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ARTIFICIAL INTELLIGENCE AND HUMAN JOBS

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The development of artificial intelligence (AI) does influence human jobs but not necessarily in a negative way. Although labor force participation rates and firms' job vacancies for human labor decline, the unemployment rate may be lower than that in an economy without AI. In an economy with heterogeneously skilled workers, the invention of AI usually has a negative effect on the skilled labor market but a positive effect on the unskilled labor market. The overall unemployment rate may decline as AI develops.

Keywords: Artificial Intelligence, Search and Matching, Unemployment

1. INTRODUCTION

The purpose of this paper is to explore the impact of the invention of artificial intelligence (AI) on the labor market. The original contribution of this paper is that we construct the AI accumulation function in a search model with labor market frictions, enabling us to investigate how the development of AI changes the behavior of households and firms in the labor market. Thus, we can discuss the impacts on both skilled and unskilled workers and analyze the influence of adopting AI in an economy along the transitional dynamics path and the balanced growth path (BGP).

People's patterns of interaction and their economic activities are being changed dramatically by AI. Unlike traditional machines, which replaced the use of human and animal labor for simple manual work and heavy or dangerous activities, AI-related inputs may change the type of work done by humans in a comprehensive way. As computer processing has become faster and cheaper, it has become cost-effective for robots to do more routine tasks that were previously done by humans. Therefore, a major concern with the development of AI is that, if robots (or automation) displace large numbers of human workers, massive unemployment and poverty will result. In addition, AI (as deep machine learning and reinforcement) can perform and replace humans not only in routine tasks but also in tasks that require thinking.

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Nowadays, AI is an important input to production in many industries and it is expected to eventually replace human resources. Frey and Osborne (2017) used a Gaussian process classifier to estimate the probability of computerization for 702 detailed US occupations and found that about 47% of total US employment is at risk and could be automated relatively soon. Morikawa (2017) collected original survey data from more than 3000 Japanese firms operating in both the manufacturing and service sectors to investigate their views regarding the impacts that AI may have on business and employment in the future. He found that firms expect AI and robotics to have favorable impacts on their businesses and that they generally consider that upgrading human capital is important. That is, the current literature indicates that employment patterns will change in future in response to the development of AI.

Acemoglu and Restrepo (2018a) considered the situation in which AI replaces labor and discussed the impact of automation on labor demand, wages, and employment. In addition, they examined the mismatch between skills and technologies, which will reduce the productivity gains from new technologies. Berg et al. (2018) built models with different elasticities of substitution between robots and labor and analyzed how automation may transform the labor market in different situations. Both studies suggested that AI may replace human labor.

Agrawal et al. (2019) discussed the impact of AI, especially its prediction abilities, on specific related human jobs, and found that the effects on the demand for labor in these jobs are ambiguous. Autor (2015), Pratt (2015), Acemoglu and Restrepo (2018b), and Acemoglu and Restrepo (2019) also discussed the relationship between AI and human jobs. Acemoglu and Restrepo (2017) considered low- and high-skill automation and discussed wage inequality. Recently, some studies have considered the impact of automation on growth accounting. For example, Autor and Salomons (2018) examined embodied automation in total factor productivity growth, and Prettnerc (2019) and Heer and Irmen (2019) introduced automation capital into the Solow model. Both studies estimated the pattern of the labor share in production.

In contrast to the above studies, we introduce AI's ability for self-accumulation and its nonrival characteristics into the models of Diamond (1982), Mortensen (1982), and Pissarides (1984), with labor market frictions, and investigate the impact of the invention and development of AI on the labor market.In this paper, we simulate an economy transitioning from a state without AI to one with AI, and investigate the impact of the development of AI using transitional dynamics and the BGP.

In the benchmark model in this paper, workers are identical and have the same skills. We find that the development of AI does indeed dramatically affect human work because firms provide fewer job vacancies and fewer people participate in the labor market. Undoubtedly, the resources and effort invested in the AI sector by households begins to increase. More household members devote their time to learning, as the accumulation of human capital contributes to the development of

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AI. That is, when AI is invented, people will find ways to upgrade their skills to have the capability to work with AI. It is worth noting that although the labor force participation rate is lower, AI does not necessarily reduce the employment rate. Compared with the situation in which AI does not exist, we find that, when AI is invented, the unemployment rate is lower along the transitional dynamic paths and in the long run.

Next, we extend our model to include heterogeneously skilled labor. As AI could replace skilled workers, the invention and development of AI usually has a negative effect on the skilled labor market but a positive effect on the unskilled labor market. The unemployment rate of skilled labor may be higher and that of unskilled labor may be lower than in the situation where AI does not exist. Importantly, even if the overall labor force participation rate declines, the overall unemployment rate may decline as a result of the development of AI.

The transitional dynamics analysis in this paper can help us to understand the problems that the current labor market will face as programmers and scientists around the world work diligently to develop AI. The long-run comparative static analysis enables us to understand the effects of different structural parameters and functional forms on the labor market. In addition, it enables us to compare an economy with AI with an economy without AI as a robustness check to identify the benefits and disadvantages of developing AI. Furthermore, it can help us to comprehend the implications of AI for the labor market. For example, jobs that require physical interaction or knowledge and skills related to AI still require human labor, whereas in the case of tasks that are easy to replicate through automation, there will be fewer jobs for human labor.

Recently, Leduc and Liu (2019) used the Diamond–Mortensen–Pissarides labor search model to analyze the impact of automation on the labor market. They assumed that firms can use a robot instead of a worker to produce output if a job opening is not filled by a worker. This assumption strengthens the firms' bargaining power when negotiating wages with job seekers and amplifies fluctuations in unemployment and job vacancies. The authors found that automation leads to a countercyclical labor share of production. They did not discuss AI accumulation in detail in their model. In contrast, in our paper, in addition to considering the resources and effort invested in the development of AI, we emphasize AI's learning and problem-solving abilities (e.g., deep machine learning and reinforcement). In addition, Leduc and Liu (2019) did not consider heterogeneously skilled labor, which is one of the contributions of our paper.

The remainder of the paper is structured as follows. In Section 2, we construct a benchmark labor search model with AI and provide the equilibrium equations. In Section 3, we analyze the impact of AI along the transitional dynamics path and provide some comparative statics analysis in the long run. Section 4 discusses the situation in an economy with heterogeneously skilled labor. Section 5 offers some concluding remarks. Technical details are provided in the Appendix.

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2. THE MODEL

We consider a discrete-time model with a continuum of identical infinitely lived firms, a continuum of identical infinitely lived large households, and a fiscal authority. We consider a large household setup, such that there is no heterogeneity in the welfare between the employed and the unemployed. Employment at a given time is predetermined and changes only gradually, as those who are unemployed find new employment and as old jobs are dissolved.

2.1. Households

The representative large household has unified preferences and pools all resources and enjoyment from its members. In period t, fraction e_t of the large household's members are employed, fraction s_t is searching for jobs, fraction n_t is improving its skills, fraction a_t is developing AI, and the remaining fraction $1 - e_t - s_t - n_t - a_t$ is outside the three sectors, including the labor force, the investment in the household itself, and AI (we refer to this fraction as pursuing leisure, following Arseneau and Chugh (2012)). The household allocates a fraction u_t of physical capital to developing AI and the remaining fraction $1 - u_t$ to producing final goods. We denote k_t as physical capital, h_t as human capital, and A_t as the stock of AI.

The level of employment from the household's perspective is given by the following process:

$$e_{t+1} = (1 - \psi)e_t + \mu_t s_t, \tag{1}$$

where μ_t denotes the (endogenous) job-finding rate and ψ is the (exogenous) job separation rate. Thus, the change in employment $(e_{t+1} - e_t)$ is equal to the inflow of workers into the employment pool $(\mu_t s_t)$ net of the outflow as a result of separation (ψe_t).

We assume that the stock of AI, which is a type of capital, accumulates by itself as a result of its use, just as human capital does. That is, when more resources are devoted to developing AI, more advanced, higher-level AI is generated. The process of AI accumulation is as follows:

$$A_{t+1} = D(a_t h_t)^{\phi}(u_t k_t)^{\theta} A_t^{1-\phi-\theta} + A_t, \tag{2}$$

where $\phi, \theta, 1 - \phi - \theta \in (0, 1)$ and $D > 0$ measures the efficiency of the process of AI accumulation, that is, the self-accumulation ability of AI. The development of AI requires machines, such as computers, and humans to write programs. Therefore, the accumulation of AI depends on the quantity of resources and effort devoted to developing AI. Thus, the accumulation equation for AI contains the physical and human capital allocated to the development of AI. In addition, AI can accumulate itself by relying on deep machine learning and reinforcement. That is, the accumulation equation should also include the AI stock itself.

According to Lucas (1988), a household's human capital is accumulated via a learning activity as follows:

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$$h_{t+1} = Bn_t h_t + h_t, \tag{3}$$

where $B > 0$ measures the efficiency of the process of human capital accumulation. Note that when AI is invented and used in the production process, people need to upgrade their knowledge to have the capability to work with AI. In our large household setup, the household will accumulate related abilities when it devotes the fractions a_t and n_t of its members to accumulating AI and human capital.

We use w_t, r_t, and r_t^A to denote the wage rate, the rental rate, and the return to AI, respectively. The representative household's budget constraint is

$$k_{t+1} = k_t + w_t e_t h_t + r_t(1 - u_t)k_t + r_t^A A_t + \pi_t - c + b_t s_t - T_t, \tag{4}$$

where π_t is the firm's profits, given that households own the firm's shares, b_t is unemployment compensation, and T_t is lump-sum taxes. The budget constraint indicates that unspent income is used to accumulate physical capital. To simplify the model, we assume that the depreciation rates of physical capital, human capital, and AI are zero. Note that the household does not allocate different fractions of AI to different sectors, that is, AI is a nonrival input. This is because AI is similar to certain types of technology or knowledge, in that its use in the production of final goods does not reduce its ability to accumulate. This feature also makes AI a different type of capital from physical and human capital.

The representative large household's utility is

$$u(c_t, 1 - e_t - s_t - n_t - a_t) = \ln(c_t) + \chi \frac{(1 - e_t - s_t - n_t - a_t)^{1-\sigma}}{1 - \sigma}, \tag{5}$$

where c_t is consumption and the parameter $\chi > 0$ measures the importance of leisure relative to consumption in utility. In (5), we use a conventional additively separable utility between consumption and leisure, with a unit intertemporal elasticity of substitution (IES) for consumption and the Frisch labor supply elasticity equal to $(1 - e_t - s_t - n_t - a_t)/[\sigma(e_t + s_t)]$. According to King and Rebelo (1999), the IES for consumption must be unity to be consistent with a BGP in this separable utility.

The household's dynamic programming problem is written as the following Bellman equation:

$$\begin{aligned} &U(k_t, h_t, e_t, A_t) \\ &\quad = \max[u(c_t, 1 - e_t - s_t - n_t - a_t) + \frac{1}{1 + \rho} U(k_{t+1}, h_{t+1}, e_{t+1}, A_{t+1})], \end{aligned}$$

subject to the constraints (1), (2), (3), and (4), taking as given the factor prices, firm profits, unemployment compensation, taxes, and the initial levels of employment (e_0), AI (A_0), human (h_0), and physical capital (k_0), where $\rho > 0$ is the time preference rate.

The first-order conditions with respect to c_t, s_t, n_t, a_t, and u_t, respectively, are

$$u_c(c_t, 1 - e_t - s_t - n_t - a_t) = \frac{1}{1 + \rho} U_k(k_{t+1}, h_{t+1}, e_{t+1}, A_{t+1}), \tag{6a}$$

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$$-u_s(c_t, 1-e_t-s_t-n_t-a_t) = \frac{1}{1+\rho}[U_k(k_{t+1}, h_{t+1}, e_{t+1}, A_{t+1})b_t + U_e(k_{t+1}, h_{t+1}, e_{t+1}, A_{t+1})\mu_t], \quad \textbf{(6b)}$$

$$-u_n(c_t, 1-e_t-s_t-n_t-a_t) = \frac{1}{1+\rho}U_h(k_{t+1}, h_{t+1}, e_{t+1}, A_{t+1})Bh_t, \quad \textbf{(6c)}$$

$$-u_a(c_t, 1-e_t-s_t-n_t-a_t) = \frac{1}{1+\rho}U_A(k_{t+1}, h_{t+1}, e_{t+1}, A_{t+1}) \times \phi D a_t^{\phi-1} h_t^{\phi}(u_t k_t)^{\theta} A_t^{1-\phi-\theta}, \quad \textbf{(6d)}$$

$$U_k(k_{t+1}, h_{t+1}, e_{t+1}, A_{t+1})r_t k_t = U_A(k_{t+1}, h_{t+1}, e_{t+1}, A_{t+1})\theta D(a_t h_t)^{\phi} u_t^{\theta-1} k_t^{\theta} A_t^{1-\phi-\theta}. \quad \textbf{(6e)}$$

The Benveniste–Scheinkman conditions (envelope conditions) for k_t, e_t, h_t, and A_t, respectively, are

$$U_k(k_t, h_t, e_t, A_t) = \frac{1}{1+\rho}\{U_k(k_{t+1}, h_{t+1}, e_{t+1}, A_{t+1})[1+r_t(1-u_t)] + U_A(k_{t+1}, h_{t+1}, e_{t+1}, A_{t+1})\theta D(a_t h_t)^{\phi} u_t^{\theta} k_t^{\theta-1} A_t^{1-\phi-\theta}\}, \quad \textbf{(6f)}$$

$$U_e(k_t, h_t, e_t, A_t) = u_e(c_t, 1-e_t-s_t-n_t-a_t) + \frac{1}{1+\rho}[U_k(k_{t+1}, h_{t+1}, e_{t+1}, A_{t+1})w_t h_t + U_e(k_{t+1}, h_{t+1}, e_{t+1}, A_{t+1})(1-\psi)], \quad \textbf{(6g)}$$

$$U_h(k_t, h_t, e_t, A_t) = \frac{1}{1+\rho}[U_k(k_{t+1}, h_{t+1}, e_{t+1}, A_{t+1})w_t e_t + U_h(k_{t+1}, h_{t+1}, e_{t+1}, A_{t+1})(Bn_t+1) + U_A(k_{t+1}, h_{t+1}, e_{t+1}, A_{t+1})\phi D a_t^{\phi} h_t^{\phi-1}(u_t k_t)^{\theta} A_t^{1-\phi-\theta}], \quad \textbf{(6h)}$$

$$U_A(k_t, h_t, e_t, A_t) = \frac{1}{1+\rho}\{U_k(k_{t+1}, h_{t+1}, e_{t+1}, A_{t+1})r_t^A + U_A(k_{t+1}, h_{t+1}, e_{t+1}, A_{t+1})[(1-\phi-\theta)D(a_t h_t)^{\phi}(u_t k_t)^{\theta} A_t^{-\phi-\theta} + 1]\}. \quad \textbf{(6i)}$$

By combining (6a), (6d), and (6e), along with (5), we can derive the following relationship:

$$\chi(1-e_t-s_t-n_t-a_t)^{-\sigma} = \frac{\phi}{\theta}\frac{u_t}{a_t}\frac{1}{c_t}r_t k_t. \quad \textbf{(7a)}$$

Note that (6d) implies that the loss of marginal utility equates to the marginal benefit of developing AI, and (6e) equates the marginal benefits of allocating physical capital between the AI sector and the goods sector. That is, (7a) indicates the marginal benefits and marginal disadvantages of using resources in the AI sector in terms of units of marginal utility of consumption.

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In addition, (6a), (6e), and (6f) together yield the following consumption Euler equation:

$$u_c(c_t, 1-e_t-s_t-n_t-a_t) = \frac{1}{1+\rho}u_c(c_{t+1}, 1-e_{t+1}-s_{t+1} - n_{t+1}-a_{t+1})(1+r_{t+1}). \quad \textbf{(7b)}$$

(7b) can be rewritten as $1/c_t = (1+r_{t+1})/[(1+\rho)c_{t+1}]$.

Moreover, using (6a) and (6b), (6g) implies that the leisure utility (which here is the marginal utility of not participating in the labor force) equals the marginal benefit of participating in the labor force, considering unemployment compensation and the discounted marginal value of employment when job matching. The equation is as follows:

$$\chi(1-e_t-s_t-n_t-a_t)^{-\sigma} = \frac{b_t}{c_t} + \frac{\mu_t}{1+\rho}\left[\frac{1-\psi-\mu_{t+1}}{\mu_{t+1}}\chi(1-e_{t+1}-s_{t+1}-n_{t+1}-a_{t+1})^{-\sigma} + \frac{h_{t+1}w_{t+1}}{c_{t+1}} - \frac{b_{t+1}(1-\psi)}{c_{t+1}\mu_{t+1}}\right]. \quad \textbf{(7c)}$$

Furthermore, (6h), along with (6a) and (6c)–(6e), indicates that the leisure utility (which here is the marginal utility of not accumulating human capital) equals the marginal benefit of accumulating human capital, considering the discounted marginal value of future wages and the related benefit of assisting with the human capital and AI accumulation. The equation is as follows:

$$\chi(1-e_t-s_t-n_t-a_t)^{-\sigma} = \frac{Bh_t}{(1+\rho)c_{t+1}}\left[w_{t+1}e_{t+1} + \left(n_{t+1}+\frac{1}{B}+a_{t+1}\right)r_{t+1}\frac{u_{t+1}k_{t+1}}{a_{t+1}h_{t+1}}\frac{\phi}{\theta}\right]. \quad \textbf{(7d)}$$

Finally, (6i), along with (6a) and (6d)–(6e), shows that the leisure utility (the marginal utility of not accumulating AI) equals the marginal benefit of accumulating AI, considering the discounted marginal value of future AI returns and the related benefit of assisting with AI accumulation and final goods production. The equation is as follows:

$$\chi(1-e_t-s_t-n_t-a_t)^{-\sigma} = \frac{\phi D a_t^{\phi-1}h_t^{\phi}(u_tk_t)^{\theta}A_t^{1-\phi-\theta}}{(1+\rho)c_{t+1}} \left[r_{t+1}^A + r_{t+1}\frac{u_{t+1}k_{t+1}}{A_{t+1}}\frac{1-\phi-\theta}{\theta} + \frac{r_{t+1}}{\theta D}(a_{t+1}h_{t+1})^{-\phi}(u_{t+1}k_{t+1})^{1-\theta}A_{t+1}^{\phi+\theta-1}\right]. \quad \textbf{(7e)}$$

2.2. Firms

The representative firm produces output and creates and maintains multiple job vacancies. The firm produces a single final good y_t by renting physical capital and

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AI and employing labor under the following production technology:

$$y_t = F[(1-u_t)k_t]^{1-\alpha-\varepsilon} A_t^{\varepsilon}[(1-\tau)(e_t h_t)^{\beta} + \tau A_t^{\beta}]^{\frac{\alpha}{\beta}}, \tag{8}$$

where $F > 0$ is productivity, $\alpha \in (0, 1)$ is the labor share, $\varepsilon \in (0, 1)$ is the AI share, and thus $1 - \alpha - \varepsilon \in (0, 1)$ is the physical capital share. Because AI is widely used in production, it is one of the key production inputs of firms. That is, we set the share of AI in the production function as ε. In addition, as introduced in the Introduction, AI is expected to eventually replace human resources. Besides, this paper mainly discusses the impact of AI on the labor market. Therefore, we use a constant elasticity of substitution (CES) production function where AI and human labor can replace each other. Note that $\beta = 1 - 1/\beta_{h,A} \in (-\infty, 1]$, in which $\beta_{h,A} \geq 0$ is the elasticity of substitution between human labor and AI, and $\tau \in (0, 1)$ is the share parameter. The bigger the τ is, the more the AI is used (compared with human labor) for production.

Note that when $\varepsilon = 0$ and $a = 0$, our model is a combination of the Lucas (1988) two-sector endogenous growth model and the standard labor search model. This is because the households have no incentive to develop AI when it does not assist with the production of final goods in this situation.

Moreover, the representative firm creates and maintains multiple job vacancies to recruit workers. The hiring cost is $\Lambda(v_t)$, where v_t is the number of vacancies with $\Lambda'(v_t) > 0$. We follow the setting in Domeij (2005) and Arseneau and Chugh (2006) and assume that the hiring cost is linear in terms of vacancies, that is, $\Lambda(v_t) = \lambda_t v_t$, where λ_t denotes the unit hiring cost. To sustain perpetual growth, following Pissarides (2000), we set $\lambda_t = \lambda \bar{y}_t$, where $\lambda > 0$ and $\bar{y}_t$ is average output in the economy, both of which are taken as given by the representative firm.

From the firm's perspective, employment in the next period is

$$e_{t+1} = (1-\psi)e_t + \eta_t v_t, \tag{9}$$

where η_t is the (endogenous) recruitment rate. Thus, the change in employment is equal to the inflow of employees ($\eta_t v_t$), net of the outflow (ψe_t).

The firm's profit is

$$\pi_t = y_t - w_t e_t h_t - r_t(1-u_t)k_t - r_t^A A_t - \lambda_t v_t. \tag{10}$$

The firm's dynamic programming problem is written as the following Bellman equation:

$$\Pi(e_t) = \max\left[\frac{\pi_t}{\bar{y}_t} + \frac{1}{1+r_t}\Pi(e_{t+1})\right],$$

subject to the constraint (9). Note that the instantaneous profit π_t is unbounded because of perpetual growth. To ensure a concave firm's dynamic programming, we divide the instantaneous profit by average output in the economy.

The first-order conditions with respect to k_t, A_t, and v_t, respectively, are

$$(1-\alpha-\varepsilon)\frac{y_t}{(1-u_t)k_t} = r_t, \tag{11a}$$

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$$\frac{y_t}{A_t}\left[\varepsilon+\frac{\alpha\tau A_t^{\beta}}{(1-\tau)(e_t h_t)^{\beta}+\tau A_t^{\beta}}\right]=r_t^A, \tag{11b}$$

$$\lambda=\frac{1}{1+r_t}\Pi_e(e_{t+1})\eta_t. \tag{11c}$$

The Benveniste–Scheinkman condition (envelope condition) for e_t is

$$\Pi_e(e_t)=\frac{1}{\bar{y}_t}\left[\frac{\alpha y_t}{e_t}\frac{(1-\tau)(e_t h_t)^{\beta}}{(1-\tau)(e_t h_t)^{\beta}+\tau A_t^{\beta}}-w_t h_t\right]+\frac{1}{1+r_t}\Pi_e(e_{t+1})(1-\psi). \tag{11d}$$

While (11a) and (11b) are standard, (11c) equates a firm's marginal value of recruitment in the next period to the marginal hiring cost. The firm's marginal value of recruitment is given by (11d), which is the sum of the firm's surplus from a successful match and the savings of tomorrow's discounted marginal cost of vacancy creation and maintenance if separation does not subsequently occur. Note that, with search frictions, the bargained wage is smaller than the marginal product of labor; therefore, profits are positive.

2.3. Labor Matching and Wage Bargaining

The labor market exhibits search frictions. The creation of new jobs requires that firms post vacancies and that the unemployed search for job opportunities. According to Diamond (1982), the new jobs are generated by the following constant-returns matching technology:

$$M_t=m(s_t)^{\xi}(v_t)^{1-\xi}, \tag{12a}$$

where $m>0$ measures the degree of matching efficacy and $\xi\in(0,1)$ is the contribution of a job seeker in the formation of a match.

The effective wage rate is determined by Nash bargaining, which maximizes the product of the firm's and the worker's surpluses from a match. The worker's surplus acquired from a successful match is evaluated by its augmented value of supplying an additional worker $U_e(k_t,h_t,e_t,A_t)$ in (6g). The firm's surplus gained from a successful match is gauged by its added value from recruiting an extra worker $\Pi_e(e_t)$ in (11d). Thus, the wage at time t solves the following cooperative bargaining game:

$$\max_{w_t}\ [U_e(k_t,h_t,e_t,A_t)]^{\gamma}[\Pi_e(e_t)]^{1-\gamma},$$

where $\gamma\in(0,1)$ is the worker's bargaining share. The first-order condition is

$$\frac{\gamma}{U_e(k_t,h_t,e_t,A_t)}\frac{dU_e(k_t,h_t,e_t,A_t)}{dw_t}=-\frac{1-\gamma}{\Pi_e(e_t)}\frac{d\Pi_e(e_t)}{dw_t}. \tag{12b}$$

2.4. The Government, Aggregate Resources, and Search Equilibrium

The government levies lump-sum taxes to finance unemployment compensation and meets the following budget constraint:

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$$T_t = b_t s_t. \tag{13a}$$

We set $b_t = b\bar{y}_t$ with $b > 0$ to sustain perpetual growth. To simplify the model, we assume that the government has no other public expenditure.

Using (6g) and (11d), along with (6a), (6b), (11c), and (7a), we can rewrite (12b) as follows:

$$w_t = \frac{y_t}{h_t}\left\{\gamma\left[\frac{\alpha(1-\tau)e_t^{\beta-1}}{(1-\tau)e_t^{\beta}+\tau(A_t/h_t)^{\beta}}+\frac{\lambda(1-\psi)}{\eta_t}\right]+(1-\gamma)\left[\frac{b(1-\psi)}{\mu_t}-\frac{1-\psi-\mu_t}{\mu_t}\frac{\phi u_t(1-\alpha-\varepsilon)}{\theta a_t(1-u_t)}\right]\right\}. \tag{13b}$$

Unlike the labor market, the goods market is frictionless. Using the household's budget constraint, (4), the firm's profit function, (10), and the government's balanced budget constraint, (13a), we obtain the following aggregate goods market constraint:

$$k_{t+1} = y_t + k_t - c_t - \lambda\bar{y}_t v_t = k_t + (1-\lambda v_t)y_t - c_t. \tag{14}$$

Note that $\bar{y}_t = y_t$ in equilibrium.

The matching number is equal to the search inflow into the employment pool and to the newly occupied vacancies, that is, $m(s_t)^{\xi}(v_t)^{1-\xi} = \mu_t s_t = \eta_t v_t$ in equilibrium. Thus, the employment equilibrium condition is as follows:

$$e_{t+1} = (1-\psi)e_t + m(s_t)^{\xi}(v_t)^{1-\xi}. \tag{15}$$

A search equilibrium consists of the households' and the firms' choices $\{c_t, s_t, n_t, a_t, u_t, v_t, k_t, e_t, h_t, A_t\}$, prices $\{r_t, r_t^A, w_t\}$, and matching rates $\{M_t, \mu_t, \eta_t\}$, such that: (i) households optimize; (ii) firms optimize; (iii) the employment evolution conditions hold; (iv) the labor market matching and wage bargaining conditions are met; (v) the government's budget is balanced; and (vi) all markets clear.

Using (2), (3), (7a)–(7e), (11d), (14), and (15), along with (11a)–(11c) and (13b), we can obtain that all the equilibrium equations are functions of $s_i, n_i, a_i, u_i, v_i, e_i, k_i/h_i, A_i/h_i$, and c_i/h_i, where $i = t, t+1$. To analyze the equilibrium, we transform the perpetually growing variables of consumption, physical capital, AI, and human capital into the ratios c/h, k/h, and A/h, respectively. Denote $q_i = k_i/h_i$, $x_i = A_i/h_i$, and $z_i = c_i/h_i$. Using those dynamic equations, we can derive the time paths of $s_t, n_t, a_t, u_t, v_t, e_t, q_t, x_t$, and z_t.[1]

A long-run equilibrium is the BGP along which the proportion of the household's members devoted to labor market, learning, and developing AI (e, s, n, and a, respectively), the proportion of the household's physical capital (u) allocated to developing AI, and the firm's job vacancies (v) are all constant, and consumption, physical capital, human capital, and AI all grow at the same rate, denoted by Γ (which is also the growth rate of the economy in the long run). That is, q, x, and z are all constant.[2] To understand the impact of AI on macroeconomic

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performance along the transitional dynamics path and in the long run more clearly, we undertake a numerical analysis in the next section.

3. NUMERICAL ANALYSIS

Here, we conduct a numerical analysis to discuss the effect of AI along the transitional dynamics path and the BGP. We consider an economy without AI as the benchmark model, that is, initially AI has not been invented. Then, we can simulate the economy's shift from a state without AI to one with AI. Exploring the impact along the transitional dynamics path is especially important because it helps understand and predict the future impact of AI.

3.1. Calibration

The initial situation is an economy where AI has not been invented. To quantify the results, we calibrate the model without AI along the BGP to reproduce the key features of the US economy at annual frequencies. When $A_0 = 0$, $\varepsilon = 0$, and $\tau = 0$, that is, AI does not exist, our model is the combination of the Lucas (1988) two-sector endogenous growth model and the standard labor search model, that is, the production function is $y_t = Fk_t^{1-\alpha}(e_t h_t)^{\alpha}$, and $a_t = u_t = x_t = 0$.

We use data for the period 2000–2014.[3] According to the Penn World Tables (version 9.0), the growth rate of gross domestic product, the human capital index, the consumption—output ratio, and the total employment—population ratio in the US during 2000–2014 were 1.8963%, 3.6559, 0.6993, and 0.4760, respectively.[4] As the human capital index is based on years of schooling and returns to education, we assume that the index represents the human capital to output ratio, h/y. Thus, we set initial $h/y = 3.6559$, $c/y = 0.6993$, $e = 0.4760$, and the economic growth rate at $\Gamma = 1.8963\%$. That is, $z = (c/y)/(h/y) = 0.1913$.

According to Shimer (2005), the monthly separation rate is given by 0.034 and the monthly job-finding rate by 0.45. Based on these data, we calculate the annual separation rate $\psi = 1 - (1 - 0.034)^{12} = 0.3397$ and the annual job-finding rate $\mu = 1 - (1 - 0.45)^{12} = 0.9992$. Along the BGP, (1), (9), and (15) yield the long-run matching equilibrium condition $\mu s = \eta v = ms^{\beta} v^{1-\beta} = \psi e$. Then, we calibrate the fraction of labor engaged in search activities as $s = 0.1618$. In addition, we follow Hagedorn and Manovskii (2008), who find that monthly labor market tightness is 0.634 and the annual labor market tightness is $1 - (1 - 0.634)^{12} = 1$, that is, the vacancy-searching worker ratio is $v/s = 1$. Thus, we use the long-run matching equilibrium condition to calibrate and obtain $v = 0.1618$ and $\eta = 0.9992$. Therefore, the participation and unemployment rates are $e + s = 0.6378$ and $s/(e + s) = 0.2537$, respectively.

In addition, we set the physical capital share in the production function of final goods at 0.36, following Kydland and Prescott (1982); therefore, $\alpha = 1 - 0.36 = 0.64$. Following Kydland and Prescott (1991), who used 4% as the annual rate of time preference, we set $\rho = 4\%$. Using the production function (7b) and (11a) in

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TABLE 1. Calibration results for economies transitioning from no AI to AI

Participation	e	s	n	a	u	v
(A) Initial economy without AI						
0.6378	0.4760	0.1618	0.1928	0	0	0.1618
(B) An economy with AI						
0.3780	0.2878	0.0901	0.4588	0.0613	0.1537	0.1014
Unemployment	x	q	z	Γ	U	
(A) Initial economy without AI						
0.2537	0	1.6488	0.1913	0.0190	−25.7166	
(B) An economy with AI						
0.2385	0.5077	0.5114	0.1047	0.0451	−26.6980	

the long run, we can calibrate $q = 1.6488$ and $F = 0.3674$, respectively. We use (14) in the long run to calibrate $\lambda = 1.1517$. This value is similar to the cost of a vacancy, 1.1636, in Shi and Wen (1999, p. 471, the value for b in their paper). In addition, the firms pay $\lambda v\bar{y}/\bar{y} = 0.1864$ to hire workers, which is around the value of 0.1572 used in Shimer (2005, p. 38, the value of c/μ in his paper). That is, our calibration result is consistent with the existing literature.

The value of γ is in the commonly used range of 0.3–0.6 (e.g., see Andolfatto (1996), Shi and Wen (1999), and Domeij (2005)). We set the worker's bargaining share at 0.3 as our benchmark case. Furthermore, we equate bargaining power and the elasticity of the matching function to internalize the externality generated by the search friction, that is, the Hosios condition is met; therefore, $\xi = \gamma = 0.3$. Then, we use the long-run matching equilibrium condition to calibrate and obtain $m = 0.9992$.

By combining (11c) and (11d) in the long run, we can derive $w = 0.2419$. Note that when AI does not exist, $w = \gamma(y/h)[\alpha/e + \lambda(1-\psi)/\eta] + (1-\gamma)[b(1-\psi)(y/h) - (1-\psi-\mu)z\chi(1-e-s-n)^{-\sigma}]/\mu$. Furthermore, the long-run growth rate of the economy is $\Gamma = Bn$. Therefore, using the above relationships and (7c) in the long run, we can calibrate $b = 0.1902$, $B = 0.0984$, and $n = 0.1928$.

The IES for labor ranges from close to 0 (MaCurdy (1981)) to 3.8 (Imai and Keane (2004)). Following Hansen and Imrohoroğlu (2009), we choose a mid-range value of the Frisch labor supply elasticity at 1.9 as our benchmark case, which implies that $\sigma = 0.1398$. Then, we use the above wage equation to calibrate the degree of leisure relative to consumption in utility as $\chi = 0.7733$. Using the above parameters and variables, we can obtain the household's welfare as $U = -25.7166$ under the assumption that $h_0 = 1$. We summarize the related macroeconomic variables in Table 1 (case A).

Next, we simulate the economy with AI. We first make assumptions about the parameters related to AI and then conduct sensitivity analysis, that is, robustness

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checks, in the following sections. To date, there are no data on the elasticity of substitution between human labor and AI. According to Boskin (1978, S17), the elasticity of substitution between labor and machines is around 0.45. We can calibrate $\beta = -1.2222$.[5] As for other parameters, we set that $\tau = 0.5$, $D = 0.1$, $\varepsilon = 0.1$, $\phi = 0.2$, and $\theta = 0.2$. That is, we can calibrate variables in this economy with AI as follows: $e = 0.2878$, $s = 0.0901$, $n = 0.4588$, $a = 0.0613$, $u = 0.1537$, $v = 0.1014$, $x = 0.5077$, $q = 0.5114$, $z = 0.1047$, $\Gamma = 0.0451$, and $U = -26.6980$. In addition, the participation and unemployment rates are 0.3780 and 0.2385, respectively. For comparison, we show an economy in which AI has not been invented (case A) and an economy in which AI is being developed (case B) in Table 1.

The above result implies that the household will assign members and resources to developing AI when AI is present but will not reduce the proportion of household members devoted to the accumulation of human capital, instead reducing the members participating in the labor market. In addition, the firm will reduce the provision of traditional job vacancies. In our simulation, the household assigns more members to learning (accumulating human capital), and this will help with AI development, as human capital is one of the indispensable elements in the development of AI. Although the resources devoted to final goods production $(1 - u)$ are reduced, the long-term economic growth rate increases because the development of AI and the accumulation of human capital are beneficial to production. Note that household welfare declines because the ratio of consumption to human capital and leisure are lower when AI is invented.

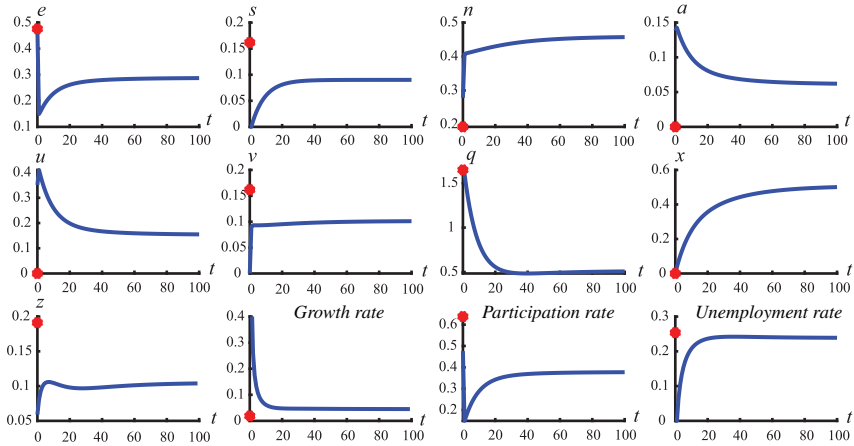
The development of AI will indeed significantly affect the type of work done by humans. Following the introduction of AI, labor force participation in traditional work declines and firms' job vacancies are reduced. Instead, household members will focus on increasing their own human capital accumulation and participate in the development of AI. However, although fewer people participate in the labor market, the existence of AI is not necessarily harmful to the employment rate. We obtain a lower unemployment rate when AI is invented.

Although we set specific parameters related to AI, using different parameters provides consistent results. The development of AI may reduce the unemployment rate. Below, as a sensitivity analysis, we conduct a long-term comparative static analysis. In addition, we note that the above simulations analyzed only the long-term results. However, as programmers around the world are currently working diligently to develop AI, we have not yet reached the long-term equilibrium. Therefore, studying the transitional dynamics of the development of AI is also very important to help us understand the current environment. We address this in the next section.

3.2. Transitional Dynamics

In this section, we examine the transition from an economy without AI to one with AI, that is, the dynamic effects of the development of AI. We take the economy without AI (case A in Table 1) as the initial situation and investigate the dynamic

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Note: The initial point at time 0 is the economy without AI, that is, case (A) in Table 1. The economy with AI is case (B) in Table 1.

FIGURE 1. Transitional dynamics when shifting from an economy without AI to one with AI.

paths of all variables from the initial situation to the situation with AI (case B in Table 1).

Using (2), (3), (7a)–(7e), (11d), (14), and (15), along with (11a)–(11c) and (13b), we can simplify the above equilibrium conditions into an eight-dimensional dynamic system with state vector $(s_t, n_t, a_t, u_t, v_t, e_t, q_t, x_t)$. To investigate the dynamic effects of the invention of AI, we start by taking a linear Taylor's expansion of the dynamical system in the neighborhood of the unique BGP and obtain a Jacobian matrix.

We must find three eigenvalues lying inside the unit circle and five eigenvalues lying outside the unit circle associated with the Jacobian matrix to guarantee a unique equilibrium path toward the BGP. Then, the values of s_t, n_t, a_t, u_t, v_t, e_t, q_t, and x_t in the unique equilibrium path are each represented by the sum of their own new BGP and three products of three components: (i) a coefficient, (ii) an eigenvalue that lies inside the unit circle to the power of t, and (iii) the corresponding eigenvector of the eigenvalue that lies inside the unit circle. The coefficient is determined by the boundary conditions in which the state variables, e_0, q_0, and x_0 are predetermined. Then, the values of z_t are derived from (7a).[6] The results are shown in Figure 1. To clearly understand the impact of the moment when AI is invented, Figure 1 only shows the changes for the first 100 periods.

Through these transitional dynamic paths, we can understand how the development of AI dramatically affects the labor market. The invention of AI changes the pattern of traditional work. Initially, employment declines significantly and people who are not working are less willing to search for jobs. That is, the labor force participation rate significantly declines. However, the

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unemployment rate decreases as s goes to 0. At that time, firms also substantially reduce job vacancies.

In addition, the household reduces its resources (physical capital) used in the goods sector and reallocates resources to the AI sector. Some members of the large household begin to develop AI, and the number of members accumulating human capital increases further because the accumulation of human capital assists with the development of AI. Therefore, the ratio of physical to human capital decreases and the ratio of AI to human capital increases. However, output does not decrease because AI contributes to production.

Using different parameter values and model settings results in consistent outcomes. To save space, we only illustrate the dynamic path from case (A) to case (B) in Table 1 and list the results derived from changing the parameter values in the next section using long-term comparative static analysis. Note that whenever a parameter is changed, there will be different time paths. However, the conclusions remain consistent. It is worth noting that although the participation rate in the labor market declines, the development of AI is not necessarily harmful to the employment rate.

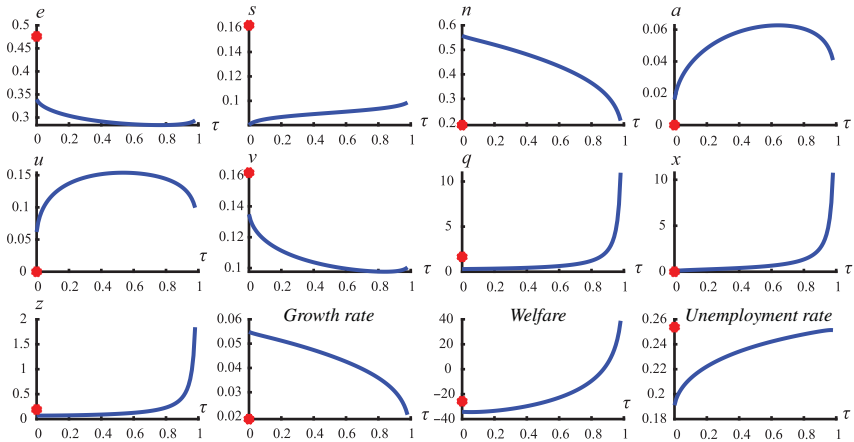
3.3. Long-Run Comparative Static Analysis

Now, we investigate the long-run effects of different parameter values on those variables related to AI and the labor market. The parameter values related to AI cannot be calibrated from existing data. Therefore, comparative static analysis is performed for these parameters. In the following comparative static exercises, we use the parameter settings in case (B) in Table 1 as the benchmark and then change one parameter at a time. Note that in the following figures, we do not show the graph of labor force participation rates because it is equal to $e + s$, but we analyze the impact on long-term welfare.

First, we discuss the effects of different production functions. We change the value of τ, with the results shown in Figure 2. Note that when $\tau = 0$, the production function is of Cobb–Douglas form, and all three inputs, k, A, and h, are necessary production factors. When $\tau \in (0, 1)$, the production function is of CES form. All three inputs, k, A, and h, are still required to produce final goods, but AI can replace human labor. When τ is larger, AI is more important and human labor is less important for production. When $\tau = 1$, the production function returns to Cobb–Douglas form, with only k and A as necessary production factors. In this case, the households still accumulate human capital as it is one of the necessary elements for developing AI.

Figure 2 indicates that when τ is not very large, the household will devote more of its members and resources to developing AI when AI becomes more important for production; however, if τ is very large, the situation will be reversed. Furthermore, the household will devote fewer of its members to learning because human capital becomes less important for production when τ increases. That is, the ratios of AI, physical capital, and consumption to human capital are increasing

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Note: The point on the vertical axis is the economy without AI (case A in Table 1).

FIGURE 2. The long-run effects of different production functions.

in τ, as is household welfare. As fewer members of the household accumulate human capital, the growth of the economy is decreasing in τ. As for the labor market, the number of vacancies and employment in traditional jobs will decrease, and the unemployment rate will rise when τ increases. However, the overall unemployment rate remains smaller than that in the economy without AI. Thus, the development of AI is not harmful to the employment rate.

We also analyze the impact of varying the elasticity of substitution between human labor and AI (β). The graph illustrating such comparative statics is similar to Figure 2. Thus, to save space, we show the results in Table 2. Intuitively, firms can directly use AI-related inputs as long as they pay rent, whereas, when hiring labor, in addition to paying wages, they also need to bargain with job seekers in the labor market to set wages and they need to pay recruitment costs. Because the use of AI as a production input is simpler compared with hiring human labor, the impact of an increase in β (an increase in the elasticity of substitution between human labor and AI) is similar to that of an increase in τ.

Next, we study the effects of different AI shares (ε) on the production function. The results are listed in Table 2. Note that in the following comparative static analysis, if the impact of changing a parameter on variables related to the labor market and the AI accumulation function is monotone, we only list the results in Table 2 and do not provide figures to save space. The number of its members and resources that the household devotes to developing AI and the ratio of AI to human capital increase as ε increases. AI becomes more important for production as ε increases; therefore, the household assigns more members to accumulating human capital because this assists with the accumulation of AI. This (higher n) also increases the long-run economic growth rate. Note that household welfare is correlated to the ratio of consumption to human capital. As the household devotes

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TABLE 2. The results of the comparative statics in the long run

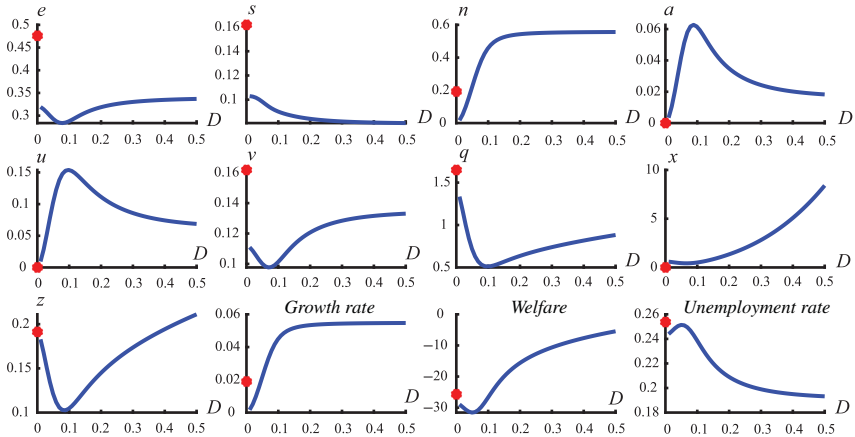
	e	s	n	a	u	v
τ	↓↑	↑	↓	↑↓	↑↓	↓↑
β	↓↑	↑	↓	↑↓	↑↓	↓↑
ε	↓	↓	↑	↑	↑	↑
D	↓↑	↓	↑	↑↓	↑↓	↓↑
ϕ	↓	↓	↓↑	↑	↑	↓
θ	↓	↑	↓	↓	↑	↓
γ	↓	↑	↑	↑↓	↑↓	↓
ξ	↓	↑	↓	↓	↓	↑
b	↑	↑	↓	↓	↓	↓
	q	x	z	Γ	U	Unemployment
τ	↑	↑	↑	↓	↑	↑
β	↑	↑	↑	↓	↑	↑
ε	↓	↑	↓↑	↑	↓↑	↓
D	↓↑	↓↑	↓↑	↑	↓↑	↑↓
ϕ	↓	↓	↓	↓↑	↓	↑↓
θ	↑	↓	↓	↓	↓	↑
γ	↓	↓	↓	↑	↓	↑
ξ	↓	↑	↓	↓	↓	↑
b	↑	↑	↑	↓	↑	↑

Notes: ↑ and ↓ indicate that the effects of changing the parameters on related variables are monotonically increasing and decreasing, respectively. ↑↓ indicates that the impact of changing the parameters on related variables is first rising and then falling, whereas ↓↑ indicates the opposite scenario.

more members to learning and developing AI, the members participating in the labor market decrease. However, the unemployment rate declines because the extent of the reduction in unemployed job seekers (who exit the labor force) is greater than the reduction in employment.

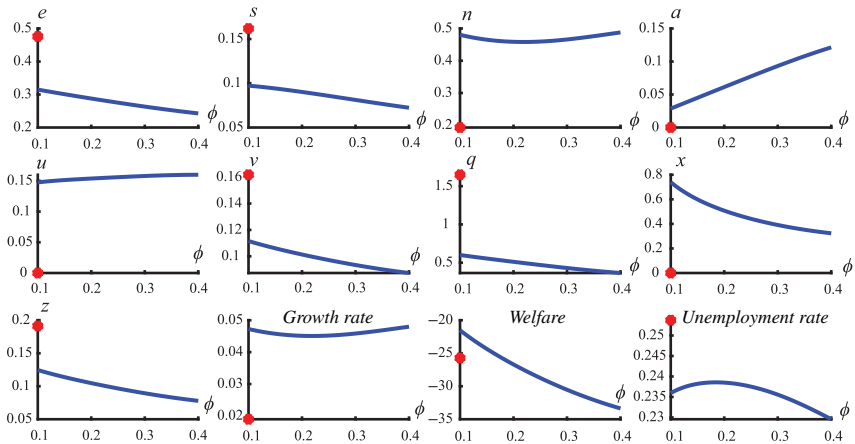
Now, we analyze the effects of different AI self-accumulation abilities, and of different labor and physical capital shares in the AI accumulation function. The results are presented in Figures 3 and 4, and Table 2, respectively. A higher D implies that AI has higher self-accumulation abilities. Therefore, the household devotes more members to accumulating human capital, one of the necessary elements for developing AI; therefore, the long-run economic growth rate increases as D increases. For other variables, the impact of changing D is not monotone. Except for the case where D is very small, the household does not have

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Note: The point on the vertical axis is the economy without AI (case A in Table 1), in which the AI accumulation function (2) does not exist.

FIGURE 3. The long-run effects of different AI self-accumulation abilities.



Note: The point on the vertical axis is the economy without AI (case A in Table 1), in which the AI accumulation function (2) does not exist.

FIGURE 4. The long-run effects of different labor shares in the AI sector.

to invest much effort or resources into the AI sector because the self-accumulation ability of AI is sufficient to support the development of AI when D increases. As AI assists production, firms are more willing to provide more vacancies; thus, employment increases and the unemployment rate declines. Moreover, the ratios of AI, physical capital, and consumption to human capital are increasing in D, as is household welfare. However, if D is very small, the foregoing results will be reversed.

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In addition, according to Figure 4, the household devotes more of its members to developing AI when the labor share in the AI accumulation function increases. Although the effects of changing ϕ on n (and on the economic growth rates) are not monotone, compared with the economy without AI, the household devotes more of its members to accumulating human capital; therefore, the economic growth rate with AI is larger than that without AI. Thus, the ratios of AI, physical capital, and consumption to human capital are decreasing in ϕ, as is household welfare. As the household assigns more members to the AI and education sectors, the proportion of its members devoted to the goods sector is reduced as ϕ increases. Firms create fewer job vacancies and there are fewer unemployed persons searching for job opportunities when ϕ increases. Although the impact of changing ϕ on the unemployment rates is not monotone, the overall unemployment rate is still smaller than that in an economy without AI.

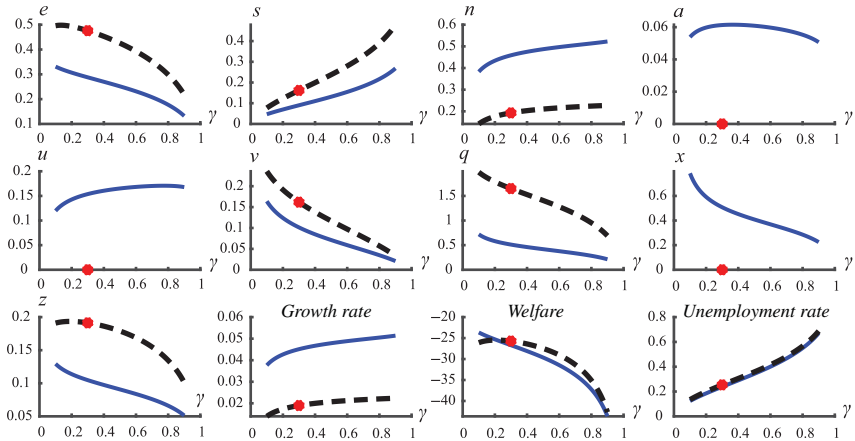
As for the impact of changing physical capital shares (θ) in the AI accumulation function, the household devotes more resources (physical capital, i.e., u) to the AI sector. As physical capital becomes more important to the development of AI, the household assigns fewer members to the AI and education sectors and accumulates more physical capital. Thus, we obtain that n, a, and growth rates are decreasing, whereas q is increasing in θ. Overall, θ has a negative impact on the ratios of AI and consumption to human capital (x and z), as it does on welfare. As for the labor market, changing θ influences the household's labor force participation, the creation of job vacancies by firms, and the unemployment rate, but the impact is not significant.

The above comparative static analysis shows that the conclusion remains consistent even when there are changes to structural parameters or functional forms. In all cases, if we compare the results to those of the economy without AI, we find that, except for the case of a very small D, the long-run economic growth rates are larger in the economy with AI. However, labor force participation rates and firm vacancies are lower. Thus, the invention of AI does influence human jobs but not necessarily in a negative way. More people devote their labor to developing AI and accumulating self-ability, as they need to upgrade their capabilities enable them to work with AI. In addition, we find that unemployment rates in all the above exercises are lower in an economy with AI.

As well as the above comparative static analysis, we also discuss the situation if the Hosios condition is not met and the impact of altering the unemployment compensation offered by the government. The results are shown in Figure 5 and Table 2, respectively. Note that to compare the economy where AI is invented with the economy without AI, we illustrate the two cases under a different γ in Figure 5. Our conclusion holds, regardless of the changes to the levels of γ, ξ, or b, that is, we continue to find that the economic growth rates in an economy with AI are larger than those without AI, and that the unemployment rate in an economy with AI is smaller than that in an economy without AI.

It is worth noting that our model can obtain results that are consistent with the standard labor search model, in which the unemployment rate is increasing

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Note: The point at $\gamma = 0.3$ is case (A) in Table 1. The dashed line is the economy without AI and the solid line is the economy with AI.

FIGURE 5. The long-run effects of different workers' bargaining power.

in γ and b. Intuitively, when workers' wage bargaining power (γ) increases, job seekers can obtain higher salaries when they are employed, which will increase the incentives for the unemployed to find a job. However, this will reduce the incentives for firms to provide vacancies because labor costs have increased. The unemployment rate increases because more people are looking for jobs but fewer vacancies are provided. When b increases, there are incentives for people to remain unemployed. In addition, according to (13b), wages will rise with the increase in b. Similar to the impact of an increase in γ, more people are searching for jobs and fewer vacancies are provided; therefore, the unemployment rate increases as b increases.

4. AN ECONOMY WITH HETEROGENEOUSLY SKILLED WORKERS

As we discussed in the Introduction, the invention of AI may change the type of work done by humans in a comprehensive way. Unlike traditional machines, which replace the use of low-skilled labor, AI-related inputs may affect high-tech labor. To further discuss the impact of AI on labor with different abilities, in this section, we extend the model to incorporate heterogeneously skilled labor.

To simplify the analysis, we continue to use a large household setup. However, the representative large household now consists of two differently skilled workers. Assume that a fraction p of the members of the large household are skilled workers and the remaining fraction $1 - p$ is unskilled. Only skilled people can accumulate human capital and develop AI. In period t, the fractions n_t and a_t of the skilled household members are improving their skills and developing AI,

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respectively, the fraction e_t^1 (e_t^2) of the skilled (unskilled) members is employed, the fraction s_t^1 (s_t^2) is searching for jobs, and the remaining fraction $1-e_t^1-s_t^1-n_t-a_t$ ($1-e_t^2-s_t^2$) are engaged in leisure. Hereafter, the superscripts $i=1,2$ denote the related variables for skilled and unskilled members, respectively.

Thus, the level of employment of the skilled (unskilled) members from the household's perspective becomes the following process: $e_{t+1}^i=(1-\psi^i)e_t^i+\mu_t^i s_t^i$. The household's utility function and budget constraint are changed to: $u(c_t, 1-e_t^1-s_t^1-n_t-a_t, 1-e_t^2-s_t^2) = p[\ln(c_t) + \chi^1\frac{(1-e_t^1-s_t^1-n_t-a_t)^{1-\sigma^1}}{1-\sigma^1}] + (1-p)[\ln(c_t)+\chi^2\frac{(1-e_t^2-s_t^2)^{1-\sigma^2}}{1-\sigma^2}]$, and $k_{t+1}=k_t+w_t^1 pe_t^1 h_t+w_t^2(1-p)e_t^2 h_t+r_t(1-u_t)k_t+r_t^A A_t+\pi_t-c_t+b_t^1 ps_t^1+b_t^2(1-p)s_t^2-T_t$, respectively. As only skilled people can develop AI and accumulate human capital, both accumulation functions are $A_{t+1}=D(pa_t h_t)^\phi(u_t k_t)^\theta A_t^{1-\phi-\theta}+A_t$ and $h_{t+1}=B(pn_t h_t)+h_t$, respectively. The Bellman equation related to the household's dynamic programming problem now becomes as follows: $U(k_t,h_t,e_t^1,e_t^2,A_t)=\max[u(c_t,1-e_t^1-s_t^1-n_t-a_t,1-e_t^2-s_t^2)+\frac{1}{1+\rho}U(k_{t+1},h_{t+1},e_{t+1}^1,e_{t+1}^2,A_{t+1})]$. Note that in this large household setup, everyone has the same initial technology (h_t), but creative talent differs. Thus, although the unskilled workers have the basic knowledge relevant to their jobs, they do not have the ability to create new knowledge or develop AI.

As for the setting of the firm, the production technology is now:

$$y_t=F\{(1-\tau^2)[(1-p)e_t^2 h_t]^\delta + \tau^2[(1-u_t)k_t]^\delta\}^{\frac{1-\alpha-\varepsilon}{\delta}}A_t^\varepsilon\{(1-\tau^1)(pe_t^1 h_t)^\beta+\tau^1 A_t^\beta\}^{\frac{\alpha}{\beta}},$$

where $\delta\in(-\infty,1]$ and $\tau^1,\tau^2\in(0,1)$. We use this setting to emphasize the fact that traditional machines can only replace unskilled labor, but AI can replace skilled labor. Note that $\alpha\gg 1-\alpha-\varepsilon$ because the productivity of the skilled labor is larger than that of the unskilled labor. When $\tau^2=1$, the above production function is the same as (8), and the model will return to the economy without heterogeneously skilled labor. Note that we regard a machine that can only do routine and repeated tasks as a form of physical capital, so it can replace unskilled workers, whereas AI has deep machine learning and problem-solving abilities; thus, AI can replace skilled workers.

Employment for the skilled (unskilled) labor from the firm's perspective is $e_{t+1}^i=(1-\psi^i)e_t^i+\eta_t^i v_t^i$. The firm's profit becomes $\pi_t=y_t-w_t^1 pe_t^1 h_t-w_t^2(1-p)e_t^2 h_t-r_t(1-u_t)k_t-r_t^A A_t-\lambda_t^1 v_t^1-\lambda_t^2 v_t^2$, where $\lambda_t^1=\lambda^1\bar{y}_t$ and $\lambda_t^2=\lambda^2\bar{y}_t$, with $\lambda^1>0$ and $\lambda^2>0$. The Bellman equation related to the firm's dynamic programming problem is changed to the following: $\Pi(e_t^1,e_t^2)=\max[\frac{\pi_t}{\bar{y}_t}+\frac{1}{1+r_t}\Pi(e_{t+1}^1,e_{t+1}^2)]$.

In addition, the matching technology now becomes $M_t^i=m^i(s_t^i)^{\xi^i}(v_t^i)^{1-\xi^i}$. The two wage rates are changed to solve the following cooperative bargaining games:

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$$\max_{w_t^1}[U_{e^1}(k_t, h_t, e_t^1, e_t^2, A_t)]^{\gamma^1}[\Pi_{e^1}(e_t^1, e_t^2)]^{1-\gamma^1},$$
$$\text{and} \quad \max_{w_t^2}[U_{e^2}(k_t, h_t, e_t^1, e_t^2, A_t)]^{\gamma^2}[\Pi_{e^2}(e_t^1, e_t^2)]^{1-\gamma^2}.$$

Moreover, the government's flow budget constraint is now: $b_t^1 p s_t^1 + b_t^2(1-p)s_t^2 = T_t$. Similarly, we set $b_t^1 = b^1\bar{y}_t$ and $b_t^2 = b^2\bar{y}_t$, with $b^1 > 0$ and $b^2 > 0$, to sustain perpetual growth.

Using the same steps used for the calculation in the benchmark model, we can derive the equilibrium equations and the BGP. To understand the impact of the invention of AI more clearly, we also undertake a numerical analysis.

4.1. Calibration

Again, we consider an economy without AI as the benchmark calibration model, that is, initially AI has not been invented. Then, we can simulate the economy as it moves from the situation without AI to the situation with AI. When AI has not been invented, $A_0 = 0$, $\varepsilon = 0$, $\tau^1 = 0$, the production function is $y_t = F\{(1-\tau^2)[(1-p)e_t^2 h_t]^\delta + \tau^2[(1-u_t)k_t]^\delta\}^{\frac{1-\alpha}{\delta}}(pe_t^1 h_t)^\alpha$, and $a_t = u_t = x_t = 0$.

To calibrate the model, we first use the same data and parameters as in Section 3.1, where $h/y = 3.6559$, $c/y = 0.6993$, $e^1 + e^2 = 0.4760$, $\Gamma = 1.8963\%$, $\alpha = 0.64$, $\rho = 4\%$, $v^i/s^i = 1$, $\xi^1 = \gamma^1 = 0.3$, and $\beta = \delta = -1.2222$. In addition, according to Hagedorn et al. (2016), the average monthly job-finding rate is 0.3618 for skilled workers and 0.4185 for unskilled workers. In addition, the separation rate for workers who leave jobs and become unemployed, not adjusted for time aggregation, equals 0.0097 for the skilled and 0.0378 for the unskilled. Based on these data, we calculate the annual separation rate for skilled and unskilled workers to be $\psi^1 = 1-(1-0.0097)^{12} = 0.1104$ and $\psi^2 = 1-(1-0.0378)^{12} = 0.3702$, respectively. The annual job-finding rates for skilled and unskilled workers are calculated to be $\mu^1 = 1-(1-0.3618)^{12} = 0.9954$ and $\mu^2 = 1-(1-0.4185)^{12} = 0.9985$, respectively.

We further assume that $p = 0.7$, $\lambda^1 = \lambda^2$, and $b^1/w^1 = b^2/w^2$, and that the unit costs of vacancy creation for different skilled workers are the same and the ratios of unemployment compensation to wages for skilled and unskilled workers are also the same.[7] Using the same steps as in the benchmark model, we can calibrate other parameters as follows: $m^1 = 0.9954$, $m^2 = 0.9985$, $\tau^2 = 0.9757$, $F = 0.8922$, $\lambda^1 = \lambda^2 = 2.6934$, $b^1 = 0.1525$, $b^2 = 0.0170$, $\gamma^2 = \xi^2 = 0.0164$, $B = 0.1476$, $\sigma^1 = 0.6348$, $\sigma^2 = 2.1627$, $\chi^1 = 1.5706$, and $\chi^2 = 0.1517$. The key macroeconomic variables are summarized in Table 3 (case A).

It is worth noting that, compared with the weight of physical capital in the CES production function, we obtain a very small weight for unskilled workers ($1-\tau^2 = 0.0243$), which is calibrated by the model. Intuitively, in addition to paying wages, those hiring workers are also required to pay search costs (vacancy creation costs), and they need to negotiate wages with job seekers. If the unskilled workers can be replaced by a machine, the firms may not rely much on unskilled

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TABLE 3. Calibration results in an economy with heterogeneously skilled labor

Participation	e^1	e^2	s^1	s^2	n	a	v^1	v^2
(A) Initial economy without AI								
0.3178	0.3332	0.1428	0.0369	0.0529	0.1835	0	0.0369	0.0529
(B) An economy with AI								
0.3099	0.3316	0.1255	0.0380	0.0453	0.0761	0.0041	0.0363	0.0466

Unemployment	u	x	q	z	Γ	U	Un^1	Un^2
(A) Initial economy without AI								
0.1314	0	0	0.8447	0.1913	0.0190	26.1887	0.0998	0.2705
(B) An economy with AI								
0.1296	0.0437	2.2086	1.0494	0.3350	0.0079	38.3107	0.1027	0.2652

Note: $Un^1 = s^1/(e^1 + s^1)$ and $Un^2 = s^2/(e^2 + s^2)$ are the unemployment rates in the skilled and unskilled labor markets, respectively.

labor, preferring to use the machine when producing goods. Because the importance of unskilled labor in production is small, the bargaining power of unskilled workers ($\gamma^2 = 0.0164$), calibrated by the model, is also low.

Now, we simulate the economy with AI. Similarly, we first make assumptions about the parameters related to AI and then conduct sensitivity analysis in the following sections. We set $\tau^1 = 0.5$, $D = 0.0644$, $\varepsilon = 0.1$, $\phi = 0.2$, and $\theta = 0.2$.[8] The key macroeconomic variables are summarized in Table 3 (case B). The results are consistent with those from the economy without heterogeneously skilled labor. As the self-accumulation ability of AI (D) is slight, the long-run growth rate is lower than in the situation without AI. Furthermore, although the labor force participation rate declines, the overall unemployment rate falls when AI exists. Household welfare increases when AI is invented because the ratio of consumption to human capital and leisure for both skilled and unskilled workers are higher. Note that the overall labor force participation rate and unemployment rate are $p(e^1 + s^1) + (1-p)(e^2 + s^2)$ and $[ps^1 + (1-p)s^2]/[p(e^1 + s^1) + (1-p)(e^2 + s^2)]$, respectively.

Specifically, when AI is invented, job vacancy creation for skilled labor is reduced because the firm can use AI to replace skilled labor. The invention of AI increases production (which is $y/h = 0.4418$ in an economy with AI and $y/h = 0.2735$ in an economy without AI), and the wage rate ($w^1 = 0.8781$ in an economy with AI and $w^1 = 0.5708$ in an economy without AI). Therefore, skilled laborers who are unemployed search harder for jobs in the economy with AI. The unemployment rate in the skilled labor market increases because more people are searching for jobs and fewer vacancies are provided.

The invention of AI affects both skilled and unskilled labor. There are two opposing effects on unskilled labor. One effect is similar to that for skilled labor,

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in that unskilled people are more willing to search for jobs as the wage rate increases ($w^2 = 0.1037$ in an economy with AI and $w^2 = 0.0637$ in an economy without AI). The other effect is that if firms use more physical capital, they reduce their use of unskilled labor, which is then reflected in the firm's unskilled labor vacancies. Unemployed people who are unskilled are less likely to find a job that matches their skills because there are fewer unskilled labor vacancies. Therefore, their incentives to search for jobs are reduced. In our simulation, the latter effect dominates the former; therefore, fewer unskilled unemployed people look for a job compared with the situation in the economy without AI and the unemployment rate in the unskilled labor market declines. We obtain similar results if we increase the importance of unskilled workers relative to physical capital in production (i.e., when we decrease τ^2).

In the following sections, we study the transitional dynamics of the invention of AI and conduct a long-term comparative static analysis. We not only analyze comparative statics under different parameters but also compare the results in an economy with AI to those in an economy without AI. We obtain consistent results. The development of AI is not usually conducive to the employment of skilled workers, but it is usually beneficial in terms of the employment of unskilled workers. Furthermore, even if the overall labor force participation rate declines, the overall unemployment rate falls in the economy with AI.

4.2. Transitional Dynamics

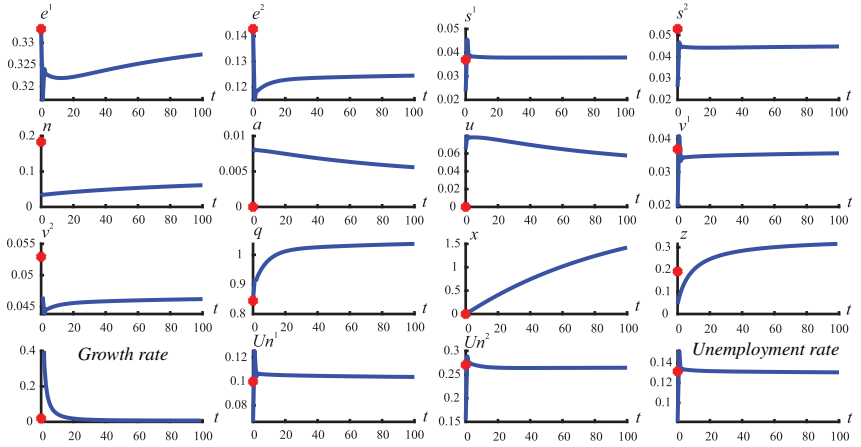
In this section, we examine the dynamic effects of the invention of AI, that is, the transition from an economy without AI to one with AI. Again, we take the economy without AI (case A in Table 3) as the initial situation and investigate the dynamic paths of all variables from the initial situation to the situation with AI (case B in Table 3). The results are shown in Figure 6.

Here, we focus on the effect of AI on the labor market. At the moment that AI is invented, firms adjust their production processes and begin to produce with AI, which reduces vacancies for both skilled and unskilled jobs. Employment for skilled and unskilled labor falls significantly and people who are not working are less willing to search for jobs. Therefore, the labor force participation rate declines significantly, and the unemployment rate decreases, as s^1 and s^2 are low.

Subsequently, firms increase vacancies in skilled jobs but continue to reduce vacancies in unskilled jobs. Skilled and unskilled workers are more willing to search for jobs as the development of AI increases production and, in turn, wage rates. We find that the unemployment rates in both labor markets increase and are even greater than their levels in an economy without AI.

Firms adjust production processes and households adjust their resource and effort allocations over time. Because AI can replace skilled labor and physical capital can replace unskilled labor (our results show that the ratio of physical capital to human capital increases over time), vacancies provided by firms in both skilled and unskilled jobs are lower than their levels in an economy without AI,

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Note: The initial point at time 0 is the economy without AI, that is, case (A) in Table 3. An economy with AI is case (B) in Table 3.

FIGURE 6. Transitional dynamics from an economy without AI to one with AI.

and matched employment for both skilled and unskilled labor is also lower. Over time, as job search incentives for skilled workers remain higher than those in an economy without AI (i.e., the unemployed skilled workers do not leave the labor market), the unemployment rate in the skilled labor market remains higher than the case in the economy without AI. In contrast, for unskilled workers, job search incentives and the unemployment rate in the unskilled labor market are thus lower than those in the economy without AI.

The invention of AI does dramatically affect the labor market. Along the transitional dynamic paths, especially in the case of skilled labor, job vacancies, but also job seekers and the unemployment rate may exceed their levels in an economy without AI. However, the development of AI does not have entirely negative consequences for labor market performance because, although fewer people participate in the labor market, the unemployment rate for unskilled workers and even the overall unemployment rate may decline.

4.3. Long-Run Comparative Static Analysis

Now, we investigate the long-run comparative statics. As above, we use the parameter settings in case (B) in Table 3 as the benchmark and then change one parameter at a time. To save space, we only list the results in Table 4.

Our results are consistent with our previous findings in that the economic growth rate is higher if the households devote more of their members to accumulating human capital. Our focus is on the effect of AI on the labor market. As for an increase in p, when there are more skilled people in the economy, fewer skilled workers can find matching employment, which reduces their incentives to search for jobs when unemployed. Therefore, the unemployment rate in the skilled labor

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TABLE 4. The results of the comparative statics in the long run

	e^1	e^2	s^1	s^2	n	a	u	v^1	v^2
p	↓	↑	↓	↑	↑	↑	↑	↓	↑
τ^1	↓	↑	↑	↑	↓	↓	↑	↓	↑
τ^2	↑	↓	↓	↓	↓	↓	↓	↑	↓
β	↓	↑	↑	↑	↓	↓	↓	↓	↑
ε	↑	↓	↓	↓	↓	↑	↑	↑	↓
D	↑	↓	↓	↑	↑	↑	↑	↑	↓
ϕ	↓	↑	↑	↓	↓	↑	↓	↓	↑
θ	↓	↑	↑	↓	↓	↓	↑	↓	↑
γ^1	↓	↓	↑	↓	↑	↑	↑	↓	↓
γ^2	↓	↓	↓	↑	↑	↑	↑	↓	↓
ξ^1	↑	↓	↓	↓	↑	↑	↑	↓	↓
ξ^2	↑	↓	↑	↑	↑	↑	↑	↑	↑
b^1	↑	↑	↑	↓	↓	↓	↓	↓	↑
b^2	↓	↑	↓	↑	↑	↑	↑	↓	↓

	q	x	z	Γ	U	Un^1	Un^2	Unemployment
p	↓	↓	↓	↑	↑	↓	↑	↓
τ^1	↑	↑	↑	↓	↑	↑	↓	↑
τ^2	↑	↑	↑	↓	↑	↓	↑	↓
β	↑	↑	↑	↓	↑	↑	↓	↑
ε	↓	↑	↑	↓	↑	↓	↑	↓
D	↓	↑	↑	↑	↑	↓	↑	↓
ϕ	↓	↓	↓	↓	↓	↑	↓	↑
θ	↑	↓	↓	↓	↓	↑	↓	↑
γ^1	↓	↓	↓	↑	↓	↑	↓	↑
γ^2	↓	↓	↓	↑	↓	↓	↑	↑
ξ^1	↓	↓	↑	↑	↑	↓	↓	↓
ξ^2	↓	↓	↓	↑	↓	↑	↑	↑
b^1	↑	↑	↑	↓	↑	↑	↓	↑
b^2	↑	↓	↑	↑	↓	↓	↑	↑

Note: The meaning of the arrows is the same as those in Table 2.

market declines because there are fewer skilled job seekers. The situation in the unskilled labor market is the opposite. The overall labor market unemployment rate decreases as p increases.

Next, a higher τ^1 implies that there is a reduction in the importance of skilled labor in relation to AI. Thus, firms provide fewer vacancies for skilled jobs, the matched employment of skilled labor decreases, and the unemployment rate in

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the skilled labor market increases. In contrast, a higher τ^1 assists in the employment of unskilled workers. Similarly, a larger τ^2 infers that the weight of unskilled labor related to physical capital declines; therefore, firms provide fewer vacancies for unskilled jobs. Then, the matched employment of unskilled labor decreases, and its unemployment rate increases. However, a larger τ^2 can decrease the unemployment rate in the skilled labor market. In addition, the impact of changing β on the labor market is the same as that of changing τ^1. The intuition for this is consistent with that in the model without heterogeneous labor (Section 3.3).

The above results indicate some of the implications of AI for human labor, as follows. If workers perform only routine and repeated tasks, that is, tasks that are easy to automate, they may be easily replaced by machines. Workers without AI-related knowledge and skills or workers who cannot create new technologies or ideas may be replaced easily by AI when it has evolved sufficiently to be more widely used in production.

Regarding an increase in ε, when the AI share in the production function (ε) increases, firms will mainly use AI inputs to increase production rather than using it to replace skilled labor. Therefore, similar to the influence of a decrease in τ^1, firms hire more skilled workers and the unemployment rate for skilled labor declines, but that for unskilled labor rises. As for the AI accumulation function, firms provide more vacancies for skilled jobs, the matched employment of skilled labor increases as AI self-accumulation abilities (D) increase, and the unemployment rate in the skilled labor market declines. However, the impact on the unskilled labor market is the reverse of that on the skilled market. It is worth noting that n is increasing in D. Intuitively, the higher is the self-accumulation ability of AI, the faster is the development of AI, and the more that people need to upgrade their technology (human capital) to enable them to work with AI.

When the labor share in the AI accumulation function (ϕ) increases, the households devote more of their members to developing AI, that is, a increases; and when the physical capital share in the AI accumulation function (θ) increases, the households allocate more physical capital to developing AI, that is, u increases. These changes having a negative (positive) effect on the skilled (unskilled) labor market because firms provide fewer (more) vacancies for skilled (unskilled) jobs, the matched employment of skilled (unskilled) labor decreases (increases), and the unemployment rate in the skilled labor market rises (declines).

Regarding the effects of changing the Hosios condition, or unemployment compensation, the results are similar to those in an economy without heterogeneously skilled workers (Section 3.3). When workers' wage bargaining power increases, the unemployed are more willing to search for jobs, whereas firms are less willing to hire workers. Thus, the unemployment rate increases. Our results show that s^1 increases, v^1 decreases, e^1 decreases, and Un^1 increases when γ^1 increases; and s^2 increases, v^2 decreases, e^2 decreases, and Un^2 increases when γ^2 increases. Similarly, wages will rise with an increase in unemployment compensation, so s^1 increases, v^1 decreases, and Un^1 increases when b^1 increases; and s^2 increases, v^2 decreases, and Un^2 increases when b^2 increases.

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To check the robustness of the results, we compare all cases in Table 4 with the situation where AI has not been invented. The results show that the invention of AI usually has a negative effect on the skilled labor market but a positive effect on the unskilled labor market. The unemployment rate of skilled labor may increase and that of unskilled labor may decrease when AI is invented. However, even if the overall labor force participation rate declines, the overall unemployment rate may decline due to the development of AI.

In other words, for skilled workers, although the development of AI seems to be detrimental to their job search activities in the labor market, they can use their resources and efforts to accumulate their own abilities and develop AI. Thus, they can obtain higher salaries when they are employed (given that wages are higher when AI is invented) as well as rewards for developing AI (r^A). Further, we find that y/h and welfare increase when AI exists. AI inputs can increase production with reduced human labor inputs; therefore, AI can increase people's happiness.

4.4. The Result When the Share of Skilled Labor is Endogenous

In the above economy with heterogeneously skilled labor, we assume that the share of skilled (and hence, unskilled) labor is exogenous and fixed at a given value p. Now we analyze the result when such a share is endogenous. When p_t is endogenous and can be chosen by the household, the household has another first-order condition with respect to p_t. Using the same calibration steps in Section 4.1 and using the household's first-order condition with respect to p_t to calibrate σ^2, we obtain that $\sigma^2 = 0.9578$ and $\chi^2 = 0.1973$. All other parameters are the same as those in Section 4.1 and the key macroeconomic variables are summarized in Table 5 (case A).

Now we use the same steps to simulate the economy with AI. We obtain $p = 0.9035$ and all other key macroeconomic variables are summarized in Table 5 (case B). If the households can choose the proportion of skilled members, they will increase the number of skilled workers. In this case, because there is more skilled labor, the household can expend more effort on accumulating human capital, and so n increases. Note that only skilled labor can invent and accumulate AI. That is, when AI is invented, there is not just higher welfare, but we also obtain a higher economic growth rate.

As for the labor market, the result is consistent with the case where p is exogenous. Although the overall labor force participation rate declines, the overall unemployment rate falls when there is AI. However, the unemployment rates in different skilled labor markets are different from those in Table 3. When p_t is endogenous, skilled members expend more effort on accumulating human capital and inventing AI, and so fewer people go to search for jobs, and thus the unemployment rate in the skilled labor market declines instead. The unemployment rate in the unskilled labor market increases because more people are searching for jobs and fewer vacancies are provided.

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TABLE 5. Calibration results in an economy with heterogeneously skilled labor and endogenous p_t

Participation	e^1	e^2	s^1	s^2	n	a	v^1	v^2
(A) Initial economy without AI ($p = 0.7$)								
0.3178	0.3332	0.1428	0.0369	0.0529	0.1835	0	0.0369	0.0529
(B) An economy with AI ($p = 0.9035$)								
0.2301	0.2097	0.1171	0.0198	0.1190	0.2931	0.1127	0.0249	0.0427
Unemployment	u	x	q	z	Γ	U	Un^1	Un^2
(A) Initial economy without AI ($p = 0.7$)								
0.1314	0	0	0.8447	0.1913	0.0190	63.6701	0.0998	0.2705
(B) An economy with AI ($p = 0.9035$)								
0.1277	0.5921	0.6472	0.5726	0.1627	0.0391	95.1073	0.0863	0.5041

Note: $Un^1 = s^1/(e^1 + s^1)$ and $Un^2 = s^2/(e^2 + s^2)$ are the unemployment rates in the skilled and unskilled labor markets, respectively.

Note that even if the household wants to internalize the proportion of different skilled members, it is not easy to do so in fact. This is because unskilled people have to make extra efforts to become skilled people and may not even be able to do so at all. Even if unskilled members can do this, their extra effort and human capital accumulation equation will be much more complicated than in this model, and this is another issue beyond the scope of this paper. However, this sample exercise implies that when the households have the opportunity, they also have incentives to allocate more members to be skilled workers.

5. CONCLUDING REMARKS

This paper introduces AI accumulation into the labor search model and investigates the impact of the invention of AI on macroeconomic performance, with a focus on labor market impacts. Furthermore, we consider an economy with heterogeneously skilled labor, explore the transitional dynamics when shifting from an economy without AI to one with AI, and analyze the related long-run comparative statics. Our results show that although the invention of AI does influence human jobs and lower labor force participation rates and firm vacancies, it also lowers the unemployment rate compared with an economy without AI. In addition, when considering heterogeneously skilled workers, we find that the invention of AI usually has a negative effect on the skilled labor market, but a positive effect on the unskilled labor market. Overall, the unemployment rate may decline as AI develops. Changes in the values of the structural parameters or using different functional forms in our model do not change the main results.

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The contribution of this paper is that our model can simultaneously analyze the effects of the development of AI along the transitional dynamics path and in the long run. Furthermore, as we compare economies with homogenous and heterogeneous skilled labor, we can more clearly understand the impact of the development of AI on workers with different abilities at different points in time.

It is possible to extend our model to incorporate more complex settings. For example, we could adapt the model with skilled and unskilled workers to consider two kinds of skilled workers, where one group has abilities to deal with AI-related technology and the other has not. Based on our simulation results, we can predict that the demand for the skilled labor with AI-related abilities will increase because those workers are required for and complementary with AI. Therefore, the impact of inventing AI on those workers will be similar to the situation for the skilled labor market under a very low β in Table 4, whereas the impact on the skilled labor group without AI-related abilities will be similar to the situation for the skilled labor market under a higher β. This also implies that when AI is invented, human workers are still required for the high-skill jobs that require knowledge or skills related to AI, or those low-skill jobs that require physical interaction and cannot be automated (e.g., food preparation and serving, or cleaning and janitorial services). However, the jobs that are easy to replicate through automation will be diminished in an economy with AI. Although we could have used a wide variety of more complex models, our simple model can help us to clearly identify the impact of the invention of AI on the labor market.

In this paper, we set the law of motion for physical capital so that it does not change with the introduction of AI. However, it could certainly change as AI requires a very different physical capital, which may accumulate also differently. If we internalize such interaction, the model's setting will become very complicated, and this is another issue beyond the scope of the paper. In addition, we did not consider intergenerational transfers in this paper. Sachs and Kotlikoff (2012) built an overlapping generation model with smart machines and emphasized the distributional conflict across generations introduced by a form of capital owned by the old that can self-replicate and substitute for the labor of the young. It would be interesting to study the implications for income distribution across generations in this framework. In addition, as AI can be widely used in the provision of long-term care, its implications for societies experiencing population aging is an interesting topic. We defer such analyses to future research.

NOTES

1. The calculation details are presented in the Appendix.
2. A variable without subscript t represents its value in the long run.
3. Note that the term Industry 4.0 originated in 2011 from a project in the high-tech strategy of the German government. In 2015, the European Commission started the international Horizon 2020 research project CREMA (providing Cloud-based Rapid Elastic Manufacturing based on the XaaS and Cloud model) as a major initiative to foster the Industry 4.0 topic. Therefore, we use data before 2014 as an economy where AI has not been invented.
4. Data are from https://www.rug.nl/ggdc/productivity/pwt/.
5. Our main results still hold under the different levels of β.

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6. Details of the linear Taylor's expansion used to calculate the dynamic paths are available on request. The BGP is a saddle in all our numerical exercises.

7. We initially set $b^1/w^1 = b^2/w^2$ as unemployment benefits are usually a percentage of wages rather than an absolute number. Besides, we set $p = 0.7$ to obtain the reasonable result that the values of all ratios are between 0 and 1, and all macroeconomic variables are positive.

8. This setting allows us to obtain the reasonable result that the values of all ratios are between 0 and 1, and that all macroeconomic variables are positive.

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APPENDIX A

A.1. EQUILIBRIUM EQUATIONS IN THE BENCHMARK MODEL

First, (3) suggests that $h_{t+1}/h_t = 1 + Bn_t$. Next, using (14), (15), (2), (7a)–(7e), and (11d), along with (3), (11a)–(11c), and (13b), we can obtain the following equilibrium equations:

$$q_{t+1}(1 + Bn_t) = q_t + (1 - \lambda v_t)\frac{y_t}{h_t} - z_t, \tag{A1a}$$

$$e_{t+1} = (1 - \psi)e_t + m(s_t)^{\xi}(v_t)^{1-\xi}, \tag{A1b}$$

$$x_{t+1}(1 + Bn_t) = x_t + Da_t^{\phi}u_t^{\theta}q_t^{\theta}x_t^{1-\phi-\theta}, \tag{A1c}$$

$$z_t = \frac{\phi}{\theta\chi}(1 - e_t - s_t - n_t - a_t)^{\sigma}\frac{u_t}{a_t}\frac{1 - \alpha - \varepsilon}{1 - u_t}\frac{y_t}{h_t}, \tag{A1d}$$

$$z_{t+1}(1 + Bn_t)(1 + \rho) = z_t + z_t\frac{1 - \alpha - \varepsilon}{1 - u_t}\frac{1}{q_t}\frac{y_t}{h_t}, \tag{A1e}$$

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$$\frac{\phi}{\theta}\frac{u_t}{a_t}\frac{1-\alpha-\varepsilon}{1-u_t}\frac{y_t}{h_t}\frac{z_{t+1}}{z_t}\frac{(1+Bn_t)(1+\rho)}{B}=\frac{y_{t+1}}{h_{t+1}}\Big\{\gamma\Big[\frac{\alpha(1-\tau)e_{t+1}^{\beta}}{(1-\tau)e_{t+1}^{\beta}+\tau x_{t+1}^{\beta}}+\frac{\lambda(1-\psi)e_{t+1}}{\eta_{t+1}}\Big]$$
$$+(1-\gamma)\Big[\frac{b(1-\psi)e_{t+1}}{\mu_{t+1}}-\frac{1-\psi-\mu_{t+1}}{\mu_{t+1}}\frac{\phi}{\theta}\frac{u_{t+1}}{a_{t+1}}\frac{(1-\alpha-\varepsilon)e_{t+1}}{1-u_{t+1}}\Big]$$
$$+\left(n_{t+1}+\frac{1}{B}+a_{t+1}\right)\frac{\phi}{\theta}\frac{u_{t+1}}{a_{t+1}}\frac{1-\alpha-\varepsilon}{1-u_{t+1}}\Big\}, \tag{A1f}$$

$$\frac{1-\alpha-\varepsilon}{\theta}\frac{u_t}{1-u_t}\frac{y_t}{h_t}\frac{z_{t+1}(1+Bn_t)(1+\rho)}{z_t}=\left(1+Bn_t-\frac{x_t}{x_{t+1}}\right)\frac{y_{t+1}}{h_{t+1}}$$
$$\Big\{\varepsilon+\frac{\alpha\tau x_{t+1}^{\beta}}{(1-\tau)e_{t+1}^{\beta}+\tau x_{t+1}^{\beta}}+\frac{(1-\alpha-\varepsilon)u_{t+1}}{1-u_{t+1}}\frac{1-\phi-\theta}{\theta}$$
$$+\frac{(1-\alpha-\varepsilon)}{1-u_{t+1}}\frac{1}{\theta D}a_{t+1}^{-\phi}u_{t+1}^{1-\theta}q_{t+1}^{-\theta}x_{t+1}^{\phi+\theta}\Big\}, \tag{A1g}$$

$$\left(\frac{\phi}{\theta}\frac{u_t}{a_t}\frac{1-\alpha-\varepsilon}{1-u_t}-b\right)\frac{y_t}{h_t}\frac{1+\rho}{\mu_t}\frac{z_{t+1}}{z_t}$$
$$=\frac{y_{t+1}}{h_{t+1}}\gamma\Big[\frac{1-\psi-\mu_{t+1}}{\mu_{t+1}}\frac{\phi}{\theta}\frac{1-\alpha-\varepsilon}{a_t}\frac{u_{t+1}}{1-u_{t+1}}+\frac{\alpha(1-\tau)e_{t+1}^{\beta-1}}{(1-\tau)e_{t+1}^{\beta}+\tau x_{t+1}^{\beta}}$$
$$+\frac{\lambda(1-\psi)}{\eta_{t+1}}-\frac{b(1-\psi)}{\mu_{t+1}}\Big], \tag{A1h}$$

$$\left[1+\frac{1-\alpha-\varepsilon}{(1-u_t)q_t}\frac{y_t}{h_t}\right]\frac{\lambda}{\eta_t}\frac{1}{1-\gamma}=\frac{\alpha(1-\tau)e_{t+1}^{\beta-1}}{(1-\tau)e_{t+1}^{\beta}+\tau x_{t+1}^{\beta}}+\frac{\lambda(1-\psi)}{\eta_{t+1}}-\frac{b(1-\psi)}{\mu_{t+1}}$$
$$+\frac{1-\psi-\mu_{t+1}}{\mu_{t+1}}\frac{\phi}{\theta}\frac{1-\alpha-\varepsilon}{a_t}\frac{u_{t+1}}{1-u_{t+1}}, \tag{A1i}$$

where $y_t/h_t=F[(1-u_t)q_t]^{1-\alpha-\varepsilon}x_t^{\varepsilon}[(1-\tau)e_t^{\beta}+\tau x_t^{\beta}]^{\alpha/\beta}$, $\mu_t=ms_t^{\xi-1}v_t^{1-\xi}$, and $\eta_t=ms_t^{\xi}v_t^{-\xi}$.

(A1d) indicates that z is a function of e, s, n, a, u, q, and x. Using the above dynamic equations (A1a)–(A1c) and (A1e)–(A1i), along with (A1d), we can derive the time paths of s_t, n_t, a_t, u_t, v_t, e_t, q_t, and x_t. Then, the time path of z_t can be derived from (A1d).

A.2. CALCULATION DETAILS IN THE MODEL WITH HETEROGENEOUS LABOR

The household's dynamic programming problem is written as the following Bellman equation:

$$U(k_t,h_t,e_t^1,e_t^2,A_t)=\max\Big[u(c_t,1-e_t^1-s_t^1-n_t-a_t,1-e_t^2-s_t^2)$$
$$+\frac{1}{1+\rho}U(k_{t+1},h_{t+1},e_{t+1}^1,e_{t+1}^2,A_{t+1})\Big],$$

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subject to the constraints:

$$e^1_{t+1} = (1-\psi^1)e^1_t + \mu^1_t s^1_t, \tag{A2a}$$

$$e^2_{t+1} = (1-\psi^2)e^2_t + \mu^2_t s^2_t, \tag{A2b}$$

$$A_{t+1} = D(pa_t h_t)^{\phi}(u_t k_t)^{\theta} A_t^{1-\phi-\theta} + A_t, \tag{A2c}$$

$$h_{t+1} = B(pn_t h_t) + h_t, \tag{A2d}$$

and

$$\begin{aligned} k_{t+1} = {} & k_t + w^1_t p e^1_t h_t + w^2_t (1-p) e^2_t h_t \\ & + r_t(1-u_t)k_t + r^A_t A_t + \pi_t - c_t + b^1_t p s^1_t + b^2_t (1-p) s^2_t - T_t, \end{aligned} \tag{A2e}$$

taking as given the factor prices, firm's profits, unemployment compensation, taxes, and the initial levels of employment, AI, human capital, and physical capital, e^1_0, e^2_0, A_0, h_0, and k_0.

The first-order conditions with respect to c_t, s^1_t, s^2_t, n_t, a_t, and u_t, respectively, are

$$u_c(c_t, 1-e^1_t-s^1_t-n_t-a_t, 1-e^2_t-s^2_t) = \frac{1}{1+\rho} U_k(k_{t+1}, h_{t+1}, e^1_{t+1}, e^2_{t+1}, A_{t+1}), \tag{A3a}$$

$$\begin{aligned} -u_{s^1}(c_t, 1-e^1_t-s^1_t-n_t-a_t, 1-e^2_t-s^2_t) = {} & \frac{1}{1+\rho}[U_k(k_{t+1}, h_{t+1}, e^1_{t+1}, e^2_{t+1}, A_{t+1}) p b^1_t \\ & + U_{e^1}(k_{t+1}, h_{t+1}, e^1_{t+1}, e^2_{t+1}, A_{t+1})\mu^1_t], \end{aligned} \tag{A3b}$$

$$\begin{aligned} -u_{s^2}(c_t, 1-e^1_t-s^1_t-n_t-a_t, 1-e^2_t-s^2_t) = {} & \frac{1}{1+\rho}[U_k(k_{t+1}, h_{t+1}, e^1_{t+1}, e^2_{t+1}, A_{t+1})(1-p) b^2_t \\ & + U_{e^2}(k_{t+1}, h_{t+1}, e^1_{t+1}, e^2_{t+1}, A_{t+1})\mu^2_t], \end{aligned} \tag{A3c}$$

$$-u_n(c_t, 1-e^1_t-s^1_t-n_t-a_t, 1-e^2_t-s^2_t) = \frac{1}{1+\rho} U_h(k_{t+1}, h_{t+1}, e^1_{t+1}, e^2_{t+1}, A_{t+1}) B p h_{t+1}, \tag{A3d}$$

$$\begin{aligned} & -u_a(c_t, 1-e^1_t-s^1_t-n_t-a_t, 1-e^2_t-s^2_t) \\ & = \frac{1}{1+\rho} U_A(k_{t+1}, h_{t+1}, e^1_{t+1}, e^2_{t+1}, A_{t+1}) \phi D a_t^{\phi-1} (p h_t)^{\phi} (u_t k_t)^{\theta} A_t^{1-\phi-\theta}, \end{aligned} \tag{A3e}$$

$$\begin{aligned} & U_k(k_{t+1}, h_{t+1}, e^1_{t+1}, e^2_{t+1}, A_{t+1}) r_t k_t \\ & = U_A(k_{t+1}, h_{t+1}, e^1_{t+1}, e^2_{t+1}, A_{t+1}) \theta D (a_t p h_t)^{\phi} u_t^{\theta-1} k_t^{\theta} A_t^{1-\phi-\theta}, \end{aligned} \tag{A3f}$$

and the Benveniste–Scheinkman conditions (envelope conditions) for k_t, e^1_t, e^2_t, h_t, and A_t, respectively, are

$$\begin{aligned} U_k(k_t, h_t, e^1_t, e^2_t, A_t) = {} & \frac{1}{1+\rho}\{U_k(k_{t+1}, h_{t+1}, e^1_{t+1}, e^2_{t+1}, A_{t+1})[1 + r_t(1-u_t)] \\ & + U_A(k_{t+1}, h_{t+1}, e^1_{t+1}, e^2_{t+1}, A_{t+1}) \theta D (a_t p h_t)^{\phi} u_t^{\theta} k_t^{\theta-1} A_t^{1-\phi-\theta}\}, \end{aligned} \tag{A3g}$$

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$$U_{e^1}(k_t, h_t, e_t^1, e_t^2, A_t) = u_{e^1}(c_t, 1 - e_t^1 - s_t^1 - n_t - a_t, 1 - e_t^2 - s_t^2) + \frac{1}{1+\rho}[U_k(k_{t+1}, h_{t+1}, e_{t+1}^1, e_{t+1}^2, A_{t+1}) w_t^1 p h_t + U_{e^1}(k_{t+1}, h_{t+1}, e_{t+1}^1, e_{t+1}^2, A_{t+1})(1 - \psi^1)], \tag{A3h}$$

$$U_{e^2}(k_t, h_t, e_t^1, e_t^2, A_t) = u_{e^2}(c_t, 1 - e_t^1 - s_t^1 - n_t - a_t, 1 - e_t^2 - s_t^2) + \frac{1}{1+\rho}[U_k(k_{t+1}, h_{t+1}, e_{t+1}^1, e_{t+1}^2, A_{t+1}) w_t^2 (1-p) h_t + U_{e^2}(k_{t+1}, h_{t+1}, e_{t+1}^1, e_{t+1}^2, A_{t+1})(1 - \psi^2)], \tag{A3i}$$

$$U_h(k_t, h_t, e_t^1, e_t^2, A_t) = \frac{1}{1+\rho}[U_k(k_{t+1}, h_{t+1}, e_{t+1}^1, e_{t+1}^2, A_{t+1})[w_t^1 p e_t^1 + w_t^2 (1-p) e_t^2] + U_h(k_{t+1}, h_{t+1}, e_{t+1}^1, e_{t+1}^2, A_{t+1})(B p n_t + 1) + U_A(k_{t+1}, h_{t+1}, e_{t+1}^1, e_{t+1}^2, A_{t+1}) \phi D (a_t p)^{\phi} h_t^{\phi-1} (u_t k_t)^{\theta} A_t^{1-\phi-\theta}], \tag{A3j}$$

$$U_A(k_t, h_t, e_t^1, e_t^2, A_t) = \frac{1}{1+\rho}\{U_k(k_{t+1}, h_{t+1}, e_{t+1}^1, e_{t+1}^2, A_{t+1}) r_t^A + U_A(k_{t+1}, h_{t+1}, e_{t+1}^1, e_{t+1}^2, A_{t+1})[(1-\phi-\theta) D (a_t p h_t)^{\phi} (u_t k_t)^{\theta} A_t^{-\phi-\theta} + 1]\}. \tag{A3k}$$

By combining (A3a), (A3e), and (A3f), along with the household's utility function, we can derive the following relationship:

$$p \chi^1 (1 - e_t^1 - s_t^1 - n_t - a_t)^{-\sigma^1} = \frac{\phi}{\theta} \frac{u_t}{a_t} \frac{1}{c_t} r_t k_t. \tag{A4a}$$

In addition, (A3a), (A3f), and (A3g) together yield the following consumption Euler equation:

$$\frac{1}{c_t} = \frac{1}{1+\rho} \frac{1}{c_{t+1}} (1 + r_{t+1}). \tag{A4b}$$

Moreover, using (A3a) and (A3b), (A3h) implies

$$\chi^1 (1 - e_t^1 - s_t^1 - n_t - a_t)^{-\sigma^1} = \frac{b_t^1}{c_t} + \frac{\mu_t^1}{1+\rho}\left[\frac{1 - \psi^1 - \mu_{t+1}^1}{\mu_{t+1}^1} \chi^1 (1 - e_{t+1}^1 - s_{t+1}^1 - n_{t+1} - a_{t+1})^{-\sigma^1} + \frac{h_{t+1} w_{t+1}^1}{c_{t+1}} - \frac{b_{t+1}^1 (1 - \psi^1)}{c_{t+1} \mu_{t+1}^1}\right]. \tag{A4c}$$

Using (A3a) and (A3c), (A3i) implies:

$$\chi^2 (1 - e_t^2 - s_t^2)^{-\sigma^2} = \frac{b_t^2}{c_t} + \frac{\mu_t^2}{1+\rho}\left[\frac{1 - \psi^2 - \mu_{t+1}^2}{\mu_{t+1}^2} \chi^2 (1 - e_{t+1}^2 - s_{t+1}^2)^{-\sigma^2} + \frac{h_{t+1} w_{t+1}^2}{c_{t+1}} - \frac{b_{t+1}^2 (1 - \psi^2)}{c_{t+1} \mu_{t+1}^2}\right]. \tag{A4d}$$

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Furthermore, (A3j), along with (A3a) and (A3d)–(A3f), indicates

$$p\chi^1(1-e_t^1-s_t^1-n_t-a_t)^{-\sigma^1}=\frac{Bph_t}{(1+\rho)c_{t+1}}\left[w_{t+1}^1pe_{t+1}^1+w_{t+1}^2(1-p)e_{t+1}^2 + (n_{t+1}+\frac{1}{Bp}+a_{t+1})r_{t+1}\frac{u_{t+1}k_{t+1}}{a_{t+1}h_{t+1}}\frac{\phi}{\theta}\right]. \tag{A4e}$$

Finally, (A3k), along with (A3a) and (A3e)–(A3f), yields

$$p\chi^1(1-e_t^1-s_t^1-n_t-a_t)^{-\sigma^1}=\frac{\phi Da_t^{\phi-1}(ph_t)^{\phi}(u_tk_t)^{\theta}A_t^{1-\phi-\theta}}{(1+\rho)c_{t+1}} \left[r_{t+1}^A+r_{t+1}\frac{u_{t+1}k_{t+1}}{A_{t+1}}\frac{1-\phi-\theta}{\theta}+\frac{r_{t+1}}{\theta D}(pa_{t+1}h_{t+1})^{-\phi}(u_{t+1}k_{t+1})^{1-\theta}A_{t+1}^{\phi+\theta-1}\right]. \tag{A4f}$$

The firm's dynamic programming problem is written as the following Bellman equation:

$$\Pi(e_t^1,e_t^2)=\max\left[\frac{\pi_t}{\bar{y}_t}+\frac{1}{1+r_t}\Pi(e_{t+1}^1,e_{t+1}^2)\right],$$

subject to the constraints:

$$e_{t+1}^1=(1-\psi^1)e_t^1+\eta_t^1v_t^1, \tag{A5a}$$

and

$$e_{t+1}^2=(1-\psi^2)e_t^2+\eta_t^2v_t^2. \tag{A5b}$$

The first-order conditions with respect to k_t, A_t, v_t^1, and v_t^2, respectively, are

$$\frac{(1-\alpha-\varepsilon)y_t\tau^2[(1-u_t)k_t]^{\delta-1}}{(1-\tau^2)[(1-p)e_t^2h_t]^{\delta}+\tau^2[(1-u_t)k_t]^{\delta}}=r_t, \tag{A6a}$$

$$\frac{y_t}{A_t}\left[\varepsilon+\frac{\alpha\tau^1A_t^{\beta}}{(1-\tau^1)(e_t^1ph_t)^{\beta}+\tau^1A_t^{\beta}}\right]=r_t^A, \tag{A6b}$$

$$\lambda^1=\frac{1}{1+r_t}\Pi_{e^1}(e_{t+1}^1,e_{t+1}^2)\eta_t^1, \tag{A6c}$$

$$\lambda^2=\frac{1}{1+r_t}\Pi_{e^2}(e_{t+1}^1,e_{t+1}^2)\eta_t^2, \tag{A6d}$$

and the Benveniste–Scheinkman conditions (envelope conditions) for e_t^1 and e_t^2, respectively, are

$$\Pi_{e^1}(e_t^1,e_t^2)=\frac{1}{\bar{y}_t}\left[\frac{\alpha y_t}{e_t^1}\frac{(1-\tau^1)(e_t^1ph_t)^{\beta}}{(1-\tau^1)(e_t^1ph_t)^{\beta}+\tau^1A_t^{\beta}}-w_t^1ph_t\right]+\frac{1}{1+r_t}\Pi_{e^1}(e_{t+1}^1,e_{t+1}^2)(1-\psi^1), \tag{A6e}$$

$$\Pi_{e^2}(e_t^1,e_t^2)=\frac{1}{\bar{y}_t}\left[\frac{(1-\alpha-\varepsilon)y_t}{e_t^2}\frac{(1-\tau^2)[e_t^2(1-p)h_t]^{\delta}}{(1-\tau^2)[e_t^2(1-p)h_t]^{\delta}+\tau^2[(1-u_t)k_t]^{\delta}}-w_t^2(1-p)h_t\right] +\frac{1}{1+r_t}\Pi_{e^2}(e_{t+1}^1,e_{t+1}^2)(1-\psi^2). \tag{A6f}$$

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In equilibrium, wage rates can be rewritten as follows

$$w_t^1 = \frac{y_t}{h_t}\left\{\gamma^1\left[\frac{\alpha(1-\tau^1)(pe_t^1)^{\beta-1}}{(1-\tau^1)(e_tp)^\beta+\tau^1x_t^\beta}+\frac{\lambda^1(1-\psi^1)}{p\eta_t^1}\right]\right.$$
$$+(1-\gamma^1)\left[\frac{b^1(1-\psi^1)}{\mu_t^1}-\frac{1-\psi^1-\mu_t^1}{\mu_t^1}\frac{\phi u_t(1-\alpha-\varepsilon)}{\theta a_tp}\right.$$
$$\left.\left.\times\frac{\tau^2(1-u_t)^{\delta-1}q_t^\delta}{(1-\tau^2)[(1-p)e_t^2]^\delta+\tau^2[(1-u_t)q_t]^\delta}\right]\right\}, \tag{A7a}$$

$$w_t^2 = \gamma^2\frac{y_t}{h_t}\left\{\frac{(1-\alpha-\varepsilon)(1-\tau^2)[(1-p)e_t^2]^{\delta-1}}{(1-\tau^2)[e_t(1-p)]^\delta+\tau^2[(1-u_t)q_t]^\delta}+\frac{\lambda^2(1-\psi^2)}{(1-p)\eta_t^2}\right]\right\}$$
$$+(1-\gamma^2)\left[\frac{b^2(1-\psi^2)}{\mu_t^2}\frac{y_t}{h_t}-\frac{1-\psi^2-\mu_t^2}{\mu_t^2}z_t\chi^2(1-e_t^2-s_t^2)^{-\sigma^2}\right]. \tag{A7b}$$

Similarly, (A2d) suggests that $h_{t+1}/h_t = 1+Bpn_t$. Next, using (A2a)–(A2c), (A2e), (A4a)–(A4f), and (A6e)–(A6f), along with (A2d), (A6a)–(A6d), and (A7a)–(A7b), we can obtain the following equilibrium equations:

$$q_{t+1}(1+Bpn_t) = q_t+(1-\lambda^1v_t^1-\lambda^2v_t^2)\frac{y_t}{h_t}-z_t, \tag{A8a}$$

$$e_{t+1}^1 = (1-\psi^1)e_t^1+m^1(s_t^1)^{\xi^1}(v_t^1)^{1-\xi^1}, \tag{A8b}$$

$$e_{t+1}^2 = (1-\psi^2)e_t^2+m^2(s_t^2)^{\xi^2}(v_t^2)^{1-\xi^2}, \tag{A8c}$$

$$x_{t+1}(1+Bpn_t) = x_t+D(pa_t)^\phi u_t^\theta q_t^\theta x_t^{1-\phi-\theta}, \tag{A8d}$$

$$z_t = \frac{\phi}{\theta p\chi^1}(1-e_t^1-s_t^1-n_t-a_t)^{\sigma^1}\frac{u_t}{a_t}\frac{y_t}{h_t}\frac{(1-\alpha-\varepsilon)\tau^2(1-u_t)^{\delta-1}q_t^\delta}{(1-\tau^2)[(1-p)e_t^2]^\delta+\tau^2[(1-u_t)q_t]^\delta}, \tag{A8e}$$

$$z_{t+1}(1+Bpn_t)(1+\rho) = z_t+z_t\frac{y_t}{h_t}\frac{(1-\alpha-\varepsilon)\tau^2[(1-u_t)q_t]^{\delta-1}}{(1-\tau^2)[(1-p)e_t^2]^\delta+\tau^2[(1-u_t)q_t]^\delta}, \tag{A8f}$$

$$\frac{(1+\rho)(1+Bpn_t)u_t}{\theta D(pa_t)^\phi(u_tq_t)^\theta x_t^{1-\psi-\theta}}\frac{z_{t+1}}{z_t}\frac{y_t}{h_t}\frac{(1-\alpha-\varepsilon)\tau^2(1-u_t)^{\delta-1}q_t^\delta}{(1-\tau^2)[(1-p)e_t^2]^\delta+\tau^2[(1-u_t)q_t]^\delta}$$
$$=\frac{y_{t+1}}{h_{t+1}x_{t+1}}\left\{\varepsilon+\frac{\alpha\tau^1x_{t+1}^\beta}{(1-\tau^1)(pe_{t+1}^1)^\beta+\tau^1x_{t+1}^\beta}\right.$$
$$+\frac{1-\phi-\theta}{\theta}\frac{(1-\alpha-\varepsilon)u_{t+1}\tau^2(1-u_{t+1})^{\delta-1}q_{t+1}^\delta}{(1-\tau^2)[(1-p)e_{t+1}^2]^\delta+\tau^2[(1-u_{t+1})q_{t+1}]^\delta}$$
$$\left.+\frac{\tau^2(1-u_{t+1})^{\delta-1}q_{t+1}^\delta}{(1-\tau^2)[(1-p)e_{t+1}^2]^\delta+\tau^2[(1-u_{t+1})q_{t+1}]^\delta}\frac{(1-\alpha-\varepsilon)u_{t+1}^{1-\theta}x_{t+1}^{\phi+\theta}}{\theta D(pa_{t+1})^\phi q_{t+1}^\theta}\right\}, \tag{A8g}$$

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$$\begin{aligned}
&\frac{(1+\rho)(1+Bpn_t)\phi u_t}{Bp\theta a_t}\frac{z_{t+1}}{z_t}\frac{y_t}{h_t}\frac{(1-\alpha-\varepsilon)\tau^2(1-u_t)^{\delta-1}q_t^{\delta}}{(1-\tau^2)[(1-p)e_t^2]^{\delta}+\tau^2[(1-u_t)q_t]^{\delta}}\\
&\quad=\frac{y_{t+1}}{h_{t+1}}\left\{\frac{\gamma^1\alpha(1-\tau^1)(pe_{t+1}^1)^{\beta}}{(1-\tau^1)(pe_{t+1}^1)^{\beta}+\tau^1x_{t+1}^{\beta}}+\frac{\gamma^1\lambda^1(1-\psi^1)e_{t+1}^1}{\eta_{t+1}^1}\right.\\
&\qquad+pe_{t+1}^1(1-\gamma^1)\left[\frac{(1-\psi^1)b^1}{\mu_{t+1}^1}-\frac{(1-\psi^1-\mu_{t+1}^1)\phi u_{t+1}}{p\mu_{t+1}^1\theta a_{t+1}}\right.\\
&\qquad\quad\left.\times\frac{(1-\alpha-\varepsilon)\tau^2(1-u_{t+1})^{\delta-1}q_{t+1}^{\delta}}{(1-\tau^2)[(1-p)e_{t+1}^2]^{\delta}+\tau^2[(1-u_{t+1})q_{t+1}]^{\delta}}\right]\\
&\qquad+\frac{\gamma^2(1-\alpha-\varepsilon)(1-\tau^2)[(1-p)e_{t+1}^2]^{\delta}}{(1-\tau^2)[(1-p)e_{t+1}^2]^{\delta}+\tau^2[(1-u_{t+1})q_{t+1}]^{\delta}}+\frac{\gamma^2\lambda^2(1-\psi^2)e_{t+1}^2}{\eta_{t+1}^2}\\
&\qquad+(1-p)e_{t+1}^2(1-\gamma^2)\frac{1-\psi^2}{\mu_{t+1}^2}b^2\\
&\qquad\left.+(n_{t+1}+\frac{1}{Bp}+a_{t+1})\frac{u_{t+1}}{a_{t+1}}\frac{\phi}{\theta}\frac{(1-\alpha-\varepsilon)\tau^2(1-u_{t+1})^{\delta-1}q_{t+1}^{\delta}}{(1-\tau^2)[(1-p)e_{t+1}^2]^{\delta}+\tau^2[(1-u_{t+1})q_{t+1}]^{\delta}}\right\}\\
&\quad-(1-p)e_{t+1}^2(1-\gamma^2)\frac{1-\psi^2-\mu_{t+1}^2}{\mu_{t+1}^2}z_{t+1}\chi^2(1-e_{t+1}^2-s_{t+1}^2)^{-\sigma^2},
\end{aligned}\tag{A8h}$$

$$\begin{aligned}
&\frac{y_t}{h_tz_t}\frac{1+\rho}{\mu_t^1\gamma^1}\left[\frac{\phi u_t}{\theta a_tp}\frac{(1-\alpha-\varepsilon)\tau^2(1-u_t)^{\delta-1}q_t^{\delta}}{(1-\tau^2)[(1-p)e_t^2]^{\delta}+\tau^2[(1-u_t)q_t]^{\delta}}-b^1\right]\\
&\quad=\frac{y_{t+1}}{h_{t+1}z_{t+1}}\left\{\frac{\alpha(1-\tau^1)(pe_{t+1}^1)^{\beta-1}}{(1-\tau^1)(pe_{t+1}^1)^{\beta}+\tau^1x_{t+1}^{\beta}}\right.\\
&\qquad+\frac{\lambda^1(1-\psi^1)}{\eta_{t+1}^1p}+\frac{1-\psi^1-\mu_{t+1}^1}{\mu_{t+1}^1}\frac{\phi u_{t+1}}{\theta a_{t+1}p}\frac{(1-\alpha-\varepsilon)\tau^2(1-u_{t+1})^{\delta-1}q_{t+1}^{\delta}}{(1-\tau^2)[(1-p)e_{t+1}^2]^{\delta}+\tau^2[(1-u_{t+1})q_{t+1}]^{\delta}}\\
&\qquad\left.-\frac{1-\psi^1}{\mu_{t+1}^1}b^1\right\},
\end{aligned}\tag{A8i}$$

$$\begin{aligned}
&\frac{1+\rho}{\mu_t^2\gamma^2}\left[\chi^2(1-e_t^2-s_t^2)^{-\sigma^2}-b^2\frac{y_t}{h_tz_t}\right]=\chi^2(1-e_{t+1}^2-s_{t+1}^2)^{-\sigma^2}\frac{1-\psi^2-\mu_{t+1}^2}{\mu_{t+1}^2}\\
&\quad+\frac{y_{t+1}}{h_{t+1}z_{t+1}}\left[\frac{\lambda^2(1-\psi^2)}{\eta_{t+1}^2(1-p)}+\frac{(1-\alpha-\varepsilon)(1-\tau^2)[e_{t+1}^2(1-p)]^{\delta-1}}{(1-\tau^2)[(1-p)e_{t+1}^2]^{\delta}+\tau^2[(1-u_{t+1})q_{t+1}]^{\delta}}-\frac{1-\psi^2}{\mu_{t+1}^2}b^2\right],
\end{aligned}\tag{A8j}$$

$$\begin{aligned}
&\frac{\lambda^1}{\eta_t^1(1-\gamma^1)p}\left\{1+\frac{y_t}{h_t}\frac{(1-\alpha-\varepsilon)\tau^2[(1-u_t)q_t]^{\delta-1}}{(1-\tau^2)[(1-p)e_t^2]^{\delta}+\tau^2[(1-u_t)q_t]^{\delta}}\right\}=\frac{\alpha(1-\tau^1)(pe_{t+1}^1)^{\beta-1}}{(1-\tau^1)(pe_{t+1}^1)^{\beta}+\tau^1x_{t+1}^{\beta}}\\
&\quad+\frac{\lambda^1(1-\psi^1)}{\eta_{t+1}^1p}+\frac{1-\psi^1-\mu_{t+1}^1}{\mu_{t+1}^1}\frac{\phi u_{t+1}}{\theta a_{t+1}p}\frac{(1-\alpha-\varepsilon)\tau^2(1-u_{t+1})^{\delta-1}q_{t+1}^{\delta}}{(1-\tau^2)[(1-p)e_{t+1}^2]^{\delta}+\tau^2[(1-u_{t+1})q_{t+1}]^{\delta}}\\
&\quad-\frac{1-\psi^1}{\mu_{t+1}^1}b^1,
\end{aligned}\tag{A8k}$$

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$$\frac{\lambda^1}{\eta_t^2(1-\gamma^2)(1-p)}\left\{1+\frac{y_t}{h_t}\frac{(1-\alpha-\varepsilon)\tau^2[(1-u_t)q_t]^{\delta-1}}{(1-\tau^2)[(1-p)e_t^2]^\delta+\tau^2[(1-u_t)q_t]^\delta}\right\}$$
$$=\chi^2(1-e_{t+1}^2-s_{t+1}^2)^{-\sigma^2}\frac{1-\psi^2-\mu_{t+1}^2}{\mu_{t+1}^2}\frac{h_{t+1}z_{t+1}}{y_{t+1}}$$
$$+\frac{\lambda^2(1-\psi^2)}{\eta_{t+1}^2(1-p)}+\frac{(1-\alpha-\varepsilon)(1-\tau^2)[e_{t+1}^2(1-p)]^{\delta-1}}{(1-\tau^2)[(1-p)e_{t+1}^2]^\delta+\tau^2[(1-u_{t+1})q_{t+1}]^\delta}-\frac{1-\psi^2}{\mu_{t+1}^2}b^2, \quad \textbf{(A8l)}$$

where $\frac{y_t}{h_t}=Fx_t^\varepsilon[(1-\tau^1)(pe_t^1)^\beta+\tau^1x_t^\beta]^{\frac{\alpha}{\beta}}\quad\{(1-\tau^2)[(1-p)e_t^2]^\delta+\tau^2[(1-u_t)q_t]^\delta\}^{\frac{1-\alpha-\varepsilon}{\delta}}$, $\mu_t^i=m^i(s_t^i)^{\xi i-1}(v_t^i)^{1-\xi i}$, and $\eta_t^i=m^i(s_t^i)^{\xi^i}(v_t^i)^{-\xi^i}$.

(A8e) shows that z is a function of e^1, e^2, s^1, n, a, u, q, and x. By combining (A8i)–(A8l), along with (A8e), we can obtain that v^2 is a function of e^1, e^2, s^1, s^2, n, a, u, q, x, and v^1, as follows:

$$v_t^2=\left[\frac{h_tz_t}{y_t}\chi^2(1-e_t^2-s_t^2)^{-\sigma^2}-b^2\right]\frac{\lambda^1(1-\gamma^2)\gamma^1s_t^2v^1(1-p)}{\lambda^2\gamma^2(1-\gamma^1)s_t^1}$$
$$\times\left[\frac{(1-\alpha-\varepsilon)\tau^2(1-u_t)^{\delta-1}q_t^\delta}{(1-\tau^2)[(1-p)e_t^2]^\delta+\tau^2[(1-u_t)q_t]^\delta}\frac{\phi u_t}{\theta a_t}-b^1p\right]^{-1}. \quad \textbf{(A9)}$$

Using the above dynamic equations (A8a)–(A8d) and (A8f)–(A8k), along with (A8e) and (A9), we can derive the time paths of s_t^1, s_t^2, n_t, a_t, u_t, v_t^1, e_t^1, e_t^2, q_t, and x_t. Then, the time paths of z_t and v_t^2 can be derived from (A8e) and (A9), respectively.

Note that when p is endogenous and can be chosen by the household, the household has another first-order conditions with respect to p_t as follows:

$$u_p(c_t,1-e_t^1-s_t^1-n_t-a_t,1-e_t^2-s_t^2,p_t)$$
$$+\frac{1}{1+\rho}U_h(k_{t+1},h_{t+1},e_{t+1}^1,e_{t+1}^2,A_{t+1})Bn_th_t$$
$$+U_A(k_{t+1},h_{t+1},e_{t+1}^1,e_{t+1}^2,A_{t+1})\phi Dp_t^{\phi-1}(a_th_t)^\phi(u_tk_t)^\theta A_t^{1-\phi-\theta}$$
$$+U_k(k_{t+1},h_{t+1},e_{t+1}^1,e_{t+1}^2,A_{t+1})[w_t^1e_t^1h_t-w_t^2e_t^2h_t+b_t^1s_t^1-b_t^2s_t^2]=0. \quad \textbf{(A3l)}$$

Besides, by using (A3a), (A3d), and (A3e), (A3l) becomes

$$\chi^1\frac{(1-e_t^1-s_t^1-n_t-a_t)^{1-\sigma^1}}{1-\sigma^1}-\chi^2\frac{(1-e_t^2-s^2)^{1-\sigma^2}}{1-\sigma^2}$$
$$+\chi^1(1-e_t^1-s_t^1-n_t-a_t)^{-\sigma^1}(n_t+a_t)$$
$$+w_t^1e_t^1\frac{h_t}{c_t}-w_t^2e_t^2\frac{h_t}{c_t}+b_t^1s_t^1\frac{y_t}{c_t}-b_t^2s_t^2\frac{y_t}{c_t}=0. \quad \textbf{(A4g)}$$

In equilibrium, (A4g) becomes as follows:

$$\chi^1\frac{(1-e_t^1-s_t^1-n_t-a_t)^{1-\sigma^1}}{1-\sigma^1}-\chi^2\frac{(1-e_t^2-s^2)^{1-\sigma^2}}{1-\sigma^2}$$
$$+\chi^2(1-e_t^2-s^2)^{-\sigma^2}\frac{(1-\gamma^2)(1-\psi^2-\mu_t^2)e_t^2}{\mu_t^2}$$
$$+\chi^1(1-e_t^1-s_t^1-n_t-a_t)^{-\sigma^1}\left[n_t+a_t-\frac{(1-\gamma^1)(1-\psi^1-\mu_t^1)e_t^1}{\mu_t^1}\right]$$

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$$+\frac{y_t}{h_t z_t}\left\{\frac{\gamma^1}{p_t}\left[\frac{\alpha(1-\tau^1)(p_t e_t^1)^\beta}{(1-\tau^1)(p_t e_t^1)^\beta+\tau^1 x_{t+1}^\beta}+\frac{\lambda^1(1-\psi^1)e_t^1}{\eta_t^1}\right]\right.$$

$$+\frac{(1-\gamma^1)(1-\psi^1)b^1 e_t^1}{\mu_t^1}-\frac{\gamma^2}{1-p_t}\left[\frac{(1-\alpha-\varepsilon)(1-\tau^2)[(1-p_t)e_t^2]^\delta}{(1-\tau^2)[(1-p_t)e_t^2]^\delta+\tau^2[(1-u_t)q_t]^\delta}\right.$$

$$\left.\left.+\frac{e_t^2\lambda^2(1-\psi^2)}{\eta_t^2}\right]-\frac{(1-\gamma^2)(1-\psi^2)b^2 e_t^2}{\mu_t^2}+b^1 s_t^1-b^2 s_t^2\right\}=0. \quad \textbf{(A8n)}$$

All other equations in this section are the same except that p becomes p_t and p_{t+1} at time t and $t+1$, respectively.

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