Robotica (2011) volume 29, pp. 221–232. © Cambridge University Press 2010
doi:10.1017/S026357471000007X

Task space control of mobile manipulators

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(Received in Final Form: February 12, 2010. First published online: March 22, 2010)

SUMMARY
This study offers the solution of the end-effector trajectory tracking problem subject to state constraints, suitably transformed into control-dependent ones, for mobile manipulators. Based on the Lyapunov stability theory, a class of controllers fulfilling the above constraints and generating the mobile manipulator trajectory with (instantaneous) minimal energy, is proposed. The problem of manipulability enforcement is solved here based on an exterior penalty function approach which results in continuous mobile manipulator controls even near boundaries of state constraints. The numerical simulation results carried out for a mobile manipulator consisting of a non-holonomic unicycle and a holonomic manipulator of two revolute kinematic pairs, operating in a two-dimensional task space, illustrate the performance of the proposed controllers.

KEYWORDS: Mobile manipulator; End-effector trajectory tracking; Lyapunov stability.

1. Introduction

Mobile manipulators are recently applied to useful practical tasks such as laser cutting, arc welding or painting a complex surface. A mobile manipulator is a mechanical system composed of a platform subject to non-holonomic constraints arising from the platform movement under the no-slip condition and a holonomic manipulator (mounted on the platform) with holonomic constraints being its kinematic equations. Constraints on the velocity of the platform are not integrable to equivalent platform position constraints. As a result of this condition, the instantaneous velocity of the platform under non-holonomic constraints is limited to certain directions. Hence, no smooth time invariant state feed-back controllers exist for the platform accomplishing a point-to-point task.[1,2] However, by combining mobility of the non-holonomic platform with manipulability of the holonomic manipulator, performance characteristics of the mobile manipulator are improved which enable it to accomplish complicated tasks especially the end-effector trajectory tracking in, for example, complex task spaces including many obstacles. The main advantage of mobile manipulators compared to the stationary ones is the ability of performing the conventional tasks and simultaneously other useful objectives such as non-holonomic manipulability enforcement (singularity avoidance). Application of mobile manipulators to the tasks mentioned above complicates their performance because such manipulators provide, in general, non-unique solutions. Consequently, some objective criteria should be specified to solve the mobile manipulator tasks uniquely. Several approaches may be distinguished in such a context.

Most of the existing literature deals with the end-effector trajectory tracking using only kinematic equations of the mobile manipulator. The approach in works[3,4] uses pseudo-inverse of the Jacobian matrix and the secondary tasks to resolve the kinematic redundancy when accomplishing those tasks. Using the configuration control approach, a set of augmented differential kinematic equations is formed in the work[5] which are then solved to obtain the mobile manipulator motion. Nevertheless, its determination requires the knowledge of additional user-specified consistent trajectories (tasks) which is not a trivial problem. In the work,[6] a solution to the inverse kinematic problem is yielded by defining the endogenous configuration space and then using so called a general Jacobian pseudo-inverse algorithm. Recently, based on optimization of certain criteria, a motion planning method avoiding both collisions with obstacles and singular configurations, and producing continuous velocities has been proposed in refs. [7, 8]. Egerstedt and Hu in the work[9] presented a coordinated virtual vehicle solution to the trajectory following problem for mobile manipulators. A trajectory for the mobile platform is planned in such a manner that the desired end-effector position is within the specified workspace of the arm. Using a secondary cost function for the non-holonomic platform and the transfer function approach, Fruchard *et al.*[10] have proposed a technique for determining separately the holonomic manipulator velocity and the platform velocity in such a way that the minimization of the secondary cost function is to bring the platform to a domain where the manipulation task can be successfully carried out. An exhaustive overview of the existing techniques for trajectory generation carried out at the kinematic level may be found in.[7,10]

There are only few references dealing with the end-effector trajectory tracking subject to mobile manipulator dynamic equations. Desai and Kumar[11] have addressed the problem of motion planning for multiple mobile manipulators. Based on the calculus of variations, they have obtained optimal trajectories in the presence of obstacles. Sub-optimal

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https://doi.org/10.1017/S026357471000007X Published online by Cambridge University Press

trajectory planning of mobile manipulator has been proposed by Mohri *et al.*[12] The planning problem was formulated in ref. [12] as an optimal control problem. To solve it, the authors used the concept of the order of priority. A gradient-based iterative algorithm which synthesizes the gradient function in a hierarchical manner based on the order of priority has then been used. A method for coordinating vehicle motion planning and manipulator motion planning including platform stability has been proposed in the work.[13] The optimal robot motion task formulated as a problem of non-linear programming has been then solved in ref. [13] by using a generalized projection method.

Although all the aforementioned algorithms produce optimal solutions, they are not suitable to real-time implementations due to their computational complexity. Therefore, it is natural to attempt alternative techniques for generating the mobile manipulator trajectories in real time. An extended Jacobian transpose control method to compensate for dynamic interactions between the manipulator and the base has been developed by Hootsmans and Dubowsky in the work.[14] Mobile manipulators with soft suspensions were shown in ref. [14] to be stable with limited vehicle sensory data. However, only holonomic motion of the mobile platform was considered. Liu and Lewis have developed in the work[15] a decentralized robust controller for trajectory tracking of the mobile manipulator end-effector. However, they considered only translational motion of the mobile platform. Yamamoto and Yun[16,17] addressed the problem of motion coordination simultaneously at the kinematic and dynamic level. The proposed solution exploits full non-holonomic characteristics of the platform kinematics and drives the platform towards a location providing the manipulator with desirable manipulability. Nevertheless, this approach does not utilize manipulator redundancy during the movement which may be troublesome by, for example, collision avoidance. The same authors introduced in ref. [18] kinematic and dynamic task space ellipsoids for mobile manipulators characterizing in a unified way mobility of the platform and manipulability of the manipulators. Similar ideas of motion coordination have appeared in the work.[19] Chung *et al.* have resolved in works[20,21] the kinematic redundancy by decomposing the mobile manipulator into two subsystems: the mobile platform and the manipulator. Based on the coordination scheme used to resolve the kinematic redundancy, a robust interaction control algorithm, in which suitable controllers are designed for the two subsystems, were developed. The adverse effect of the wheel slip on the tracking of commanded motion is additionally discussed in ref. [21]. The control schemes proposed in the works[22,23] require the reference trajectories for both the holonomic manipulator and non-holonomic platform. The control algorithm from [22]uses a full knowledge of mobile manipulator dynamics whereas that from the study[23] does not need the knowledge of dynamic parameters. In the paper,[24] a specific task for mobile manipulators is accomplished by coordinating the platform and the manipulator. The kinematic redundancy resolution makes it possible the coordinated control of the mobile platform and the robot arm such that the holonomic manipulator dexterity is preserved and the robot is kept away from its singular configuration. Nevertheless, the mobile platform considered in ref. [24] is holonomic. A resolved acceleration control and proportional-integral active force control has been proposed in the work[25] as an approach for the robust motion control of a mobile manipulator comprising a differentially driven wheeled mobile platform with a two-link planar arm mounted on the top of the platform. However, the control law from[25] requires the reference trajectories for both the holonomic manipulator and non-holonomic platform.

The present work addresses the problem of controlling the mobile manipulator subject to different state variable constraints. In our approach, platform and manipulator kinematics are represented as a driftless control system and holonomic constraints, respectively and mobile manipulator dynamic equations are given in the form derived from the d'Alembert Principle. Kinematic and dynamic parameters of the mobile manipulator can be obtained with sufficient accuracy by means of calibration and identification technique.[26,27] Consequently, they are assumed further on to be fully known. The task of the mobile manipulator is to track a desirable (reference) trajectory by the end-effector (no platform reference trajectory is required to accomplish the end-effector task). On account of non-holonomic constraints, the mobile manipulator usually becomes a redundant mechanism even when its holonomic part is non-redundant. Therefore, the trajectory tracking problem is transformed here into an optimization task of some criterion function subject to both holonomic and non-holonomic (state) equality constraints (expressed in control variables) which result from the imposed condition of asymptotically stable end-effector movement. The criterion function is constructed in such a way as to (instantaneously) minimize the whole robot energy. This optimization is particularly important for heavy both the mobile platform and the manipulator whose end-effector carries a payload. Such approach seems to be advantageous as compared to control algorithms known from the literature where mainly the redundancy is resolved, in general not optimally in any sense, to coordinate platform and manipulator motions. Most frequently met approaches from the literature construct control algorithms under mild condition that non-holonomic constraints are fulfilled at the initial time instant of the trajectory tracking. However, this assumption may not be true due to, for example, random external disturbances acting on the mobile manipulator. If this is the case, we modify non-holonomic constraints in such a way as to provide platform motions (asymptotically) fulfilling these constraints.

Moreover, it is important to track the desired trajectory with as large as possible manipulability measure of the mobile manipulator (introduced by Bayle *et al.*[3]) whose precise definition is given in Section 3.2. If the manipulability measure were sufficiently small, then the end-effector could loose the tracking capabilities of a relatively complex desired trajectory due to numerical instabilities appearing in neighbourhoods of (possible) kinematic singularities met on the mobile manipulator trajectory. The requirement of large manipulability measure during the tracking imposes additional state inequality constraint which is suitably transformed here in control-dependent one. Furthermore, it is shown herein how through the use of exterior penalty

https://doi.org/10.1017/S026357471000007X Published online by Cambridge University Press

function method, this additional transformed inequality constraint is incorporated in our algorithm. As is known, exterior penalty function methods take into account only active state constraints, which decreases computational burden, and generate bounded torques/forces even in neighbourhoods of boundaries of the state constraints. By fulfilment of practically reasonable assumptions, the control scheme derived, incorporating state constraints suitably transformed into control-dependent ones, is shown to be asymptotically stable. Furthermore, the resulting manipulator controls are bounded and present continuous mappings with respect to time. The remainder of the paper is organized as follows. Section 2 formulates the trajectory tracking control problem as a constrained optimization task. Section 3 provides us with a class of controllers solving the trajectory tracking problem subject to control-dependent constraints. Section 4 presents a computer example of the trajectory tracking by the end-effector for a mobile manipulator consisting of a non-holonomic unicycle and a holonomic manipulator of two revolute kinematic pairs, operating in a two-dimensional task space. Finally, some concluding remarks are made in Section 5.

2. Problem Formulation

Consider a mobile manipulator composed of a non-holonomic platform. It is described by the vector of generalized coordinates $\boldsymbol{x} \in \mathbb{R}^l$ (e.g. for exemplary platform of type (2, 0) such coordinates mean: platform posture $x_{1,c}\ x_{2,c}\ \theta$ and angles of driving wheels $\alpha_1\ \alpha_2$ (see Fig. 1), where θ is the angle measured in the task space; $x_{1,c},\ x_{2,c}$ mean coordinates of unicycle centre), where $l \geq 1$ and a holonomic manipulator (mounted on the platform) with joint coordinates $\boldsymbol{y} \in \mathbb{R}^n$, where n is the number of its kinematic pairs. The platform motion is usually subject to $1 \leq k < l$ non-holonomic constraints in the so-called Pfaffian form

$$\boldsymbol{A}(\boldsymbol{x})\dot{\boldsymbol{x}} = 0, \tag{1}$$

where $\boldsymbol{A}(\boldsymbol{x})$ stands for the $k \times l$ matrix of the full rank (i.e. $rank(\boldsymbol{A}(\boldsymbol{x})) = k$) depending on $\boldsymbol{x}$ analytically. Suppose that $Ker(\boldsymbol{A}(\boldsymbol{x}))$ is spanned by vector fields $\boldsymbol{a}_1(\boldsymbol{x}), \ldots, \boldsymbol{a}_{l-k}(\boldsymbol{x})$. Then, the kinematic constraint (1) can be equivalently expressed by an analytic driftless dynamic system

$$\dot{\boldsymbol{x}} = \boldsymbol{N}(\boldsymbol{x})\dot{\boldsymbol{\alpha}}, \tag{2}$$

where $\boldsymbol{N}(\boldsymbol{x}) = [\boldsymbol{a}_1(\boldsymbol{x}), \ldots, \boldsymbol{a}_{l-k}(\boldsymbol{x})]$; $rank(\boldsymbol{N}(\boldsymbol{x})) = l - k$ and vector $\dot{\boldsymbol{\alpha}} \in \mathbb{R}^{l-k}$ denotes (scaled) angular velocities of the platform driving wheels. The position and orientation of the end-effector with respect to an absolute coordinate system is described by a mobile manipulator kinematic equations $\boldsymbol{p}(\boldsymbol{x}, \boldsymbol{y}) = (p_1, \ldots, p_m)^T$, where $\boldsymbol{p} : \mathbb{R}^l \times \mathbb{R}^n \longrightarrow \mathbb{R}^m$ denotes the m-dimensional (in general, non-linear with respect to $\boldsymbol{x}$ and $\boldsymbol{y}$) mapping; vector $\boldsymbol{q} = (\boldsymbol{x}^T\ \boldsymbol{y}^T)^T$ represents the configuration of the mobile manipulator and m stands for the dimension of the task space. The dynamics of a mobile manipulator is given by

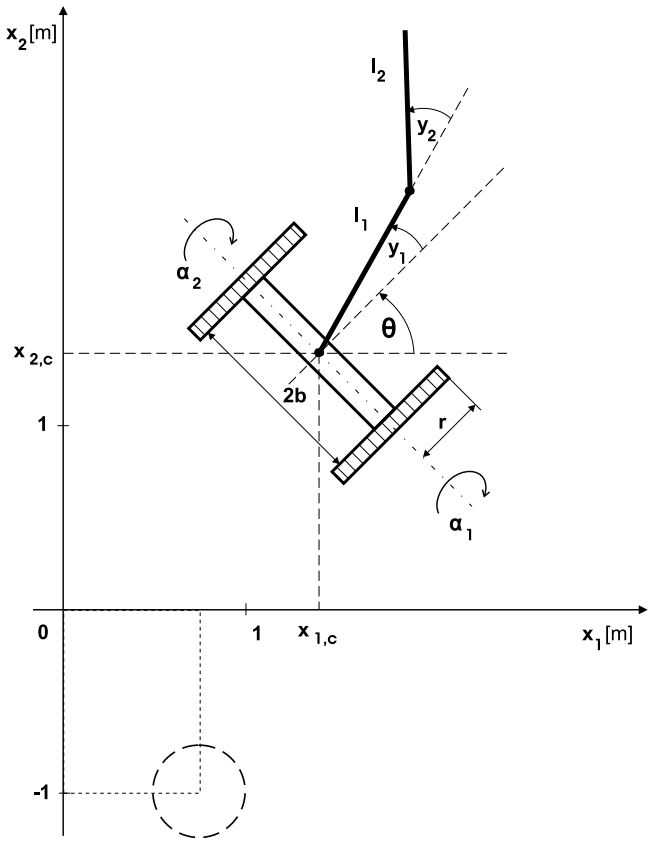


Fig. 1 Kinematic scheme of the mobile manipulator and the task to be accomplished.

the following general equation[28]:

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{F}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + [\boldsymbol{A}(\boldsymbol{x})\ 0_{k\times n}]^T \boldsymbol{\Lambda} = \boldsymbol{B}\boldsymbol{v}, \tag{3}$$

where $\boldsymbol{M}(\boldsymbol{q})$ denotes the $(n + l) \times (n + l)$ inertia matrix; $\boldsymbol{F}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ is the $(n + l)$-dimensional vector representing centrifugal, Coriolis and gravity forces; $0_{k\times n}$ denotes the $k \times n$ zero matrix; $\boldsymbol{\Lambda}$ is the vector of Lagrange multipliers corresponding to non-holonomic constraints (1); $\boldsymbol{B}$ stands for the $(l + n) \times (n + l - k)$ matrix describing which state variables of the platform and holonomic manipulator are directly driven by the actuators and $\mathbb{R}^{n+l-k} \ni \boldsymbol{v}$ is the vector of controls (torques/forces).

As is known, a violation of non-holonomic constraints has been analysed in works,[21,29,30] where the authors have proposed relatively complicated first order in refs. [21, 29] and second order in ref. [30] differential equations of slippage in wheels based both on Dugoff's tire friction model[31] and assumption about dissipative nature of interaction forces in ref. [30]. The potential drawback of this approach is that the tire friction may not be exactly known in practice. The authors of the aforementioned works concentrate mainly on the problem of motion with skid control. Since our aim is to eliminate a slippage (and not to control it) we will not be interested in analysing the reasons of the skid phenomenon. Hence, it suffices at this stage to take into account only direct results of skidding. Consequently, we impose (or require) a specific platform movement (described by the differential equations of the second order), which results in (asymptotic)

https://doi.org/10.1017/S026357471000007X Published online by Cambridge University Press

fulfilment of non-holonomic constraints (2), and then search for controls generating such a platform movement. To be specific, if the non-holonomy condition given by Eqs. (1) or (2) is violated, that is, $\dot{\boldsymbol{x}} - \boldsymbol{N}(\boldsymbol{x})\dot{\boldsymbol{\alpha}} \neq 0$ then the platform is required to move according to the following equation

$$\ddot{\boldsymbol{x}} = \boldsymbol{N}(\boldsymbol{x})\ddot{\boldsymbol{\alpha}} + \dot{\boldsymbol{N}}(\boldsymbol{x})\dot{\boldsymbol{\alpha}} - \lambda\left(\dot{\boldsymbol{x}} - \boldsymbol{N}(\boldsymbol{x})\dot{\boldsymbol{\alpha}}\right), \tag{4}$$

where $\lambda > 0$ is a given constant coefficient. Let us note that for a non-holonomic platform motion (Eq. (2) holds true) Eq. (4) is equivalent to Eq. (2). When equality (2) is not satisfied, platform motion described by Eq. (4) is realized in such a way as to (asymptotically) fulfil non-holonomic constraints (2). Combining $\ddot{\boldsymbol{q}}$ with (4) results in the following relation

$$\ddot{\boldsymbol{q}} = \boldsymbol{C}\begin{pmatrix}\ddot{\boldsymbol{\alpha}}\\ \ddot{\boldsymbol{y}}\end{pmatrix} + \begin{pmatrix}\dot{\boldsymbol{N}}(\boldsymbol{x})\dot{\boldsymbol{\alpha}} - \lambda\left(\dot{\boldsymbol{x}} - \boldsymbol{N}(\boldsymbol{x})\dot{\boldsymbol{\alpha}}\right)\\ 0\end{pmatrix}, \tag{5}$$

where $\boldsymbol{C} = \left[\begin{smallmatrix}\boldsymbol{N}(\boldsymbol{x}) & 0\\ 0 & \mathbb{I}_n\end{smallmatrix}\right]$; $\mathbb{I}_n$ denotes the $n \times n$ identity matrix. Let us note, that $\boldsymbol{C}$ is a full-rank matrix ($rank(\boldsymbol{C}) = n + l - k$). Our purpose is to simplify dynamic equations (3) using dependence (5). Multiplying (3) by $\boldsymbol{C}^T$ and noting that $\boldsymbol{A}(\boldsymbol{x})\boldsymbol{N}(\boldsymbol{x}) = 0$, we obtain

$$\boldsymbol{C}^T\boldsymbol{M}\ddot{\boldsymbol{q}} = \boldsymbol{C}^T\left(\boldsymbol{B}\boldsymbol{v} - \boldsymbol{F}\right). \tag{6}$$

Inserting the right-hand side of (5) into (6), we have

$$\boldsymbol{D}\begin{pmatrix}\ddot{\boldsymbol{\alpha}}\\ \ddot{\boldsymbol{y}}\end{pmatrix} = \boldsymbol{E}\boldsymbol{v} - \boldsymbol{z}, \tag{7}$$

where

$$\boldsymbol{D} = \boldsymbol{C}^T\boldsymbol{M}\boldsymbol{C}; \quad \boldsymbol{E} = \boldsymbol{C}^T\boldsymbol{B};$$
$$\boldsymbol{z} = \boldsymbol{C}^T\boldsymbol{F} + \boldsymbol{C}^T\boldsymbol{M}\begin{pmatrix}\dot{\boldsymbol{N}}(\boldsymbol{x})\dot{\boldsymbol{\alpha}} - \lambda\left(\dot{\boldsymbol{x}} - \boldsymbol{N}(\boldsymbol{x})\dot{\boldsymbol{\alpha}}\right)\\ 0\end{pmatrix}.$$

Hence

$$\begin{pmatrix}\ddot{\boldsymbol{\alpha}}\\ \ddot{\boldsymbol{y}}\end{pmatrix} = \boldsymbol{D}^{-1}\boldsymbol{E}\boldsymbol{v} - \boldsymbol{D}^{-1}\boldsymbol{z}. \tag{8}$$

Applying the transformation $\boldsymbol{v} = (\boldsymbol{D}^{-1}\boldsymbol{E})^{-1}(\boldsymbol{u} + \boldsymbol{D}^{-1}\boldsymbol{z})$ (let us note that matrix $\boldsymbol{E}$ is the (4×4) identity matrix multiplied by a constant for a unicycle depicted in Fig. 1), where $\boldsymbol{u}$ is a new control vector, we simplify dynamic equations (8) and express it in partial state variables as follows:

$$\begin{pmatrix}\ddot{\boldsymbol{\alpha}}\\ \ddot{\boldsymbol{y}}\end{pmatrix} = \boldsymbol{u}. \tag{9}$$

Dependence (9) presents an input-state linearizable equation of the mobile manipulator. However, form (9) seems to be practically cumbersome since it does not directly comprise all the coordinates in which the trajectory tracking task is expressed. A task accomplished by a mobile manipulator is to track by the end-effector a desired trajectory $\boldsymbol{d}(t) \in \mathbb{R}^m$ ($t \geq 0$) given in the task space (for many practical tasks such as assembly on production line or arc welding, it seems necessary to define desired trajectory for all the task coordinates, that is, desired position and orientation of the end-effector). Mapping $\boldsymbol{d}(\cdot)$ is assumed to be at least twice continuously differentiable, that is, $\boldsymbol{d}(\cdot) \in C^2$.

By introducing the end-effector tracking error $\boldsymbol{e}$ defined as $\boldsymbol{e} = \boldsymbol{p}(\boldsymbol{q}) - \boldsymbol{d}(t)$, we may formally express our task by means of the following dependencies:

$$\lim_{t\to\infty} \boldsymbol{e}(t) = 0, \tag{10}$$

$$\lim_{t\to\infty} \dot{\boldsymbol{e}}(t) = 0, \tag{11}$$

and additional relation presenting (asymptotical) stabilizing the non-holonomic platform motion

$$\lim_{t\to\infty} \left(\dot{\boldsymbol{x}} - \boldsymbol{N}(\boldsymbol{x})\dot{\boldsymbol{\alpha}}\right) = 0. \tag{12}$$

In general, condition (10) does not necessarily imply relation (11). Dependencies (10), (11) make it possible to specify precision of both position and velocity task accomplishment required, for example, in arc welding. Particularly, initial condition $\dot{\boldsymbol{q}}(0)$ is assumed not necessarily to fulfil non-holonomic constraints (2) (as opposed to assumptions usually made in the literature). In order to make it possible to track desired trajectory $\boldsymbol{d}(\cdot)$, it is reasonable to assume that $m \leq n$ (holonomic part should have enough degrees of freedom to accomplish complex end-effector movements). If this is the case, the mobile manipulator becomes redundant even when $m = n$. Consequently, there exist, in general, infinitely many controls $\boldsymbol{u}$ which lead to tracking desired trajectory $\boldsymbol{d}(\cdot)$. In order to determine a unique solution, the following performance index is introduced

$$I(\boldsymbol{u}) = \frac{1}{2}\|\boldsymbol{u}\|^2, \tag{13}$$

where $\|\cdot\|$ is the Euclidean norm. Formula (13) presents an (instantaneous) energy lost by the mobile manipulator to track $\boldsymbol{d}(\cdot)$. Expressions (9)–(13) formulate the mobile manipulator task as an (instantaneously) optimal control problem. Our aim is to find control $\boldsymbol{u}(t)$ which asymptotically tracks desired trajectory $\boldsymbol{d}(t)$ and simultaneously (instantaneously) minimizes performance index (13). Existing the state constraints makes the solution of this problem difficult. The next section will present an approach to the solution (provided that it exists) of the control problem (9)–(13) making use of the Lyapunov stability theory.

3. Generating the Controls of the Mobile Manipulator

Combining Eqs. (5) and (9), we obtain a simplified form of mobile manipulator dynamic equations expressed in all the state variables

$$\ddot{\boldsymbol{q}} = \boldsymbol{C}\boldsymbol{u} + \boldsymbol{r}, \tag{14}$$

where $\boldsymbol{r} = \left(\begin{smallmatrix}\dot{\boldsymbol{N}}(\boldsymbol{x})\dot{\boldsymbol{\alpha}} - \lambda\left(\dot{\boldsymbol{x}} - \boldsymbol{N}(\boldsymbol{x})\dot{\boldsymbol{\alpha}}\right)\\ 0\end{smallmatrix}\right)$. Thus, the mobile manipulator task may now be reformulated as follows. Find a control $\boldsymbol{u}$ which moves the mobile manipulator from initial

https://doi.org/10.1017/S026357471000007X Published online by Cambridge University Press

configuration and velocity $\boldsymbol{q}(0)$, $\dot{\boldsymbol{q}}(0)$ (which do not necessarily satisfy non-holonomic constraints (1) or (2)) such that equalities $\boldsymbol{e} = 0$, $\dot{\boldsymbol{e}} = 0$ and $\dot{\boldsymbol{x}} - \boldsymbol{N}\dot{\boldsymbol{\alpha}} = 0$ are asymptotically fulfilled. Moreover, $\boldsymbol{u}$ should minimize (13).

3.1. Singularity-free manipulator motion

In order to determine mobile manipulator controls, matrix $\mathbb{J} = \boldsymbol{J}\boldsymbol{C}$, where $\boldsymbol{J} = \frac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}} = \frac{\partial \boldsymbol{p}}{\partial \boldsymbol{q}}$ stands for the $m \times (n + l)$ Jacobian mobile manipulator matrix, is assumed to be non-singular during the movement in a region of our interest. Singular cases are discussed in the next section. Based on Eqs. (13)–(14), a (simple) control law is proposed, as follows:

$$\boldsymbol{u} = \mathbb{J}^{\#}\left(-\boldsymbol{J}\boldsymbol{r} - \dot{\boldsymbol{J}}\dot{\boldsymbol{q}} + \ddot{\boldsymbol{d}}(t) - 2k_0\dot{\boldsymbol{e}} - \frac{k_2}{k_1}\boldsymbol{e}\right), \tag{15}$$

where $\mathbb{J}^{\#} = \mathbb{J}^T(\mathbb{J}\mathbb{J}^T)^{-1}$; k_2, k_0 and k_1 stand for positive coefficients (controller gains) which could be replaced by diagonal matrices of positive constants without affecting the stability results obtained further on (this should lead to improved performance). Here, scalar constants are chosen for simplicity of presentation. The closed-loop error dynamics is obtained by inserting the right-hand side of Eq. (15) into Eq. (14):

$$\ddot{\boldsymbol{q}} = \boldsymbol{C}\mathbb{J}^{\#}\left(-\boldsymbol{J}\boldsymbol{r} - \dot{\boldsymbol{J}}\dot{\boldsymbol{q}} + \ddot{\boldsymbol{d}}(t) - 2k_0\dot{\boldsymbol{e}} - \frac{k_2}{k_1}\boldsymbol{e}\right) + \boldsymbol{r}. \tag{16}$$

The aim is to provide conditions on controller gains k_2, k_0 and k_1 guaranteeing asymptotic stability of the tracking for the trajectory $\boldsymbol{d}$ with its time derivative $\dot{\boldsymbol{d}}$ and non-holonomic constraints $\dot{\boldsymbol{x}} - \boldsymbol{N}\dot{\boldsymbol{\alpha}} = 0$ in the sense of fulfilment of relations (10)–(12). Applying the Lyapunov stability theory we derive the following result.

Theorem 1. *If $\mathbb{J}$ is the full-rank matrix in a (closed) region of the task space and*

$$k_1,\ k_2 > 0, \tag{17}$$

$$k_0 k_1 > \frac{1}{4}, \tag{18}$$

$$k_0 > \frac{k_2}{4} + \frac{1}{4k_1}, \tag{19}$$

then the control law (15) guarantees both asymptotic stability of the origin $(\boldsymbol{e},\ \dot{\boldsymbol{e}},\ \dot{\boldsymbol{x}} - \boldsymbol{N}\dot{\boldsymbol{\alpha}}) = (0,\ 0,\ 0)$ and minimization of performance index (13).

Proof. The proof of Theorem 1. will be based on the Barbalat lemma.[32] Consider the following Lyapunov function candidate:

$$V = k_1\langle \dot{\boldsymbol{e}},\ \dot{\boldsymbol{e}}\rangle + k_0\langle \boldsymbol{e},\ \boldsymbol{e}\rangle + \langle \dot{\boldsymbol{e}},\ \boldsymbol{e}\rangle + \frac{1}{2}\langle \dot{\boldsymbol{x}} - \boldsymbol{N}(\boldsymbol{x})\dot{\boldsymbol{\alpha}}, \dot{\boldsymbol{x}} - \boldsymbol{N}(\boldsymbol{x})\dot{\boldsymbol{\alpha}}\rangle. \tag{20}$$

Using conditions (17) and (18) on parameters k_0, k_1, it is easy to see that the sum of the first three terms of function V is non-negative. Hence, V is also non-negative. Differentiating function V with respect to time results in the following expression:

$$\dot{V} = \langle \ddot{\boldsymbol{e}},\ 2k_1\dot{\boldsymbol{e}} + \boldsymbol{e}\rangle + 2k_0\langle \dot{\boldsymbol{e}},\ \boldsymbol{e}\rangle + \langle \dot{\boldsymbol{e}},\ \dot{\boldsymbol{e}}\rangle + \langle \dot{\boldsymbol{x}} - \boldsymbol{N}(\boldsymbol{x})\dot{\boldsymbol{\alpha}},\ \ddot{\boldsymbol{x}} - \dot{\boldsymbol{N}}(\boldsymbol{x})\dot{\boldsymbol{\alpha}} - \boldsymbol{N}(\boldsymbol{x})\ddot{\boldsymbol{\alpha}}\rangle.$$

It follows from the definition of $\boldsymbol{e}$ that $\ddot{\boldsymbol{e}} = \boldsymbol{J}\ddot{\boldsymbol{q}} + \dot{\boldsymbol{J}}\dot{\boldsymbol{q}} - \ddot{\boldsymbol{d}}(t)$. Hence,

$$\dot{V} = \langle \boldsymbol{J}\ddot{\boldsymbol{q}} + \dot{\boldsymbol{J}}\dot{\boldsymbol{q}} - \ddot{\boldsymbol{d}}(t),\ 2k_1\dot{\boldsymbol{e}} + \boldsymbol{e}\rangle + 2k_0\langle \dot{\boldsymbol{e}},\ \boldsymbol{e}\rangle + \langle \dot{\boldsymbol{e}},\ \dot{\boldsymbol{e}}\rangle + \langle \dot{\boldsymbol{x}} - \boldsymbol{N}(\boldsymbol{x})\dot{\boldsymbol{\alpha}},\ \ddot{\boldsymbol{x}} - \dot{\boldsymbol{N}}(\boldsymbol{x})\dot{\boldsymbol{\alpha}} - \boldsymbol{N}(\boldsymbol{x})\ddot{\boldsymbol{\alpha}}\rangle. \tag{21}$$

Substituting $\ddot{\boldsymbol{q}}$ for the right-hand side of (16) and inserting the right-hand side of (4) into time-derivative of V given by Eq. (21) we have after simple algebraic computations that

$$\dot{V} = -\left\langle \begin{bmatrix} 4k_0k_1 - 1 & k_2 \\ k_2 & \frac{k_2}{k_1} \end{bmatrix} \begin{pmatrix} \dot{\boldsymbol{e}} \\ \boldsymbol{e} \end{pmatrix}, \begin{pmatrix} \dot{\boldsymbol{e}} \\ \boldsymbol{e} \end{pmatrix} \right\rangle - \lambda\langle \dot{\boldsymbol{x}} - \boldsymbol{N}\dot{\boldsymbol{\alpha}},\ \dot{\boldsymbol{x}} - \boldsymbol{N}\dot{\boldsymbol{\alpha}}\rangle.$$

It follows from (19) that matrix

$$\begin{bmatrix} 4k_0k_1 - 1 & k_2 \\ k_2 & \frac{k_2}{k_1} \end{bmatrix}$$

is positive definite. Hence $\dot{V} \leq 0$. The time-derivative of $\dot{V}$ equals

$$\ddot{V} = -2\left\langle \begin{bmatrix} 4k_0k_1 - 1 & k_2 \\ k_2 & \frac{k_2}{k_1} \end{bmatrix} \begin{pmatrix} \dot{\boldsymbol{e}} \\ \boldsymbol{e} \end{pmatrix}, \begin{pmatrix} \ddot{\boldsymbol{e}} \\ \dot{\boldsymbol{e}} \end{pmatrix} \right\rangle - 2\lambda\langle \dot{\boldsymbol{x}} - \boldsymbol{N}\dot{\boldsymbol{\alpha}},\ \ddot{\boldsymbol{x}} - \dot{\boldsymbol{N}}\dot{\boldsymbol{\alpha}} - \boldsymbol{N}\ddot{\boldsymbol{\alpha}}\rangle.$$

Due to non-positiveness of $\dot{V}$, vector functions $\boldsymbol{e}$, $\dot{\boldsymbol{e}}$ and $\dot{\boldsymbol{x}} - \boldsymbol{N}\dot{\boldsymbol{\alpha}}$ are bounded. Moreover, multiplying both sides of (16) by $\boldsymbol{J}$ and then using identity $\ddot{\boldsymbol{e}} = \boldsymbol{J}\ddot{\boldsymbol{q}} + \dot{\boldsymbol{J}}\dot{\boldsymbol{q}} - \ddot{\boldsymbol{d}}(t)$, it is easy to see that $\ddot{\boldsymbol{e}} = -2k_0\dot{\boldsymbol{e}} - \frac{k_2}{k_1}\boldsymbol{e}$. Hence, we conclude that $\ddot{\boldsymbol{e}}$ is bounded, too. Furthermore, it follows from (4) that $\ddot{\boldsymbol{x}} - \dot{\boldsymbol{N}}\dot{\boldsymbol{\alpha}} - \boldsymbol{N}\ddot{\boldsymbol{\alpha}} = -\lambda(\dot{\boldsymbol{x}} - \boldsymbol{N}\dot{\boldsymbol{\alpha}})$. Hence, all the terms appearing in $\ddot{V}$ are bounded. Consequently, $\ddot{V}$ is bounded, too. Finally, it follows from Barbalat's lemma[32] that $\dot{V}$ tends asymptotically to approach zero, that is, $\dot{\boldsymbol{e}} \to 0$, $\boldsymbol{e} \to 0$ and $\dot{\boldsymbol{x}} - \boldsymbol{N}\dot{\boldsymbol{\alpha}} \to 0$ as $t \to \infty$.

It remains to be shown that control law (15) minimizes performance index (13). For this purpose, using identity $\ddot{\boldsymbol{e}} = \boldsymbol{J}\ddot{\boldsymbol{q}} + \dot{\boldsymbol{J}}\dot{\boldsymbol{q}} - \ddot{\boldsymbol{d}}(t)$ and (14), we reformulate equation $\ddot{\boldsymbol{e}} = -2k_0\dot{\boldsymbol{e}} - \frac{k_2}{k_1}\boldsymbol{e}$ to the following convenient form

$$\boldsymbol{J}(\boldsymbol{C}\boldsymbol{u} + \boldsymbol{r}) + \dot{\boldsymbol{J}}\dot{\boldsymbol{q}} - \ddot{\boldsymbol{d}}(t) = -2k_0\dot{\boldsymbol{e}} - \frac{k_2}{k_1}\boldsymbol{e}. \tag{22}$$

Equation (22) only ensures the mobile manipulator to asymptotically and stably track desired trajectory $\boldsymbol{d}$ for

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arbitrary (continuous) mapping (control) $\boldsymbol{u}$. By treating the vector functions $\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{e}, \dot{\boldsymbol{e}}, \ddot{\boldsymbol{d}}, \boldsymbol{J}, \dot{\boldsymbol{J}}$ and $\boldsymbol{C}$ as known quantities, one may formulate the following (instantaneous) optimal control problem. Minimize

$$I(\boldsymbol{u}) = \frac{1}{2}\|\boldsymbol{u}\|^2 \tag{23}$$

subject to equality constraint

$$\mathbb{J}\boldsymbol{u} = -\boldsymbol{J}\boldsymbol{r} - \dot{\boldsymbol{J}}\dot{\boldsymbol{q}} + \ddot{\boldsymbol{d}}(t) - 2k_0\dot{\boldsymbol{e}} - \frac{k_2}{k_1}\boldsymbol{e}. \tag{24}$$

Introducing the Lagrange function

$$L = \frac{1}{2}\|\boldsymbol{u}\|^2 + \left\langle \mathbb{J}\boldsymbol{u} - \left(-\boldsymbol{J}\boldsymbol{r} - \dot{\boldsymbol{J}}\dot{\boldsymbol{q}} + \ddot{\boldsymbol{d}}(t) - 2k_0\dot{\boldsymbol{e}} - \frac{k_2}{k_1}\boldsymbol{e}\right),\ \Lambda \right\rangle,$$

where $\Lambda \in \mathbb{R}^m$ stands for the vector of Lagrange's multipliers, it is easy to analytically solve for $\boldsymbol{u}$ optimal control problem (23)–(24). Its solution equals control (15) what proves also the optimality of Eq. (15).

□

3.2. Avoidance of singular manipulator configurations

In this section we shall discuss the case of mobile manipulator movement in which determinant of matrix $\mathbb{J}\mathbb{J}^T$ tends to approach zero. For this purpose, a small neighbourhood (a singular neighbourhood) around the singular manifold is defined, based on a user-specified small number. When the mobile manipulator starts to penetrate the singular neighbourhood, it is subject to acting a singular control whose magnitude becomes greater as the manipulator moves closer to the singular manifold. Singularity avoidance is accomplished by continuously perturbing manipulator control (15) so as to escape from the singular neighbourhood and (possibly) to attain by determinant of matrix $\mathbb{J}\mathbb{J}^T$ prescribed value S_0, specified also by the user. It is practically important to track $\boldsymbol{d}(\cdot)$ with a desirably large manipulability measure (defined as a valuation of difficulty of the mobile manipulator operation, proposed by ref. [3]). From the reason of numerical implementation, we omit the square root in the definition of manipulability measure.[3] Hence, manipulability S is defined as

$$S = S(\boldsymbol{q}) = det\left(\mathbb{J}\mathbb{J}^T\right). \tag{25}$$

If a manipulability value were small, then the end-effector could loose a tracking ability of a relatively complex desired trajectory $\boldsymbol{d}(\cdot)$. To be precise, let $\boldsymbol{U} = \{\boldsymbol{q} : S(\boldsymbol{q}) < \epsilon\}$, where ϵ is a user-defined sufficiently small number, define the singular neighbourhood around singular manifold $S(\boldsymbol{q}) = 0$. Moreover, hypersurface $S(\boldsymbol{q}) = 0$ is assumed to be non-degenerated, that is, $\frac{\partial S}{\partial \boldsymbol{q}} \neq 0$. The non-degeneracy assumption is not restrictive from the practical point of view since in the regular case (the Jacobian matrix $\frac{\partial^2 S}{\partial \boldsymbol{q}^2}$ is full rank at $S(\boldsymbol{q}) = 0$) at most a finite number of configurations may only not fulfil this assumption and therefore it is unlikely to meet such configurations during the manipulator movement. By continuity of $S(\boldsymbol{q})$ with respect to $\boldsymbol{q}$, $\frac{\partial S}{\partial \boldsymbol{q}}$ does not also equal zero in $\boldsymbol{U}$ (for sufficiently small ϵ). Without loss of generality, $\boldsymbol{q}_0$ is assumed further on to fulfil the following inequality $S(\boldsymbol{q}_0) > 0$. If a penetration into $\boldsymbol{U}$ is not detected ($S(\boldsymbol{q}) \geq \epsilon$), then the control given by Eq. (15) realizes a singularity-free motion. As soon as configuration $\boldsymbol{q}$ enters the singular neighbourhood ($S(\boldsymbol{q}) < \epsilon$), a singular control is generated whose magnitude is proportional to the extent of penetration. Let us note that on manifold $S(\boldsymbol{q}) - S_0 = 0$, where by definition $S_0 < \max_{\boldsymbol{q}}\{S(\boldsymbol{q})\}$, gradient $\frac{\partial S}{\partial \boldsymbol{q}} \neq 0$ provided that S_0 is not a (local) extreme of S. Consequently, $\frac{\partial S}{\partial \boldsymbol{q}}$ does not equal zero in a (small) neighbourhood of $S - S_0 = 0$. In order to attain specified value S_0 of the manipulability measure we propose the following control law provided that $\frac{\partial S}{\partial \boldsymbol{q}} \notin Ker(\boldsymbol{C}^T)$ (in the sequel, we comment how to fulfil this condition)

$$\boldsymbol{u}_s = \boldsymbol{C}^T\frac{\partial S}{\partial \boldsymbol{q}} \times \frac{-\left\langle \frac{\partial S}{\partial \boldsymbol{q}}, \boldsymbol{r}\right\rangle - \left\langle \frac{\partial^2 S}{\partial \boldsymbol{q}^2}\dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}\right\rangle - \lambda_1\left\langle \frac{\partial S}{\partial \boldsymbol{q}},\ \dot{\boldsymbol{q}}\right\rangle - \lambda_0(S - S_0)}{\left\langle \boldsymbol{C}^T\frac{\partial S}{\partial \boldsymbol{q}},\ \boldsymbol{C}^T\frac{\partial S}{\partial \boldsymbol{q}}\right\rangle}, \tag{26}$$

where λ_0, λ_1 denote constant positive coefficients. The following Lyapunov function candidate is defined to show the aforementioned property of control $\boldsymbol{u}_s$

$$V_s = \frac{\lambda_0}{2}(S - S_0)^2 + \frac{1}{2}\left(\frac{\mathrm{d}}{\mathrm{d}t}(S - S_0)\right)^2.$$

Differentiating V_s with respect to time yields

$$\dot{V}_s = \frac{\mathrm{d}}{\mathrm{d}t}(S - S_0)\left(\lambda_0(S - S_0) + \frac{\mathrm{d}^2}{\mathrm{d}t^2}(S - S_0)\right).$$

Choosing function $\frac{\mathrm{d}^2}{\mathrm{d}t^2}(S - S_0)$ as follows:

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}(S - S_0) = -\lambda_0(S - S_0) - \lambda_1\frac{\mathrm{d}}{\mathrm{d}t}(S - S_0)$$

or equivalently

$$\left\langle \frac{\partial S}{\partial \boldsymbol{q}},\ \ddot{\boldsymbol{q}}\right\rangle = -\left\langle \frac{\partial^2 S}{\partial \boldsymbol{q}^2}\dot{\boldsymbol{q}},\ \dot{\boldsymbol{q}}\right\rangle - \lambda_1\left\langle \frac{\partial S}{\partial \boldsymbol{q}},\ \dot{\boldsymbol{q}}\right\rangle - \lambda_0(S - S_0) \tag{27}$$

and then setting the right-hand side of (14) into Eq. (27) results (after simple algebraic calculations) in control $\boldsymbol{u}_s$. Combining tracking control $\boldsymbol{u}$ given by (15) with singular control (26) we obtain generalized control law $\boldsymbol{u}_{ge}$ with

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singularity avoidance of the form

$$\boldsymbol{u}_{ge} = \mathbb{J}^{\#}\left(-\boldsymbol{J}\boldsymbol{r} - \dot{\boldsymbol{J}}\dot{\boldsymbol{q}} + \ddot{\boldsymbol{d}}(t) - 2k_0\dot{\boldsymbol{e}} - \frac{k_2}{k_1}\boldsymbol{e}\right) + \left(\mathbb{I}_{n+l-k} - \mathbb{J}^{\#}\mathbb{J}\right) w(\boldsymbol{q})\boldsymbol{u}_s, \tag{28}$$

where $w(\boldsymbol{q}) = \begin{cases} c\,(S(\boldsymbol{q}) - \epsilon)^4 & \text{for } \boldsymbol{q} \in \boldsymbol{U} \\ 0 & \text{otherwise} \end{cases}$; $\mathbb{I}_{n+l-k}$ stands for the $(n+l-k)\times(n+l-k)$ identity matrix; c is positive gain coefficient. Let us note that more correct form for control $\boldsymbol{u}_s$ should be

$$\boldsymbol{u}_s = (\boldsymbol{C}\mathbb{P})^T \frac{\partial S}{\partial \boldsymbol{q}} \times \frac{-\left\langle \frac{\partial S}{\partial \boldsymbol{q}}, \boldsymbol{r}\right\rangle - \left\langle \frac{\partial^2 S}{\partial \boldsymbol{q}^2}\dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}\right\rangle - \lambda_1\left\langle \frac{\partial S}{\partial \boldsymbol{q}}, \dot{\boldsymbol{q}}\right\rangle - \lambda_0(S - S_0)}{\left\langle (\boldsymbol{C}\mathbb{P})^T \frac{\partial S}{\partial \boldsymbol{q}},\ (\boldsymbol{C}\mathbb{P})^T \frac{\partial S}{\partial \boldsymbol{q}}\right\rangle}, \tag{29}$$

where $\mathbb{P} = \mathbb{I}_{n+l-k} - \mathbb{J}^{\#}\mathbb{J}$ is the $(n+l-k)\times(n+l-k)$ projection matrix. It may be derived from Eq. (27) by taking into account $\mathbb{P}\boldsymbol{u}_s$ in (14) instead of $\boldsymbol{u}_s$. However, control (29) seems to be more complex than expression (26) and, as the numerical simulations have shown, it does not improve the manipulability measure S. As is seen from formula (28) control $\boldsymbol{u}_{ge}$ is continuous, asymptotically tracks desired end-effector trajectory $\boldsymbol{d}$ and avoids singular configurations if $S(\boldsymbol{q}) - \epsilon < 0$ or equivalently $\boldsymbol{q} \in \boldsymbol{U}$. Moreover, $\boldsymbol{u}_{ge}$ asymptotically stabilizes non-holonomic constraints (1) or (2) when, for example, initial manipulator velocity violates them. For a small value of c, a contribution of component $\mathbb{P}w(\boldsymbol{q})\boldsymbol{u}_s$ compared to $\boldsymbol{u}$ is expected to be small. Hence, control (28) seems also (instantaneously) to minimize mobile manipulator energy lost to track $\boldsymbol{d}(\cdot)$, as the computer simulations carried out in the next section show. In order to fulfil condition $\frac{\partial S}{\partial \boldsymbol{q}} \notin Ker(\boldsymbol{C}^T)$, let us determine distance $l(\boldsymbol{q})$ between vector $\frac{\partial S}{\partial \boldsymbol{q}}$ and its orthogonal projection onto the null space of $\boldsymbol{C}^T$. It is given as follows:

$$l(\boldsymbol{q}) = \left\| \frac{\partial S}{\partial \boldsymbol{q}} - \left(\mathbb{I}_{n+l} - \boldsymbol{C}(\boldsymbol{C}^T\boldsymbol{C})^{-1}\boldsymbol{C}^T\right)\frac{\partial S}{\partial \boldsymbol{q}}\right\|.$$

We propose the following control law to enforce condition $\frac{\partial S}{\partial \boldsymbol{q}} \notin Ker(\boldsymbol{C}^T)$:

$$\boldsymbol{u}_a = \Lambda_1 \frac{\mathrm{d}}{\mathrm{d}t}\left(\boldsymbol{W}\frac{\partial l}{\partial \boldsymbol{q}}\right) - \Lambda_0\left(\begin{pmatrix}\dot{\boldsymbol{\alpha}} \\ \dot{\boldsymbol{y}}\end{pmatrix} - \Lambda_1 \boldsymbol{W}\frac{\partial l}{\partial \boldsymbol{q}}\right),$$

where $\boldsymbol{W} = \begin{bmatrix} (\boldsymbol{N}^T\boldsymbol{N})^{-1}\boldsymbol{N}^T & 0 \\ 0 & \mathbb{I}_n \end{bmatrix}$; Λ_0, Λ_1 denote constant positive coefficients. The following Lyapunov function candidate is defined to show the aforementioned property of control $\boldsymbol{u}_a$

$$V_a = \frac{1}{2}\left(\begin{pmatrix}\dot{\boldsymbol{\alpha}} \\ \dot{\boldsymbol{y}}\end{pmatrix} - \Lambda_1 \boldsymbol{W}\frac{\partial l}{\partial \boldsymbol{q}}\right)^2.$$

As is seen from definition of V_a, the mobile manipulator moves for $V_a = 0$ in ascent direction of distance function $l(\boldsymbol{q})$. As is not difficult to see (we omit detailed derivation due to its analogy with the proof of Theorem 1.), control $\boldsymbol{u}_a$ (asymptotically and stably) enforces such a movement. As is also seen from the definition of set $\boldsymbol{U}$, it is straightforward to extend our control problem to a collision avoidance task since collision avoidance conditions may be expressed as a system of non-linear (with respect to vector $\boldsymbol{q}$) algebraic inequalities.

4. Computer Example

This section demonstrates on a few chosen mobile manipulator tasks, the performance of controllers given by Eqs. (15) and (28). For this purpose, a mobile manipulator, schematically shown in Fig. 1, is considered. In all numerical simulations, the SI units are used.

The holonomic part (a Selective Compliant Articulated Robot Arm (SCARA) type stationary manipulator) with two revolute kinematic pairs ($n = 2$) operating for simplicity of computations in two-dimensional task space ($m = 2$), is mounted on a unicycle which is subject to the following one holonomic h_1 and two non-holonomic nh_2, nh_3 constraints ($k = 3$):

$$\begin{aligned} h_1 &= \dot{\theta} - \frac{r}{2b}\dot{\alpha}_1 + \frac{r}{2b}\dot{\alpha}_2 = 0, \\ nh_2 &= \dot{x}_{1,c} - \frac{r}{2}\cos\theta\cdot\dot{\alpha}_1 - \frac{r}{2}\cos\theta\cdot\dot{\alpha}_2 = 0, \\ nh_3 &= \dot{x}_{2,c} - \frac{r}{2}\sin\theta\cdot\dot{\alpha}_1 - \frac{r}{2}\sin\theta\cdot\dot{\alpha}_2 = 0 \end{aligned} \tag{30}$$

where θ is the angle measured in the task space; $x_{1,c}$, $x_{2,c}$ mean coordinates of unicycle centre; r denotes the radius of the driving wheel; $2b$ stands for the distance of the two wheels; $\boldsymbol{x} = (\theta\ x_{1,c}\ x_{2,c}\ \alpha_1\ \alpha_2)^T$ ($l = 5$). The unicycle is assumed to be physically driven by two wheels of angular velocities $\dot{\boldsymbol{\alpha}} = (\dot{\alpha}_1,\ \dot{\alpha}_2)^T$. Matrix $\boldsymbol{N}(\boldsymbol{x})$ corresponding to constraints (30) is equal to

$$\boldsymbol{N}(\boldsymbol{x}) = \begin{bmatrix} \frac{r}{2b} & -\frac{r}{2b} \\ \frac{r}{2}\cos\theta & \frac{r}{2}\cos\theta \\ \frac{r}{2}\sin\theta & \frac{r}{2}\sin\theta \\ 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{31}$$

The kinematic equations of the mobile manipulator equal

$$\begin{aligned} p_1 &= \cos\theta\cdot(l_1\cdot\cos y_1 + l_2\cdot\cos(y_1+y_2)) - \sin\theta \\ &\quad\cdot(l_1\cdot\sin y_1 + l_2\cdot\sin(y_1+y_2)) + x_{1,c} \\ p_2 &= \sin\theta\cdot(l_1\cdot\cos y_1 + l_2\cdot\cos(y_1+y_2)) + \cos\theta \\ &\quad\cdot(l_1\cdot\sin y_1 + l_2\cdot\sin(y_1+y_2)) + x_{2,c}, \end{aligned} \tag{32}$$

where $l_1 = l_2 = 1$ stand for link lengths of its holonomic part; y_1, y_2 denote joint coordinates of the holonomic

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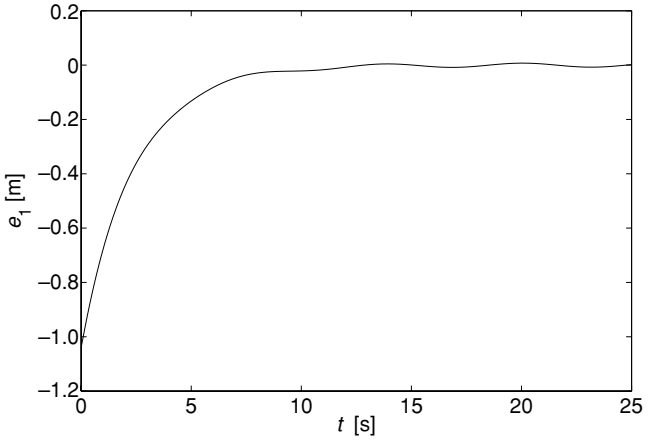


Fig. 2 Tracking error e_1 for controller (15) with $\lambda = 0$.

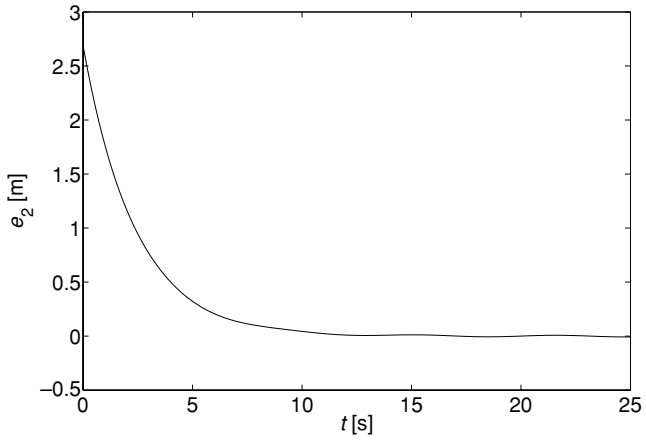


Fig. 3 Tracking error e_2 for controller (15) with $\lambda = 0$.

manipulator; $\boldsymbol{y} = (y_1,\ y_2)^T$. Hence, vector of generalized coordinates $\boldsymbol{q}$ takes the form $\boldsymbol{q} = (\theta\ x_{1,c}\ x_{2,c}\ \alpha_1\ \alpha_2\ y_1\ y_2)^T$. Without loss of computations generality, the simplified form of mobile manipulator dynamic Eqs. (14) is taken for simulation, where

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{N}(\boldsymbol{x}) & 0 \\ 0 & \mathbb{I}_2 \end{bmatrix}, \tag{33}$$

and $\boldsymbol{u} = (u_1\ u_2\ u_3\ u_4)^T$; u_i denote transformed controls of non-holonomic platform and holonomic part, respectively, $i = 1:4$. The initial configuration and velocity of the mobile manipulator are chosen in such a way as not to fulfil non-holonomic constraint nh_3 in (30), that is, $\boldsymbol{q}(0) = (0\ -1\ 0\ 0\ 0\ \pi/4\ \pi/6)^T$, $\dot{\boldsymbol{q}}(0) = (0\ 0\ 0.2\ 0\ 0\ 0\ 0)^T$, respectively. Controller gains k_0, k_1, k_2 take the following values $k_0 = 31$, $k_1 = 1$ and $k_2 = 26$.

The task of the mobile manipulator is to track by the end-effector desired trajectory $\boldsymbol{d}(t) = (d_1(t),\ d_2(t))^T = (0.75 + 0.25\cos(t),\ -1 + 0.25\sin(t))^T$. On account of the fact that $l + n - (k + m) = 2$, the mobile manipulator becomes redundant by accomplishment of the aforementioned tracking task. Let us introduce the following errors:

$$e_1 = p_1(\boldsymbol{q}(t)) - d_1(t), \quad e_2 = p_2(\boldsymbol{q}(t)) - d_2(t), \tag{34}$$

respectively, to evaluate the performance of controllers (15) and (28).

Firstly, the mobile manipulator motion with control not stabilizing the non-holonomic constraint (30) (i.e. $\lambda = 0$ in Eq. (4)) is considered. The results of computer simulations are presented in Figs. 2–8. As is seen from Figs. 2–3, controller (15) with $\lambda = 0$ asymptotically and stably tracks desired trajectory $\boldsymbol{d}(\cdot)$. Nevertheless, Fig. 4 shows that non-holonomic constraint nh_3 is not fulfilled through the planning session. Figures 5–8 present mobile manipulator controls. As is seen, they take small values which is the result of minimum norm property of controller (15). In order to asymptotically stabilize non-holonomic constraint (30), the second experiment has been performed based on controller (15) with $\lambda = 8$. Figures 9–12 show the results of computer simulations. Figures 9 and 10 show that control law (15)

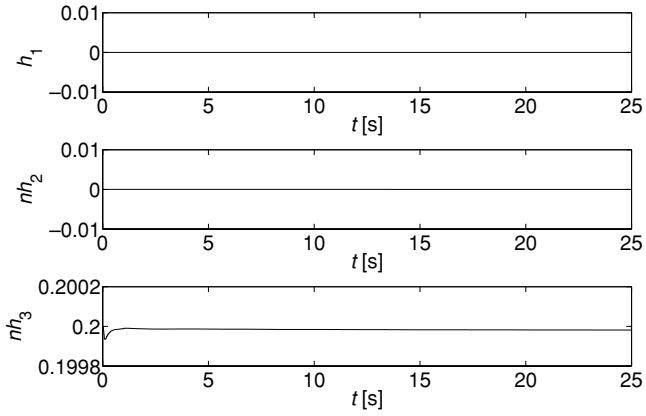


Fig. 4 Non-holonomic constraints for controller (15) with $\lambda = 0$.

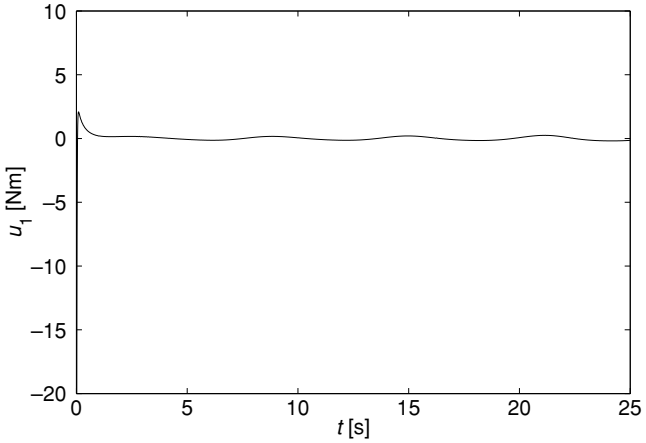


Fig. 5 Input torque u_1 for controller (15) with $\lambda = 0$.

results in asymptotic tracking of desired trajectory $\boldsymbol{d}(\cdot)$. As is seen from Fig. 11, controller (15) indeed asymptotically stabilizes constraint (30). Moreover, Fig. 12 presents time course of $S(\boldsymbol{q})$ based on controller (15).

Thirdly, the mobile manipulator considered in this section was used to solve the same tracking trajectory problem as in the second experiment, however with the requirement of improving the manipulability measure (or at least avoiding singular configurations during the movement). In order to accomplish this task, controller (28) has been applied in

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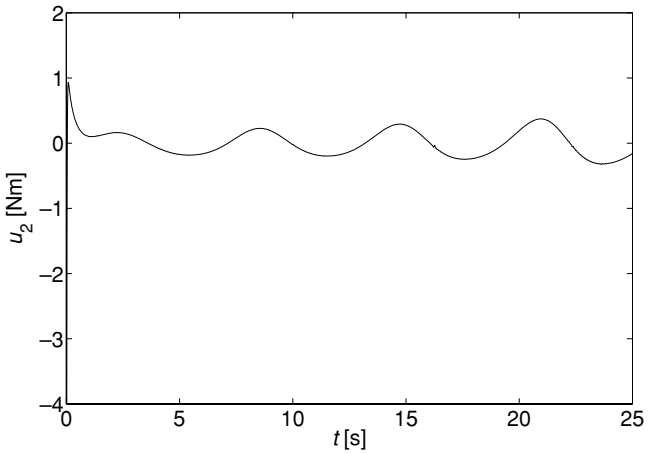


Fig. 6 Input torque u_2 for controller (15) with $\lambda = 0$.

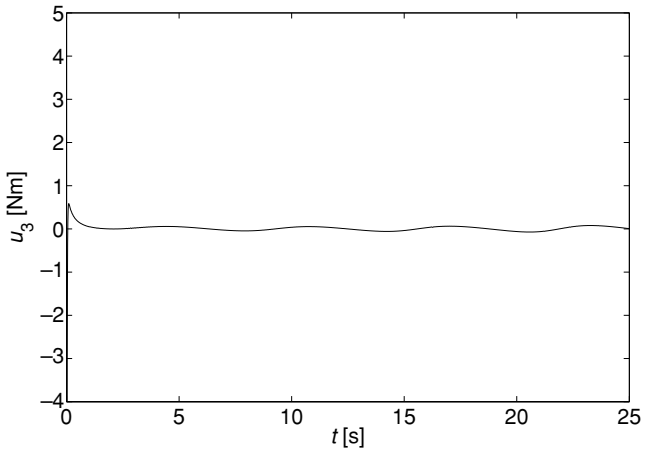


Fig. 7 Input torque u_3 for controller (15) with $\lambda = 0$.

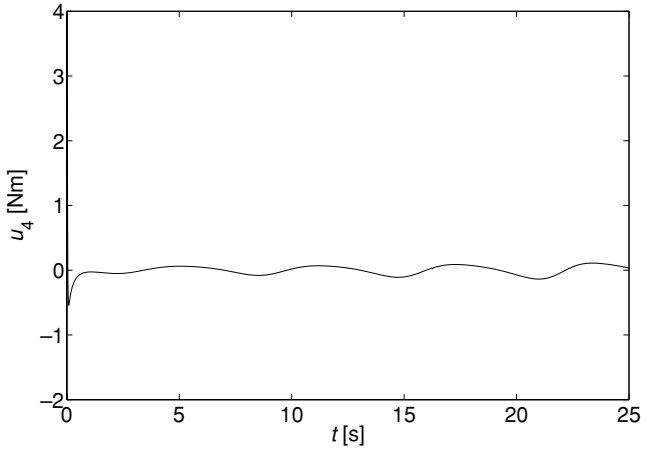


Fig. 8 Input torque u_4 for controller (15) with $\lambda = 0$.

this experiment. Parameters S_0, and ϵ are chosen in such a way as to activate the singularity avoidance constraint through the planning session. Hence, they take the following values $S_0 = \epsilon = 8.95$, $c = 10^{-4}$, $\lambda_0 = \lambda_1 = 1$. The results of computer simulations for controller (28) are presented in Figs. 13–18. On account of controller (28), the tracking errors e_1 and e_2 are the same as in the second experiment (control $\boldsymbol{u}_s$ multiplied by projection matrix $(\mathbb{I}_{n+l-k} - \mathbb{J}^{\#}\mathbb{J})$ does not

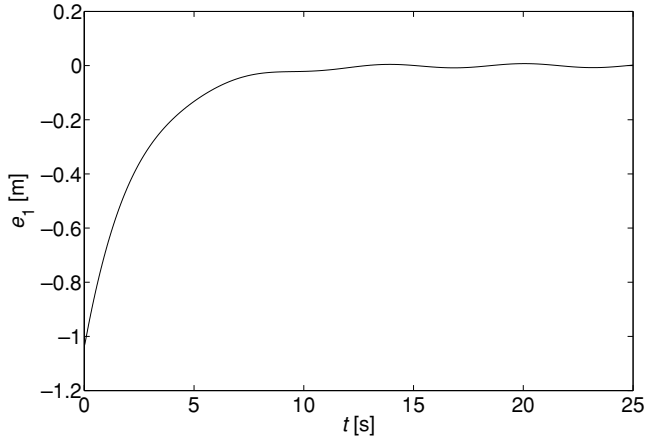


Fig. 9 Tracking error e_1 for controller (15) with $\lambda \neq 0$.

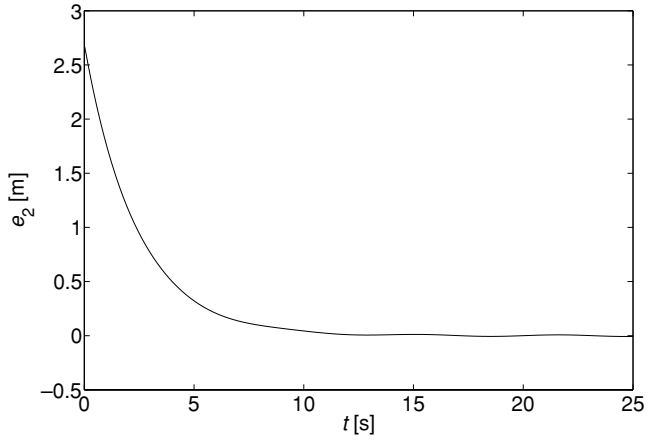


Fig. 10 Tracking error e_2 for controller (15) with $\lambda \neq 0$.

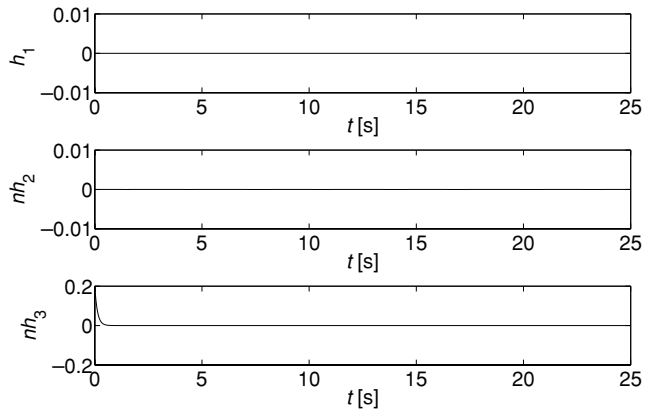


Fig. 11 Non-holonomic constraints for controller (15) and $\lambda \neq 0$.

change the end-effector movement). Moreover, control law (28) also asymptotically stabilizes (as is seen from Fig. 13) non-holonomic constraint (30). By comparing Fig. 12 from the second simulation and 14 we can see that controller (28) improves manipulability measure when tracking trajectory $\boldsymbol{d}(\cdot)$. Figures 15–18 present mobile manipulator controls. As is seen, they take small values which is also the result of minimum norm property of controller (15) and small values of $(\mathbb{I}_{n+l-k} - \mathbb{J}^{\#}\mathbb{J})w(\boldsymbol{q})\boldsymbol{u}_s$ due to small value of parameter c taken in simulations.

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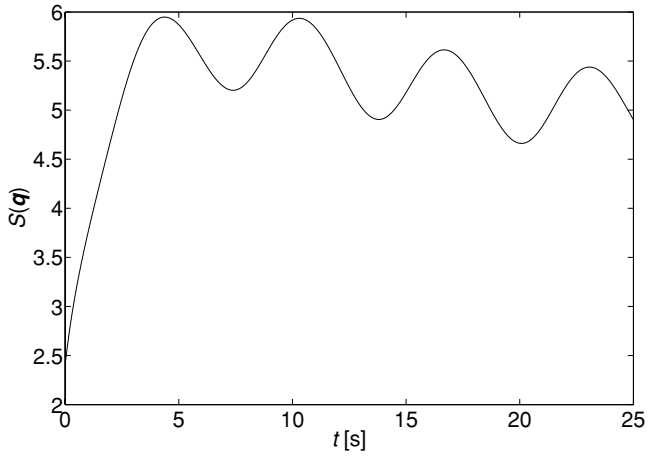


Fig. 12 Time course of $S(\boldsymbol{q})$ for controller (15) and $\lambda \neq 0$.

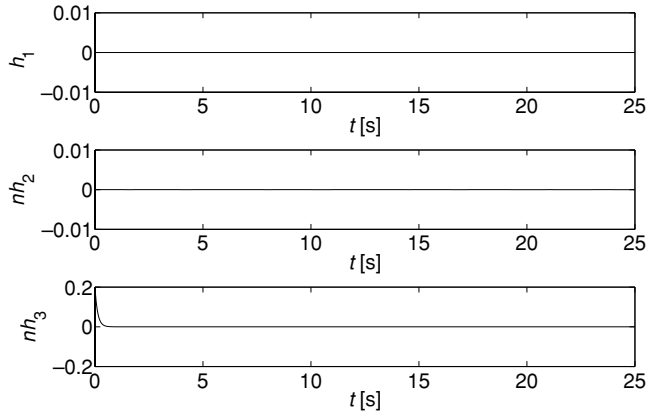


Fig. 13 Non-holonomic constraints for controller (28) and $\lambda \neq 0$.

Alternative methods of trajectory tracking by the end-effector of mobile manipulators, based on dynamic equations, have also been studied in works.[16,17] Based on coordinating the locomotion and manipulation and input-output linearization, the end-effector trajectory tracking algorithm has been proposed in refs. [16,17], which positions the holonomic manipulator at the preferred configurations measured by its manipulability. On the other hand, controller (28) tries to maximize manipulability measure of the whole mobile manipulator. The controls in both algorithms are determined based on quite different kinds of information, which makes the performance of these algorithms, in fact, numerically incomparable. This is due to the fact that, the control algorithm proposed herein fully exploits the mechanism of redundancy by the accomplishment of the end-effector task (vectors $\boldsymbol{q}$, $\dot{\boldsymbol{q}}$ should be known) whereas the mobile manipulator from refs. [16, 17] is not redundant with respect to its task (only vectors $\boldsymbol{\alpha}$, $\dot{\boldsymbol{\alpha}}$ are required). Moreover, controllers (15), (28) (instantaneously) minimize the energy lost to track the end-effector trajectory whereas the algorithm from refs. [16, 17] is not optimal in any sense. Finally, it is straightforward to adapt controllers (15), (28) to collision avoidance tasks which is not possible for control law from ref. [16, 17].

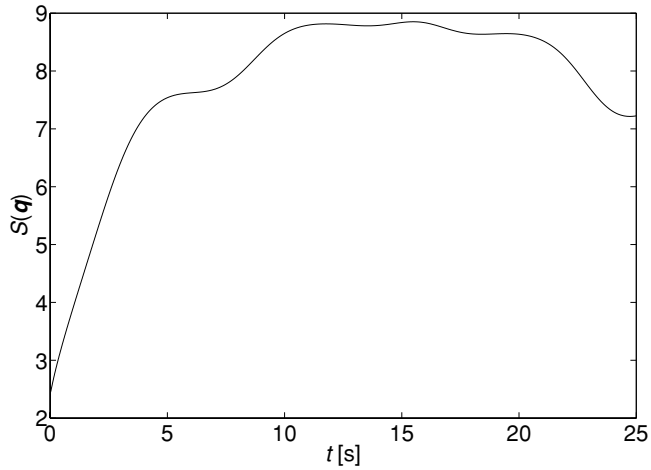


Fig. 14 Time course of $S(\boldsymbol{q})$ for controller (28) and $\lambda \neq 0$.

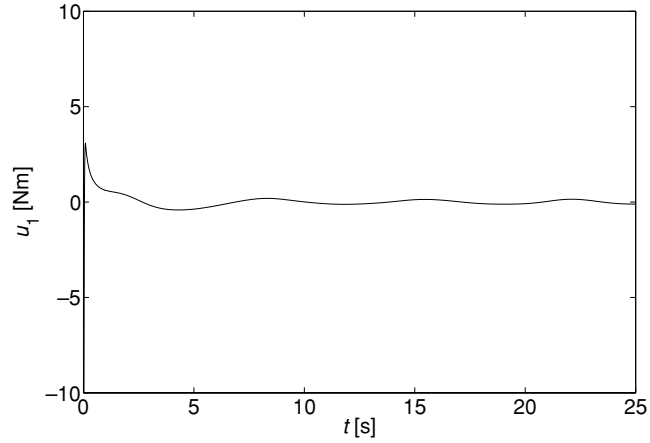


Fig. 15 Input torque u_1 for controller (28) and $\lambda \neq 0$.

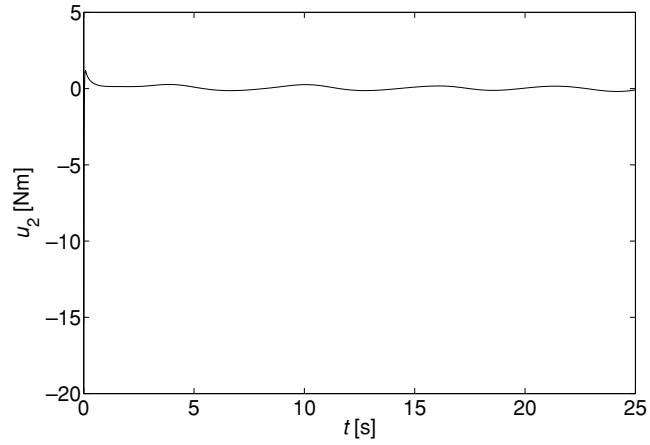


Fig. 16 Input torque u_2 for controller (28) and $\lambda \neq 0$.

5. Conclusions

In this paper, the end-effector trajectory tracking problem subject to state equality and inequality constraints, for mobile manipulators, has been discussed. Using the Lyapunov stability theory, a class of controllers generating the mobile manipulator trajectory fulfilling state constraints transformed into control-dependent ones, has been derived. The control scheme proposed in this paper generates continuous and

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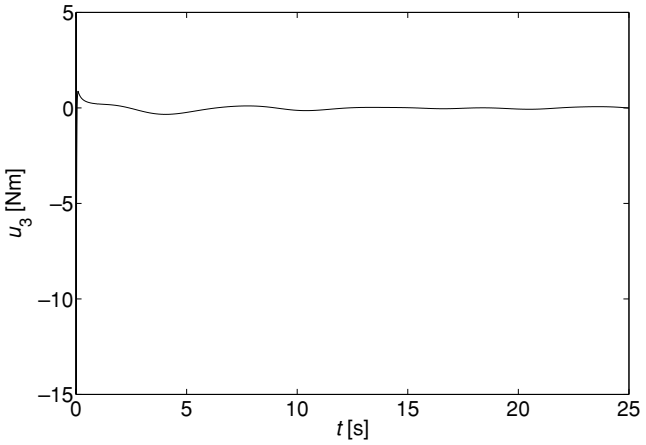


Fig. 17 Input torque u_3 for controller (28) and $\lambda \neq 0$.

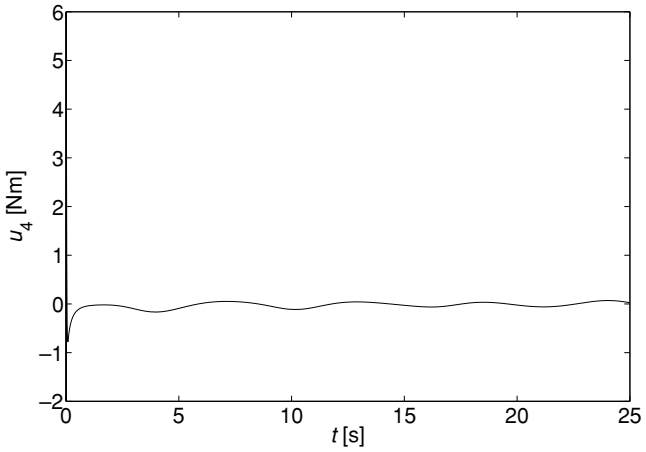


Fig. 18 Input torque u_4 for controller (28) and $\lambda \neq 0$.

bounded controls (even near boundaries of state constraints) which is a desirable property in an online control. Moreover, the approach presented forces the mobile manipulator trajectory to escape from singular configurations which makes our control algorithm numerically stable. Numerical simulations carried out on an exemplary mobile manipulator consisting of a non-holonomic unicycle and a holonomic manipulator of two revolute kinematic pairs have confirmed theoretical results obtained in Section 3. The advantage of using the method proposed is the possibility to implement it in an online trajectory tracking. Our future research will focus on application of controllers (15), (28) to collision avoidance tasks.

Acknowledgements

This work was supported by the DFG Ga 652/1–1,2.

References

1. R. W. Brockett, "Asymptotic Stability and Feedback Stabilization," *Proceedings of Conference Differential Geometric Control Theory, Progress in Mathematics*, Boston, MA, Birkhauser, vol. 27 (1983) pp. 181–208.
2. A. M. Bloch, M. Reyhanoglu and N. H. McClamroch, "Control and stabilization of nonholonomic dynamic systems," *IEEE Trans. Automat. Control* **37**(11), 1746–1757 (1992).
3. B. Bayle, J.-Y. Fourquet and M. Renaud, "Manipulability Analysis for Mobile Manipulators," *Proceedings of the IEEE International Conference on Robotics and Automation*, Seoul, Korea (2001) pp. 1251–1256.
4. B. Bayle, J.-Y. Fourquet and M. Renaud, "Manipulability of wheeled mobile manipulators: Application to motion generation," *Int. J. Robot. Res.* **22**(7–8), 565–581 (2003).
5. H. Seraji, "A unified approach to motion control of mobile manipulators," *Int. J. Robot. Res.* **17**(2), 107–118 (1998).
6. K. Tchon and J. Jakubiak, "Endogenous configuration space approach to mobile manipulators: A derivation and performance assessment of jacobian inverse kinematics algorithms," *Int. J. Control* **76**(14), 1387–1419 (2003).
7. M. Galicki, "Inverse kinematics solution to mobile manipulators," *Int. J. Robot. Res.* **22**(12), 1041–1064 (2003).
8. M. Galicki, "Control-based solution to inverse kinematics for mobile manipulators using penalty functions," *J. Intell. Robot. Syst.* **42**, 213–238 (2005).
9. M. Egersted and X. Hu, "Coordinated Trajectory Following for Mobile Manipulation," *Proceedings of the IEEE International Conference on Robotics and Automation*, San Francisco, USA (2000) pp. 3479–3484.
10. M. Fruchard, P. Morin and C. Samson, "A framework for the control of nonholonomic mobile manipulators," *Int. J. Robot. Res.* **25**(8), 745–780 (2006).
11. J. P. Desai and V. Kumar, "Nonholonomic Motion Planning for Multiple Mobile Manipulators," *Proceedings of the IEEE International Conference on Robotics and Automation*, Albuquerque, USA (1997) pp. 3409–3414.
12. A. Mohri, S. Furuno, M. Iwamura and M. Yamamoto, "Sub-optimal Planning of Mobile Manipulator," *Proceedings of the IEEE Conference on Robotics and Automation*, Seoul, Korea (2001) pp. 1271—1276.
13. Q. Huang, K. Tanie and S. Sugano, "Coordinated motion planning for a mobile manipulator considering stability and manipulation," *Int. J. Robot. Res.* **19**(8), 732–742 (2000).
14. N. A. M. Hootsmans and S. Dubowsky, "Large Motion Control of Mobile Manipulators Including Vehicle Suspension Characteristics," *Proceedings of the IEEE Conference on Robotics and Automation* (1991) pp. 2336–2341.
15. K. Liu and F. L. Lewis, "Control of Mobile Robot With Onboard Manipulator," *Proceedings of the International Symposium on Robotics and Manufacturing*, Cincinnati, USA (1992) pp. 539–546.
16. Y. Yamamoto and X. Yun, "Coordinating locomotion and manipulation of a mobile manipulator," *IEEE Trans. Autom. Control* **39**(6), 1326–1332 (1994).
17. Y. Yamamoto and X. Yun, "Effect of the dynamic interaction on coordinated control of mobile manipulators," *IEEE Trans. Robot. Autom.* **12**(5), 816–824 (1996).
18. Y. Yamamoto and X. Yun, "Unified Analysis on Mobility and Manipulability of Mobile Manipulators," *Proceedings of the IEEE International Conference on Robotics and Automation*, Detroit, USA (1999) pp. 1200–1206.
19. C. S. Tzafestas and S. G. Tzafestas, "Full-state modelling, motion planning and control of mobile manipulators," *Stud. Inform. Control* **10**(2) 2001).
20. J. H. Chung, S. A. Velinsky and R. A. Hess, "Interaction control of a redundant mobile manipulator," *Int. J. Robot. Res.* **17**(12), 1302–1309 (1998).
21. J. H. Chung and S. A. Velinsky, "Robust interaction control of a mobile manipulator – dynamic model based coordination," *J. Intell. Robot. Syst.* **26**, 47–63 (1999).
22. A. Mazur, "New Approach to the Control Problem of Mobile Manipulators," *Proceedings Third Workshop on Robot Motion and Control*, Bukowy Dworek, Poland (2002) pp. 297–301.

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23. S. Lin and A. A. Goldenberg, "Robust damping control of mobile manipulators," *IEEE Trans. Syst. Man Cybern.* 1–6 (2002).
24. J. Tan, N. Xi and Y. Wang, "Integrated task planning and control for mobile robots," *Int. J. Robot. Res.* **22**(5), 337–354 (2003).
25. M. Mailah, E. Pitowarno and H. Jamaluddin, "Robust motion control for mobile manipulator using resolved acceleration and proportional-integral active force control," *Int. J. Adv. Robot. Syst.* **2**(2), 125–134 (2005).
26. C. H. An, C. G. Atkenson and J. M. Hollerbach, *Model-Based Control of a Robot Manipulator* (Mit Press, Cambridge, MA, 1988).
27. J. M. Renders, E. Rossignal, M. Becquet and R. Hanus, "Kinematic calibration and geometrical parameter identification for robots," *IEEE RA* **7**(6), 721–732 (1991).
28. F. L. Lewis, C. T. Abdallach and D. M. Dawson, *Control of Robotic Manipulators* (Macmillan, New York, 1993).
29. M. Michalek, "VFO Control for Mobile Vehicles in the Presence of Skid Phenomenon," **In**: *Lecture Notes in Control and Information Sciences* (Springer-Verlag London, 2007) pp. 57–66.
30. B. D'Andrea-Novel, G. Campion and G. Bastin, "Control of wheeled mobile robots not satisfying ideal velocity constraints: A singular perturbation approach," *Int. J. Robust Nonlinear Control* **5**, 243–267, (1995).
31. F. D. Boyden and S. A. Velinsky, "Dynamic Modelling of Wheeled Mobile Robots for High Load Applications," *Proceedings of the IEEE International Conference on Robotics and Automation*, Washington, USA (1994) pp. 3071–3078.
32. M. Krstic, I. Kanellakopoulos and P. Kokotovic, *Nonlinear and Adaptive Control Design* (J. Wiley and Sons, New York, 1995).

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