy switches to permanent barter, it would be sufficient to assume that $y[u(q^*) - q^*]$ is comparatively small. This makes barter a poor alternative.

18. The other side of this case is also bizarre. If $\lambda_{s's''}$ decreases, the price level in state s' increases. This is the case where "the Swiss have gone too far" (details are available from the author upon request).

REFERENCES

Aiyagari, R., N. Wallace & R. Wright (1996) Coexistence of money and interest-bearing securities. *Journal of Monetary Economics* 37, 397–419.

Azariadis, C. & R. Guesnerie (1986) Sunspots and cycles. *Review of Economic Studies* 53, 725–737.

Bhattacharya, J., M. Guzman & K. Shell (1998) Price-level volatility: A simple model of money, taxes, and sunspots. *Journal of Economic Theory* 81, 401–430.

Cass, D. & K. Shell (1983) Do sunspots matter? *Journal of Political Economy* 91, 193–227.

Coles, M. & R. Wright (1998) A dynamic equilibrium model of search, bargaining, and money. *Journal of Economic Theory* 78, 32–54.

Ennis, H. (1998) Search, Monetary Equilibria and Sunspots. CAE working paper 98-12, Cornell University.

Ennis, H. (1999) Bargaining When Sunspots Matter. CAE working paper 99-03, Cornell University.

Farmer, R. & M. Woodford (1997) Self-fulfilling prophecies and the business cycle. *Macroeconomic Dynamics* 1, 740–769.

Kiyotaki, N. & R. Wright (1993) A search-theoretic approach to monetary economics. *American Economic Review* 83, 63–77.

Li, Y. & R. Wright (1998) Policy analysis in search-based models of money. *Journal of Economic Theory* 81, 290–313.

Shi, S. (1995) Money and prices: A model of search and bargaining. *Journal of Economic Theory* 67, 467–496.

Trejos, A. & R. Wright (1995) Search, bargaining, money and prices. *Journal of Political Economy* 103, 118–141.

Wallace, N. (1997) Absence-of-double-coincidence models of money: A progress report. Federal Reserve Bank of Minneapolis *Quarterly Review* 21, 2–20.

Wright, R. (1994) A note on sunspot equilibria in search models of fiat money. *Journal of Economic Theory* 64, 234–241.

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