These are essentially the only questions for which T′ differs from T or L′ differs from L. However, the welcomers must also be able to answer arbitrary Boolean combinations of other questions together with these ones. A precise way to do this is for T′ and L′ both to imagine that T has been interchanged with T′ and L with L′. If they do so, then the anthropologist cannot distinguish between this imagined situation and the actual one, and so cannot possibly sort them out in any number of questions.

doi:10.1017/mag.2015.99

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99.38 Proof without words: Cotangent expressed as a series of cosecants of multiple angles in powers of two

$$\cot x - \cot\left(2^n x\right) = \operatorname{cosec} 2x + \operatorname{cosec} 4x + \operatorname{cosec} 8x + \ldots + \operatorname{cosec}\left(2^n x\right).$$

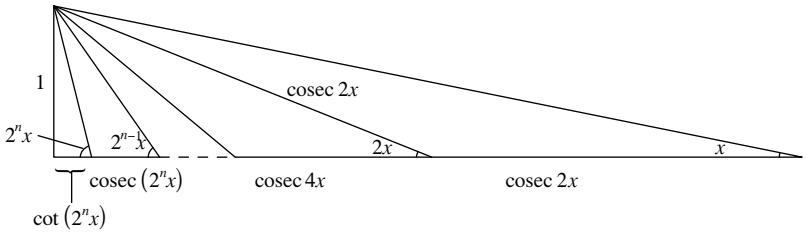


FIGURE 1

doi:10.1017/mag.2015.100

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99.39 Proof without words: A delightful water pouring problem

A version of the following little-known problem can be found in [1], a Soviet psychology research project used to gauge the mathematical abilities of middle school students. Although the problem is elementary, it gives rise to an enjoyable and elegant visual solution.

Problem: There are two identical glasses: glass 1 is completely filled with water, and glass 2 is empty. Water is poured back and forth between the glasses as follows. On the first pour, half of the water in glass 1 is poured into glass 2. On the second pour, a third of the water in glass 2 is poured back into glass 1. On the third pour, a fourth of the water in glass 1 is poured into glass 2, and so on. How much water is in glass 2 after 2015 pours?

https://doi.org/10.1017/mag.2015.101 Published online by Cambridge University Press

Proof:

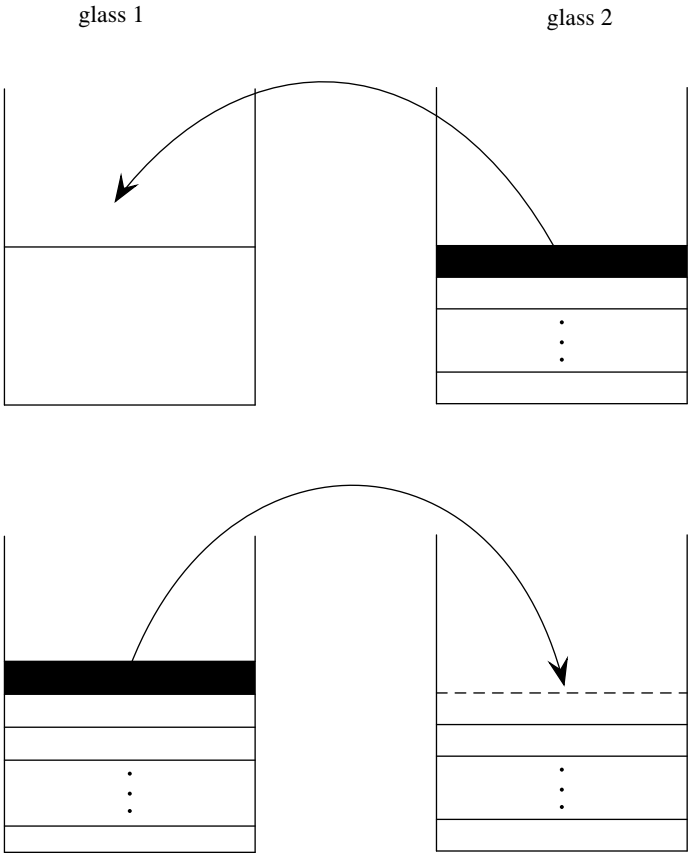


FIGURE 1

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doi:10.1017/mag.2015.101

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https://doi.org/10.1017/mag.2015.101 Published online by Cambridge University Press