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# Generating controllable velocity fluctuations using twin oscillating hydrofoils

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An experiment apparatus has been previously developed with the ability to independently control the instantaneous flow velocity in a water flume. This configuration, which uses two pitching hydrofoils to generate the flow fluctuations, allows the unsteady response of submerged structures to be studied over a wide range of driving frequencies and conditions. Linear unsteady lift theory has been used to calculate the instantaneous circulation about two pitching hydrofoils in uniform flow. A vortex model is then used to describe the circulation in the wakes that determine the velocity perturbations at the centreline between the foils. This paper introduces how the vortex model can be discretized to allow the inverse problem to be solved, such that the foil motions required to recreate a desired velocity time series can be determined. The results of this model are presented for the simplified cases of oscillatory velocity fluctuations in the vertical and stream-wise directions separately, and also simultaneously. The more general case of two-dimensional aperiodic velocity fluctuations is also presented, which demonstrates the capability of configuration between the suggested frequency limits of $0.06 \leqslant k \leqslant 1.9$.

**Key words:** vortex flows, vortex streets, wakes/jets

## 1. Introduction

Unsteady structural responses resulting from dynamic flow conditions are important in a range of engineering applications, and generating such flow conditions in the laboratory enables the dynamic loads to be measured experimentally. Therefore, controlling the behaviour of the time-varying fluid velocity becomes a prerequisite challenge. Although some experimental methods elect to introduce relative motion between the fluid and structure by studying a moving body in a steady flow environment, this does not capture the effects of flow variations with length scales less than that of the body. These effects constitute an important part of the diminishing aero/hydrodynamic admittance function at high frequencies (Sears 1941; Liepmann 1952). In response, this paper examines a laboratory set-up that will allow the user to generate predefined flow fluctuations in the stream-wise and vertical direction along the centreline. This configuration has been shown to be effective in creating sinusoidal variations in vertical flow velocities; however, this paper demonstrates the ability to reproduce an aperiodic predetermined velocity time series through the use of two oscillating lifting surfaces. Such a facility allows full-scale velocity measurements to be reproduced in a controlled environment to enable the improved experimental derivation of unsteady response characteristics.

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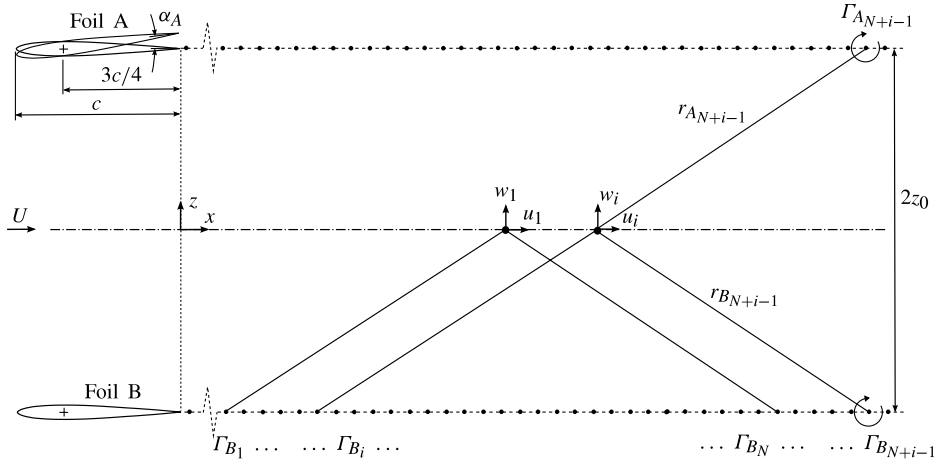



FIGURE 1. Schematic of velocity contributions from two parallel hydrofoils.

The use of oscillating lifting surfaces for the independent generation of oscillatory perturbations in longitudinal and vertical velocities was originally proposed by Ham, Bauer & Lawrence (1974), who describe a method that uses elliptical lifting surfaces with a jet slot in the trailing edge to alter the circulation about the foil in air. The direction of the jet is controlled by a cam in a round plenum at the aft of the lifting surface. Perturbations of up to 20 % of the free-stream velocity of the air have been achieved using a similar method of gust generation (Jancauskas & Melbourne 1980). Subsequently, pitching hydrofoils have been introduced to generate gusts in a single direction in wind tunnels (Tang, Cizmas & Dowell 1996; Passmore, Richardson & Imam 2001), and at high reduced frequencies in a water flume (Delpero 1992; Horwich 1993).

The benefit of using two independently controlled oscillating foils, shown in figure 1, is that the perturbations can be generated in the horizontal direction, $x$, and vertical direction, $z$, independently along the centreline between them. When the hydrofoil motion is harmonic, the motion of Foil A is denoted $\alpha_A = \alpha_0 e^{i\omega t}$, and that of Foil B is denoted $\alpha_B = \alpha_0 e^{i(\omega t+\theta)}$. If the two foils are oscillating in phase ($\theta = 0°$), the longitudinal components of the induced velocity, $u$, cancel along the centreline, and the vertical components, $w$, are additive. When the foils are oscillating out of phase ($\theta = 180°$), the $w$ components of the perturbations cancel along the centreline, and the $u$ components are additive (Ham *et al.* 1974). A phase difference of $0° < \theta < 180°$ will result in both stream-wise and vertical perturbations, with relative magnitudes dependent on the phase.

This paper presents a numerical model that allows the velocity perturbations to be calculated for a given time series of foil motions. This model is then manipulated to allow the reverse problem to be solved, such that the required time series of foil motions to generate a specified velocity time series can be calculated. The model is validated against existing experimental results, and the output of the reversed process is demonstrated for a range of combinations of vertical and stream-wise velocity fluctuations.

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## 2. Mathematical model

The simplifying assumptions used in the approach described in this paper are:

(*a*) the aerofoils are flat plates undergoing small-angle oscillations about the quarter-chord location;

(*b*) the vorticity imparted to the wake from the trailing edge of the hydrofoil is convected along a horizontal line at the velocity of the free-stream flow (planar wake assumption);

(*c*) the location of the specified velocity time series is at a sufficient distance from the foils that the bound circulation is assumed to have negligible influence;

(*d*) the Kutta–Joukowski condition applies at all times.

### 2.1. *Circulation about a pitch-oscillating hydrofoil*

The transient development of circulation about a foil undergoing pitching motion is accounted for by calculating the 'effective' angle of attack, which can then be used in the quasi-steady calculation of the bound circulation of the foil through the Kutta–Joukowski theorem. The present analysis uses the non-dimensionalized time variable given by $s = 2Ut/c$, where $U$ is the steady horizontal free-stream velocity, $c$ is the foil chord length and $t$ is the time. The relationship between the effective angle of attack for an oscillating foil, $\alpha_e(s)$, and the actual instantaneous angle of attack, $\alpha_a(s)$, can be described using the Duhamel integral (Bisplinghoff, Ashley & Halfman 1996; Leishman 2006):

$$\alpha_e(s) = \alpha_a(0)\phi(s) + \int_0^s \frac{\mathrm{d}\alpha_a(\sigma)}{\mathrm{d}\sigma}\phi(s-\sigma)\,\mathrm{d}\sigma, \tag{2.1}$$

where $\phi(s)$ represents the Wagner function for the response to an indicial change in angle of attack (Garrick 1938) and $\sigma$ is the time variable of integration. When the indicial lift response is approximated by the form of $\phi = 1 - A_1\mathrm{e}^{-b_1 s} - A_2\mathrm{e}^{-b_2 s}$, (2.1) can be solved using the one-step recursive solution of Algorithm-D1 (see Leishman 2006, p. 461):

$$\alpha_e(s) = \alpha_a(s) - X(s) - Y(s), \tag{2.2a}$$

$$X(s) = X(s-\Delta s)\mathrm{e}^{-b_1\Delta s} - A_1[\alpha_a(s) - \alpha_a(s-\Delta s)], \tag{2.2b}$$

$$Y(s) = Y(s-\Delta s)\mathrm{e}^{-b_2\Delta s} - A_2[\alpha_a(s) - \alpha_a(s-\Delta s)]. \tag{2.2c}$$

For the Wagner function, Jones (1940) gives the values of $A_1 = 0.165$, $A_2 = 0.335$, $b_1 = 0.045$ and $b_2 = 0.300$.

Using the Kutta–Joukowski theorem, the instantaneous bound circulation per unit span for each foil is given by $\Gamma_b(s) = UC_{L\alpha}\alpha_e(s)c/2$, where $C_{L\alpha} = \mathrm{d}C_L/\mathrm{d}\alpha_e$ and $C_L$ is the instantaneous quasisteady lift coefficient of the foil. The vorticity advected into the wake of the foils at each time step is $\Gamma(s) = \Gamma_b(s-\Delta s) - \Gamma_b(s)$.

### 2.2. *Contributions of twin hydrofoil wakes*

By locating the origin of the $(x, z)$ coordinate system on the centreline between the two foils and applying a planar wake simplification, the magnitude of the $z$-ordinate of the wakes can be defined as half the vertical distance between the two foils, $z_0$, such that $z_{A_j} = z_0$ and $z_{B_j} = -z_0$.

From figure 1 it can be seen that the component of the induced velocity in the longitudinal direction by a clockwise vortex is always negative from Foil A ($u_A < 0$) and always positive from Foil B ($u_B > 0$). As such, the magnitude of the perturbation

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in the longitudinal velocity is effectively controlled by the difference between the wake vorticity of Foil A and Foil B. Denoting the circulation per unit length of the wake as $\gamma$, the Biot–Savart law gives

$$u(X,t)=\frac{z_0}{2\pi}\int_0^{\infty}\frac{\gamma_B(x)-\gamma_A(x)}{(X-x)^2+z_0^2}\,\mathrm{d}x, \tag{2.3}$$

where d$x$ is a segment of the wake.

Furthermore, it can be seen that Foil A and Foil B have an additive effect on the induced velocity in the vertical direction. As such, the magnitude of the perturbation in the vertical velocity is effectively controlled by the sum of the wake vorticity of Foil A and Foil B, such that

$$w(X,t)=\frac{-1}{2\pi}\int_0^{\infty}\frac{(X-x)\left[\gamma_B(x)+\gamma_A(x)\right]}{(X-x)^2+z_0^2}\,\mathrm{d}x. \tag{2.4}$$

## 3. Numerical model

### 3.1. *Discretization*

As the model is intended to use arbitrary time series of $\gamma_A$ and $\gamma_B$, the solution to the vortex model is implemented in discrete form for ease of numerical integration. This is also shown to be beneficial when the reverse problem is addressed in § 3.2. Using the notation defined in figure 1, the combined effect of $\Gamma_A$ and $\Gamma_B$ on the induced velocity at location $(x_i, 0)$ can be calculated as a finite summation given by

$$u_i=\frac{z_0}{2\pi}\sum_{j=i}^{N+i-1}\left(\frac{1}{r_{ij}^2}\right)(\Gamma_{B_j}-\Gamma_{A_j}), \tag{3.1a}$$

$$w_i=\frac{-1}{2\pi}\sum_{j=i}^{N+i-1}\left(\frac{X_i-x_j}{r_{ij}^2}\right)(\Gamma_{B_j}+\Gamma_{A_j}), \tag{3.1b}$$

where $r_{ij}^2=(X_i-x_j)^2+z_0^2$.

The spatial range of the summation, $L$, which includes $N$ discrete vortices, must be large enough to capture all of the induced velocities from the wake. A sensitivity study was carried out for the required range, with the results summarized in figure 2. In response to this analysis, the summation was made over the range of $L=30c$ for the results presented in this paper. It was found that at ordinates of $x\geqslant 15c$ the perturbations induced by the wake vorticity were translationally independent. As such, the model was implemented to solve the velocity at coordinates of $(15c,\ 0)$, and the effect of the bound vorticity of the oscillating foils was ignored. The numerical model uses the same foil configuration as the experiments of Stapountzis (1982), where $2z_0=2.3c$ and $c=0.203$ m.

Letting $\boldsymbol{f}=z_0/2\pi[z_0^2+(X_i-x_j)^2]$ and $\boldsymbol{g}=-(X_i-x_j)/2\pi[z_0^2+(X_i-x_j)^2]$, (3.1$b$) can be expressed in the form of a matrix product by

$$\boldsymbol{u}=\boldsymbol{f}\cdot(\Gamma_B-\Gamma_A)\,, \tag{3.2a}$$

$$\boldsymbol{w}=\boldsymbol{g}\cdot(\Gamma_A+\Gamma_B)\,. \tag{3.2b}$$

The time series of the velocity fluctuations can then be reconstructed using the assumption that the velocity fluctuations are being advected at the free-stream velocity.

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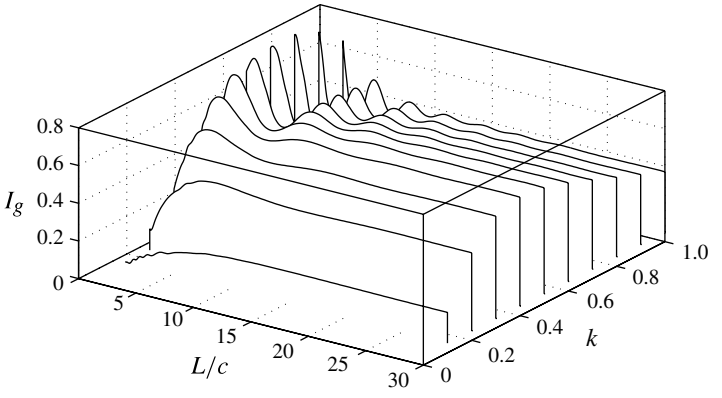



FIGURE 2. Sensitivity study for gust intensity, $I_g$, as a function of integral length, $L$, for a range of reduced frequencies, $k$.

### 3.2. *Foil motions for a desired velocity time series*

The mathematical model described in § 2.2 can be reversed, such that the foil motions required to generate a specified velocity time series can be calculated. Although the $i \times j$ matrices of $\boldsymbol{f}$ and $\boldsymbol{g}$ do not have a unique inverse, the pseudo-inverse calculated using a 'least-squares' best fit can be determined using the Moore–Penrose method (Penrose 1956). Denoting the pseudo-inverse of these matrices as $\boldsymbol{f}^+$ and $\boldsymbol{g}^+$, the required wake circulation to achieve a given velocity time series is calculated as

$$(\Gamma_B - \Gamma_A) = \boldsymbol{f}^+ \boldsymbol{u}, \tag{3.3a}$$

$$(\Gamma_A + \Gamma_B) = \boldsymbol{g}^+ \boldsymbol{w}. \tag{3.3b}$$

Applying a windowing function, such as a Tukey window, to the velocity time series reduced instabilities in the solution of the pseudo-inverse (Tukey 1967). The circulation matrices are then solved as simultaneous equations and (2.2*c*) can then be rearranged and solved for $\alpha_a(t)$.

## 4. Results

### 4.1. *Model verification*

The results of the numerical model of the twin oscillating foil configuration are summarized in figure 3, which shows the outputs of four representative simulations to demonstrate the operating principle of the foil arrangement, and to verify the inverse solution procedure. The top row (figure 3*a,e*) demonstrates the common situation of vertical velocity perturbations only, where the two hydrofoils are required to follow identical time series. The second row (figure 3*b,f*) illustrates the foil motion required to generate an oscillatory fluctuation in stream-wise velocity only, which is shown to be produced by sinusoidal foil oscillation with a phase difference of $\theta = 180°$, as expected. The third row (figure 3*c,g*) illustrates the case of stream-wise and vertical flow fluctuations of equal magnitudes and a phase difference of 180°. This represents the case where the effective angle of the incident flow is the greatest, which is the 'worst-case' combination of flow fluctuations in structural and aerospace applications such as long-span bridges and aerofoils. As such, the ability for this configuration of twin hydrofoils to be able to generate such a flow scenario in the laboratory is of special interest.

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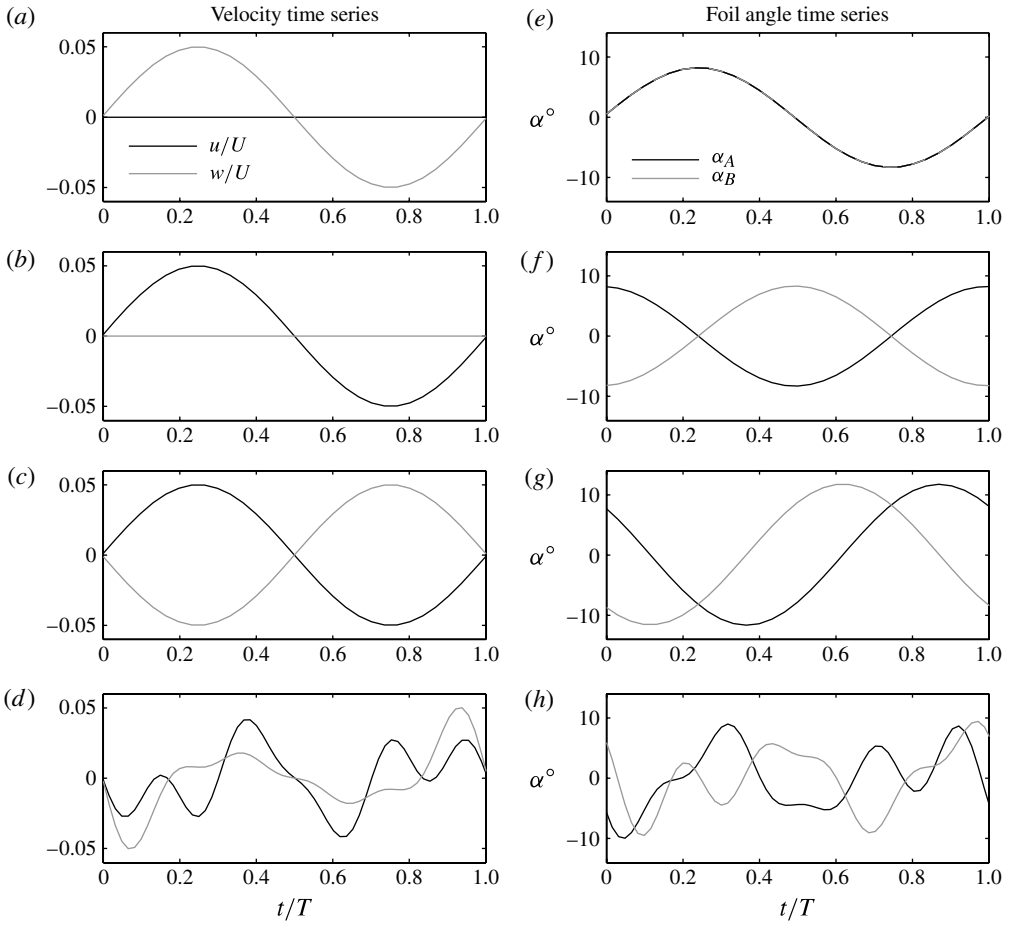



FIGURE 3. Representative results of the vortex model. The left column (*a–d*) shows the time series of the desired velocity perturbations and the right column (*e–h*) shows the corresponding time series of the angular positions of the foils required.

### 4.2. *Model validation*

A comparison with the experimental results presented by Stapountzis (1982) is made in figure 4 for the case of two sinusoidally oscillating foils with $\theta = 0°$. The metric of dimensionless gust intensity used to quantify the magnitude of the velocity fluctuations in response to the sinusoidal motion of the oscillating foils is given by (Stapountzis 1982)

$$I_g = \frac{w_0}{U\alpha_0}, \tag{4.1}$$

where $w_0$ is the amplitude of the vertical velocity fluctuations and $\alpha_0$ is the amplitude of the foil pitching motion. The gust intensity varies as a function of reduced frequency, $k = \omega c/2U$, where $\omega$ is the driving frequency of the velocity fluctuation in radians per second. The numerical results show close agreement with the experimental results, which were available for the range of $0.05 \leqslant k \leqslant 0.67$.

The gust intensities diminish as $k \to 0$, as the circulation about the foil is shed so slowly that the resulting induced velocities become negligible. The reduction in

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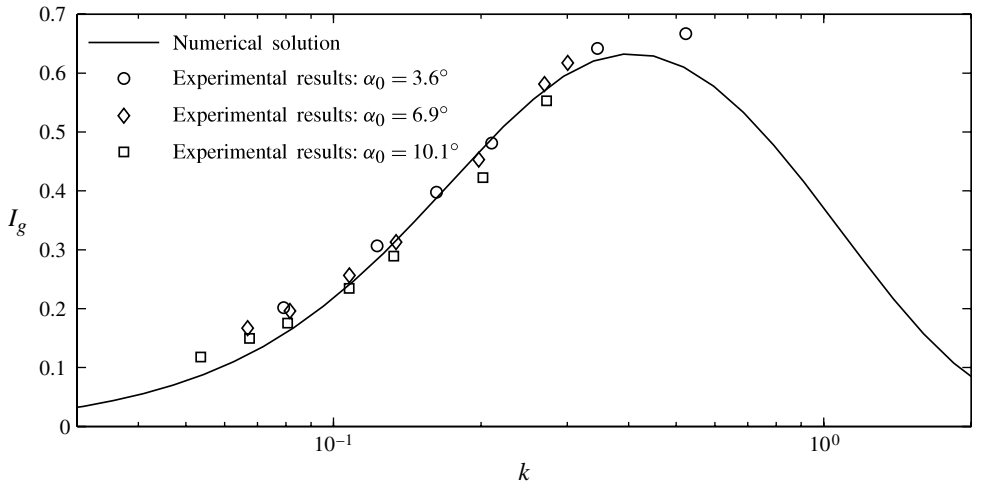



FIGURE 4. Comparison of numerical results using the method described in this paper with experimental results from Stapountzis (1982), for the case of in-phase sinusoidal foil oscillations at $Re = 4 \times 10^5$.

gust intensities as $k \to \infty$ is a result of unsteady lift theory, which accounts for the transient development of circulation about a pitching foil, as originally proposed Theodorsen (1935). As the frequency of pitch oscillation is increased, the reduced circulation about the foil has less time to develop, resulting in an increasing deficit between the actual circulation about the foil and that given by quasisteady lift theory (Ashley & Landahl 1956). This effect is implemented in the numerical model through the indicial response given by the Wagner function; however, experimental values were not available for frequencies greater than $k = 0.52$.

Experimental values are only available for comparison against the simplified case of in-phase oscillations of the foils.

### 4.3. *New results*

The novel contribution of this analysis is the ability to calculate the foil motion required to generate a desired velocity time series along the centreline. Figure 3(*d*,*h*) shows the case of temporally controlled flow fluctuations in both stream-wise and vertical directions, and the corresponding calculated time series of the foil angles. This example demonstrates the flexibility of the numerical model, as the required angular motion of the two foils is shown to be calculated for a time series with a range of driving frequencies. This situation also demonstrates the capability of the twin-foil configuration to recreate a complex flow environment. The frequency range of the velocity time series that can be produced is bounded by the approximate range of $0.06 \leqslant k \leqslant 1.9$, as shown in figure 4. Outside of this range, $I_g < 0.1$ and the relative velocity fluctuations become very small, such that $u/U < 1.7\,\%$ when $\alpha_0 = 10°$. It is also important to note that the steady Kutta condition used in the numerical model has been shown to be invalid at frequencies of $k > 2.0$ (Poling & Telionis 1986). As such, the velocity time series is required to be band-pass filtered to within this frequency range prior to implementation in the numerical model in order to achieve realistic results.

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## 5. Conclusions

The use of two oscillating hydrofoils to generate arbitrary velocity fluctuations on the centreline of an experimental flume facility has been investigated. A numerical model has been developed to calculate these velocity fluctuations given the angular motion of the foils. This modelling approach incorporates unsteady lift effects through the use of the Wagner function and uses a simplified vortex model to calculate the induced velocities between the hydrofoil wakes.

The discrete formulation of the numerical model facilitates the novel development of the inverse solution procedure, whereby the required foil motions to generate a desired velocity fluctuation time series along the centreline can be calculated. In this way, velocity perturbations can be created to reproduce a wide range of environmental conditions in the laboratory.

For the generation of oscillatory vertical velocity perturbations, gust intensities greater than $I_g = 0.1$ are expected to be achievable for a wide range of reduced frequencies, from $0.06 \leqslant k \leqslant 1.9$. The driving frequencies that give the maximum velocity fluctuations for a given $\alpha_0$ are in the range of $0.4 \leqslant k \leqslant 0.6$, and are dependent on the geometry of the configuration. However, it has been demonstrated that the combination of stream-wise and vertical velocity fluctuations is not limited to oscillatory motions, and predetermined combinations of velocity perturbations at a point can also be introduced through more complex motions of the two foils.

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