Macroeconomic Dynamics, **16**, 2012, 576–604. Printed in the United States of America.
doi:10.1017/S1365100510000751

PRICING POLICIES AND INFLATION DYNAMICS

Luis Felipe Cespedes
Universidade Adolfo Ibáñez

Michael Kumhof
International Monetary Fund

Eric Parrado
Universidad Adolfo Ibáñez

The paper proposes a New Keynesian monetary model where firms' pricing policies feature a forward-looking optimal choice of the rate of price growth. The model can be thought of as a reduced form model of rational inattention that has only one additional state variable relative to the Calvo model [*Journal of Monetary Economics* 12 (1983), 383–398]. Like the model of backward-looking automatic indexation to past aggregate inflation, it generates a gradual and persistent reaction of the inflation rate, and output losses, following highly persistent monetary shocks. But unlike that model, its price setters behave very similarly to Calvo price setters when shocks are transitory.

Keywords: Inflation Inertia, Inflation Persistence, Price Setting

1. INTRODUCTION

A large and growing body of research in monetary theory uses the assumption of nominal rigidities embedded in dynamic general equilibrium models with rational expectations. Comprehensive surveys of this literature and its successes can be found in Taylor (1998), Clarida et al. (1999), Lane (2001), Galí (2003), and Woodford (2003). The resurgence of this model class is based both on much-improved theoretical foundations and on empirical arguments. The time-dependent price adjustment formulations of Taylor (1980), Rotemberg (1982), and Calvo (1983) made it possible to incorporate nominal rigidities into rational expectations models with forward-looking optimizing agents. Empirical support came from evidence showing that monetary policy has significant short-run real effects, as reported by Christiano et al. (1996, 1998) and Leeper et al. (1996).

The authors thank two referees of this paper, Ariel Burstein, Guillermo Calvo, Chris Erceg, Petra Geraats, Charles Goodhart, Andre Kurmann, Andrew Levin and Zheng Liu for helpful comments. A major part of this research was completed while Michael Kumhof visited the Research Department of the IDB, whose support is gratefully acknowledged. Address correspondence to: Michael Kumhof, International Monetary Fund, Washington, DC 20431, USA; e-mail: mkumhof@imf.org.

© 2011 Cambridge University Press 1365-1005/11

https://doi.org/10.1017/S1365100510000751 Published online by Cambridge University Press

Whether the inflation specifications derived from this model class can fully account for the short-run empirical properties of inflation and output has recently been much debated.[1] Mankiw (2001) notes that they do not generate the empirically observed delayed and gradual initial response of inflation to monetary policy shocks, a phenomenon that we will refer to as *inflation inertia*. Fuhrer and Moore (1995) show that they also do not generate the observed very prolonged steady state deviations of inflation following shocks, a phenomenon that is generally referred to as *inflation persistence*.[2] Persistence is a general property of the inflation process, whereas inertia has mainly been identified in responses to monetary policy shocks. Finally, this model class typically implies that inflation-stabilization policies have minimal real costs. This is also inconsistent with a large body of empirical evidence [Gordon (1982, 1997)], which shows that disinflationary policies give rise to *recessions*.

In this paper we use a tractable generalization of the Calvo (1983) staggered pricing model first introduced in an open-economy continuous-time setting by Calvo et al. (2002, 2007). The model contains the basic Calvo (1983) model as a special case. But it is also capable of generating inflation inertia, inflation persistence, and recessionary disinflations. Its main innovation is in its specification of firms' price-setting behavior, in which we assume that once a firm receives an opportunity to change its pricing policy, it jointly and optimally chooses an initial price level and an unconditional rate at which it will update its price in the future. In this specification, which we refer to as the optimal log-linear price paths model, an unexpected and credibly permanent decline in the steady state inflation rate targeted by monetary policy entails a slow and drawn-out inflation response and output losses. The main reason is that for such a shock the model's inflation process is mainly driven by a slow-moving state variable, the weighted average of historically chosen updating rates. The real–interest rate increase induced by the slow inflation response gives rise to a recession.

Any model adopting the Calvo (1983) timing assumption for pricing policies is a model of inattention, of random duration, to news about the state of the economy. This is so because firms do not revise their pricing policies in response to new information that arrives between random reoptimization opportunities. The seminal work of Reis (2006) on rational inattention improves dramatically on random inattention, first by making the end date of a given pricing policy part of the firms' optimization problem (which features a fixed cost of reoptimizing), and second by allowing agents to choose arbitrary, rather than log-linear, price paths. But the benefit of the more restricted optimal log-linear price paths model, with random inattention and log-linear firm specific price paths, is that the state space of the economy is dramatically simplified. Standard linearization methods can therefore be applied to solve for the equilibrium dynamics in a fully microfounded stochastic model. Similar comments apply in relation to the work of Mankiw and Reis (2002), on sticky information, where the beliefs of groups of agents that last updated their information at different times (arbitrarily far) in the past are state variables.[3] The number of state variables is therefore infinite, and standard solution methods of the

https://doi.org/10.1017/S1365100510000751 Published online by Cambridge University Press

type used in this paper cannot be used.[4] We will show in this paper that, despite our simplifying assumptions about the timing and form of pricing decisions, some of the most important macroeconomic phenomena that we seek to explain, inflation inertia, inflation persistence, and recessionary disinflations, do arise in the optimal log-linear price paths model. The critical assumption leading to this result is the existence, rather than the precise form, of firm-specific price paths. This assumption implies that inflation will at all times be determined not only by current price setters but also by the firm-specific inflation rates chosen by historic price setters who have not updated their information about the stance of monetary policy.

In the existing literature there are a number of alternative approaches to specifying the forms of pricing policies permitted by the model. We focus our comparison on two especially popular ones. The first and earlier approach [Yun (1996)], which we refer to as the Calvo–Yun model, has firms choosing only their initial price level, and thereafter updating their prices at the current steady state inflation rate. This implies that only the aggregate price level is sticky, whereas inflation is flexible. The model therefore exhibits no inflation inertia whatsoever, and disinflations are not recessionary.

This led the literature [Christiano et al. (2005)] to adopt another specification where firms again choose only their initial price level, but where they subsequently update their prices at the lagged aggregate inflation rate. We will refer to this approach, which does produce highly inertial and persistent inflation and costly disinflations, as the backward indexation model.[5] We will show that the backward indexation model exhibits inflation inertia and persistence that is not only high but also almost invariant to the nature and persistence of the underlying shocks. The optimal log-linear price-paths model behaves very similarly to the backward indexation model under highly persistent monetary shocks, as in a disinflation, whereas when monetary shocks are transitory, or under technology shocks, optimizing price setters behave very similarly to Calvo–Yun agents.[6]

An a priori choice between the backward-indexation model and the optimal log-linear price-paths model on the basis of plausibility or realism is not obvious, as the arguments cut both ways. On the one hand, if firms could indeed change prices each period, they would ideally like to update their prices by the actual rate of inflation. If this is not possible because price-level data are released with a lag, indexation to the lagged value of the aggregate price level may be the most realistic approach. But on the other hand, in such a world an individual firm clearly would have an incentive to deviate by choosing an individually optimal nonlinear price path rather than an indexed path. Restricting the price path to be log-linear reduces that incentive, but it does not in general eliminate it. On the other hand, no such incentive to deviate exists in a world where all firms choose individually optimal price paths, except again that deviation to a fully nonlinear price path would have additional benefits that the model does not allow for.[7]

We do recognize that all of these approaches, including optimal log-linear price paths, are somewhat ad hoc in terms of their microfoundations. The philosophy

https://doi.org/10.1017/S1365100510000751 Published online by Cambridge University Press

we adopt here, and that we share with a good portion of the monetary economics literature, is that a useful macroeconomic model of price stickiness should not primarily try to closely match features of the microeconomic data, but should instead be thought of as a reduced-form model of rational inattention. There is, however, debate about whether this should take the form of models with continuous quarterly updating, as in the three models investigated here, or of models with price paths that feature no price changes between reoptimizations. We return to this issue in Section 2.

The predictions of our model for a disinflation share an important feature with models of learning about monetary policy. In particular, a measure of expected inflation lags actual inflation for a long period during a disinflation, as in the influential paper of Erceg and Levin (2003). Under learning, the reason is a lack of stability and credibility of monetary policy, whereas under optimal log-linear price paths, the reason is the high persistence of monetary policy shocks. Given that periods of unstable or noncredible monetary policies are often characterized by persistent changes in the inflation target, we see learning and optimal log-linear price paths as highly complementary approaches that can contribute to explaining similar phenomena.

The rest of the paper is organized as follows. Section 2 discusses the arguments in favor of and against models with and without quarterly updating. Section 3 contains a detailed exposition of the optimal log-linear price-paths model, and a brief discussion of the pricing blocks of the alternative Calvo–Yun and backward indexation models. Section 4 discusses model calibration, impulse responses, and general model properties including steady states, determinacy, and welfare. Section 5 compares inflation inertia and inflation persistence across the three models. Section 6 concludes. A separate Technical Appendix contains detailed derivations of optimality conditions.

2. QUARTERLY UPDATING VERSUS FLAT PRICE PATHS

The approach taken by this paper is based on the idea that a reduced-form model of rational inattention can be a useful macroeconomic model of price stickiness. The rational inattention literature following Reis (2006) postulates that price setters optimally determine price paths, but that they only do so at certain points in time when they rationally choose to collect information and make optimal decisions. That literature does allow these time paths to be time-varying rather than flat, but if their form is completely unrestricted, standard techniques cannot be used to solve the model. A large part of the literature has therefore opted for short cuts that make the solution more tractable. This includes the indexation schemes of the Calvo–Yun model, the backward indexation model, and models positing a hybrid New Keynesian Phillips curve. But in each of these models price setters only optimally choose their initial price level, whereas subsequent price changes are prescribed by a nonoptimized indexation rule. The optimal log-linear price paths model proposed in this paper has price setters choose their entire price path,

https://doi.org/10.1017/S1365100510000751 Published online by Cambridge University Press

including an initial price level and an updating rate, in a forward-looking manner, and therefore more closely resembles the price-setting scheme under rational inattention. In this case it is the restriction to log-linear paths that simplifies the model's state space and makes the solution tractable. But crucially, all of these literatures assume that price setters change their prices every quarter.

This is a departure from the original intent of the New Keynesian macroeconomics literature, which stressed the importance of rigidities in the level of prices as a source of monetary nonneutralities. That is why another part of the literature avoids indexation and instead assumes that prices remain fixed between reoptimization dates.

The assumption of flat price paths has support in the empirical literature on product prices such as Golosov and Lucas (2003), Bils and Klenow (2004), and Klenow and Kryvtsov (2005), who find an average frequency of price changes at the micro level of once every 1.5 quarters for consumer prices in the United States, and close to once every 2 quarters if one-period sales are accounted for. The European Control Bank (ECB) (2005) documents even longer frequencies for Europe.

This approach is, however, not without its own drawbacks. Most importantly, strict consistency with the microeconomic evidence on the frequency of price changes does not necessarily imply that the resulting model fits the macroeconomic data better. An excellent discussion of this tension can be found in Ascari (2004), who analyzes the effects of trend inflation on model performance. With zero trend inflation there can be little argument that the assumption of fixed prices between reoptimizations is a good one, and in fact a significant part of that literature has stuck to that assumption. But as soon as even moderate trend inflation is introduced, models following the Calvo (1983) scheme of time-dependent pricing, but without allowing for quarterly updating, generate surprisingly large, and unrealistic, effects on steady state output and on dynamic impulse responses. In other words, the model is no longer a good candidate to fit the macroeconomic data. Ascari (2004) shows that indexation gets rid of these problems, but he is skeptical, for the reasons mentioned above. Instead he prefers the Taylor (1980) model of price setting, where trend inflation has less extreme effects. But most authors in the empirical literature have instead opted for some form of indexation to fit the data. Indeed, this was the main justification of Christiano et al. (2005) for choosing the backward indexation approach. We have done so elsewhere with the optimal log-linear price paths model, with good results.[8]

3. THE MODEL

The economy consists of a continuum of measure one of identical price-taking infinitely lived households, a continuum indexed by $j \in [0, 1]$ of monopolistically competitive infinitely lived firms, and a government. All our main results can be obtained using a simple baseline model with sticky prices and flexible wages. We will at one point briefly discuss an economy with both sticky wages and sticky

https://doi.org/10.1017/S1365100510000751 Published online by Cambridge University Press

prices, but to conserve space the algebra for that case is only presented in the Technical Appendix.

3.1. Households

Households maximize lifetime utility, which depends on their per capita consumption c_t and labor effort L_t.[9] Their objective function is

$$\text{Max} \quad E_t \Sigma_{k=0}^{\infty} \beta^k \left\{ \ln (C_{t+k}) - \frac{\kappa}{1+\gamma} L_{t+k}^{1+\gamma} \right\}, \tag{1}$$

where E_t is the expectations operator, β is the discount factor, and γ is the inverse of the wage elasticity of the labor supply. Households exhibit internal habit persistence in consumption, with habit parameter ν:[10]

$$C_t = c_t - \nu c_{t-1}. \tag{2}$$

Consumption c_t is a CES aggregator over individual varieties $c_t(j)$, with elasticity of substitution $\sigma > 1$:

$$c_t = \left[\int_0^1 c_t(j)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}. \tag{3}$$

The aggregate price index P_t is the consumption-based price index associated with this consumption aggregator,

$$P_t = \left[\int_0^1 P_t(j)^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}}, \tag{4}$$

where $P_t(j)$ is the price of variety j. In addition to money, households hold one-period nominal government bonds B_t with gross nominal return i_t. Their income consists of nominal wage income $W_t L_t$ determined in a competitive labor market, lump-sum profit redistributions from firms $\int_0^1 \Pi_t(j) dj$, and lump-sum transfers from the government $P_t \tau_t$. Households' budget constraint is

$$B_t = i_{t-1} B_{t-1} + W_t L_t + \int_0^1 \Pi_t(j) dj + P_t \tau_t - P_t c_t. \tag{5}$$

We denote the multiplier of this budget constraint by Λ_t and let $\lambda_t = \Lambda_t P_t$. Then the first-order conditions with respect to $c_t(j)$, c_t, L_t, and B_t are

$$c_t(j) = c_t \left[\frac{P_t(j)}{P_t} \right]^{-\sigma}, \tag{6}$$

$$C_t^{-1} - \beta \nu E_t C_{t+1}^{-1} = \lambda_t, \tag{7}$$

$$\kappa L_t^{\gamma} = \lambda_t w_t, \tag{8}$$

$$\lambda_t = \beta i_t E_t \left(\frac{\lambda_{t+1}}{\pi_{t+1}} \right), \tag{9}$$

https://doi.org/10.1017/S1365100510000751 Published online by Cambridge University Press

where $w_t = W_t/P_t$ and $\pi_t = P_t/P_{t-1}$. We proceed to linearize conditions (7)–(9) and (2) around the steady state. A circumflex above a variable denotes its log deviation from steady state; e.g., $\hat{x}_t = \ln(x_t/\bar{x})$, where $\bar{x}$ is the steady state of x_t. We assume that the inflation target π_t^* of the central bank follows a unit root process $\pi_t^* = \pi_{t-1}^* \varepsilon_t^{\pi^*}$, and note that linearization of inflation rates is always performed around the current inflation target; for example, $\hat{\pi}_{t+1} = \ln(\pi_{t+1}) - \ln(\pi_t^*)$. The linearized first-order conditions are

$$\frac{\beta\nu}{1-\nu} E_t \hat{c}_{t+1} = \frac{1+\beta\nu^2}{1-\nu}\hat{c}_t - \frac{\nu}{1-\nu}\hat{c}_{t-1} + (1-\beta\nu)\hat{\lambda}_t, \tag{10}$$

$$E_t(\hat{\lambda}_{t+1} - \hat{\pi}_{t+1}) = \hat{\lambda}_t - \hat{\imath}_t, \tag{11}$$

$$\hat{w}_t + \hat{\lambda}_t = \gamma \hat{L}_t. \tag{12}$$

3.2. Firms

Each firm $j \in [0, 1]$ sells a distinct product variety. Heterogeneity in price setting decisions and therefore in demand for individual products arises because each firm receives its price changing opportunities at different, random points in time. Following Calvo (1983), it is assumed that these opportunities follow a geometric distribution, so that the probability $(1-\delta)$ of a firm's receiving a new opportunity is independent of how long ago it was last able to change its price. It is also independent across firms, so that it is straightforward to determine the aggregate distribution of prices. We assume that firms' output $y_t(j)$ is produced via linear production functions in labor input $\ell_t(j)$:

$$y_t(j) = z_t \ell_t(j). \tag{13}$$

Aggregate output y_t is defined as

$$y_t = \left[\int_0^1 y_t(j)^{\frac{\sigma-1}{\sigma}} dj\right]^{\frac{\sigma}{\sigma-1}}. \tag{14}$$

For productivity z_t we assume $\bar{z} = 1$ and the following law of motion:

$$\hat{z}_t = \rho^z \hat{z}_{t-1} + \hat{\varepsilon}_t^z. \tag{15}$$

Firms have market power and therefore set the prices of their varieties $P_t(j)$ to maximize profits taking into account consumers' demand for their variety (6):

$$y_t(j) = c_t \left[\frac{P_t(j)}{P_t}\right]^{-\sigma}. \tag{16}$$

Under optimal log-linear price paths, when a firm j gets an opportunity to decide on its pricing policy, it chooses both its current price level V_t^j and the rate v_t^j at which it will update its price from today onward until the time it is next allowed

https://doi.org/10.1017/S1365100510000751 Published online by Cambridge University Press

to change its policy. At any time $t+k$ when the time t policy is still in force, its price is therefore

$$P_{t+k}(j) = V_t^j (v_t^j)^k. \tag{17}$$

Firms discount nominal profits expected in period $t+k$ by the intertemporal marginal rate of substitution and by δ^k, the probability that their period-t pricing policy will still be in force k periods from t. Nominal revenue at t equals $P_t(j)y_t(j)(1+\xi)$, where ξ is an output subsidy that for most of our analysis, except an illustrative welfare analysis, will be set equal to zero. The nominal wage bill at t equals $W_t \ell_t(j)$. The firms' problem is therefore

$$\underset{V_t^j, v_t^j}{\text{Max}} \quad E_t \Sigma_{k=0}^{\infty} (\delta\beta)^k \lambda_{t+k} \left[\frac{P_{t+k}(j) y_{t+k}(j) (1+\xi)}{P_{t+k}} - w_{t+k} \ell_{t+k}(j) \right], \tag{18}$$

subject to (13), (16), and (17). Note that in the statement of the optimality conditions, the firm, specific index j can be dropped, because all firms that get a price changing opportunity at time t will behave identically. Let $p_t \equiv V_t / P_t$. We scale all inflation rates by the inflation target π_t^*, with notation $\check{v}_t = v_t / \pi_t^*$, $\check{\pi}_t = \pi_t / \pi_t^*$. Using this, we can define cumulative aggregate normalized inflation as $\check{\Pi}_{t,k} = \Pi_{j=1}^{k} \check{\pi}_{t+j}$ for $k \geq 1$ ($\equiv 1$ for $k = 0$). The first-order condition with respect to V_t is then

$$p_t = \mu \frac{E_t \Sigma_{k=0}^{\infty} (\delta\beta)^k \lambda_{t+k} y_{t+k}(j) \dfrac{w_{t+k}}{z_{t+k}}}{E_t \Sigma_{k=0}^{\infty} (\delta\beta)^k \lambda_{t+k} y_{t+k}(j) \left(\dfrac{(\check{v}_t)^k}{\check{\Pi}_{t,k}} \right)}, \tag{19}$$

and with respect to v_t we have[11]

$$p_t = \mu \frac{E_t \Sigma_{k=0}^{\infty} (\delta\beta)^k k \lambda_{t+k} y_{t+k}(j) \dfrac{w_{t+k}}{z_{t+k}}}{E_t \Sigma_{k=0}^{\infty} (\delta\beta)^k k \lambda_{t+k} y_{t+k}(j) \left(\dfrac{(\check{v}_t)^k}{\check{\Pi}_{t,k}} \right)}, \tag{20}$$

where $\mu = \sigma / [(\sigma - 1)(1+\xi)]$. Before analyzing these conditions further, we need to describe government policy and define equilibrium.

3.3. Government

The government's fiscal policy is assumed to be Ricardian. In particular, we assume that the government budget is balanced period by period through lump-sum taxes/transfers, and that the initial stock of government bonds is zero. For the case of zero output subsidies the budget constraint is therefore simply

$$\tau_t = \frac{M_t - M_{t-1}}{P_t}, \tag{21}$$

https://doi.org/10.1017/S1365100510000751 Published online by Cambridge University Press

whereas for $\xi > 0$, aggregate output subsidies have to be deducted from the right-hand side of (21). We assume that the central bank pursues the following interest rate rule for its policy instrument i_t:

$$i_t = \frac{\pi_t^*}{\beta} E_t \left(\frac{\pi_{t+1}}{\pi_t^*} \right)^{\phi} \left(\frac{y_t}{y_t^n} \right)^{\theta} (h_t)^{\phi - 1}. \tag{22}$$

The first component on the right-hand side equals the long-run gross nominal interest rate at the current inflation target π_t^*. Permanent disinflations correspond to shocks $\varepsilon_t^{\pi^*}$ to the unit root process of this target. The nominal interest rate responds to expected one–period ahead[12] deviations of inflation from its target, and to deviations of output from the natural or flexible price rate of output y_t^n, with response coefficients ϕ and θ. The natural rate of output is given by the solution of the flexible price version of our model economy for output. Finally, h_t is an autocorrelated monetary policy shock, with $\bar{h} = 1$ and with law of motion

$$\hat{h}_t = \rho^h \hat{h}_{t-1} + \hat{\varepsilon}_t^h. \tag{23}$$

The coefficient of this shock in the monetary policy rule $(\phi - 1)$ ensures that a permanent positive shock to h_t of given size has, holding π_t^* constant, the same long-run effect on inflation as a permanent negative shock of the same size to the inflation target π_t^*. After the nominal interest rate is normalized by the inflation target ($\check{i}_t = i_t / \pi_t^*$), the rule (22) can be linearized (noting that $E_t \hat{\varepsilon}_{t+1}^{\pi^*} = 0$) as

$$\hat{\imath}_t = \phi E_t \hat{\pi}_{t+1} + \theta \left(\hat{y}_t - \hat{y}_t^n \right) + (\phi - 1) \hat{h}_t. \tag{24}$$

In the literature an alternative to this rule is very popular, namely one that adds a lag of the interest rate, also known as interest rate smoothing, as in Clarida et al. (1999). We have therefore checked the robustness of our results to that alternative, and found that all our main results are unaffected, except for one detail concerning inflation persistence. For that one case we will work with the following alternative specification:

$$\hat{\imath}_t = \rho^i \hat{\imath}_{t-1} + (1 - \rho^i) \left[\phi E_t \hat{\pi}_{t+1} + \theta \left(\hat{y}_t - \hat{y}_t^n \right) + (\phi - 1) \hat{h}_t \right]. \tag{25}$$

Furthermore, rule (24) is supported empirically by the work of Rudebusch (2002, 2006), who shows that interest rate smoothing would imply a large amount of forecastable variation in interest rates at horizons of more than three months, which is contradicted by evidence from the term structure of interest rates. Highly persistent shocks are shown not to imply a large forecastable variation. Rudebusch and Wu (2007) estimate a monetary policy rule that allows for both interest rate smoothing and autocorrelated shocks, after embedding that rule in a macro-finance model that directly exploits such term-structure information. They find an interest rate–smoothing parameter near zero and a coefficient of monetary policy shock autocorrelation very close to one.[13]

https://doi.org/10.1017/S1365100510000751 Published online by Cambridge University Press

We define a *government policy* as a set of stochastic processes $\{i_s, \tau_s\}_{s=t}^{\infty}$ such that, given stochastic processes $\{M_s, P_s, y_s, z_s, h_s\}_{s=t}^{\infty}$, the conditions (21) and (22) hold for all $s \geq t$.

3.4. Equilibrium

A list of stochastic processes $\{B_s, M_s, c_s, L_s, y_s, y_s^n, c_s(j), \ell_s(j), y_s(j), j \in [0, 1]\}_{s=t}^{\infty}$ is an *allocation*, and a list of stochastic processes $\{P_s, W_s, P_s(j), V_s^j, v_s^j, j \in [0, 1]\}_{s=t}^{\infty}$ is a *price system*. *Shock processes* are a list of stochastic processes $\{\varepsilon_t^{\pi^*}, \varepsilon_s^h, \varepsilon_s^z\}_{s=t}^{\infty}$. Then equilibrium is defined as follows:

An equilibrium given initial conditions π_{-1}^*, h_{-1}, z_{-1} and P_{-1} is an allocation, a price system, a government policy, and shock processes such that

(a) given the government policy, the price system, and shock processes, the allocation solves the household's problem of maximizing (1) subject to (2), (3), and (5);
(b) given the government policy, shock processes, the restrictions on price setting, and the sequences $\{P_s, W_s, c_s\}_{s=0}^{\infty}$, the sequences $\{V_s^j, v_s^j, j \in [0, 1]\}_{s=0}^{\infty}$ solve firms' problem of maximizing (18);
(c) the goods market clears for all goods and at all times, $y_t(j) = c_t(j) \quad \forall t, \forall j \in [0, 1]$;
(d) the labor market clears at all times, $L_t = \int_0^1 \ell_t(j)dj \quad \forall t$;
(e) the bond market clears at all times, $B_t = 0 \quad \forall t$.

It is straightforward to show that $\bar{L} = \bar{c} = \bar{y}$ and that

$$\hat{z}_t + \hat{L}_t = \hat{y}_t = \hat{c}_t. \tag{26}$$

3.5. Linearized Price Dynamics

Aggregate price dynamics can be derived by linearizing the price index (4) to obtain

$$\hat{\pi}_t = \frac{1-\delta}{\delta}\hat{p}_t + \hat{\psi}_t, \tag{27}$$

$$\hat{\psi}_t = \delta\hat{\psi}_{t-1} + (1-\delta)\hat{v}_{t-1} - \hat{\varepsilon}_t^{\pi^*}. \tag{28}$$

The state variable $\hat{\psi}_t$ is, in deviation form and allowing for changes in the inflation target, the weighted average of all the past firm-specific inflation rates that are still in force between periods $t-1$ and t and that therefore enter into period-t aggregate inflation.[14]

The two components of equation (27) reflect the reasons for inflation inertia in this model. Large jumps in current prices $\hat{p}_t$ can in principle cause large and immediate jumps in inflation. But when shocks to monetary policy are persistent, new price setters optimally respond mainly through changes in their updating rate $\hat{v}_t$. In that case inflation is dominated by the state variable $\hat{\psi}_t$, which moves slowly because it represents the continuing effects of price updating decisions made prior to the shock.

The complete pricing block of the optimal log-linear price paths model can be derived by combining (27) and (28) with linearized versions of the optimality

https://doi.org/10.1017/S1365100510000751 Published online by Cambridge University Press

conditions (19) and (20). Instead of the usual one-equation New Keynesian Phillips curve, it consists of three equations, (28) for the state variable $\hat{\psi}_t$ and the following two difference equations for $\hat{\pi}_t$ and $\hat{v}_t$:

$$E_t\hat{v}_{t+1} = \hat{v}_t + \frac{(1-\delta\beta)^2}{(\delta\beta)^2}\frac{\delta}{1-\delta}(\hat{\psi}_t - \hat{\pi}_t) + \frac{(1-\delta\beta)^2}{(\delta\beta)^2}(\hat{w}_t - \hat{z}_t), \tag{29}$$

$$E_t\hat{\pi}_{t+1} = \left(\frac{2}{\beta} - \delta\right)\hat{\pi}_t + (1-\delta)(1+\delta)\hat{v}_t + \left[\delta(1+\delta) - \frac{2}{\beta}\right]\hat{\psi}_t - \frac{2(1-\delta)(1-\delta\beta)}{(\delta\beta)}(\hat{w}_t - \hat{z}_t). \tag{30}$$

To summarize, the dynamic behavior of the economy can be characterized by the aggregate demand block (10)–(12), the aggregate supply block (28)–(30), the market clearing condition (26), the monetary policy rule (24), and the exogenous shock processes (15), (23), and $\hat{\varepsilon}_t^{\pi^*}$.

Except for the aggregate supply block, the Calvo–Yun and backward indexation models are identical to the above. The New Keynesian Phillips curve for the Calvo–Yun case is

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \frac{(1-\delta\beta)(1-\delta)}{\delta}(\hat{w}_t - \hat{z}_t). \tag{31}$$

Note that this is unaffected by unit root shocks to the inflation target, because this model variant assumes that following such shocks, all agents immediately start updating at the same rate as the new target. Such shocks therefore do not give rise to any transitional dynamics.[15] On the other hand, for the backward indexation case we obtain

$$(\hat{\pi}_t - \hat{\pi}_{t-1}) = \beta E_t(\hat{\pi}_{t+1} - \hat{\pi}_t) + \frac{(1-\delta\beta)(1-\delta)}{\delta}(\hat{w}_t - \hat{z}_t) - \hat{\varepsilon}_t^{\pi^*}. \tag{32}$$

The main additional equations for the case of sticky wages consist of a system of equations for wage inflation identical in form to (28)–(32), but with the marginal cost term being replaced by the difference between the marginal rate of substitution and the real wage.

4. MODEL PROPERTIES

4.1. Calibration

To study the properties of the model we calibrate parameter values for the quarterly frequency. We follow the literature in assuming $\beta = 0.99$. The value for the habit parameter $\nu = 0.7$ follows Boldrin et al. (2001). The parameter γ, which represents the inverse of the wage elasticity of labor supply, is set equal to 1.[16] We assume that firms on average review their pricing policies every five quarters, or

https://doi.org/10.1017/S1365100510000751 Published online by Cambridge University Press

$\delta = 0.8$, and similarly, for the sticky wages case, we assume that $\delta_w = 0.8$. This is close to the estimates reported for comparable models in Smets and Wouters (2003) and Christiano et al. (2005). As can be seen in equations (28)–(30), the markup of price over marginal cost does not affect the linearized dynamics of the model. For the sticky wages case we assume a wage markup parameter μ_w of 1.2, similar to the estimates of Rotemberg and Woodford (1997) and Amato and Laubach (1999). For the monetary policy rule we adopt the coefficients originally used by Taylor (1993), namely $\phi = 1.5$ and $\theta = 0.5$. The Taylor principle directs the central bank to respond to positive deviations of inflation from its target by raising nominal interest rates more than proportionately, and therefore raising real interest rates, meaning $\phi > 1$. Rudebusch (2006) estimates coefficients similar to those in Taylor (1993) for an interest rate rule without smoothing. We present one illustrative example for the case of interest rate smoothing, where we will assume $\rho^i = 0.7$. For the monetary policy shock baseline we assume a persistence of $\rho^h = 0.98$, again based on Rudebusch (2006). But because ρ^h is a key parameter in our model, we will explore the sensitivity of our results to other values. We illustrate the technology shock for a persistence of $\rho^z = 0.98$. This is close to the unit root assumption that is becoming more common in applied work.[17]

We first use impulse responses to study the dynamic response of the economy to permanent shocks to the inflation target $\hat{\varepsilon}_t^{\pi^*}$, to monetary policy shocks $\hat{\varepsilon}_t^h$, and to technology shocks $\hat{\varepsilon}_t^z$. We then study key model properties, including steady states, equilibrium determinacy, and welfare properties. In the figures below, the solid lines represent the results of the optimal log-linear price paths model and are labeled OPT, the dotted lines represent those of the Calvo–Yun model and are labeled CY, and the dashed lines represent those of the backward indexation model and are labeled BWI.

4.2. Impulse Responses

The first monetary policy shock we consider, in Figure 1, is a permanent change in π_t^* from 5% to 4% per annum. In the Calvo–Yun model, because firms are assumed to start updating their prices immediately at the new steady state inflation rate, inflation instantaneously drops to its new target level. It is therefore neither inertial nor persistent. And because the ex ante real interest rate never changes, consumption, output, and employment remain flat; i.e., disinflations are not recessionary.

In contrast, in the optimal log-linear price paths model, inflation exhibits both inertia and persistence. The key factor generating this result is that, following the shock, aggregate inflation is expected to be lower than its initial value for a long period of time, in this example forever. The importance of this factor derives from the fact that, given the concavity of their profit function, firms maximize profits by keeping the deviations of their price path from the market average price path as small as possible over the expected length of the policy.[18] When, as in Figure 1, firms expect a long-run change in trend inflation rather than a short-lived jump in inflation, they minimize the deviations of their price path by changing

https://doi.org/10.1017/S1365100510000751 Published online by Cambridge University Press

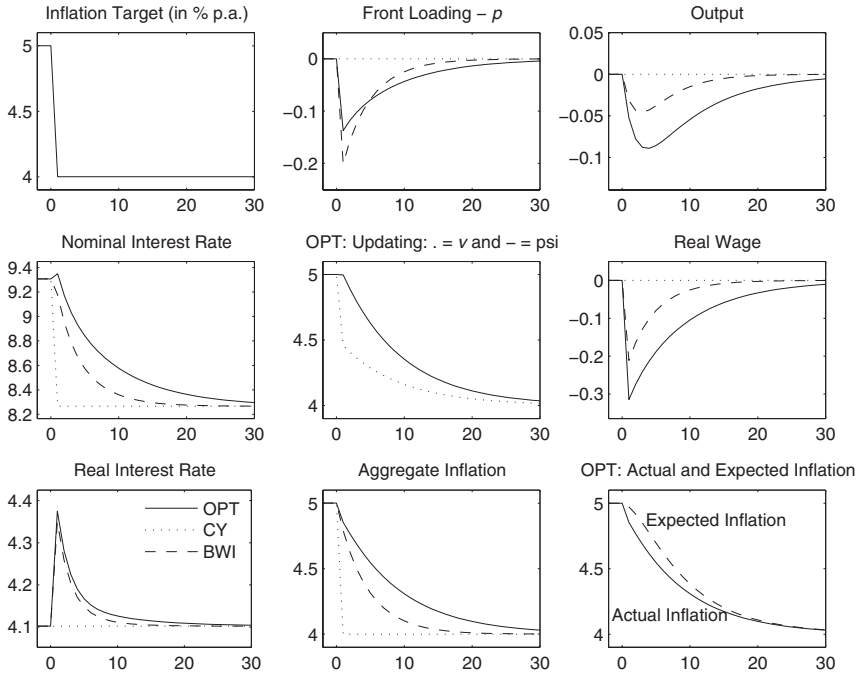


FIGURE 1. Permanent disinflation in the baseline model.

the long-run component of their pricing policies v_t rather than their initial price level p_t. That, in turn, slows down the response of aggregate inflation, because v_t enters the inflation process through ψ_t, a slow-moving state variable. This reason for inertia is different from another one that is commonly stressed in the literature, which relies on a slow response of marginal cost to shocks. In our model inflation is inertial and persistent despite the fact that marginal cost w_t/z_t is perfectly flexible. Adding nominal wage rigidities, as in Erceg and Levin (2003), does, however, help in one important respect. The consensus view is that an initial increase in the nominal interest rate is required to start to bring down inflation, but in the baseline model in Figure 1 that increase is small at only 5 basis points. Figure 2 therefore explores the case of sticky wages and prices, and here we find that nominal interest rates initially have to rise by 35 basis points for a 100–basis-point reduction in inflation. The remainder of this paper returns to the flexible wage case.

The initially high nominal interest rate is required due to the large initial deviation of inflation from its new steady state combined with a high response coefficient ϕ to such deviations in the monetary policy rule. Given the slow adjustment of inflation, the immediate consequence is a steep rise in the real interest rate. This causes consumption, output, and therefore labor demand to drop; i.e., we observe the recession that is associated with disinflations in the data.[19] This in turn lowers the real wage, which exerts downward pressure on prices, so that inflation begins

https://doi.org/10.1017/S1365100510000751 Published online by Cambridge University Press

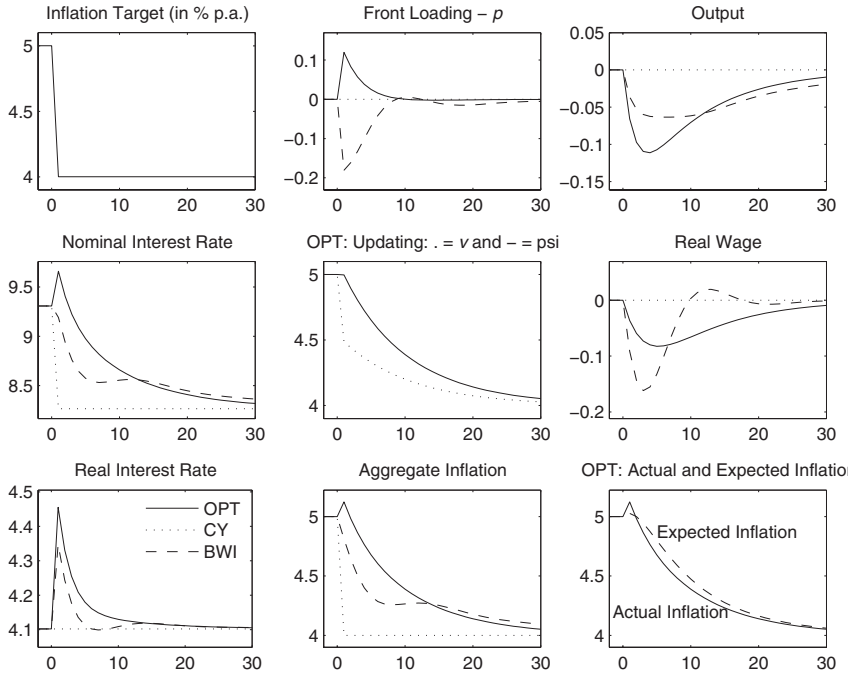


FIGURE 2. Permanent disinflation under sticky wages and prices.

to fall. Both the drop in inflation and the recession then start to lower the nominal and real interest rates. Once this process is complete, the recession ends and inflation drops to its new target. An output sacrifice is therefore unavoidable in bringing down inflation. The sacrifice ratio in this deliberately simple version of the model is modest at only 0.30%, but adding nominal wage rigidities, capital accumulation with investment adjustment costs, and a higher parameter δ can produce much larger real effects, although still displaying qualitatively identical inflation dynamics.

Finally, backward indexation displays dynamics similar to that of optimal log-linear price paths. In fact inflation is less inertial and persistent, and output losses are smaller, with a sacrifice ratio of only 0.10%. The reason is that under optimal log-linear price paths, price setting exhibits a substantially smaller reliance on an initial price level drop p_t than under backward indexation. Instead, firms choose an intermediate updating rate v_t of around 4.5% for their price path that strikes a balance between initially high aggregate inflation of 5%, and later low aggregate inflation approaching 4%.[20]

The bottom right panel of Figure 1 (and of subsequent figures) compares actual inflation to a measure of expected inflation under optimal log-linear price paths, defined as

$$\hat{\pi}_t^e = (1-\delta)\Sigma_{k=0}^{\infty}\delta^k E_{t-k}\hat{\pi}_t. \tag{33}$$

https://doi.org/10.1017/S1365100510000751 Published online by Cambridge University Press

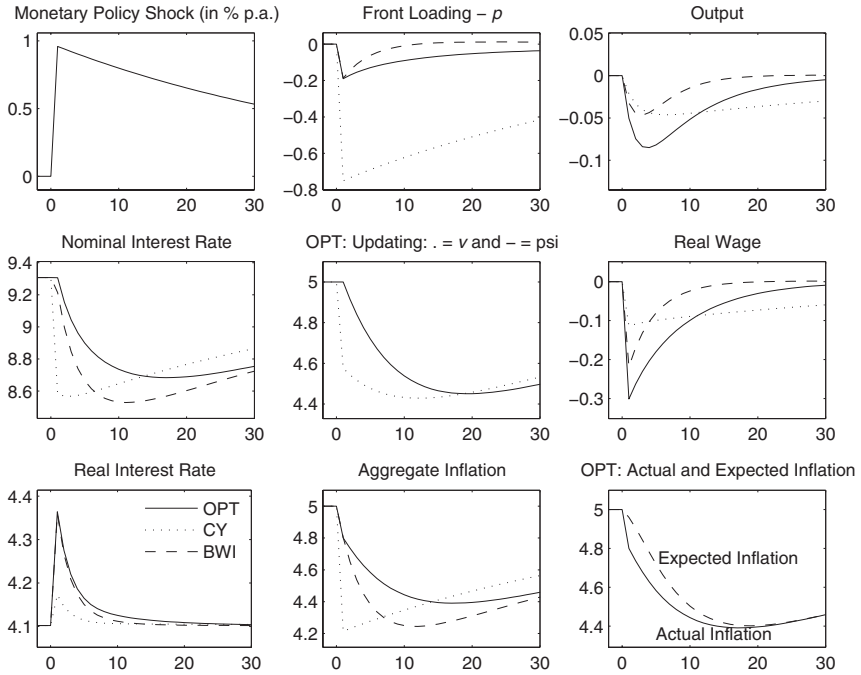


FIGURE 3. Persistent monetary policy shock $\rho^h = 0.98$.

This formula averages the expectations of cohorts of agents who last reset their pricing policies k periods ago, and who therefore last updated their information and expectations for today's inflation at that time. The weights attached to each cohort in (33) correspond to the fraction of such pricing policies that are currently still in force. These old expectations matter under optimal log-linear price paths, as they are still reflected in current updating rates of old pricing policies. Although the expectation $\hat{\pi}_t^e$ is not directly comparable to standard measures of inflation expectations, it behaves quite similarly in one key respect—it lags actual inflation for a long time during the disinflation. Its dynamics in Figure 1 therefore closely resembles the learning dynamics in Erceg and Levin (2003).[21] The reason in the case of learning is unstable or noncredible monetary policy rather than persistent changes in the inflation target per se. But the two are clearly closely related, and we therefore see the two models as highly complementary.

The importance of a prolonged expected change in aggregate inflation suggests that the effect of a highly persistent monetary policy shock should be very similar to that of a permanent shock. In our second monetary policy experiment, we therefore assume that the steady state inflation rate remains at 5% per annum, whereas there is a persistent ($\rho^h = 0.98$) monetary policy shock of equal impact size to the inflation target shock of Figure 1. Figure 3 shows that for the optimal log-linear price paths and backward indexation models, the economy indeed responds similarly to the

https://doi.org/10.1017/S1365100510000751 Published online by Cambridge University Press

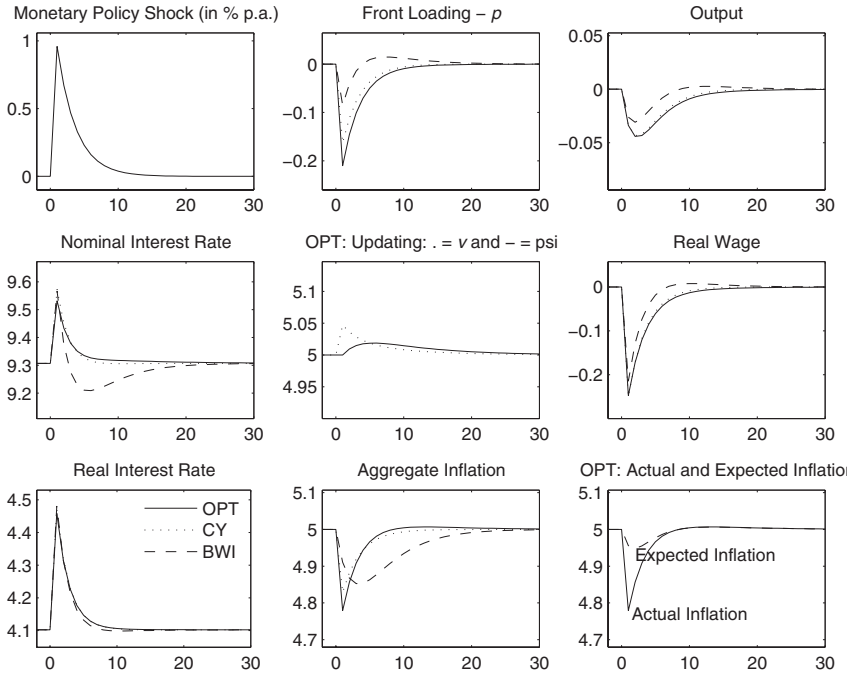


FIGURE 4. Transitory monetary policy shock $\rho^h = 0.7$.

previous case, except of course in the very long run. But the Calvo–Yun model now performs very differently, exhibiting not only prolonged inflation deviations but also output deviations from steady state. This discontinuity arises because the presence of a discount factor in the New Keynesian Phillips curve (31) makes the model behave very differently depending on whether there are persistent inflation deviations from steady state, or whether the steady state itself has changed. This feature follows directly from the rigidity of the updating assumption in that model. Furthermore, the Calvo–Yun model again does not imply an inertial response of inflation, with the maximum jump in inflation occurring in the period of the shock.

Figure 4 displays impulse responses for a more transitory monetary policy shock, with $\rho^h = 0.7$. Because in this case the expected change in inflation is of very short duration, price setters in the optimal log-linear price paths model react mostly through the short-term initial–price level component of their pricing policies. This is identical to the behavior of agents in the Calvo–Yun model, whose only option is to react through this variable. As a result pricing, and the real effects of the shock, are almost identical for the two models. We observe a recession that is both more shallow and shorter than that for a highly persistent shock of equal impact size. It is the backward indexation model that displays very different

https://doi.org/10.1017/S1365100510000751 Published online by Cambridge University Press

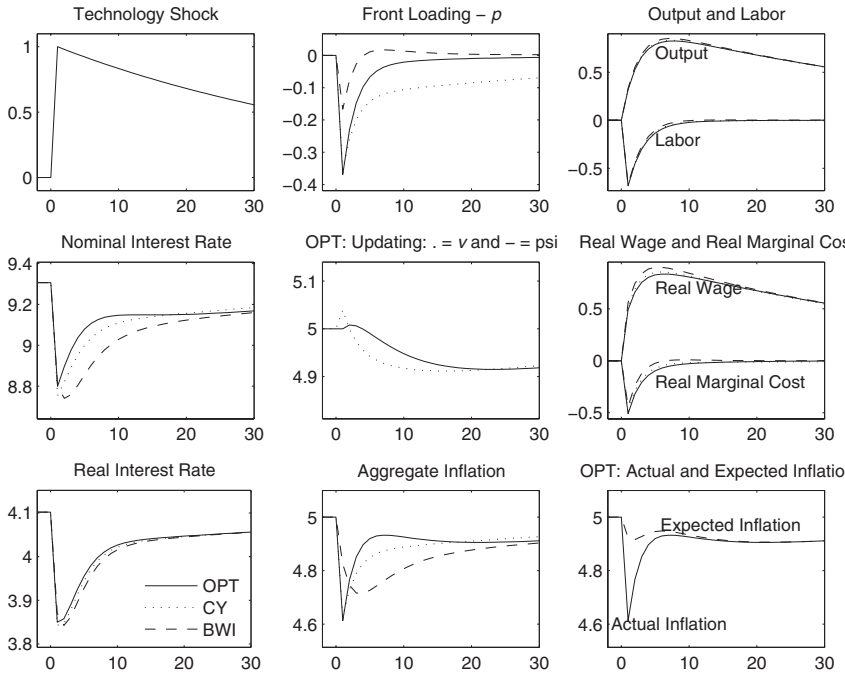


FIGURE 5. Persistent technology shock $\rho^z = 0.98$.

dynamics for this case. The key observation is that inflation inertia and persistence are built into this model, regardless of the persistence of the underlying shock, thereby denying price setters the option to take the transitory nature of a shock into account.

The effects of different assumptions about pricing policies must affect the response of inflation not only to monetary policy shocks but also to all shocks that affect marginal cost. In Figure 5 we therefore study the effects of a persistent ($\rho^z = 0.98$) 1% increase in the level of technology z_t. The real effects of technology shocks are familiar and qualitatively similar across all models. Because the technology shock is highly persistent, there is a similarly persistent increase in output, which thereafter returns to its long-run level in a very gradual fashion. Consequently the real interest rate gradually approaches its long-run level from below. This is in turn reflected in the longer-run behavior of the nominal interest rate and of inflation, which is highly persistent in all three models. But as we will see below, the optimal log-linear price paths model again implies much less inflation persistence for more transitory shocks. Moreover, the behavior of inflation inertia differs significantly across models even for high levels of shock persistence. The shock immediately reduces real marginal cost, the driving force of inflation. But whereas for both the Calvo–Yun model and the optimal log-linear price paths

https://doi.org/10.1017/S1365100510000751 Published online by Cambridge University Press

model, the maximum response of inflation occurs in the first period, the backward indexation model again displays built-in inertia.[22]

4.3. Steady States, Determinacy, and Welfare

As shown by Ascari (2004) and Yun (2005), the steady state of sticky price models that follow the Calvo (1983) scheme for reoptimization opportunities is not invariant to trend inflation if price setters cannot update their prices between reoptimizations.[23] Specifically, the higher the trend rate of inflation, the lower is steady state output. The reason is that under higher trend inflation, price setters will set their prices at a greater premium over the current average market price, in anticipation of faster increases in future marginal costs. This results in higher price dispersion and therefore, given nonlinearities in production functions, in output losses. Ascari (2004) points out that full indexation (partial indexation is not sufficient) following the Calvo–Yun or the backward-indexation scheme can overcome these problems, so that the steady state equations of the model become invariant to trend inflation. Optimal log-linear price paths achieve the same result, but in this case this is the result of price setters' choices, rather than being hardwired into the model.

Woodford (2003, pp. 90–94) shows that the Taylor principle $\phi > 1$ guarantees equilibrium determinacy in the standard New Keynesian model. Figure 6 conducts a numerical analysis of determinacy regions for the three models considered in this paper. We follow Woodford (2003) in setting $\theta = 0$, and search over inflation coefficients ϕ and interest rate smoothing coefficients ρ^i. The black region of the parameter space indicates determinate equilibria. The results confirm Woodford's result for all three models; in other words, the stability properties of our model are the same as those of standard New Keynesian alternatives.

Finally, we present a simple illustration of the three models' welfare properties. To do so, we assume that the output subsidy is set at $\xi = 1/(\sigma - 1)$ to eliminate steady state distortions, and then evaluate a second-order approximation of the model's utility function using the first-order approximation of the laws of motion of the economy's competitive equilibrium. The model is much too simple for a full empirically based calibration of shocks, but to nevertheless fix reasonable magnitudes, we set the standard deviation of productivity and monetary policy shocks to reproduce the standard deviation of U.S. linearly detrended GDP for the period 1984Q1 through 2007Q4. Not surprisingly, the welfare results are primarily driven by the productivity shocks, which are calibrated with $\rho^z = 0.9$ and a standard deviation of 0.0125. Figure 7 shows the Lucas (1987) compensating consumption variations of a welfare comparison of the optimal log-linear price paths and backward indexation models with the Calvo–Yun model. These represent the permanent increases in consumption (in percent) that households would have to receive under the two pricing regimes in order to be indifferent between those regimes and the Calvo–Yun regime. In order to show how these welfare results depend on the ability of households to reoptimize, we plot results against δ on the

https://doi.org/10.1017/S1365100510000751 Published online by Cambridge University Press

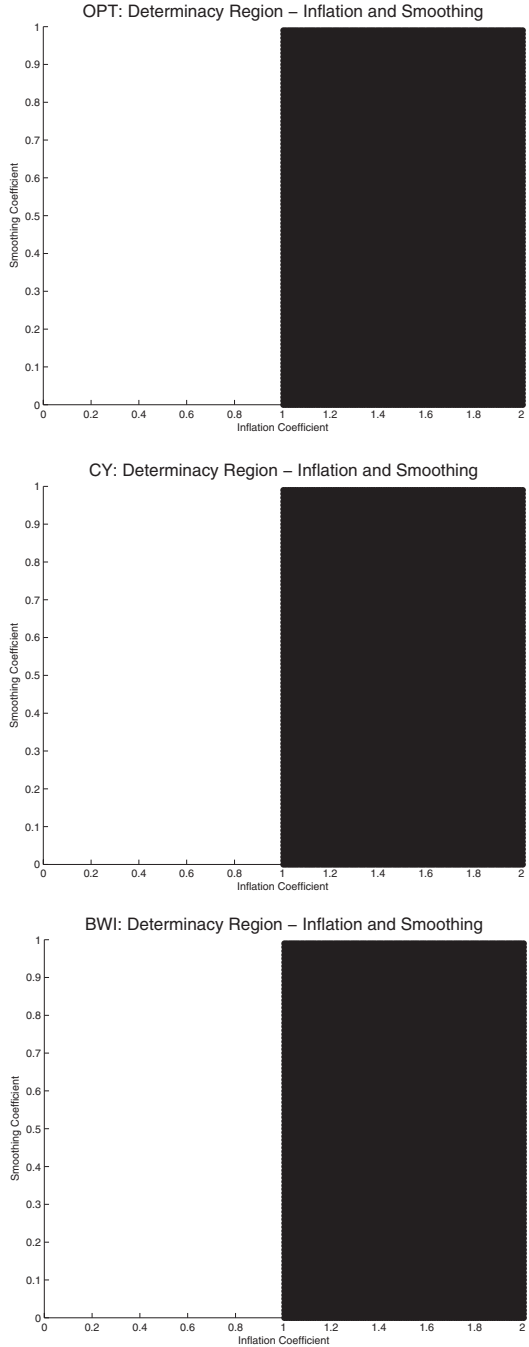


FIGURE 6. Determinacy regions.

https://doi.org/10.1017/S1365100510000751 Published online by Cambridge University Press

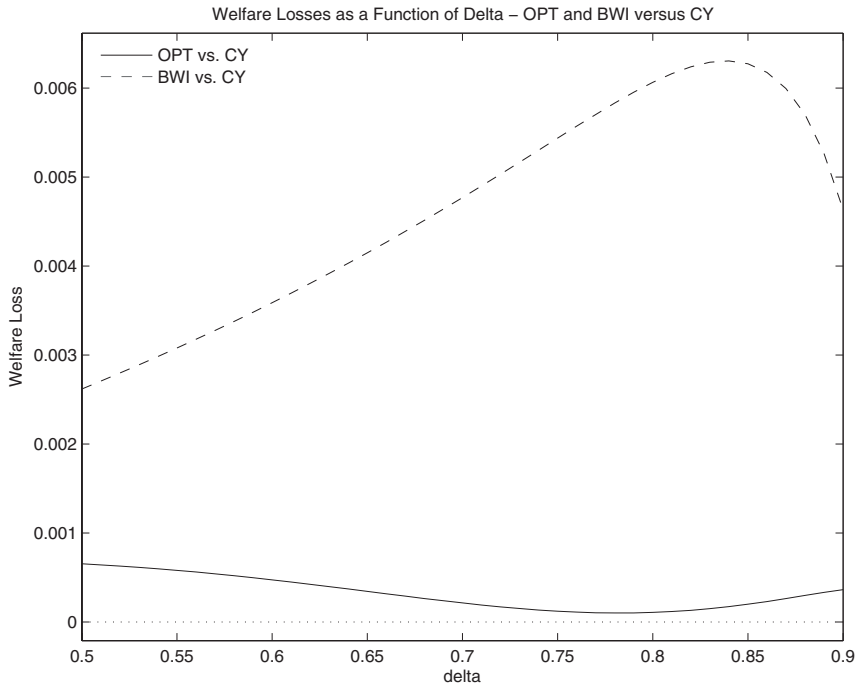


FIGURE 7. Welfare.

horizontal axis. We vary δ between values that have been found reasonable in the empirical literature, $\delta \in [0.5, 0.9]$.

Under temporary shocks, backward-looking indexation or forward-looking optimal log-linear price paths give rise to welfare losses relative to indexation to (unchanging) steady state inflation. The reason is that, for a given interest rate rule, the more persistent response of inflation to shocks exacerbates real interest rate movements that lead to more volatility. But under optimal log-linear price paths the additional inflation persistence is very small if shocks are temporary, so that the associated welfare losses are very small. Under backward indexation, the additional inflation persistence is much larger, and consequently the welfare losses are an order of magnitude larger than under optimal log-linear price paths, with a range of 0.0027–0.0062 versus a range of 0.0002–0.0007.

5. INFLATION DYNAMICS

This section studies two aspects of inflation dynamics in more detail—inflation persistence and inflation inertia. Inflation persistence refers to the time it takes for inflation to return to its long-run value following a shock. We therefore plot the autocorrelation functions of inflation under the different models as a function of shock persistence. Inflation inertia refers to a delayed and gradual initial change

https://doi.org/10.1017/S1365100510000751 Published online by Cambridge University Press

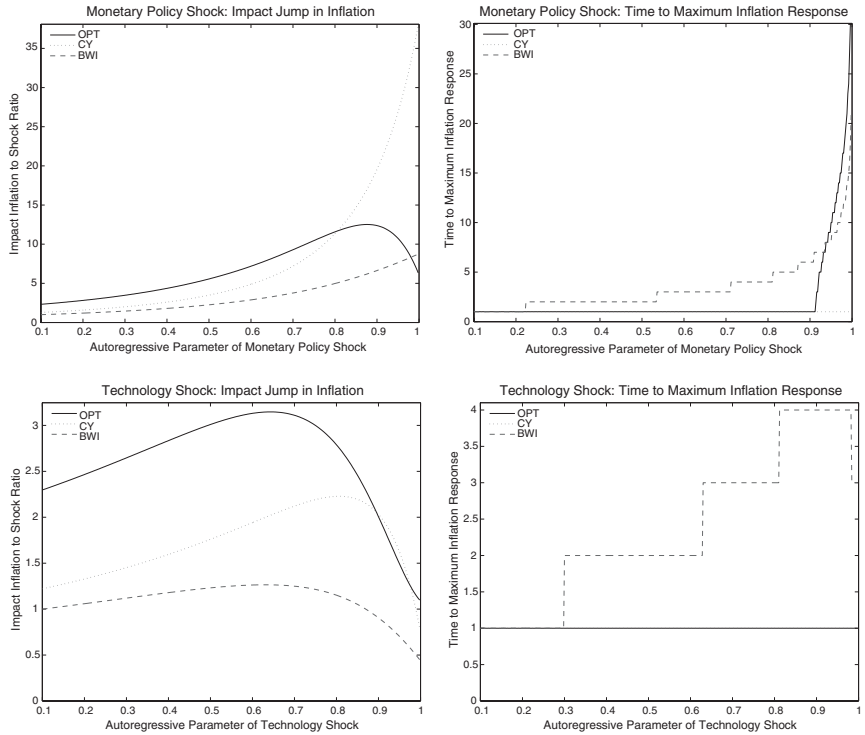


FIGURE 8. Inflation inertia.

in inflation following a shock. The presence of inertia cannot be reliably inferred from autocorrelation functions, in that whereas high inertia will generally imply high autocorrelation, the reverse is not true. Instead we capture inflation inertia by displaying two measures that capture the initial behavior of inflation as a function of the persistence of the shock. The first is the first-period jump in inflation, scaled by the size of the shock. High inertia implies a small initial jump in inflation, whereas in models without any inertia the maximum jump in inflation occurs in the period of the shock. This leads us to the second measure, the number of periods that elapses before inflation attains its maximum response to the shock. Here higher inertia is associated with this number of periods being larger.

5.1. Inflation Inertia

Figure 8 studies the extent of inflation inertia generated by the three models. The left column shows how the ratio of the impact jump in inflation to the size of the shock changes with increasing degrees of monetary policy or technology shock persistence, whereas the right column shows the time period in which the maximum inflation response occurs, again as a function of shock persistence. The

https://doi.org/10.1017/S1365100510000751 Published online by Cambridge University Press

top row shows monetary policy shocks, whereas the bottom row shows technology shocks.

For monetary policy shocks, in the top left panel we observe that, as expected from Figures 3 and 4, for low shock persistence the initial jump in inflation is small, simply because the cumulative disinflationary impulse is small. For the same reason, as shock persistence increases, the initial jump in inflation increases for all three pricing models. But it increases only little for the backward indexation model, because inertia in this model is built into the structure. At the other extreme, we find the Calvo–Yun model, where the initial jump in inflation increases monotonically and steeply in the persistence of shocks. At the limit, for permanent shocks the jump in inflation exactly equals the size of the shock, the case considered in Figure 1. In the Calvo–Yun model, there is therefore no inertia whatsoever; for any shock, the largest jump in inflation always occurs in the first period. The optimal log-linear price paths case is intermediate, reflecting the assumption that agents can respond to the persistence of shocks by making their pricing response more or less inertial. For transitory shocks, there is therefore even less inertia than in the Calvo–Yun model. This is because agents optimally respond to a short-lived dip in inflation by initially lowering their price substantially and subsequently letting it grow at a slightly higher rate than aggregate prices. But for more persistent shocks (approximately $\rho^h > 0.8$), optimal log-linear price paths generate more inertia than Calvo–Yun, and for highly persistent shocks such as those in Figures 1 and 3 it even generates more inertia than the backward indexation model.[24] As we saw in our discussion of Figure 1, this is because for such shocks it is optimal to respond mostly through the inflation updating component of pricing policies.

The top right panel shows that, for the Calvo–Yun model, the maximum inflation response always occurs in the first period, whereas under backward indexation the delay until the maximum inflation response increases gradually with shock persistence. Under optimal log-linear price paths, we observe the same type of response as in Calvo–Yun for shock persistence below $\rho^h = 0.9$, but thereafter inertia increases steeply, with the maximum inflation response occurring significantly later than under backward indexation if shock persistence exceeds $\rho^h = 0.95$.

For technology shocks, in the bottom left panel we observe similar relative levels of inflation inertia as under monetary policy shocks when shocks are transitory. But the empirically relevant range of technology shocks is at high levels of persistence, and here we observe, as suggested by Figure 5, much more inertia, in the sense of smaller initial changes in inflation (by a factor of at least two) under backward indexation. The reason is, as for monetary policy shocks, the fact that inertia is built into this model. This is also evident in the bottom right panel, which shows that under highly persistent technology shocks the maximum response of inflation under backward indexation occurs after three to four quarters, whereas under both Calvo–Yun and optimal log-linear price paths the maximum response is always in the first period.

https://doi.org/10.1017/S1365100510000751 Published online by Cambridge University Press

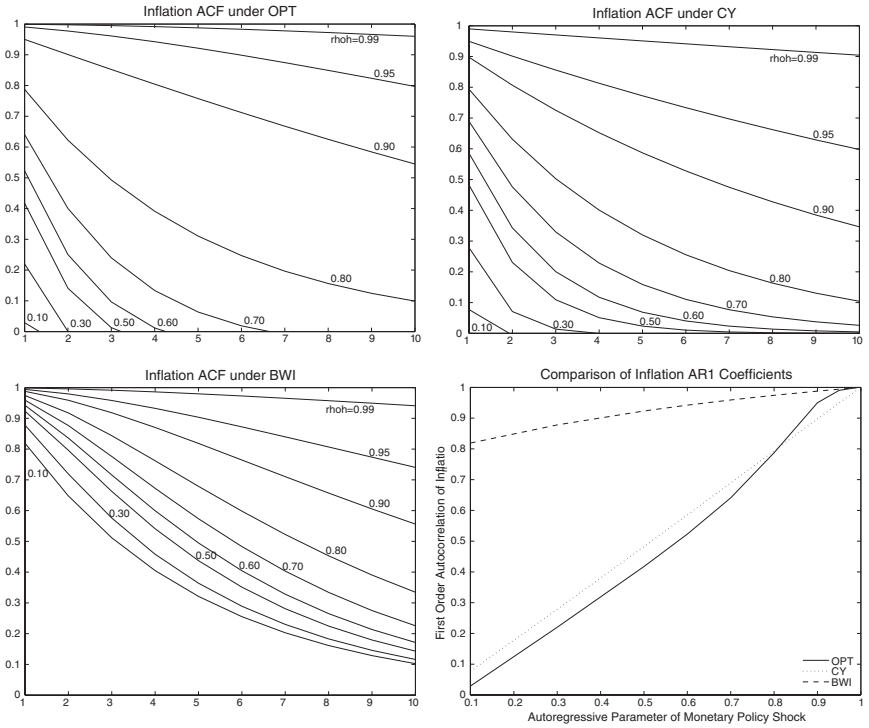


FIGURE 9. Inflation persistence under monetary policy shocks.

5.2. Inflation Persistence

Figures 9 and 10 study the degrees of inflation persistence generated by the three models. The figures use inflation autocorrelation functions (ACF) and inflation AR1 coefficients to represent persistence. Figure 9 considers monetary policy shocks and Figure 10 technology shocks. Each figure contains three plots, one for each pricing model, that show a family of ACF corresponding to shock persistence between 0.99 and 0.1 (0.5 for technology shocks). The fourth plot shows only the AR1 coefficients against shock persistence along the horizontal axis, but it does so for all three pricing models to facilitate comparison.

For monetary policy shocks in Figure 9, inflation persistence under both the optimal log-linear price paths model and the Calvo–Yun model is almost the same as the persistence of the underlying shock, whereas under backward indexation inflation persistence is highly insensitive to shock persistence. As a result, for highly persistent shocks, all three models predict highly persistent inflation, but for shocks that are nearly white noise, the AR1 coefficient of inflation under backward indexation is still at 0.82.

Similar but not identical results obtain for technology shocks. For the empirically relevant range of highly persistent technology shocks, inflation is highly

https://doi.org/10.1017/S1365100510000751 Published online by Cambridge University Press

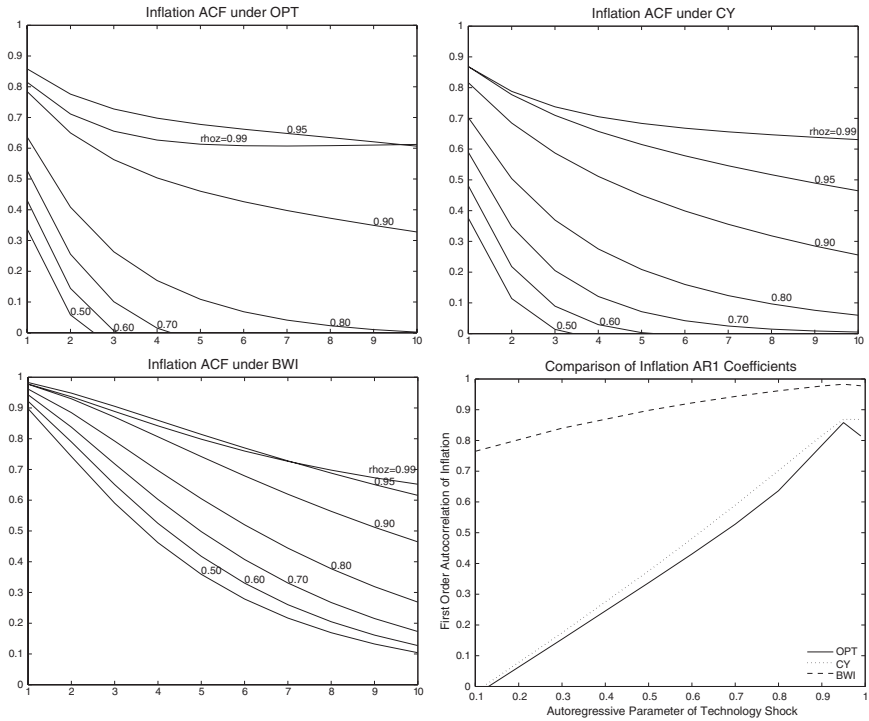


FIGURE 10. Inflation persistence under technology shocks.

persistent under backward indexation; it essentially follows a unit root process. But under optimal log-linear price paths, its maximum autocorrelation is considerably lower at around 0.86, with a much higher sensitivity to the persistence of the underlying shocks.

In Figures 9 and 10, inflation under optimal log-linear price paths and under Calvo–Yun indexation exhibits little persistence beyond the persistence of the underlying shocks. It is in this dimension that a modest degree of interest rate smoothing leads to a different conclusion. This is shown in Figure 11, which plots the AR1 coefficients for inflation in a similar fashion to Figure 9, but under the assumption that $\rho^i = 0.7$. Interest rate smoothing does generate some inflation persistence even for monetary policy shocks with very little serial correlation, but the difference to the backward indexation case is still very large, with AR1 coefficients just above 0.5 for optimal log-linear price paths and everywhere above 0.91 for backward indexation.

6. CONCLUSION

In this paper, we have presented a monetary model with nominal rigidities and optimizing, forward-looking households and firms that have full information about

https://doi.org/10.1017/S1365100510000751 Published online by Cambridge University Press

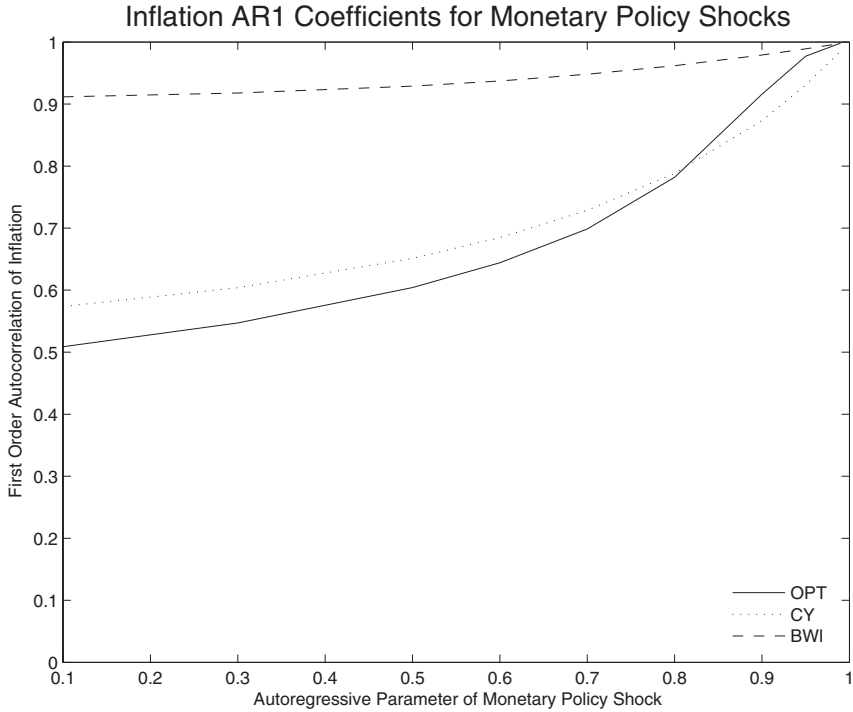


FIGURE 11. Inflation persistence under interest rate smoothing.

the state of the economy. The model differs from conventional alternatives in this class in one key respect—firms set pricing policies instead of price levels. The paper is motivated by some important shortcomings of alternative models, namely their inability to generate inflation inertia, inflation persistence, and recessionary disinflations without resorting to some variant of a backward-looking price setting specification.

The model, which we refer to as the optimal log-linear price paths model, does generate all of these effects in response to persistent shocks to monetary policy. The reason is the ability of price setters to separately determine two components of their pricing policies, their initial price level and a long-run or inflation updating component. The presence of firm-specific inflation updating introduces a state variable into aggregate inflation dynamics despite the absence of backward looking behavior. This is distinct from another frequently stressed reason for inflation inertia and persistence, a slow response of marginal cost to shocks.

The alternative Calvo–Yun model does not generate inflation inertia and recessionary disinflations, whereas the alternative backward indexation model generates high inflation inertia and persistence irrespective of both the nature and persistence of the underlying shocks.

https://doi.org/10.1017/S1365100510000751 Published online by Cambridge University Press

NOTES

1. See Rudd and Whelan (2006, 2007) for critical reviews.

2. We note that although significant inertia implies significant persistence, the reverse is not true, as in many models with a highly persistent inflation process the maximum response of inflation occurs in the initial period following the shock.

3. Mankiw and Reis (2002) can be thought of as embedding the pricing contracts of Fischer (1977), who first suggested predetermined price paths, in the Calvo (1983) structure.

4. Another example is Burstein (2006), who provides a state-dependent pricing model in which firms can set nonlinear price paths. Because of this nonlinearity, the model cannot be solved with conventional perturbation methods. Instead the paper focuses on the perfect-foresight case and uses a nonlinear solution method.

5. The "hybrid" New Keynesian model, first introduced by Clarida et al. (1999) and Galí and Gertler (1999), can be derived from the same microfoundations. It also allows an intermediate case where a fraction of agents index to steady state inflation and another fraction index to lagged aggregate inflation.

6. In this context it is interesting to note that, in an important empirical study, Benati (2008) claims that there is little or no inflation persistence under stable monetary regimes with clearly defined nominal anchors, so that hardwiring inflation persistence in macroeconomic models could be misleading.

7. This argument foreshadows our results on the welfare performance of different types of price-setting behavior in Section 4.3, where we demonstrate that optimal log-linear price paths are much less costly in terms of welfare than backward-looking indexation.

8. See Juillard et al. (2007).

9. For money we assume the cashless limit advocated by Woodford (2003).

10. Habit persistence is not essential for our main results. Its only role is to produce more reasonable, U-shaped instead of V-shaped, output responses.

11. It turns out that there is no analytically tractable way to express the numerator and denominator of (20) in nonlinear recursive form. Among other things, this rules out solving nonlinear perfect foresight versions of the model.

12. We have also considered an alternative rule where the nominal interest rate instead responds to current inflation. Although this changes some details, our main results are unaffected.

13. Rudebusch and Wu (2007) discuss such shocks as representing responses by the Fed to a variety of persistent determinants beyond inflation and output. On the other hand, Charles Goodhart pointed out to us that they may also represent the effects on interest rates of autocorrelated inflation forecast errors.

14. Note that $\hat{v}_{t-1}$ is defined as the firm-specific inflation rate from $t-1$ to t. This differs from the timing convention for the aggregate inflation rate from $t-1$ to t, which is $\hat{\pi}_t$. This convention is adopted because, unlike $\hat{\pi}_t$, $\hat{v}_{t-1}$ is decided on and therefore known at $t-1$.

15. See the Technical Appendix for details on the treatment of unit root shocks.

16. Pencavel (1986) reports that most microeconomic estimates of this elasticity are between 0 and 0.45, but in the business cycle literature it is common to assume a unitary elasticity. As discussed by Chang and Kim (2005), a low labor supply elasticity makes it difficult to explain cyclical fluctuations in hours worked, and they present a heterogenous agent model in which aggregate labor supply is considerably more elastic than individual labor supply.

17. For an example and discussion, see Juillard et al. (2007).

18. It can in fact be shown that the profit maximization problem amounts to choosing a price path that minimizes the distance between the firm-specific and the aggregate price paths in a weighted–least squares sense. See Calvo et al. (2002).

19. Ball (1994) first discussed that, in contrast to models with staggered fixed prices, models with predetermined prices cause costly disinflations and inflation inertia/persistence.

20. As in Calvo (1983) models with Yun (1996) updating, the model implies that very old pricing policies exhibit high relative prices. But this problem is far less severe because of updating. For

https://doi.org/10.1017/S1365100510000751 Published online by Cambridge University Press

example, the first cohort in Figure 1 has around a 1% chance of its pricing policy surviving for five years, but even at that time its price is only 0.4% above the average market price.

21. There are of course also significant differences to learning models. Most importantly, with learning, short-run expected inflation falls more quickly than long-run expected inflation, whereas the opposite is true under optimal indexation.

22. The literature has documented inflation inertia primarily for monetary policy shocks and not for technology shocks. For example, Dedola and Neri (2007) find no evidence of inflation inertia following technology shocks.

23. Both papers show that the dynamic behavior of such models is also not invariant to trend inflation, but we do not focus on that point here.

24. Some details of the rankings in Figure 8 depend on the specific calibration, especially on the parameters of the monetary policy rule. But the overall conclusions are robust.

REFERENCES

Amato, Jeffrey D. and Thomas Laubach (1999) Monetary Policy in an Estimated Optimization-Based Model with Sticky Wages and Sticky Prices. Research Working paper 99-09, Federal Reserve Bank of Kansas City.

Ascari, Guido (2004) Staggered prices and trend inflation: Some nuisances. *Review of Economic Dynamics* 7, 642–667.

Ball, Larry (1994) Credible disinflation with staggered price-setting. *American Economic Review* 84(1), 282–289.

Benati, Luca (2008) Investigating inflation persistence across monetary regimes. *Quarterly Journal of Economics* 123(3), 1005–60.

Bils, Mark and Peter Klenow (2004) Some evidence on the importance of sticky prices. *Journal of Political Economy* 112(5), 947–985.

Boldrin, Michele, Lawrence J. Christiano, and Jonas Fisher (2001) Habit persistence, asset returns and the business cycle. *American Economic Review* 91(1), 149–166.

Burstein, Ariel (2006) Inflation and output dynamics with state dependent pricing decisions. *Journal of Monetary Economics* 53(7), 1235–1257.

Calvo, Guillermo A. (1983) Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics* 12, 383–398.

Calvo, Guillermo A., Oya Celasun, and Michael Kumhof (2002) A theory of rational inflationary inertia. In Philippe Aghion, Roman Frydman, Joseph Stiglitz, and Michael Woodford (eds.), *Knowledge, Information and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps,* pp. 87–117. Princeton, NJ: Princeton University Press.

Calvo, Guillermo A., Oya Celasun, and Michael Kumhof (2007) Inflation inertia and credible disinflation: The open economy case. *Journal of International Economics* 73(1), 48–68.

Chang, Yongsung and Sun-Bin Kim (2005) On the aggregate labor supply. *Federal Reserve Bank of Richmond Economic Quarterly* 91(1), 21–37.

Christiano, Lawrence J., Martin Eichenbaum, and Charles Evans (1996) The effects of monetary policy shocks: Evidence from the flow of funds. *Review of Economics and Statistics* 78(1), 16–34.

Christiano, Lawrence J., Martin Eichenbaum, and Charles Evans (1998) Monetary Policy Shocks: What Have We Learned and To What End? Working paper 6400, National Bureau of Economic Research.

Christiano, Lawrence J., Martin Eichenbaum, and Charles Evans (2005) Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy* 113(1), 1–45.

Clarida, Richard, Jordi Galí, and Mark Gertler (1999) The science of monetary policy: A New Keynesian perspective. *Journal of Economic Literature* 37, 1661–1707.

Dedola, Luca and Stefano Neri (2007) What does a technology shock do? A VAR analysis with model-based sign restrictions. *Journal of Monetary Economics* 54, 512–549.

Erceg, Christopher J. and Andrew T. Levin (2003) Imperfect credibility and inflation persistence. *Journal of Monetary Economics* 50(4), 915–944.

https://doi.org/10.1017/S1365100510000751 Published online by Cambridge University Press

European Central Bank (ECB) (2005) Inflation Persistence and Price Setting Behaviour in the Euro Area. European Central Bank, Inflation Persistence Network.
Fischer, Stanley (1977) Long-term contracts, rational expectations, and the optimal money supply rule. *Journal of Political Economy* 85(1), 191–205.
Fuhrer, Jeffrey and George Moore (1995) Inflation persistence. *Quarterly Journal of Economics* 110, 127–159.
Galí, Jordi (2003) New perspectives on monetary policy, inflation and the business cycle. In Matthias Dewatripont, Lars Hansen, and Stephen Turnovsky (eds.), *Advances in Economic Theory*, Vol. III, pp. 151–197. Cambridge, UK: Cambridge University Press.
Galí, Jordi and Mark Gertler (1999) Inflation dynamics: A structural econometric analysis. *Journal of Monetary Economics* 44(2), 195-222.
Golosov, Mikhail and Robert E. Lucas (2003) Menu Costs and Phillips Curves. Working paper 10187, National Bureau of Economic Research.
Gordon, Robert J. (1982) Why stopping inflation may be costly: Evidence from fourteen historical episodes. In Robert E. Hall (ed.), *Inflation: Causes and Consequences*, pp. 11–40. Chicago: University of Chicago Press.
Gordon, Robert J. (1997) The time-varying NAIRU and its implications for economic policy. *Journal of Economic Perspectives* 11(1), 11–32.
Juillard, Michel, Ondra Kamenik, Michael Kumhof, and Douglas Laxton (2007) Optimal price setting and inflation inertia in a rational expectations model. *Journal of Economic Dynamics and Control* 32(8), 2584–2621.
Klenow, Peter and Oleg Kryvtsov (2005) State-Dependent or Time-Dependent Pricing: Does It Matter for Recent U.S. Inflation? Working paper 11043, National Bureau of Economic Research.
Lane, Philip (2001) The new open economy macroeconomics: A survey. *Journal of International Economics* 54(2), 235–266.
Leeper, Eric M., Chris Sims, and Tao Zha (1996) What does monetary policy do? *Brookings Papers on Economic Activity* 2, 1–63.
Lucas, Robert E., Jr. (1987) *Models of Business Cycles*. Oxford/New York: Basil Blackwell.
Mankiw, Gregory N. (2001) The inexorable and mysterious tradeoff between inflation and unemployment. *Economic Journal* 111, 45–61.
Mankiw, Gregory and Ricardo Reis (2002) Sticky information versus sticky prices: A proposal to replace the New Keynesian Phillips curve. *Quarterly Journal of Economics* 117(4), 1295–1328.
Pencavel, John (1986) Labor supply of men. In Orley Ashenfelter and Richard Layard (eds.), *Handbook of Labor Economics,* Vol. 1. Amsterdam: North Holland.
Reis, Ricardo (2006) Inattentive producers. *Review of Economic Studies* 73, 793–821.
Rotemberg, Julio (1982) Sticky prices in the United States. *Journal of Political Economy* 90, 1187–1211.
Rotemberg, Julio and Michael Woodford (1997) An optimization-based econometric framework for the evaluation of monetary policy. In Ben Bernanke and Julio Rotemberg (eds.), *NBER Macroeconomics Annual*, pp. 297–346. Cambridge, MA: MIT Press.
Rudd, Jeremy and Karl Whelan (2006) Can rational expectations sticky-price models explain inflation dynamics? *American Economic Review* 96(1), 303–320.
Rudd, Jeremy and Karl Whelan (2007) Modelling inflation dynamics: A critical survey of recent research. *Journal of Money, Credit and Banking* 39, 155–70.
Rudebusch, Glenn (2002) Term structure evidence on interest rate smoothing and monetary policy inertia. *Journal of Monetary Economics* 49, 1161–1187.
Rudebusch, Glenn (2006) Monetary policy inertia: Fact or fiction? *International Journal of Central Banking* 2(4), 85–135.
Rudebusch, Glenn and Tao Wu (2007) Accounting for a shift in term structure behavior with no-arbitrage and macro-finance models. *Journal of Money, Credit and Banking* 39(2–3), 395–422.
Smets, Frank and Rafael Wouters (2003) An estimated dynamic stochastic general equilibrium model of the euro area. *Journal of the European Economic Association* 1(5), 1123–1175.

https://doi.org/10.1017/S1365100510000751 Published online by Cambridge University Press

Taylor, John B. (1980) Aggregate dynamics and staggered contracts. *Journal of Political Economy* 88, 1–24.

Taylor, John B. (1993). Discretion versus policy rules in practice. *Carnegie–Rochester Conference Series on Public Policy* 39, 195–214.

Taylor, John B. (1998) Staggered Price and Wage Setting in Macroeconomics. Working paper 6754, National Bureau of Economic Research.

Woodford, Michael (2003) *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton, NJ: Princeton University Press.

Yun, Tak (1996) Nominal price rigidity, money supply endogeneity, and business cycles. *Journal of Monetary Economics* 37, 345–370.

Yun, Tak (2005) Optimal monetary policy with relative price distortions. *American Economic Review* 95(1), 89–109.

https://doi.org/10.1017/S1365100510000751 Published online by Cambridge University Press