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AN ARGUMENT FOR CONJUNCTION CONDITIONALIZATION

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Abstract. Are counterfactuals with true antecedents and consequents automatically true? That is, is Conjunction Conditionalization: $(X \wedge Y) \supset (X > Y)$ valid? Stalnaker and Lewis think so, but many others disagree. We note here that the extant arguments for Conjunction Conditionalization are unpersuasive, before presenting a family of more compelling arguments. These arguments rely on some standard theorems of the logic of counterfactuals as well as a plausible and popular semantic claim about certain semifactuals. Denying Conjunction Conditionalization, then, requires rejecting other aspects of the standard logic of counterfactuals or else our intuitive picture of semifactuals.

§1. Introduction. The standard Stalnaker–Lewis (Stalnaker, 1968; Lewis, 1986) possible worlds semantics for counterfactuals has it that the truth of A and C suffice for the truth of the corresponding counterfactual 'if A were the case, then C would be the case' $(A > C)$. That is,

Conjunction Conditionalization: $(X \wedge Y) \supset (X > Y)$

is valid. This has been seen by many as a defect in the standard account (Bennett, 1974; Fine, 1975; Kvart, 1994; McDermott, 2007) and has led some (Bennett, 2003; Bigelow, 1976; Cogburn & Roland, 2013; Gundersen, 2004; McGlynn, 2012; Nozick, 1981; Penczek, 1997) to modify the Stalnaker–Lewis semantics.[1] These authors reject Conjunction Conditionalization because of putative counterexamples like the following from McDermott (2007): a coin is to be tossed twice; before it is tossed I bet that it will come up heads both times; it does and I win. Now consider

(1) If at least one head had come up, I would have won.

(1) is true on the standard account since it follows from Conjunction Conditionalization—at least one head did come up and I did win. McDermott claims, however, that, intuitively (1) is false, and so Conjunction Conditionalization is to be rejected. Similarly, Nozick rejects Conjunction Conditionalization because he thinks that

> If it is truly a random matter which slit a photon goes through, then its going through (say) the right slit does not establish the subjunctive: 'if a photon were fired at that time from that source it would go through the right-hand slit' (Nozick, 1981, 680–681).

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[1] See Walters (2009, MS) for a discussion of these proposals.

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The prevailing attitude amongst these critics is that Conjunction Conditionalization is a minor irritant, a feature of the standard treatment that is undesirable, but which is an easily avoided commitment. (Of course, Conjunction Conditionalization is not easily avoided in Stalnaker's (1968) specific approach, since he assumes Conditional Excluded Middle (CEM), which together with Modus Ponens entails Conjunction Conditionalization. We are here, however, leaving the status of CEM open so that we can meaningfully pursue the debate over Conjunction Conditionalization.) Moreover, whether one is disposed to accept or reject Conjunction Conditionalization, it is standardly thought that not much turns on it. We show, however, that this attitude is mistaken by presenting a family of compelling arguments for Conjunction Conditionalization. Rather than being an optional extra, Conjunction Conditionalization is mandatory in a Stalnaker–Lewis possible worlds semantics, given our other commitments. So any advantage that Lewis's semantics might be thought to have over Stalnaker's with regard to Conjunction Conditionalization is illusory. Further, we argue that regardless of whether we employ a possible worlds semantics, Conjunction Conditionalization follows from independent logical and semantic commitments.

Below, we briefly consider and reject Lewis's arguments for Conjunction Conditionalization (§2). We then lay out our new arguments for Conjunction Conditionalization (§3). In §4 and §5, we defend the semantic assumption of the arguments of §3, which is accepted by Lewis and numerous theorists hostile to Conjunction Conditionalization. In §6, we close with a discussion of the theorems of counterfactual logic required for the arguments, noting that they are validated by a wide variety of semantics, and that other plausible theorems have to be rejected, if our arguments are deemed to be invalid.

§2. Lewis's arguments for conjunction conditionalization. Lewis (1986) offers two arguments for Conjunction Conditionalization. On the standard semantics, $A > C$ is non-vacuously true at w iff some $(A \wedge C)$-world is closer to w than any $(A \wedge \sim C)$-world is (and is vacuously true if there are no A-worlds). This semantics validates Conjunction Conditionalization because Lewis imposes the following constraint on the closeness ordering:

> Strong Centring: Any world is closer to itself than any other world is to it.

Lewis thinks of closeness in terms of overall similarity and this provides him with his first argument for Conjunction Conditionalization since he (1986, 15) contends that

> (2) For all worlds w, no other world is as similar to w as w itself is.

If Lewis is correct, Strong Centring holds, and given his semantics, Conjunction Conditionalization follows.

But even if we accept the standard semantics, this observation is not enough to ensure Conjunction Conditionalization. As Lewis (1979) himself admits, in response to objections from Bennett (1974) and Fine (1975), similarity is here a technical notion, which does not clearly coincide with our intuitive notion of overall similarity. As Lewis puts it:

> the relation that governs the counterfactual ... may nevertheless be a relation of overall similarity – not because it is likely to guide our explicit judgments of similarity, but rather because it is a resultant, under some system of weights or priorities, of a multitude of relations of similarity in particular respects (Lewis, 1979, 43).

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(2) is true, perhaps, if similarity is taken in the ordinary sense,[2] but it is not clear that we should accept (2) once we employ a technical notion designed to ensure that the Stalnaker–Lewis semantics provides the correct truth conditions (cf. Loewer, 1979, 112–113).[3] Indeed, one might think that Lewis has things back-to-front here. Given that similarity needs to be cashed out in terms which ensure the adequacy of the semantics (see, e.g., Lewis, 1979, 43–48), we would need to know first whether or not Conjunction Conditionalization is valid before we could determine the correct similarity metric. So considerations of similarity cannot determine whether or not Conjunction Conditionalization is valid.[4]

Lewis (1986, 27–28) offers a second argument for Conjunction Conditionalization based on a single example where Conjunction Conditionalization seems natural. But the intuitive truth of a single counterfactual with true components hardly shows that Conjunction Conditionalization is valid. Moreover, Bennett (1974, 387–388) replies, modifying Lewis's example, to provide a counterfactual with true components which is, he thinks, intuitively false. And whereas the truth of a single example is not a good reason for thinking Conjunction Conditionalization is valid, a single example which seems false is, at least prima facie, good reason for rejecting the validity of Conjunction Conditionalization.

So Lewis's defence of Conjunction Conditionalization is inadequate. In fact, Lewis (1986, 29) himself considers rejecting Strong Centring in favour of the weaker constraint

Weak Centring: Any world, w, is amongst the closest worlds to w

which does not validate Conjunction Conditionalization. Lewis, then, apparently views Conjunction Conditionalization as an inessential, although desirable feature of his semantics. As we shall show, this view is, given a modest assumption, incorrect.[5]

§3. Arguments for conditionalization. There is a family of arguments for Conjunction Conditionalization that are not subject to the difficulties discussed above. These

[2] It is not clear that even this is true, however. If our intuitive notion of similarity is a purely qualitative one, and if the actual world (@) has a qualitative duplicate, w, then on our intuitive notion of similarity, @ and w are equally similar to @. As a result, Strong Centring fails, and the falsity of 'If P had been the case, the actual world would have been realized' represents a counterexample to Conjunction Conditionalization, where P is some true, purely qualitative proposition.

[3] Stalnaker (1968, 46–47; 1987, 121) argues along similar lines, although he, like Lewis, accepts that Fine and Bennett's examples "show decisively that the intuitive notion of overall similarity between possible worlds ... is not the [notion] that is relevant to the interpretation of counterfactual conditionals" (1987, 127), and that "[a]n account of the respects of similarity that are relevant to selection [of possible worlds] might say that some respects of similarity count for nothing at all, and so should be ignored" (1987, 128). Stalnaker's position is complicated, however, by the fact that he endorses CEM, which combined with Modus Ponens, entails Conjunction Conditionalization. Stalnaker concedes though (1987, 176 n13) that if we give up CEM there is no argument from similarity to Conjunction Conditionalization. Stalnaker (1987, chap. 7) does, however, provide an argument for CEM and so this amounts to a distinct argument for Conjunction Conditionalization (see also Williams, 2010). But, as noted above, we are concerned here with Conjunction Conditionalization in settings which do not presuppose CEM.

[4] Lewis (1981, 78) repeats this questionable move even after conceding that similarity is a technical notion. All he says by way of justifying Strong Centring on this occasion is that "[i]t is reasonable to assume, further, that any other world differs more from *i* than *i* itself does".

[5] For those, like McDermott, who reject Conjunction Conditionalization on the basis of the alleged falsity of (1), adopting Weak Centring will not do (see Walters, MS).

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arguments show that Conjunction Conditionalization follows from our intuitive judgments about the truth of certain counterfactuals, together with Lewis's (1986, 132) logic for counterfactuals, VW, which is Lewis's official logic, VC, without Conjunction Conditionalization. In fact, the arguments below do not require the full force of VW. Rather, they show that Conjunction Conditionalization follows given certain packages of theorems of VW.[6] The relevant theorems of VW are:

Modus Ponens: $(X > Y) \supset (X \supset Y)$.

Necessary Consequent: if $\vdash Z$, then $X > Z$.[7]

Substitution of Logical Equivalents: if $\vdash X \equiv Y$ then $(X > Z) \equiv (Y > Z)$.[8]

Disjunction: $((X > Y) \wedge (Z > Y)) \supset ((X \vee Z) > Y)$.

LAS: $((X > Z) \wedge \sim (X > \sim Y)) \supset ((X \wedge Y) > Z)$.

VLAS: $((X > Y) \wedge (X > Z)) \wedge ((X \wedge Y) > Z)$.

Substitution: $((X > Y) \wedge (Y > X) \wedge (X > Z)) \wedge (Y > Z)$.

The following combinations of these principles can be used to provide arguments for the validity of Conjunction Conditionalization:

- Disjunction, Substitution of Logical Equivalents, LAS, and Modus Ponens.
- Disjunction, Substitution, Necessary Consequent.
- Disjunction, Substitution of Logical Equivalents, VLAS.

Consider some true A and C. If we can find a B which bears out the assumptions in the arguments below, then the packages of logical principles above validate Conjunction Conditionalization:

Argument 1

(3)	$B > C$	Assumption
(4)	$\sim B > C$	Assumption
(5)	$(B \vee \sim B) > C$	(3), (4), Disjunction
(6)	$\sim(B \vee \sim B > \sim A)$	Modus Ponens[9]
(7)	$(A \wedge (B \vee \sim B)) > C$	(5), (6), LAS
(8)	$A > C$	(7), Substitution of Logical Equivalents

[6] Or near enough. The arguments in fact fall short of establishing Conjunction Conditionalization in its full generality. We ignore this complication for the time being, but return to it in §4.

[7] Necessary Consequent can be taken as axiomatic, or else derived from Reflexivity: $X > X$, and Weakening the Consequent: if $\vdash Y \supset Z$, then $(X > Y) \supset (X > Z)$.

[8] As Williamson (2007, 295–296) notes, some delicacy is required in formulating this principle if the language contains a rigidifying actually operator, A. Plausibly, $(P \equiv AP)$ is a logical truth, but we cannot conclude from the obvious $AP > AP$, that $P > AP$, since this is false when $\sim P$, contra Substitution of Logical Equivalents. Such issues are orthogonal to our concerns, however, and in any case we can avoid such issues by replacing $\vdash$ in the formulation of this principle with a necessity operator, $\Box$.

[9] If $(B \vee \sim B) > \sim A$ were true, then by Modus Ponens we could derive $\sim A$, contrary to the assumption that A is true.

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Argument 2

(9)	$B > C$	Assumption
(10)	$\sim B > C$	Assumption
(11)	$B > A$	Assumption
(12)	$\sim B > A$	Assumption
(13)	$(B \vee \sim B) > C$	(9), (10), Disjunction
(14)	$(B \vee \sim B) > A$	(11), (12), Disjunction
(15)	$A > (B \vee \sim B)$	Necessary Consequent
(16)	$A > C$	(13), (14), (15), Substitution

Argument 3

(17)	$B > C$	Assumption
(18)	$\sim B > C$	Assumption
(19)	$B > A$	Assumption
(20)	$\sim B > A$	Assumption
(21)	$(B \vee \sim B) > C$	(17), (18), Disjunction
(22)	$(B \vee \sim B) > A$	(19), (20), Disjunction
(23)	$(A \wedge (B \vee \sim B)) > C$	(21), (22), VLAS
(24)	$A > C$	(23), Substitution of Logical Equivalents

So if, (i) we can find some B for each true A and C which bears out the assumptions in one of the arguments above, and (ii) endorse the required theorems of VW, then Conjunction Conditionalization follows. Let us take these questions in turn.

§4. Morgenbesser's coin and semifactuals. It is a commitment of our counterfactual thinking that we hold certain post-antecedent facts fixed when considering counterfactuals. The facts that we hold fixed are those, to a first approximation, that are in some sense independent of the obtaining of the antecedent.[10] It is this fact that underlies the case of Morgenbesser's coin (Slote, 1978): I toss an indeterministic coin, and whilst the coin is in midair, I offer you good odds that it will come up heads. You decline the bet and the coin lands heads. The following is then true

(25) If you had bet heads, you would have won.

What underlies the truth of (25) is the following semifactual:

(26) If you had bet heads, the coin would (still) have landed heads.

And (26) is true because the coin's landing heads is independent of your betting behaviour. More generally, we accept any semifactual whose antecedent is irrelevant to the obtaining of its consequent. As Kvart puts it:

> In considering the contrary-to-fact assumption, we consider this change, and only this change, and contemplate the possible effects of this change against the background of the actual course of events, and only these effects. Consequently, in considering this change, we are not prompted to question those actual events which have nothing to do with it (Kvart, 1986, 44).

[10] The exact nature of the required independence is disputed (see, e.g., Kvart, 1986; Noordhof, 2004; Schaffer, 2004; Won, 2009). We can here prescind from the exact details, since all that we require are the truth of the counterfactuals discussed below, which the varying accounts of independence all take as data.

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The general picture that Kvart is outlining seems to underpin our judgments regarding (25)–(26). One might then argue from this picture to Conjunction Conditionalization on abductive grounds: when we come to counterfactuals with true antecedents no changes are prompted by considering the antecedent, and we are not to consider any other changes, so no changes at all are required when considering counterfactuals with true antecedents, and so Conjunction Conditionalization holds.[11]

Whatever the merits of such an argument, the truth of (25) and (26), provide some of the materials for the deductive arguments for Conjunction Conditionalization of the last section. As well as (25) and (26), it seems that in the same situation, the following is also true, and for the same reason

(27) If you had not bet on heads, the coin would (still) have landed heads.

That is, your betting behaviour is irrelevant to the outcome of the toss, and so, whether or not you had bet heads, the coin would have landed heads. Of course, (27) is a conditional with a true antecedent, but we are not relying on Conjunction Conditionalization to support it, rather it is true because the antecedent is irrelevant to the obtaining of the true consequent.[12]

Now, (26) and (27) are of the form

(28) $\sim B > C$
(29) $B > C$

where whether or not B obtains is irrelevant to the obtaining of C. That is, given that C is true, and that there is no connection between whether or not B obtains and C's obtaining, C would not be undermined by either B or by $\sim B$. For example, we might say of a witch doctor

> 'It would not rain if he did not do a rain dance, but it would not rain if he did either.' This conjunction of two simple subjunctives expresses the lack of a connection rather than the presence of one (Pollock, 1976, 26; see also McDermott, 2007, 334).

So, if for every true C, there is a B which is irrelevant to whether or not C obtains, we can, if our logical principles are valid, establish Conjunction Conditionalization, via Argument 1. Similarly, if for every true A and C, there is a B which is irrelevant to both of them, then Arguments 2 and 3, establish Conjunction Conditionalization.

Of course, there is not such a B for every true A and C whatsoever. For instance, let C be a proposition that holds only at the world of assessment, so that if B is true, then so is $\sim B > \sim C$. Arguments 1–3, then, fall short of establishing Conjunction Conditionalization in its full generality. Rather, what they establish are the myriad instances of Conjunction Conditionalization for which there is such a B. But once we accept these instances, to reject Conjunction Conditionalization because for some special cases of A and C, we cannot find a B independent of both of them, would be extremely ill-motivated. Conjunction

[11] Kvart would reject this generalization of his thought since he himself rejects Conjunction Conditionalization (Kvart, 1994).

[12] Compare Bennett who rejects Conjunction Conditionalization but who subscribes to The Irrelevance Principle: 'When A and C are both true, and the causation of C is indeterministic, $A > C$ is true just in case A is irrelevant to the causal chain that leads to C's obtaining' (2003, 240). See also McDermott (2007) who rejects Conjunction Conditionalization but accepts (27).

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Conditionalization is, then, nonnegotiable for those who accept the truth of semifactuals with irrelevant antecedents and one of the packages of logical principles relied on above.

§5. Rejecting the semantic claim. The above arguments for Conjunction Conditionalization rely both on the sublogics of VW highlighted and on the semantic claim that semifactuals with antecedents irrelevant to the obtaining of the consequent are true. But early critics of Conjunction Conditionalization

> including Bennett (1974, 386–388), Fine (1975a, 453), and Bigelow (1976, 218), have tacitly argued on informal semantic grounds from what might be called the Connection Hypothesis: a necessary condition for the truth of a subjunctive conditional of the form $A > C$ is the existence of some sort of 'connection' (presumably, of either a logical or a causal nature) between the propositions expressed by A and C. Mere concurrent or coincident truth, so the argument runs, is either not a connection at all or at least not one of the requisite sort. Hence, since [Conjunction Conditionalization] would secure the truth of subjunctive conditionals with true but unconnected antecedents and consequents, it should be reckoned invalid, the critics maintain, for exactly this reason (Butcher, 1983, 71; see also Penczek, 1997, 80).

If this Connection Hypothesis is true, then, the semantic claim outlined in §4 and upon which our arguments rely is false. However, conditionals of the form of (25)–(27), seem like good reason to reject the Connection Hypothesis. As Pollock puts it

> it seems clear that simple subjunctives do not express a relation of necessitation [or any other connection] between their antecedent and consequent. Rather, ... [it] seems ... a simple subjunctive can be true because the consequent is already true and there is no connection between the antecedent and the consequent such that the antecedent's being true would interfere with the consequent's being true (Pollock, 1976, 26).

Of course, someone antecedently wedded to the Connection Hypothesis might reject the truth of conditionals like (25)–(27), despite their plausibility. But the Connection Hypothesis is implausible even bracketing these considerations. Nute (1980, 7) has shown that the Connection Hypothesis is incompatible with one of the most plausible principles of counterfactuals logic, and one which was not relied upon above, namely

> Weakening the Consequent: if $\vdash Y \supset Z$, then $(X > Y) \supset (X > Z)$.

This is because although the Connection Hypothesis allows 'if I were to push the button, the doorbell would ring' to be true in suitable circumstances, it does not allow 'if I were to push the button, the doorbell would exist' to be true in those same circumstances, even though the doorbell's ringing entails that the doorbell exists. But it is hard to see how Weakening the Consequent could fail. So the Connection Hypothesis must be rejected on independent grounds, and it is this rejection that allows (25)–(27) to be true, just as we ordinarily take them to be.

We should note, however, that despite the widespread acceptance of our semantic claim (Edgington, 2004; Kvart, 1986; Noordhof, 2004; Pollock, 1976, 26; Schaffer, 2004) some reject it without relying on the Connection Hypothesis. For example, Phillips (2007) rejects (25) and the intuition underlying it, not because he subscribes to the Connection Hypothesis, but because the coin toss is indeterministic. Presumably, Phillips would also

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reject the truth of irrelevant semifactuals with indeterministically caused consequents, such as (26). As such, then, our argument for Conjunction Conditionalization will not convince Phillips. But as noted above it does seem to be part of our practice to accept conditionals such as (25)–(27).[13]

Lewis himself accepts the truth of semifactuals with irrelevant antecedents. For example, Lewis (1979, 48) notes that the case of Morgenbesser's coin shows that some future facts count towards similarity. As a result then, our arguments show that despite Lewis's wavering over Conjunction Conditionalization, he is in fact committed to it as he accepts both the semantic claim and the theorems of VW required to establish Conjunction Conditionalization.

More importantly, many who want to reject Conjunction Conditionalization accept the semantic claim appealed to above (see, e.g., Bennett, 2003, chap. 1; Cogburn & Roland, 2013; McDermott, 1999, 304–305; 2007; McGlynn, 2012), and so if they are to reject Conjunction Conditionalization, they must reject the packages of theorems of VW. It is to these theorems which we now turn.

§6. The theorems of VW. Given the truth of semifactuals with irrelevant antecedents, the status of Conjunction Conditionalization turns on whether the theorems of VW employed by Arguments 1–3 are valid.

Although Gundersen (2004) and McGee (1985), for example, reject Modus Ponens, most theorists who wish to reject Conjunction Conditionalization, want to do so whilst maintaining Modus Ponens, and so would not object to our employment of that principle. In any case, as only one version of the argument relies on Modus Ponens, we are not concerned to defend it here.

Those who want to offer a nontrivial treatment of counterfactuals with impossible antecedents will presumably reject Necessary Consequent and Substitution of Logical Equivalents in their full generality. But as the arguments above employ only counterfactuals with possible antecedents, the arguments need only employ the weakened principles

> Necessary Consequent*: if $\Diamond X$, and $\vdash Z$, then $X > Z$
> Substitution of Logical Equivalents*: if $\Diamond X$, $\vdash X \equiv Y$ then $(X > Z) \equiv (Y > Z)$.

Of course, one may still reject these weakened principles, but we are content to note their intuitive appeal and that a wide variety of approaches to counterfactuals endorse both of them.

One of the more controversial theorems of VW, which is employed by Argument 1, is

$$\text{LAS: } ((X > Z) \wedge \sim(X > \sim Y)) \supset ((X \wedge Y) > Z).$$

The standard account validates LAS because it assumes that the closeness ordering of worlds is connected. That is, for all worlds u, v, and w, either w is closer to u than v is, or v is closer to u than w is, or w and v are equally close to u. But approaches which allow for incomparables, and so reject connectedness, invalidate LAS. Moreover, both advocates and opponents of Conjunction Conditionalization have been happy to reject LAS in the face of putative counterexamples (Bennett, 2003, 332–335; Hiddleston, 2005, 646–647;

[13] One of us (Walters) is happy to take appearances at face value and endorse the argument for Conjunction Conditionalization presented here. The other (Williams) is not so committed to the semantic claim relied on here.

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Pollock, 1976, 43–44), and so they will not be moved by the proof of Conjunction Conditionalization given in Argument 1. Neither Argument 2 nor Argument 3, however, rely on connectedness or LAS, and so will be of wider appeal.

Instead of LAS, Argument 2 relies on

$$\text{Substitution: } ((X > Y) \wedge (Y > X) \wedge (X > Z)) \supset (Y > Z)$$

(standard gloss on Substitution: if the closest A-worlds are B-worlds and the closest B-worlds are A-worlds, then the closest A-worlds are the closest B-worlds, and so C obtains at the former worlds iff C obtains at the latter worlds),

whereas, Argument 3, instead relies on

$$\text{VLAS: } ((X > Y) \wedge (X > Z)) \supset ((X \wedge Y) > Z)$$

(standard gloss on VLAS: if the closest A-worlds are B-worlds, then the closest (A ∧ B)-worlds are the closest A-worlds, so C is true at the former iff it is true at the latter).

Arguments 2 and 3 are simply variants of each other, since the conjunction of Substitution of Logical Equivalents and VLAS is equivalent to Substitution in a weak background counterfactual logic (see Burgess, 1981). In fact the package of principles appealed to in Argument 2 is equivalent to the package of principles appealed to in Argument 3 assuming $X > X$, Weakening the Consequent, and $((X > Y) \wedge (X > Z)) \supset (X > (Y \wedge Z))$ (proof omitted).[14]

Substitution, VLAS, and Disjunction all follow from the formal constraints that Stalnaker (1968, 46–47; 1987, 120–121) and Lewis (1971) place on the closeness ordering. For both Stalnaker and Lewis, the closeness ordering is an absolute affair, that is, it is not antecedent-relative. As Stalnaker puts it,

> the selection [of closest worlds] is to be based on two considerations: first, on which possible worlds are eligible to be selected; second, on which, among those that are eligible, is minimally different from the actual world. The only role of an antecedent of a conditional is to determine the class of eligible worlds. The relevant respects of difference between possible worlds can be assessed (it is assumed) independently of which worlds are eligible [that is to say, in an antecedent-independent manner]. This implies that if one possible world is preferred by the selection function, relative to one antecedent, it must be preferred to it relative to any antecedent for which both are eligible (Stalnaker, 1987, 121).

To illustrate the types of constraint that generate an absolute closeness ordering and the related theorems of VW, let us assume, as Lewis does not, that the Limit Assumption holds. With this assumption in place, a Stalnaker–Lewis semantics can be given in terms of a selection function, f, which selects the closest eligible worlds.[15] On a selection function

[14] The version employing VLAS, however, may be more intuitive. Bennett, despite being ambivalent about Substitution (2001, 201), thinks that '[t]here can be no doubt' about the validity of VLAS (2003, 332). Counterexamples to Substitution and VLAS, have been proposed by Ahmed (2011), Tichý (1978), and Tooley (2002). We lack the space to discuss them here, but we are unpersuaded by any of them. See Walters (2011) for a reply to Ahmed.

[15] Although Lewis (1971, 1986) presents a selection function semantics, he rejects the Limit Assumption and so his official theory cannot be presented in terms of a selection function. Nonetheless presenting things in terms of a selection function serves our purpose here.

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semantics, $A > C$ is true iff $f(A) \subseteq C$. Let us assume with Stalnaker and Lewis that we have the following constraint on the selection function: $f(A) \subseteq A$. This constraint ensures the truth of $A > A$. In order to achieve antecedent independence, Stalnaker imposes the following additional constraint on the selection function (1987, 120)

Stalnaker's Constraint: If $f(A) \subseteq B$, and $f(B) \subseteq A$, then $f(A) = f(B)$.

We can see that Stalnaker's constraint validates Substitution and VLAS from the glosses provided for them above. In Stalnaker's semantics, Stalnaker's Constraint also validates Disjunction. Since his selection function returns a single world, either $f(A \vee B) \subseteq A$, or $f(A \vee B) \subseteq B$. Stalnaker's constraint then ensures that either $f(A) = f(A \vee B)$, or $f(B) = f(A \vee B)$, given that $f(A) \subseteq (A \vee B)$ and $f(B) \subseteq (A \vee B)$. So, if both $f(A) \subseteq C$ and $f(B) \subseteq C$, then $f(A \vee B) \subseteq C$, which validates Disjunction.

In Lewis's semantics, however, Stalnaker's Constraint does not validate Disjunction. On Lewis's semantics it does not follow that either $f(A \vee B) \subseteq A$, or $f(A \vee B) \subseteq B$, and so it does not follow from Stalnaker's Constraint that $f(A) = f(A \vee B)$, or $f(B) = f(A \vee B)$. As a result, an $(A \wedge C)$-world could be preferred to any $(A \wedge \sim C)$-world relative to the antecedent A and to the antecedent B and yet not relative to the antecedent $(A \vee B)$. So, in Lewis's semantics, Stalnaker's Constraint fails to fully capture the idea of antecedent-independence.

As well as Stalnaker's Constraint, Lewis imposes the following constraint:

Lewis's Constraint: either $f(AvB) \subseteq A$ or $f(AvB) \subseteq B$ or $f(AvB) = f(A) \cup f(B)$.

which validates Disjunction: if either of the first two disjuncts of Lewis's constraint are satisfied, then the proof is as before, whereas if the final disjunct holds, Disjunction is guaranteed by the fact that then when $f(A) \subseteq C$ and $f(B) \subseteq C$, then $f(A) \cup f(B) \subseteq C$. So we can see that any selection function semantics which obeys Stalnaker and Lewis's Constraints validate the theorems of VW required for our arguments to go through.

Although Substitution, VLAS, and Disjunction are valid in antecedent-independent selection function semantics, these principles are not tied to such accounts. Unlike LAS, these principles can be accepted by those who reject connectedness. Moreover, these principles are validated by Pollock's (1976, 42–43) cotenability semantics, Adams' (1975, 61) probabilistic account, and Gärdenfors (1978, 1988) belief revision semantics, as well as by the Stalnaker/Lewis selection function semantics. And any semantics on which these principles are invalid will have to reject constraints akin to Stalnaker's and Lewis's Constraints.

One could, of course, reject antecedent-independence and the constraints that validate Substitution, VLAS and Disjunction. The problem with this is that, if we reject Stalnaker's Constraint, we also invalidate

Limited Transitivity: $((X > Y) \wedge ((X \wedge Y) > Z)) \supset (X > Z)$

since without it, we cannot move from $f(X) \subseteq (X \wedge Y)$, $f(X \wedge Y) \subseteq X$, and $f(X \wedge Y) \subseteq Z$ to $f(X) \subseteq Z$.[16]

See Lewis (1971, 1986) for details on how to secure an absolute closeness ordering in the absence of the Limit assumption.

[16] This is true, for instance, of Cross's (2009) proposal to invalidate Substitution.

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But Limited Transitivity, as well as being an intuitive and popular principle, also plays a role in explaining why instances of the stronger principle

Transitivity: $(X > Y) \wedge (Y > Z) \supset (X > Z)$

often seem good, when in fact the principle is invalid (cf. Morreau, 2009). Moreover, if we reject VLAS, then we have to give up the following plausible principle which entails it (see Appendix):

SDA*: $((X \vee Y) > Z) \supset ((X > Z) \vee (Y > Z))$.

So the costs of rejecting Substitution and VLAS are high. One might try to mitigate these costs by noting that we are often mislead by counterfactuals with disjunctive antecedents, and so SDA* might not be as innocent as it seems. For example, although

SDA: $((X \vee Y) > Z) \supset ((X > Z) \wedge (Y > Z))$

also looks good, it is invalid on the standard account, and we do not want to endorse it. There are two reasons for this. First SDA entails that Substitution of Logical Equivalents is invalid: consider some true counterfactual, $A > C$. By Substitution of Logical Equivalents we have $((A \wedge C) \vee (A \wedge \sim C)) > C$. SDA then allows us to conclude, $(A \wedge \sim C) > C$. But this is necessarily false when A and $\sim C$ are compossible. Second, there are good counterexamples to SDA. To take McKay & van Inwagen's (1977, 355) example, from

(30) If Spain had fought on either the Allied side or the Axis side, she would have fought on the Axis side

it does not follow that

(31) If Spain had fought on the Allied side, she would have fought on the Axis side.

But note that this case against SDA does not extend to SDA*. That is, SDA* does not conflict with Substitution of Logical Equivalents, nor is it subject to these putative counterexamples. So even if we do think that we are mislead by disjunctive antecedents in the case of SDA, this consideration does not tell against SDA*. Of course, this observation does not itself establish SDA*, but it does look good.[17]

Arguments 2 and 3 can also be resisted, though, by rejecting Disjunction rather than Substitution/VLAS. This would allow us to keep Stalnaker's Constraint which validates Limited Transitivity, although rejecting Disjunction means rejecting Lewis's Constraint which also validates SDA*. What can be said to such an opponent of Conjunction Conditionalization? Well, at some point we reach bedrock, and can only point to the plausibility of principles such as Disjunction, SDA* (and the rest). Burgess (1981, 77) and Pollock (1976, 42–43; 1981, 249), simply take both VLAS and Disjunction as axioms in their systems, and Pruss, a critic of the standard account, says of Disjunction

[17] SDA cannot be maintained by those who endorse Conjunction Conditionalization. *Proof:* $A \wedge C$ entails $(A \vee B) \wedge C$, giving $(A \vee B) > C$ by Conjunction Conditionalization. By (SDA), this gives $B > C$, for any B, erroneously making the truth of the consequent sufficient for the truth of a counterfactual. So SDA is false, if Conjunction Conditionalization holds (cf. Butcher, 1983, 75). But as SDA is invalid, this observation does not threaten Conjunction Conditionalization. Further 'free choice' counterexamples to Conjunction Conditionalization can be rejected as reflecting our pretheoretic commitment to principles such as SDA, principles which although initially appealing, are invalid.

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> the subjunctive conditional says what would happen were some condition realized . . . if some proposition would hold under one condition and would also hold under another condition, then it is hard to deny that it would hold under the disjunction of these two conditions (Pruss, 2007, 33–34).

Apart from agreeing with these philosophers, we should note that one reason for rejecting Disjunction does not undermine our arguments for Conjunction Conditionalization. Disjunction is suspect, one may think, because one way in which X v Y can be true is when X ∧ Y is true. So instead of endorsing Disjunction, you may only be prepared to endorse

> Disjunction*: $((X > Y) \wedge (Z > Y) \wedge (X \wedge Z > Y)) \supset ((X \vee Z) > Y)$.

But Disjunction is appealed to above, only in cases where X and Z are incompatible, and so if you accept that counterfactuals with impossible antecedents are vacuously true, then Disjunction* can be used in Arguments 2 and 3 instead of the stronger Disjunction. In any case, regardless of our attitudes to counterpossibles, Arguments 2 and 3 do not require the full force of Disjunction, but only the weaker:

> Disjunction**: $((X > Y) \wedge (Z > Y) \wedge (\Diamond(X \wedge Z) \supset ((X \wedge Z)) > Y)) \supset ((X \vee Z) > Y)$.

In fact, it is not clear that we need appeal to any theorem along the lines of Disjunction. When motivating the case for (27), we said that given that your betting behavior is irrelevant to the outcome of the coin toss, *whether or not you had bet heads, the coin would have landed heads*. That is, the conditionals of the form $(B \vee \sim B) > A$ which Arguments 1–3 rely on, are independently plausible when B is irrelevant to the obtaining of A. As a result, the argument can start from premises of that form, and so need not employ Disjunction at all. Indeed, in the arguments above, $(B \vee \sim B) > A$ is a semifactual with an antecedent irrelevant to the obtaining of its consequent. As a result, those who accept that such conditionals are true need not rely on Disjunction. VLAS alone, with our semantic claim, is sufficient to establish Conjunction Conditionalization.

§7. Conclusion. We have shown that although Lewis's own arguments for Conjunction Conditionalization fail, Lewis is nevertheless committed to Conjunction Conditionalization (or at least almost every instance of it). More generally, anyone who accepts certain sublogics of VW and the truth of semifactuals with irrelevant antecedents are committed to Conjunction Conditionalization. And as we have just argued, Conjunction Conditionalization can only be rejected if VLAS is rejected. No doubt some will be happy to reject VLAS. But such a rejection comes at a cost. Rejecting the constraint which validates VLAS in a selection function semantics also invalidates the attractive Limited Transitivity. More generally, VLAS follows from the intuitive SDA* in a weak counterfactual logic. The costs of rejecting Conjunction Conditionalization, then, are high, and we prefer not to pay them. At the very least, removing Conjunction Conditionalization is no minor surgery, but a complicated operation involving the mutilation of far more entrenched aspects of the logic of conditionals.

It would be disingenuous, however, to end before noting the intuitive pull of some of the putative counterexamples to Conjunction Conditionalization with which we started. A full defence of Conjunction Conditionalization would consist not only of an argument for Conjunction Conditionalization, but also of a well-motivated explanation of the putative

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counterexamples consistent with the truth of Conjunction Conditionalization. The latter would take us too far afield, however,[18] so we are content here to simply to offer the former.[19]

Appendix

VLAS: $((X > Y) \wedge (X > Z)) \supset ((X \wedge Y) > Z)$

follows from the principles:

Agglomeration: $((X > Y) \wedge (X > Z)) \supset X > (Y \wedge Z)$
Weakening the Consequent: if $\vdash Y \supset Z$, then $(X > Y) \supset (X > X)$
Reflexivity: $X > X$
Vacuity: $(X > \bot) \supset \Box{\sim}X$
Equivalence: $(\Box (X \equiv Y)) \supset ((X > Z) \equiv (Y > Z))$
SDA*: $((X \vee Y) > Z) \supset ((X > Z) \vee (Y > Z))$

Proof.

(32)	$A > B$	Assumption
(33)	$A > C$	Assumption
(34)	$A > (B \wedge C)$	32, 33, Agglomeration
(35)	$((A \wedge B) \vee (A \wedge {\sim}B)) > (B \wedge C)$	34, Equivalence
(36)	$((A \wedge B) > (B \wedge C)) \vee ((A \wedge {\sim}B) > (B \wedge C))$	35, SDA*
(37)	$(A \wedge {\sim}B) > (B \wedge C)$	Assumption
(38)	$(A \wedge {\sim}B) > (A \wedge {\sim}B)$	Reflexivity
(39)	$(A \wedge {\sim}B) > \bot$	37, 38, Agglomeration
(40)	$\Box{\sim}(A \wedge {\sim}B)$	39, Vacuity
(41)	$\Box (A \equiv (A \wedge B))$	40, Propositional Logic
(42)	$(A \wedge B) > (B \wedge C)$	34, 41, Equivalence
(43)	$(A \wedge B) > (B \wedge C)$	Assumption
(44)	$(A \wedge B) > (B \wedge C)$	36, 37, 42, 43, vE
(45)	$(A \wedge B) > C$	44, Weakening

We could replace Equivalence in the above proof with Substitution of Logical Equivalents, if we replaced Vacuity with

Vacuity*: if $X > \bot$, then $\vdash {\sim}X$.

If we stick with Vacuity, we can take this as primitive or else derive it from further principles. For instance, in his recent work on the link between modal and counterfactual logic, Williamson (2007, 156), assumes the following principle:

Possibility: $(X > Y) \supset (\Diamond X \supset \Diamond Y)$.

[18] See Walters (MS) for such an explanation.

[19] This paper has its origins in the argument for Conjunction Conditionalization in Walters (2009). The arguments here are variants of the ones presented in that paper but avoid reliance on claims about when we can strengthen the antecedent of the conditional. In correspondence about that earlier paper, the second author (Williams) proposed the kind of strengthenings of those arguments here investigated; the full development of these ideas as presented in this paper is due to the first author (Walters). We are grateful to two referees for comments, and also to Andrew Bacon for a conversation which led to the proof in the Appendix. Lee Walters would also like to thank The Royal Institute of Philosophy for funding his work on this paper.

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If we agree with Williamson that counterfactuals obey Possibility, Vacuity follows.

In fact, the proof does not require the full force of either Vacuity or Vacuity*, but only the following weaker principle:

$$(46)\quad ((X \wedge \sim Y) > \bot) \supset ((X > Z) \supset (X \wedge Y) > Z))$$

which from (34) and (39), yields (42). □

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