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Plasma-object diagnostics via resonant transition radiation from an electron bunch

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Abstract. The resonant transition radiation from an electron charge that moves along the concentration gradient of a cold isotropic plasma is calculated. The possibility of using this radio-emission for inhomogeneous plasma diagnostics is discussed. Measurements of this kind of radiation in a planar-stratified cold isotropic plasma give the possibility of finding out the profile function of the plasma concentration profile (parts of the plasma density profile where the density is increased over its previous maximum value along the direction of the plasma bunch velocity).

1. Introduction

Transition radiation from charged bunches and modulated electron beams in plasmas has been studied in view of active plasma–beam experiments in the ionosphere and because of the problem of transmitting electromagnetic (EM) signals in beam–plasma devices [1].

In our earlier work [2, 3], we proposed to use the transition radiation from a charged bunch (or modulated electron beam) for plasma-object diagnostics. We have investigated the possibility of finding the plasma density profile using information obtained from the non-resonant (high-frequency) part of the radiation spectrum from an electron bunch. However, the radiation in this part of the spectrum is too weak and mainly directed ahead of the electron bunch. Thus, the low-frequency radiation from the bunch – that caused by local plasma resonance regions (LPRR) – is more suitable for plasma diagnostics, since it is more intense and radiated mainly backwards. Obviously, we can place the receiver on the same side of the plasma object as the injector in this case.

In this work, we study the low-frequency (resonant) transition radiation from a charged bunch that moves along the gradient of a planar-stratified spatially confined cold plasma. We also examine the possibility of finding the plasma concentration profile from the received transition radiation spectrum.

We use the method described in [4] for calculating the transition radiation from the bunch. In this method, plasma fields are mapped by a penetrating p-polarized EM wave. This could be also done for a weakly inhomogeneous plasma, if we use the approximation of geometric optics [5], together with the precise results in the LPRR (where that approximation is not applicable), to give the whole plasma profile. Note that the LPRR could be situated in the linear part of the plasma

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density profile or close to its maximum. The transition radiation parameters are calculated assuming a given current for the bunch.

We are interested in the transition radiation components with frequencies $\omega \leqslant \omega_{p\max}$. We study two cases: (a) when the LPRR is situated in the linear part of the plasma concentration profile; (b) when LPRR is situated at the maximum of the plasma concentration profile.

2. Resonant transition radiation of an electron bunch in the cases of near and far local plasma resonance regions

Consider a charged bunch q with velocity v that gives a current density

$$\mathbf{j}^q(\mathbf{r},t) = \frac{qv}{\pi}\delta(z-vt)\frac{\delta(r)}{r}\mathbf{e}_z. \tag{2.1}$$

It moves along the gradient of a planar-stratified cold isotropic plasma ($n = n(z)$, $n(z \ll -L_{\ln},\ z \gg L_{\ln}) = 0$). The plasma object can be treated as planar-stratified if its transverse length is large relative to the transverse length of the transitional radiation formation zone [6]. The plasma profile can be considered as linear near the reflection point $z = z_0^{(1)}$ and the local plasma resonance point $z = z_p^{(1)}$:

$$n_p(z) = n_c\left(1 + \frac{z - z_p^{(1)}}{L_{\ln}}\right). \tag{2.2}$$

In this part, the dielectric permitivity of the plasma $\varepsilon(\omega, z)$ (with $\omega^2 = 4\pi n_c e^2/m$) varies

$$\varepsilon(\omega,z) = -\frac{z - z_p^{(1)}}{L_{\ln}} - i\nu, \tag{2.3}$$

where ν is the effective electron collision frequency normalized by ω. We assume $k_0 L_{\ln} \gg 1$ (with $k_0 = \omega/c$).

The total beam current density can be written as an integral of partial plane waves:

$$j_z(\mathbf{r},t) = \int_{-\infty}^{\infty} d\omega \int_0^{\infty} k_\perp\, dk_\perp \int_0^{2\pi} d\varphi\, j_m(\omega,z)\exp(i\omega t - ik_\perp r\cos\varphi), \tag{2.4}$$

where

$$j_m(\omega,z) = j_m\exp(-iKz), \qquad K = \frac{\omega}{\nu}, \qquad j_m = \frac{q}{2\pi}.$$

For the low-frequency part of the spectrum the E_y component ($y = r\cos\varphi$, $k_\perp \to k_y$) of the transition radiation field excited by the partial current wave in the far zone can be obtained from the equation

$$\frac{d}{dz}\left(\frac{\varepsilon}{\varepsilon k_0^2 - k_y^2}\frac{dE_y}{dz}\right) + \varepsilon E_y = \frac{4\pi k_y}{\omega}\frac{d}{dz}\left[\frac{j_m(\omega,z)}{\varepsilon k_0^2 - k_y^2}\right]. \tag{2.5}$$

Equation (2.5) has a solution that depends on waves coming from the LPRR:

$$E_y = E_y^{(2)}\int_z^{\infty}\frac{E_y^{(1)} f}{W}\,dz + E_y^{(1)}\int_{-\infty}^{z}\frac{E_y^{(2)} f}{W}\,dz, \tag{2.6}$$

where f is the right-hand side of (2.5), $E_y^{(1,2)}$ are solutions of (2.5) with zero right-hand side, which correspond to waves incident on the plasma and radiated away to

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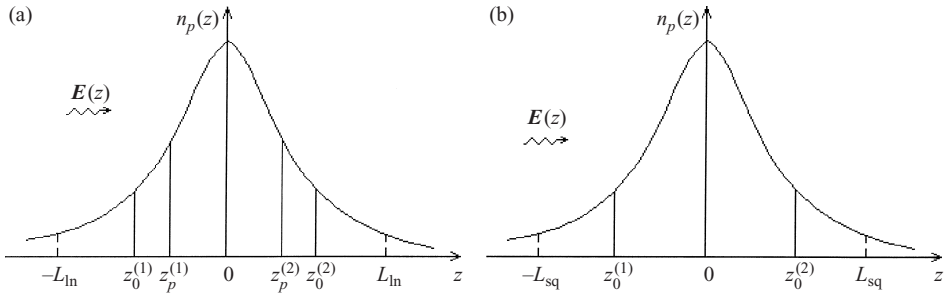


Figure 1. Characteristic points on the plasma concentration profile: (a) LPRR situated in the linear part of the plasma concentration profile; (b) LPRR situated at the maximum of the plasma concentration profile.

$z \to \infty$, and

$$W(z) = E_y^{(1)} \frac{dE_y^{(2)}}{dz} - E_y^{(2)} \frac{dE_y^{(1)}}{dz}.$$

First we study the case when two local plasma resonance points $z = z_p^{(1)}$ and $z = z_p^{(2)}$ are situated in the linear parts of the plasma density profile with different signs of the gradient function (Fig. 1a). We assume that the distance between these points is about the radiated wavelength. This means that the plasma concentration profile contains a part between the reflection points $z = z_0^{(1)}$ and $z = z_0^{(2)}$ that is opaque to EM waves. Within the context of this assumption, we can derive the expressions for $E_y^{(1,2)}$ in the approximation of geometric optics. In the range $0 > z > z_0^{(1)}$,

$$E_y^{(1,2)} = E_m^{(1,2)} \sqrt{\frac{2\kappa L_{\ln} \sqrt{k_y^2 + i\nu k_0^2}}{\pi k_0}} (e^{i\pi/12} + R_{1,2} e^{-i\pi/12}) \times \exp(-\tfrac{2}{3} k_0 L_{\ln} \sin^3\theta)\, K_0(\zeta), \tag{2.7}$$

where $E_m^{(2,1)}$ are the magnitudes of the decaying wave as $z \to \pm\infty$ respectively,

$$\kappa^2(z) = k_y^2 - \varepsilon(z) k_0^2, \qquad \zeta = (z - z_p^{(1)} + i\nu L_{\ln}) \sqrt{k_y^2 + i k_0^2}, \qquad I(a,b) = \int_a^b \kappa(z)\, dz,$$

$$R_1 = -i \frac{A_1 - B}{A_1 - B e^{i\pi/3}}, \qquad R_2 = i \frac{A_2 - B}{A_2 e^{-i\pi/3} - B},$$

$$A_{1,2} = \exp[2I(z_p^{(1)}, z_p^{(2)}) - I(z_0^{(1)}, z_0^{(2)})]\{1 - \exp[2I(z_p^{(1)}, z_p^{(2)})] \times \{\mp \exp[-\tfrac{1}{3} i\pi + 2I(z_p^{(1)}, z_0^{(1)})] - 1\}\}$$

$$B = \exp[I(z_0^{(1)}, z_0^{(2)}) - I(z_p^{(1)}, z_p^{(2)})]\{1 - \exp[\tfrac{1}{3} i\pi - 2I(z_p^{(1)}, z_0^{(1)})]\}.$$

Using (2.6), one can find the field excited by the partial wave of the current density far from the plasma ($z < z_0^{(1)}$). Note that there is a strong increase of the field in the LPRR, which causes most of the radiation. Thus we can operate with the LPRR fields in (2.6), expanding the integration region from $-\infty$ to $+\infty$.

The integrals in (2.6) can be derived in this approximation using the method of residues. Finally, for an electron bunch that moves into the plasma, one can obtain

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the magnitude of the resonant component of the transition radiation field:

$$E_y = i\frac{2(2\pi)^{3/2} j_m \sqrt{k_0 L_{\ln}\cos\theta}}{\omega}\frac{e^{i\pi/12} + R_1 e^{-i\pi/12}}{R_1 - R_2} \times \exp[-KL_{\ln}\nu - I(z_0^{(1)}, z_p^{(1)})]. \tag{2.8}$$

Note that the contribution from second integral in (2.6) can be neglected.

The expression (2.8) gives the radiation excited by the partial density wave $j_m(\omega, z)$. The total field excited by the density wave (2.1) at a frequency ω can be written as an integral of these radiated fields. Since only waves with $\kappa^2(z \to -\infty) > 0$ make contributions to the radiation field, we can vary k_r from 0 to k_0. Using the substitutions

$$k_r = k_0 \sin\Theta, \qquad k_z = k_0 \cos\Theta, \qquad r = R\sin\theta, \qquad z = R\cos\theta, \tag{2.9}$$

and the asymptotics for Bessel functions of large argument, we can derive (2.6) in the far zone ($R \to \infty$):

$$E^{\text{sum}}_{\theta\,\ln}(R, \theta, \omega) = \sqrt{8\pi k_0 L_{\ln}\cos\theta}\frac{q}{cR}\Phi(\theta) \times \exp[-L_{\ln}K\nu - ik_0 R - I(-\infty, z_p^{(1)})], \tag{2.10}$$

where

$$\Phi(\theta) = \frac{e^{i\pi/12} + R_1 e^{-i\pi/12}}{R_1 - R_2}$$

and the reflection point $z = z_0^{(1)}$ corresponds to $k_y = k_0 \sin\theta$.

Taking (2.3) into account, the radiated power at frequency ω can be written in the form

$$\Pi_{R\,\ln} = \frac{2k_0 L_{\ln}}{c}\left(\frac{q}{R}\right)^2 |\Phi(\theta)|^2 \cos\theta \exp\{-2L_{\ln}[K\nu + I(z_0^{(1)}, z_p^{(1)})]\}. \tag{2.11}$$

In particular, when the second LPRR is constrained to be far enough from the first LPRR, the plasma object is fully opaque to EM waves. In this case, we do not take into account fields located in the region $z > z_p^{(1)}$. Thus, we can assume an extremely large distance between the local plasma resonance points $z = z_0^{(1)}$ and $z = z_0^{(2)}$ in (2.10) and (2.11). In this case, the total radiation in the far zone has the form

$$E^{\text{sum}}_{\theta\,\ln}(R, \theta, \omega) = \sqrt{8\pi k_0 L_{\ln}\cos\theta}\frac{q}{cR} \times \exp[-L_{\ln}K\nu - \tfrac{1}{12}i\pi - ik_0 R - I(-\infty, z_p^{(1)})]. \tag{2.12}$$

The expression for the radiated power at frequency ω simplifies to

$$\Pi_{R\,\ln} = \frac{2k_0 L_{\ln}}{c}\left(\frac{q}{R}\right)^2 \cos\theta \exp[-2L_{\ln}(K\nu + \tfrac{2}{3}k_0 \sin^3\theta)]. \tag{2.13}$$

One can integrate (2.11) and (2.13) over the whole range of θ and $\omega < \omega_{p\,\max}$ and estimate the radiated output of the resonant transition radiation from an electron bunch in both cases.

3. LF radiation of a bunch for LPRR situated at the maximum of the plasma concentration profile

Consider a charged bunch that moves through a plasma as described in Sec. 1 and gives a current density (2.1). In the case when LPRR lies at the maximum of the

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plasma concentration profile, we can change the origin of coordinates so that

$$n_p(z) = n_c(1 - \lambda z^2), \tag{3.1}$$

where $\lambda \sim 1/L_{\text{sq}}^2$ (see Fig. 1b). Thus, near the LPRR, the dielectric permitivity can be written as

$$\varepsilon(\omega, \mathbf{r}) = \lambda(z - i\nu')^2, \tag{3.2}$$

where $v' \sim L_{\text{sq}}\sqrt{\nu}$. This expression for ε qualitatively correctly describes the restriction of plasma fields in the LPRR due to electron collisions.

Obviously we have one resonance point $z = 0$ and two reflection points $z = z_0^{(1)}$ and $z = z_0^{(2)}$ in this case. Equation (2.5) can be solved using the same method as implemented in Sec. 1. According to the geometric optics approximation, the expressions for $E_y^{(1),(2)}$ valid in the region $0 > z > z_0^{(1)}$ have the form

$$E_y^{(1,2)} = -E_m^{(1,2)}\zeta'^{-1/2}\sqrt{\frac{2\kappa\sqrt{k_y^2 + \lambda\nu'^2 k_0^2}}{\pi\lambda k_0}}(e^{i\pi/12} + R'_{1,2}e^{-i\pi/12})$$

$$\times\exp\left(\frac{\pi}{4}\frac{k_0}{\sqrt{\lambda}}\sin^2\theta\right)K_{1/2}(\zeta'), \tag{3.3}$$

where

$$\zeta' = -(z - i\nu')\sqrt{k_y^2 + \lambda\nu'^2 k_0^2}, \qquad R'_1 = -i\frac{A'_1 - B'}{A'_1 - B'e^{i\pi/3}}, \qquad R'_2 = i\frac{A'_2 - B'}{A'_2 e^{-i\pi/3} - B'},$$

$$A'_{1,2} = \exp[-I(z_0^{(1)}, z_0^{(2)})]\ \{2 \pm \exp[-\tfrac{1}{3}i\pi + 2I(z_p^{(1)}, z_0^{(1)})]\},$$

$$B' = \exp[I(z_0^{(1)}, z_0^{(2)})]\ \{1 - \exp[\tfrac{1}{3}i\pi - 2I(z_p^{(1)}, z_0^{(1)})]\}.$$

Both integrals in (2.6) contribute to the radiation in this case. Assuming the primary impact of the LPRR as before and using (2.6), one can find the field excited by the partial wave of the current density far from the plasma ($z < z_0^{(1)}$):

$$E_y = \frac{8\pi^2}{\nu_0}\sqrt{\frac{\cot\theta}{\lambda}}j_m\ \exp\left(-K\nu' - \frac{\pi}{4}\frac{k_0}{\sqrt{\lambda}}\sin^2\theta\right)F(\theta)\ \exp\left(-i\int_{z_0^{(1)}}^{-\infty}\kappa\,dz\right), \tag{3.4}$$

where

$$F(\theta) = \frac{2e^{i\pi/12} + e^{-i\pi/12}(R'_1 + R'_2)}{R'_1 - R'_2}.$$

Integrating (3.4) (see the previous case), one can derive the field of the transition radiation in the far zone:

$$E_{\theta\text{sq}}^{\text{sum}}(R, \theta, \omega) = \frac{4\pi k_0}{\nu_0 R}q\sqrt{\frac{\cot\theta}{\lambda}}F(\theta)$$

$$\times\exp\left[-K\nu' - \frac{\pi}{4}\frac{k_0}{\sqrt{\lambda}}\sin^2\theta - ik_0R + iI(-\infty, z_p^{(1)})\right]. \tag{3.5}$$

The radiated power has the form

$$\Pi_{R\text{sq}}(R, \theta, \omega) = 4\pi c\left(\frac{k_0 q}{\nu_0 R}\right)^2\frac{\cos^3\theta}{\lambda\sin\theta}|F(\theta)|^2\exp\left(-2K\nu' - \frac{\pi}{2}\frac{k_0}{\sqrt{\lambda}}\sin^2\theta\right). \tag{3.6}$$

The radiated powers given by (2.11) or (2.13) and by (3.6) are valid only for

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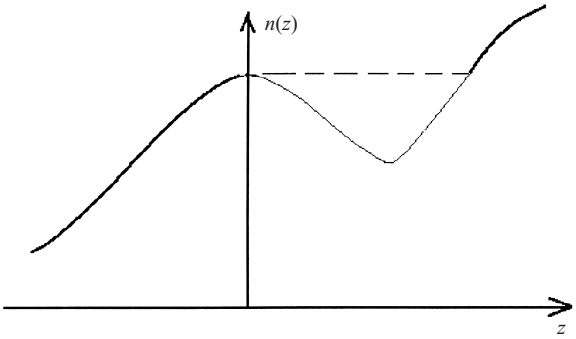


Figure 2. Plasma concentration profile and its profile function (bold line).

large enough angles $\theta(\theta \gg (L_{\ln}k_0)^{-1/3}$ and $\theta \gg \lambda^{1/4}/k_0^{1/2}$, respectively). Another requirement

$$n_c L^2 \gg \frac{mc^2}{e^2} \tag{3.7}$$

constrains the maximum electron concentration n_c, and the characteristic scale L defines the slightly inhomogeneous plasma object.

4. Discussion and numerical estimates

As shown by (2.11) and (3.6), one can obtain the spatial spectrum of the plasma density from the temporal dependence of the transition radiation from a charged bunch in the low-frequency region of the spectrum, $\Pi_R(\omega)$.

At least, the so-called profile function [7] can be found (it is formed by the parts of the plasma density profile where the concentration increases in the direction of the bunch motion over any previous maximum value; see Fig. 2). Actually, we can place a receiver on the same side of the plasma object as the injector (within a wide enough angle θ), measure the output of the transitional radiation $P_R(\omega)$, and find the value of $L_{\ln}(\omega)$ (taking into account small ν) that describes the linear part of plasma profile at the LPRR at frequency ω (see (2.2)). In the next step, we integrate the ratio for the linear plasma concentration profile (the definition of the inhomogeneity length),

$$L(\omega) = L\left(n = \frac{m\omega^2(z)}{4\pi ne^2}\right) = \left|\frac{1}{n(z)}\frac{dn(z)}{dz}\right|^{-1}, \tag{4.1}$$

considering that the characteristic scale of the inhomogeneity varies from point to point (for different frequencies). In this way, the dependence $L(z)$ can be obtained from the relation

$$\int_{\omega(z_0)}^{\omega(z)} \frac{2L(\omega)}{\omega}\, d\omega = z - z_0 \tag{4.2}$$

(obviously for a linear plasma concentration profile $L(\omega) \sim \omega^2$). Using (4.2), one can obtain the function $n(z)$

$$n(z) = n_0 \exp\left[\int \frac{dz}{L(z)}\right]. \tag{4.3}$$

Note that the magnitude of the transition radiation depends strongly on the

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position of the LPRR (in the linear part of plasma density profile or near its maximum). As shown by (2.12) and (3.5), if $L_{\ln}$ and I_{sq} are of the same order of magnitude, then

$$\left|\frac{E_{y\mathrm{sq}}^{\mathrm{sum}}}{E_{y\ln}^{\mathrm{sum}}}\right| \sim \frac{1}{\beta}\frac{k_0 L_{\mathrm{sq}}}{\sqrt{k_0 L_{\ln}}} \gg 1, \tag{4.4}$$

which means that the radiation at a frequency equal to the Langmuir frequency at the plasma density maximum is much stronger than the radiation at lower frequencies (note that a similar result was obtained in [8] for thin plasma layers with a parabolic concentration profile; the interpretation of this effect was given in [9]). So maxima in the spectrum of the transition radiation correspond to maxima of the plasma density profile. Obviously, when the function $n(z)$ has one maximum, the spectrum of the resonant transitional radiation is fully described by (2.11) or (3.6). But this spectrum is quite different if there are a few spatially separated maxima of the plasma concentration profile. As stated above, the spectrum of the transition radiation would have maxima at frequencies equal to the Langmuir frequency for each density maximum. But, in this case, the part of the spectrum outside that corresponding to the radiation from LPRR becomes irregular through the interference of radiation from different parts of the plasma profile that contain concentration maxima. These irregularities characterize the complex plasma concentration profile; one can find the variation of plasma concentration near maxima by the overlapping width of these spectra.

We estimate the output of resonant transition radiation. Consider an inhomogeneous plasma object with Gaussian concentration profile ($n_p \approx 10^8\,\mathrm{cm}^{-3}$ and $L = 100\,\mathrm{m}$). Suppose that the length of the electron bunch does not exceed the length of the radiated wave in vacuum. For example, an electron bunch with nanosecond duration and radius 30 cm has a velocity $v = 0.1c$ and current $I = 1\,\mathrm{A}$.

Figure 3(a) shows the frequency dependence for the radial component of the Poynting vector in the band of frequencies corresponding to radiation from the linear part of the plasma density profile (see Sec. 1). The radiation power for frequencies near the maximum plasma frequency greatly exceeds (by an order of magnitude or more) the power for lower frequencies. In this case, at frequencies of about $\omega_{p\,\max}$, one have to use the expressions from Sec. 2 (in particular, the full output can be calculated from (3.6)).

In our numerical estimations, we assume that the characteristic scale of plasma inhomogeneity also depends on the frequency (Fig. 3b). This means that different values of the plasma inhomogeneity length correspond to different frequencies, which themselves correspond to different points on the plasma profile.

Figure 3(c) demonstrates the full (integrated over frequency) energy of transition radiation as a function of plasma-object scale. Expansion of that scale can increase the energy of transition radiation through extending the inhomogeneity length where this radiation arrives.

Figure 3(d) shows the energy distribution of transition radiation as a function of direction angle θ and frequency ω. One can see that the radiation is confined to a small range of angles for the most part. This feature is more striking if the radiation arises at frequencies close to the maximum plasma frequency.

We have estimated the average output of transition radiation from a charged bunch. For the plasma and bunch parameters given above, the output of transition radiation is about 120 mW. This radiation can be detected by receivers placed at

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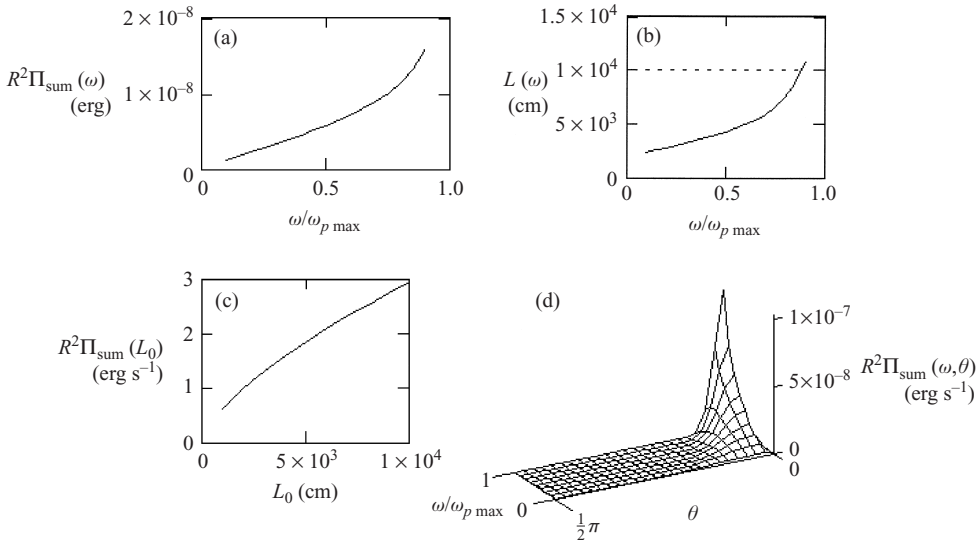


Figure 3. The energy of the transition radiation from a charged bunch: (a) frequency spectrum of the radiation; (b) frequency dependence of the plasma inhomogeneity length; (c) energy of the transition radiation as a function of the plasma object scale; (d) density of the energy flow of the transition radiation as a function of frequency and direction angle.

given angles to the direction of bunch motion. The estimated power is underestimated, because we do not take into account the radiation from the region nearest to the plasma concentration maximum.

The energy used for the excitation of the transition radiation is about 3.5% of the full energy of the bunch. We regard this efficiency as quite enough for this kind of radiation.

In our earlier work, we have shown that the average output of transition radiation radiated backwards (i.e. in the opposite direction to the bunch motion) in the case of non-resonant (high-frequency) diagnostics was only about 50 μW (for the same parameters of the bunch and plasma) [2]. Obviously, the power of the resonant radiation (at frequencies of about the maximum plasma frequency) greatly exceeds the non-resonant component.

5. Conclusions

1. Measurements of the resonant component of the transition radiation in a planar-stratified cold isotropic plasma caused by an electron bunch moving along the plasma density gradient give the possibility of determining the profile function of the plasma concentration profile (the parts of the plasma density profile where the density is increased over its previous maximum value along the direction of the plasma bunch velocity).

2. To determine the profile function, one can measure the time dependence of the backward-radiated field in some given direction, on the same side of the plasma object as the injector. In distance case unlike the high-frequency diagnostics [2], both relativistic and non-relativistic bunches can be used.

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3. The maxima of the transition radiation spectrum correspond to Langmuir frequencies at plasma concentration maxima.

References

[1] Balakirev, V. A. and Sidelnikov, G. L., *Transition Radiation from Modulated Electron Beams in an Inhomogeneous Plasma*. Kharkov: KhFTI, 1994 (in Russian).

[2] Anisimov, I. O. and Lyubich, K. I., Plasma-object diagnostics using transition radiation from a charged bunch. *Ukr. Phys. J.* **42**, 959–966 (1997) (in Ukrainian).

[3] Anisimov, I. O., Plasma-object diagnostics using transition radiation from a modulated electron beam. *Ukr. Phys. J.* **41**, 798–801 (1996) (in Ukrainian).

[4] Stefan, V. and Frolov, A. A., Magnetic field generation via linear conversion of laser radiation into leaking surface waves in laser produced plasma. *Phys. Lett.* **87A**, 398–402 (1982).

[5] Ginzburg, V. L., *Electromagnetic Wave Propagation in Plasmas*. Moscow: Nauka, 1967 (in Russian).

[6] Anisimov, I. O. and Borisov, O. A., On the tranverse length of the formation zone of transition radiation from a moving charge. *Bull. Kyiv Univ. Phys. Maths* **1**, 357–365 (1997) (in Ukrainian).

[7] Astanin, L. Yu. and Kostylev, A. A., *Basics of Superbroadband Radiolocation Measurements*. Moscow: Radio i svyaz', 1989 (in Russian).

[8] Bakunov, M. N. and Sorokin, Yu. M., On the transition radiation from a charge in a thin inhomogeneous plasma layer. *High Schools Bull. Radiophys.* **30**, 389–393 (1987) (in Russian).

[9] Anisimov, I. O., Kotlyarov, I. Yu and Levitsky, S. M., On the possibility of an increase in the transition radiation from a modulated electron stream in an inhomogeneous plasma. *High Schools Bull. Radiophys.* **32**, 1034–1038 (1989) (in Russian).

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