A NOVEL APPROACH FOR OBTAINING ASSEMBLY MODES OF A 3UPS-S FULLY SPHERICAL PARALLEL MANIPULATOR

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ABSTRACT

The 3(UPS)-S fully spherical parallel manipulator is the most famous fully spherical parallel manipulator (FSPM). In this paper, we propose a novel approach to model the forward displacement analysis of the manipulator to obtain its assembly modes. Rodrigues' formula is used as a mathematical tool to perform the proposed modeling. Utilizing geometry of the manipulator, two coupled trigonometric equations are obtained. Using Bezout's elimination method, the two coupled equations are transformed to one polynomial of degree eight. Finally, an example is given with eight real solutions. Therefore, the degree of the polynomial is minimal and the introduced modeling method is optimal. This is very important to control modelling and dynamics simulation. Also, the proposed method can be extended to the other FSPMs (*e.g.*, 3(RPSP)-S, 3(RPSP)-S and 3(RSS)-S).

Keywords: Fully spherical parallel manipulator (FSPM), Rodrigues' Formula, Forward displacement problem, Bezout's elimination method.

1. INTRODUCTION

A parallel manipulator (PM) typically is consists of a moving platform and a fixed base, connected together via closed chains. Generally, the number of the actuated joints is equal to the number of degrees of freedom (DOF) [1]. There are several usages of PM, mentioned in the papers. Telescopes, surgical instruments, rehabilitation equipment, as well as table of machining tools [2-5]. The spherical movement of a rigid body around a spherical joint or a point allows variable orientation of a rigid body or end-effector [6]. A fully spherical parallel manipulator (SPM) is a developed PM that its moving platform has rotational movement around a spherical joint.

With regard to the PMs, the kinematics and dynamics of PMs are more complicated than serial manipulators [1]. Di Gregorio introduced a new family of spherical parallel manipulators and demonstrated that the 3(RRPRR) architectures matching the geometric conditions for elementary spherical motion make the platform accomplish finite spherical motion [7]. Alici and Shirinzadeh introduced the 3(SPS)-S SPMs. Each kinematic chain of the manipulator has spherical (S), prismatic (P) and spherical (S) joints. Spherical joint of each leg on the fixed platform is connected to the spherical joint of the moving platform with a prismatic joint [8]. Innocenti and Parenti [9], Wohlhart [10] and also Vertechy and Parenti-Castelli [11] investigated the 3(UPS)-S SPMs. Bonev presented a methodology for the analytical determination and representation of the workspace boundaries of symmetrical spherical parallel mechanisms (SPMs). The methodology based on an intuitive orientation representation that is very useful for the analysis of symmetrical parallel mechanisms [12].

Basically, the first step in controlling manipulators is to determine the end-effector position with respect to the position of actuators, such as machining tools or flight simulators [13]. In general, five different methods have been employed solve the forward kinematics problem of the PMs; analytical approaches, iterative methods, extra sensors or transducers, special techniques, and neural-network-based approaches. Borràs and Di Gregorio provided an algorithm that determined all the assembly modes of two structures with the same topology that are generated from two families of mechanisms: One planar and the other spherical. The topology of these structures is constituted of nine links (one quaternary link, four ternary links, and four binary links) connected through 12 revolute pairs to form four closed loops [14]. Tsai, Shiaua, and Chang utilized the forward kinematic of a novel 3(PRS) PM [15]. They used Bezout's elimination method which leads to an equation with 64 solutions. They also categorized the 64 solutions into six groups and examined the corresponding configuration for each group. Wu, Zhou, Ai and Meng investigated the forward kinematic

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problem of a parallel manipulator based on the neuro-fuzzy method. They used the neuro-fuzzy method instead of solving complicated nonlinear equation by a common numerical iterative method such as Newton-Raphson. They also combined the inverse kinematics analysis with fuzzy theory and neural network to overcome their problems [16]. Chebbi, Affi and Romdhane presented an analytical method to solve the forward kinematic problem of the 3-(UPU) SPM [17]. Boudreau and Turkkan solved the forward displacement of a parallel manipulators by a genetic algorithm [18]. They used inverse kinematics problem to optimize their method. Zheng, Bin and Luo presented a hybrid serial-parallel manipulator that is based on two kinds of 3-UPU parallel mechanisms with six degrees of freedom. The motion of the hybrid serial-parallel mechanism can be decoupled to pure translation and pure spherical rotation. They also obtained a closed-form solution for the direct displacement problem of the hybrid manipulator [19]. Some studies have addressed the forward displacement problem for different manipulator architectures [20-25]. These works showed that the forward displacement analysis of their corresponding mechanisms can be solved in echelon form (finding all possible solutions of the forward displacement analysis).

Bai, Hansen and Angeles proposed a robust procedure to determine the platform orientation of the 3(RRR) spherical parallel manipulator using the input/output equations of spherical four-bar linkages [26]. A similar approach is used by Saafi, Laribi and Zeghloul for the spherical manipulator for surgical application. Furthermore, they improved the calculation of the direct kinematic model using an extra sensor for a passive joint [27].

In this paper, an innovative method is presented to solve the forward displacement problem of the manipulator based on the geometry of the manipulator. Solving the forward displacement problem of the 3(UPS)-S manipulator using Rodrigues' rotation formula leads to two coupled trigonometric equations with two unknown parameters. After the substitution tangent of the half angle into the trigonometric equations, we obtain two algebraic equations. Next, using Bezout's elimination method, the two algebraic equations are converted to a polynomial of degree eight with one unknown parameter. Finally, an example is given with eight real solutions and will be shown that the polynomial is minimal and the innovative method is optimal. Note that for this type of manipulator, Innocenti and Parenti-Castelli [9] derived a system of two equations, one eighth-order polynomial and one linear equation. Also, in the work of Ji and Wu [28], the forward displacement problem of the spherical parallel robot with prismatic actuators with identical pyramids was studied by resorting to three closure equations. Since the methodology of solving the forward displacement problem of a manipulator is significance for real-time path generation, real-time control, forward dynamics analysis and *etc*., the simple proposed method can be a viable alternative to other work.

2. DESCRIPTION OF THE MANIPULATOR, COORDINATES FRAMES AND ROTATION MATRIX

2.1 Description of the 3(UPS)-S Spherical Parallel Manipulator

The 3(UPS)-S fully spherical parallel manipulator consists of a fixed platform connected to the moving platform by a non-moving spherical joint at the center of the moving platform that is called fixed spherical joint and three similar legs. Each leg has Universal (U), Prismatic (P) and Spherical (S) joints that is actuated with linear actuator. See Figs. 1 ~ 3. The unit vector $\mathbf{u}_i$ is referred along OA_i. The angle between this unit vector and axis x_B is defined by λ_i. The fixed distance between the universal joint and the origin of the base coordinate frame, O, is denoted by a_i. The unit vector $\mathbf{e}_i$ is indicated along A_iB_i. The prismatic joint slides along the unit vector $\mathbf{e}_i$ and the spherical joint is located at the end of each leg. The unit vector $\mathbf{v}_i$ along SB_i is defined. The variable length of each leg is denoted by l_i. Also, we assume that all of the $\mathbf{v}_i$ vectors are on one plane. Furthermore, the fixed distance between the fixed spherical joint, S, and the center of the fixed platform, O, is equal to h. The unit vector $\mathbf{w}_i$ is defined along A_iS. The unit vector $\mathbf{m}_i$ is perpendicular to the plane A_iB_iS. Therefore, the unit vector $\mathbf{m}_i$ is normal to the unit vectors $\mathbf{w}_i$ and $\mathbf{e}_i$. The unit vector $\mathbf{n}$ is defined perpendicular to the moving platform. Therefore, this unit vector is perpendicular to the unit vectors $\mathbf{v}_i$. The unit vectors $\mathbf{w}_i$, $\mathbf{m}_i$ and $\mathbf{n}$ can be shown as

$$\mathbf{w}_i = \frac{a_i\mathbf{u}_i - \mathbf{h}}{\|a_i\mathbf{u}_i - \mathbf{h}\|} \quad \text{for} \quad i = 1,2,3 \tag{1a}$$

$$\mathbf{m}_i = \frac{\mathbf{w}_i \times \mathbf{e}_i}{\|\mathbf{w}_i \times \mathbf{e}_i\|} \quad \text{for} \quad i = 1,2,3 \tag{1b}$$

$$\mathbf{n} = \frac{\mathbf{v}_i \times \mathbf{v}_{i+1}}{\|\mathbf{v}_i \times \mathbf{v}_{i+1}\|} \quad \text{for} \quad i = 1,2,3 \tag{1c}$$

where

$$\mathbf{h} = [0 \quad 0 \quad h]^{\mathrm{T}}, \ \mathbf{u}_i = [\cos\lambda_i \quad \sin\lambda_i \quad 0]^{\mathrm{T}}, {}^{\mathrm{M}}\mathbf{v}_i = [\cos\alpha_i \quad \sin\alpha_i \quad 0]^{\mathrm{T}}$$

2.2 Coordinate Frames Description

Description of the coordinate frames is required for the forward displacement problem. For these analyses, we define three coordinate frames as depicted in Fig. 3.

- The fixed coordinate frame, {B}, is located on the center of the fixed platform and is called base coordinate frame. The x_B axis of this frame is along the unit vector $\mathbf{u}_1$. The z_B axis is defined along the vector $\mathbf{u}_1 \times \mathbf{u}_2$. The y_B axis also can be specified by the right hand rule.
- The moving coordinate frame, {M}, is attached to the moving platform that its origin is coincided on the fixed spherical joint. The x_M axis of this frame is along the unit vector $\mathbf{v}_1$.The z_M axis of this frame is along the perpendicular to the moving platform in direction vector $\mathbf{v}_1 \times \mathbf{v}_2$. The y_M axis can be specified by the right hand rule.

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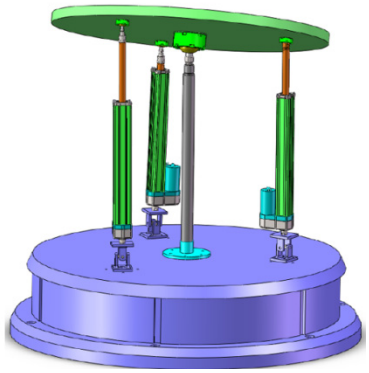


Fig. 1 Physical model with actuators.

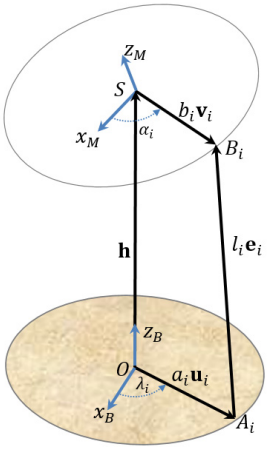


Fig. 2 Geometric parameters of i[th] branch.

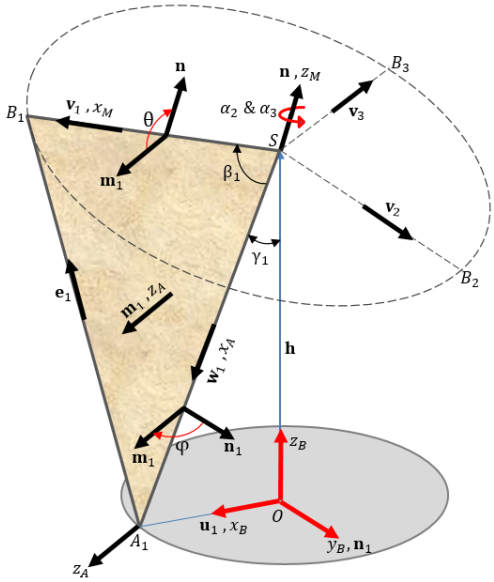


Fig. 3 Geometric parameters and the unit vectors of 1[st] branch.

- The moving coordinate frame {A} is attached to plane E that is called link coordinate frame. The x_A axis of this frame is along the unit vector $\mathbf{w}_1$. The z_A axis of this frame is along the vector $\mathbf{w}_1 \times \mathbf{e}_1$ that is perpendicular to plane E. This direction is shown by the unit vector $\mathbf{m}_1$. The y_{A_1} axis can also be specified by the right hand rule.

2.3 Rotation Matrix

To define rotation matrix, there are two common methods; the space fixed rotation and the body fixed rotation. For 3(RPSP)-S SPM the space fixed rotation is chosen. Roll(φ_x), pitch(φ_y) and yaw(φ_z) angles are rotation angles around the fixed x, y and z axes, respectively. If c means cosine and s means sine, the rotation matrix of the moving coordinate frame with respect to the base coordinate frame writes as

$$
{}^{B}_{M}\mathbf{R} = \begin{bmatrix} c\varphi_z c\varphi_y & c\varphi_z s\varphi_y s\varphi_x - s\varphi_z c\varphi_x & c\varphi_z s\varphi_y c\varphi_x + s\varphi_z s\varphi_x \\ s\varphi_z c\varphi_y & s\varphi_z s\varphi_y s\varphi_x + c\varphi_z c\varphi_x & s\varphi_z s\varphi_y c\varphi_x - c\varphi_z s\varphi_x \\ -s\varphi_y & c\varphi_y c\varphi_x & c\varphi_y c\varphi_x \end{bmatrix} \tag{2}
$$

Also, we can define ${}^{B}_{M}\mathbf{R}$ using the unit vectors $\mathbf{n}$ and $\mathbf{v}_1$. As it is shown in Fig. 3, the unit vector $\mathbf{n}$ is perpendicular to the moving platform (along axis z_M) and the unit vector $\mathbf{v}_1$ is along axis x_M. Therefore, the unit vector $\mathbf{n} \times \mathbf{v}_1$ will represent the axis y_M. Therefore, the rotation matrix of the moving platform with respect to the fixed platform can be defined as

$$
{}^{B}_{M}\mathbf{R} = [\mathbf{v}_1 \quad \mathbf{n} \times \mathbf{v}_1 \quad \mathbf{n}]^{T} \tag{3}
$$

In the forward displacement analysis of the manipulator, we find the unit vectors $\mathbf{n}$ and $\mathbf{v}_1$. Therefore, we can obtain the rotation matrix of the moving platform with respect to the fixed platform by using the above equation. Since Eqs. (2) and (3) are equivalent, we can determine the Roll, Pitch and Yaw angles as

$$
\varphi_y = Atan2\left(-{}^{B}_{M}\mathbf{R}(3,1), \left(({}^{B}_{M}\mathbf{R}(1,1))^2 + ({}^{B}_{M}\mathbf{R}(2,1))^2\right)^{0.5}\right) \tag{4a}
$$

$$
\varphi_z = Atan2\left(\left({}^{B}_{M}\mathbf{R}(2,1)\right)/c\varphi_y, \left({}^{B}_{M}\mathbf{R}(1,1)\right)/c\varphi_y\right) \tag{4b}
$$

$$
\varphi_x = Atan2\left(\left({}^{B}_{M}\mathbf{R}(3,2)\right)/c\varphi_y, \left({}^{B}_{M}\mathbf{R}(3,3)\right)/c\varphi_y\right) \tag{4c}
$$

where, the ${}^{B}_{M}\mathbf{R}(m,n)$ represents m^{th} row and n^{th} column of the rotation matrix ${}^{B}_{M}\mathbf{R}$.

Also, the rotation matrix of the moving coordinate frame {A} with respect to the base coordinate frame {B} can be defined as

$$
{}^{B}_{A}\mathbf{R} = [\mathbf{w}_1 \quad \mathbf{m}_1 \times \mathbf{w}_1 \quad \mathbf{m}_1]^{T} \tag{5}
$$

3. DISPLACEMENT ANALYSIS

3.1 Mathematical Tool

Before presenting the forward displacement problem of the manipulator, it is useful to consider matrix representation of a rotation presented by Rodrigues' rotation formula. The Rodrigues' form states that the rotation matrix can be represented as

$$
\mathbf{Q}(\mathbf{e}, \varphi) = \mathbf{I}_{3\times3} + (1 - \cos\varphi)\mathbf{E}^2 + \sin\varphi\,\mathbf{E} \tag{6}
$$

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where the unit vector $\mathbf{e}$ is the axis of rotation and φ is the angle of rotation about the unit vector $\mathbf{e}$. Also, $\mathbf{E}$ is a skew symmetric matrix that is formed by the unit vector $\mathbf{e}$ as

$$\mathbf{E} = \begin{bmatrix} 0 & -e_z & e_y \\ e_z & 0 & -e_x \\ -e_y & e_x & 0 \end{bmatrix} \tag{7}$$

where e_x, e_y, e_z are Cartesian components of the unit vector $\mathbf{e}$. Consider a vector $\mathbf{v}$ that is transformed by matrix $\mathbf{Q}(\mathbf{e}, \varphi)$. The resulting vector $\mathbf{v}'$, transformed vector, can be seen as

$$\mathbf{v}' = \mathbf{Q}(\mathbf{e}, \varphi)\mathbf{v} = \mathbf{v} + (1 - \cos\varphi)\mathbf{E}^2\mathbf{v} + \sin\varphi\,\mathbf{E}\mathbf{v} \tag{8}$$

The above equation can also be shown as

$$\mathbf{v}' = \mathbf{v} + (1 - \cos\varphi)(\mathbf{e} \times (\mathbf{e} \times \mathbf{v})) + \sin\varphi\,(\mathbf{e} \times \mathbf{v}) \tag{9}$$

Therefore, linear operator of rotation matrix can be written as follows

$$\mathbf{Q}(\mathbf{e}, \varphi) = \cos\varphi\,\mathbf{I}_{3\times3} + (1 - \cos\varphi)\mathbf{e}\mathbf{e}^{\mathrm{T}} + \sin\varphi\,\mathbf{e}\times \tag{10}$$

The above equation is a special form of Rodrigues' rotation formula. This form will be used in the next sections.

3.2 Modeling of the Forward Displacement Analysis

The forward displacement analysis, FDA, finds the orientation of the end-effector given the actuator positions. Unlike the serial manipulators, the FDA for parallel manipulators is more complex and results in a set of nonlinear equations. The nonlinear equations are usually transformed into a single high degree polynomial. For any given manipulator, there exist different modeling methods to derive the forward displacement problem. Among these methods, the method that results in the lowest order polynomial is the superior method. Furthermore, the modeling method is optimal if we can find an example where the number of real answers is equal to the order of the polynomial.

In this section, we propose a novel modeling approach to the forward displacement problem of a 3UPS-S fully spherical manipulator. For this purpose, we obtain two trigonometric equations using a special form of Rodrigues' rotation formula as well as the structure of the manipulator. Next, using Bezout's elimination method, the two trigonometric equations are transformed into a single algebraic equation. The solutions of this equation are answers of the forward displacement analysis.

Now, we define the forward displacement problem of the manipulator as

Given the constant geometric parameters $(\lambda_i, \alpha_i, b_i, a_i$ *and* $h)$ *and the variable parameters of the actuators* $(l_1,\ l_2,\ l_3)$*, obtain the rotation matrix of the moving platform with respect to the base plate,* ${}^{\mathrm{B}}_{\mathrm{M}}\mathbf{R}$.

Step 1: Obtaining the unit vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ in the moving coordinate frame, {A}, in terms of an unknown angle, θ.

See Fig. 3. First, we can represent the unit vectors $\mathbf{w}_1$, $\mathbf{m}_1$ and $\mathbf{v}_1$ in the coordinate frame {A} as

$${}^{\mathrm{A}}\mathbf{w}_1 = [1 \quad 0 \quad 0]^{\mathrm{T}} \tag{11a}$$

$${}^{\mathrm{A}}\mathbf{m}_1 = [0 \quad 0 \quad 1]^{\mathrm{T}} \tag{11b}$$

$${}^{\mathrm{A}}\mathbf{v}_1 = [cos\beta_1 \quad -sin\beta_1 \quad 0]^T \tag{11c}$$

where angle β_1 can be easily calculated from the triangle A_1SB_1. Since both the unit vectors $\mathbf{m}_1$ and $\mathbf{n}$ are perpendicular to the unit vector $\mathbf{v}_1$, we can obtain the unit vector $\mathbf{n}$ by rotating the unit vector $\mathbf{m}_1$ about the unit vector $\mathbf{v}_1$ by angle θ. According to Rodrigues' rotation formula, we can obtain the unit vector $\mathbf{n}$ as a function of the unit vectors $\mathbf{m}_1$, $\mathbf{v}_1$ and the angle θ. This function can be written in the link coordinate frame $\{A\}$ as

$$\begin{aligned} {}^{\mathrm{A}}\mathbf{n} &= \mathbf{Q}\left({}^{\mathrm{A}}\mathbf{v}_1, \theta\right){}^{\mathrm{A}}\mathbf{m}_1 = cos\theta\,{}^{\mathrm{A}}\mathbf{m}_1 \\ &\quad +(1 - cos\theta)\,{}^{\mathrm{A}}\mathbf{v}_1\,{}^{\mathrm{A}}\mathbf{v}_1^{\mathrm{T}}\,{}^{\mathrm{A}}\mathbf{m}_1 + sin\theta({}^{\mathrm{A}}\mathbf{v}_1 \times {}^{\mathrm{A}}\mathbf{m}_1) \end{aligned} \tag{12}$$

As stated before, the unit vector $\mathbf{v}_1$ is always perpendicular to the unit vector $\mathbf{m}_1$. Therefore, we have

$$\mathbf{v}_1 \perp \mathbf{m}_1 \Longrightarrow {}^{\mathrm{A}}\mathbf{n} = cos\theta\,{}^{\mathrm{A}}\mathbf{m}_1 + sin\theta({}^{\mathrm{A}}\mathbf{v}_1 \times {}^{\mathrm{A}}\mathbf{m}_1) \tag{13}$$

We now substitute ${}^{\mathrm{A}}\mathbf{m}_1$ and ${}^{\mathrm{A}}\mathbf{v}_1$ from equations (11b) and (11c), respectively, into (13) and rewrite it as

$${}^{\mathrm{A}}\mathbf{n} = [-sin\theta sin\beta_1 \quad -sin\theta cos\beta_1 \quad cos\theta]^T \tag{14}$$

Note that the unit vector ${}^{\mathrm{A}}\mathbf{n}$ is a function of an unknown angle θ. As it is shown in Fig. 3, the unit vectors $\mathbf{v_2}$ and $\mathbf{v_3}$ can be obtained by rotating the unit vector $\mathbf{v}_1$ about the unit vector $\mathbf{n}$ by angles α_2 and α_3 in the positive direction, respectively. Note that the two known angles α_2 and α_3 depend on the physical structure of the manipulator. Therefore, the unit vectors $\mathbf{v_2}$ and $\mathbf{v_3}$ in the link coordinate frame using Rodrigues' rotation formula are obtained as

$$\begin{aligned} {}^{\mathrm{A}}\mathbf{v}_2 &= \mathbf{Q}\left({}^{\mathrm{A}}\mathbf{n}, \alpha_2\right){}^{\mathrm{A}}\mathbf{v}_1 \\ &= cos\alpha_2\,{}^{\mathrm{A}}\mathbf{v}_1 + \left(1 - cos\alpha_2\right){}^{\mathrm{A}}\mathbf{n}\,{}^{\mathrm{A}}\mathbf{n}^{\mathrm{T}}\,{}^{\mathrm{A}}\mathbf{v}_1 + \\ &\quad sin\alpha_2\left({}^{\mathrm{A}}\mathbf{n} \times {}^{\mathrm{A}}\mathbf{v}_1\right) \end{aligned} \tag{15a}$$

$$\begin{aligned} {}^{\mathrm{A}}\mathbf{v}_3 &= \mathbf{Q}\left({}^{\mathrm{A}}\mathbf{n}, \alpha_3\right){}^{\mathrm{A}}\mathbf{v}_1 \\ &= cos\alpha_3\,{}^{\mathrm{A}}\mathbf{v}_1 + \left(1 - cos\alpha_3\right){}^{\mathrm{A}}\mathbf{n}\,{}^{\mathrm{A}}\mathbf{n}^{\mathrm{T}}\,{}^{\mathrm{A}}\mathbf{v}_1 + \\ &\quad sin\alpha_3({}^{\mathrm{A}}\mathbf{n} \times {}^{\mathrm{A}}\mathbf{v}_1) \end{aligned} \tag{15b}$$

The unit vector $\mathbf{n}$ is perpendicular to the unit vector $\mathbf{v_1}$. Therefore, Eqs. (15a) and (15b) can be simplified as

$$\mathbf{v}_1 \perp \mathbf{n} \Longrightarrow {}^{\mathrm{A}}\mathbf{v}_2 = cos\alpha_2\,{}^{\mathrm{A}}\mathbf{v}_1 + sin\alpha_2({}^{\mathrm{A}}\mathbf{n} \times {}^{\mathrm{A}}\mathbf{v}_1) \tag{16a}$$

$$\mathbf{v}_1 \perp \mathbf{n} \Longrightarrow {}^{\mathrm{A}}\mathbf{v}_3 = cos\alpha_3\,{}^{\mathrm{A}}\mathbf{v}_1 + sin\alpha_3({}^{\mathrm{A}}\mathbf{n} \times {}^{\mathrm{A}}\mathbf{v}_1) \tag{16b}$$

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Substitution of ${}^{\mathrm{A}}\mathbf{v}_1$ and ${}^{\mathrm{A}}\mathbf{n}$ from Eqs. (11c) and (14) into the above equations and rewriting them leads

$$ {}^{\mathrm{A}}\mathbf{v}_2 = \left[c\alpha_2 c\beta_1 + s\alpha_2 s\beta_1 c\theta \quad s\alpha_2 c\beta_1 c\theta - c\alpha_2 s\beta_1 \quad s\alpha_2 s\theta\right]^T \tag{17a} $$

$$ {}^{\mathrm{A}}\mathbf{v}_3 = \left[c\alpha_3 c\beta_1 + s\alpha_3 s\beta_1 c\theta \quad s\alpha_3 c\beta_1 c\theta - c\alpha_3 s\beta_1 \quad s\alpha_3 s\theta\right]^T \tag{17b} $$

Therefore, as shown in the above equations, both the unit vectors ${}^{\mathrm{A}}\mathbf{v}_2$ and ${}^{\mathrm{A}}\mathbf{v}_3$ are also a function of an unknown angle θ.

Step 2: Determining the unit vector $\mathbf{m}_1$ in terms of an unknown angle, φ.

Since both the unit vectors $\mathbf{n}_1$ and $\mathbf{m}_1$ are perpendicular to the unit vector $\mathbf{w}_1$, the unit vector $\mathbf{m}_1$ can be obtained by rotating the unit vector $\mathbf{n}_1$ about the unit vector $\mathbf{w}_1$ by an unknown angle φ in the positive direction. Therefore, according to Rodrigues' rotation formula, the unit vector $\mathbf{m}_1$ can be written in the link coordinate frame $\{\mathbf{A}\}$ as

$$ \mathbf{m}_1 = \mathbf{Q}(\mathbf{w}_1, \varphi)\mathbf{n}_1 = cos\varphi\,\mathbf{n}_1 + (1 - cos\varphi)\mathbf{w}_1\mathbf{w}_1^{\mathrm{T}}\mathbf{n}_1 + sin\varphi(\mathbf{w}_1 \times \mathbf{n}_1) \tag{18} $$

As mentioned before, the unit vector $\mathbf{m}_1$ is always perpendicular to the unit vector $\mathbf{w}_1$. Therefore, Eq. (18) can be simplified as

$$ \mathbf{m}_1 \perp \mathbf{w}_1 \Longrightarrow \mathbf{m}_1 = cos\varphi\,\mathbf{n}_1 + sin\varphi(\mathbf{w}_1 \times \mathbf{n}_1) \tag{19} $$

Without loss of generality, we assume $\lambda_1 = 0$ and $\mathbf{u}_1 \equiv x_B$. Therefore, we can write

$$ \mathbf{n}_1 = [0 \quad 1 \quad 0]^{\mathrm{T}} \equiv y_B, \quad \mathbf{w}_1 = [\sin\gamma_1 \quad 0 \quad -\cos\gamma_1]^{\mathrm{T}} \tag{20} $$

Substituting the unit vectors $\mathbf{w}_1$ and $\mathbf{n}_1$ from the above equation into (19), yields

$$ \mathbf{m}_1 = [sin\varphi\ \cos\gamma_1 \quad cos\varphi \quad sin\varphi\ \sin\gamma_1]^{\mathrm{T}} \tag{21} $$

Step 3: Finding the rotation matrix ${}^{\mathrm{B}}_{\mathrm{A}}\mathbf{R}$ in terms of the unknown angle φ.

For this purpose, we substitute the unit vectors $\mathbf{w}_1$ and $\mathbf{m}_1$ from Eqs. (20) and (21) into (5). Therefore, we will have

$$ {}^{\mathrm{B}}_{\mathrm{A}}\mathbf{R} = \begin{bmatrix} \sin\gamma_1 & -cos\varphi\cos\gamma_1 & sin\varphi\ \cos\gamma_1 \\ 0 & sin\varphi & cos\varphi \\ -\cos\gamma_1 & -cos\varphi\sin\gamma_1 & sin\varphi\ \sin\gamma_1 \end{bmatrix} \tag{22} $$

Step 4: Obtaining the unit vector $\mathbf{w}_i$ in the link coordinate frame {A}.

The unit vector $\mathbf{w}_i$ can be computed from Eq. (1a) in the base coordinate frame. Now, we can represent the unit vectors $\mathbf{w}_i$ in the link coordinate frame {A} using Eq. (22) in terms of the unknown angle φ as follows

$$ {}^{\mathrm{A}}\mathbf{w}_i = {}^{\mathrm{A}}_{\mathrm{B}}\mathbf{R}\,\mathbf{w}_i = \left({}^{\mathrm{B}}_{\mathrm{A}}\mathbf{R}\right)^{\mathrm{T}}\mathbf{w}_i \tag{23} $$

Step 5: Obtaining two trigonometric equations

See Fig. 3. According to the structure of the manipulator, the two trigonometric equations are now formulated by noting that

$$ \mathbf{v}_2^{\mathrm{T}}\mathbf{w}_2 = {}^{\mathrm{A}}\mathbf{v}_2^{\mathrm{T}}\,{}^{\mathrm{A}}\mathbf{w}_2 = \cos\beta_2 \tag{24a} $$

$$ \mathbf{v}_3^{\mathrm{T}}\mathbf{w}_3 = {}^{\mathrm{A}}\mathbf{v}_3^{\mathrm{T}}\,{}^{\mathrm{A}}\mathbf{w}_3 = \cos\beta_3 \tag{24b} $$

Next, we substitute the constant geometric parameters of the manipulator and ${}^{\mathrm{A}}\mathbf{w}_2$, ${}^{\mathrm{A}}\mathbf{w}_3$, ${}^{\mathrm{A}}\mathbf{v}_2$ and ${}^{\mathrm{A}}\mathbf{v}_3$ from the previous steps into Eqs. (24a) and (24b) and rewrite them as follows

$$ p_1 sin\theta + p_2 cos\theta + p_3 = 0 \tag{25a} $$

$$ q_1 sin\theta + q_2 cos\theta + q_3 = 0 \tag{25b} $$

where

$$ p_1 = p_{11} sin\varphi + p_{12} cos\varphi \tag{26a} $$

$$ p_2 = p_{21} sin\varphi + p_{22} cos\varphi + p_{23} \tag{26b} $$

$$ p_3 = p_{31} sin\varphi + p_{32} cos\varphi + p_{33} \tag{26c} $$

$$ q_1 = q_{11} sin\varphi + q_{12} cos\varphi \tag{26d} $$

$$ q_2 = q_{21} sin\varphi + q_{22} cos\varphi + q_{23} \tag{26e} $$

$$ q_3 = q_{31} sin\varphi + q_{32} cos\varphi + q_{33} \tag{26f} $$

where

$p_{11} = \sin(\alpha_2)\cos(\gamma_1)\sin(\gamma_2)\cos(\lambda_2) - \sin(\alpha_2)\sin(\gamma_1)\cos(\gamma_2)$

$p_{12} = \sin(\alpha_2)\sin(\gamma_2)\sin(\lambda_2)$

$p_{21} = \sin(\alpha_2)\cos(\beta_1)\sin(\gamma_2)\sin(\lambda_2)$

$p_{22} = -\sin(\alpha_2)\cos(\beta_1)\cos(\gamma_1)\sin(\gamma_2)\cos(\lambda_2) + \sin(\alpha_2)\cos(\beta_1)\sin(\gamma_1)\cos(\gamma_2)$

$p_{23} = \sin(\alpha_2)\sin(\beta_1)\sin(\gamma_1)\sin(\gamma_2)\cos(\lambda_2) + \sin(\alpha_2)\sin(\beta_1)\cos(\gamma_1)\cos(\gamma_2)$

$p_{31} = -\cos(\alpha_2)\sin(\beta_1)\sin(\gamma_2)\sin(\lambda_2)$

$p_{32} = \cos(\alpha_2)\sin(\beta_1)\cos(\gamma_1)\sin(\gamma_2)\cos(\lambda_2) - \cos(\alpha_2)\sin(\beta_1)\sin(\gamma_1)\cos(\gamma_2)$

$p_{33} = \cos(\alpha_2)\cos(\beta_1)\sin(\gamma_1)\sin(\gamma_2)\cos(\lambda_2) + \cos(\alpha_2)\cos(\beta_1)\cos(\gamma_1)\cos(\gamma_2) - \cos(\beta_2)$

$q_{11} = \sin(\alpha_3)\cos(\gamma_1)\sin(\gamma_3)\cos(\lambda_3) - \sin(\alpha_3)\sin(\gamma_1)\cos(\gamma_3)$

$q_{12} = \sin(\alpha_3)\sin(\gamma_3)\sin(\lambda_3)$

$q_{21} = \sin(\alpha_3)\cos(\beta_1)\sin(\gamma_3)\sin(\lambda_3)$

$q_{22} = -\sin(\alpha_3)\cos(\beta_1)\cos(\gamma_1)\sin(\gamma_3)\cos(\lambda_3) + \sin(\alpha_3)\cos(\beta_1)\sin(\gamma_1)\cos(\gamma_3)$

$q_{23} = \sin(\alpha_3)\sin(\beta_1)\sin(\gamma_1)\sin(\gamma_3)\cos(\lambda_3) + \sin(\alpha_3)\sin(\beta_1)\cos(\gamma_1)\cos(\gamma_3)$

$q_{31} = -\cos(\alpha_3)\sin(\beta_1)\sin(\gamma_3)\sin(\lambda_3)$

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$q_{32} = \cos(\alpha_3)\sin(\beta_1)\cos(\gamma_1)\sin(\gamma_3)\cos(\lambda_3) - \cos(\alpha_3)\sin(\beta_1)\sin(\gamma_1)\cos(\gamma_3)$

$q_{33} = \cos(\alpha_3)\cos(\beta_1)\sin(\gamma_1)\sin(\gamma_3)\cos(\lambda_3) + \cos(\alpha_3)\cos(\beta_1)\cos(\gamma_1)\cos(\gamma_3) - \cos(\beta_3)$ (27)

Step 6: Transforming the trigonometric equations into a set of polynomials

To simplify nonlinear Eqs. (25a) and (25b), the trigonometric equations must be transformed into a set of polynomials. This transformation can be achieved by using the following trigonometric identities

$$sin\theta = \frac{2x_1}{1+{x_1}^2}\ ,\ cos\theta = \frac{1-{x_1}^2}{1+{x_1}^2},$$

$$sin\varphi = \frac{2x_2}{1+{x_2}^2}\ ,\ cos\varphi = \frac{1-{x_2}^2}{1+{x_2}^2} \quad (28)$$

where

$$x_1 = tan(\theta/2)\ ,\qquad x_2 = tan(\varphi/2)$$

These steps are performed using MAPLE software. Result of manual computing is given as

$$(F_1x_2^2 + F_2x_2 + F_3)x_1^2 + (F_4x_2^2 + F_5x_2 + F_6)x_1 + (F_7x_2^2 + F_8x_2 + F_9) = 0 \quad (29a)$$

$$(E_1x_2^2 + E_2x_2 + E_3)x_1^2 + (E_4x_2^2 + E_5x_2 + E_6)x_1 + (E_7x_2^2 + E_8x_2 + E_9) = 0 \quad (29b)$$

where values F_1 to F_9 and E_1 to E_9 are defined as

$$F_1 = p_{22} - p_{23} - p_{32} + p_{33},\ F_2 = -2\,p_{21} + 2\,p_{31},\ F_3 = -p_{22} - p_{23} + p_{32} + p_{33}$$

$$F_4 = -2\,p_{12},\ F_5 = 4\,p_{11},\ F_6 = 2\,p_{12},\ F_7 = -p_{22} + p_{23} - p_{32} + p_{33}$$

$$F_8 = 2\,p_{21} + 2\,p_{31},\ F_9 = p_{22} + p_{23} + p_{32} + p_{33}$$

$$E_1 = q_{22} - q_{23} - q_{32} + q_{33},\ E_2 = -2\,q_{21} + 2\,q_{31},\ E_3 = -q_{22} - q_{23} + q_{32} + q_{33},$$

$$E_4 = -2\,q_{12},\ E_5 = 4\,q_{11},\ E_6 = 2\,q_{12},\ E_7 = -q_{22} + q_{23} - q_{32} + q_{33},$$

$$E_8 = 2\,q_{21} + 2\,q_{31},\ E_9 = q_{22} + q_{23} + q_{32} + q_{33} \quad (30)$$

Step 7: Reducing the two coupled polynomials to one polynomial

Bezout's elimination method is traditionally used for reducing a set of polynomials of multiple variables into a polynomial of only one variable [14]. Using Bezout's elimination method, the variable x_1 is eliminated from Eqs. (29a) and (29b). Therefore, one single variable polynomial of degree eight is computed as

$$\sum_{p=0}^{8} N_p x_2^p = 0 \quad (31)$$

where values N_0 to N_8 are defined in terms of F_1 to F_9 and E_1 to E_9. These coefficients are functions of the constant geometric parameters $(\lambda_i, \alpha_i, b_i, a_i, h)$ and the variable parameters of the actuators (l_1, l_2, l_3).

Step 8: Computing the Roll, Pitch and Yaw angles and specifying assembly modes

First, the above polynomial is solved by MATLAB software. Next, the angle of φ can be calculated using the Eq. (28). Therefore, the angle θ is easily computed by substituting the value φ into Eqs. (25a) and (25b). Now, we can compute ${}^{A}\mathbf{n}$ and ${}^{B}_{A}\mathbf{R}$ from Eqs. (14) and (22), respectively, and then determine the unit vectors $\mathbf{n}$ and $\mathbf{v}_1$ in the base coordinate frame as

$$\mathbf{n} = {}^{B}_{A}\mathbf{R}\,{}^{A}\mathbf{n} \quad (32a)$$

$$\mathbf{v}_1 = {}^{B}_{A}\mathbf{R}\,{}^{A}\mathbf{v}_1 \quad (32b)$$

Using the above equations and Eq. (3), the rotation matrix of the moving platform with respect to the fixed platform, ${}^{B}_{M}\mathbf{R}$, can be calculated. Finally, we can compute the Roll, Pitch and Yaw angles using Eqs. (4a), (4b) and (4c), and complete solving the forward displacement analysis of the manipulator.

It is important to point out that the polynomial admits eight solutions, which may be real and/or complex. If we can find an example, having eight real solutions, we can claim the polynomial is minimal which indicates the proposed modeling method for solving the forward displacement problem of the manipulator is optimal. This greatly decreases the computational time, which is necessary for dynamics modeling, control and simulation of the manipulator.

3.3 Assembly Modes (Configurations) of a Case Study

In this subsection, an example for solving the forward displacement problem of the manipulator and obtaining its assembly modes is presented. In the forward displacement problem, the constant geometric parameters $(\lambda_i, \alpha_i, b_i, a_i$ and $h)$ and the variable parameter of each actuator (l_1, l_2, l_3), are supplied. The forward displacement problem will determine the orientation of the moving platform with respect to the fixed platform, ${}^{B}_{M}\mathbf{R}$, by obtaining the two angles θ and φ.

(a) Architecture Parameters of the Manipulator

Assume that $h = 1\text{m}$, $a_i = 0.55\text{m}$, $b_i = 1\text{m}$, $\alpha_i = \lambda_i = 2(i-1)\pi/3\,\text{rad}$. Therefore, we have

$$\mathbf{u}_1 = {}^{M}\mathbf{v}_1 = [1 \quad 0 \quad 0]^{T}$$

$$\mathbf{u}_2 = {}^{M}\mathbf{v}_2 = [-1/2 \quad \sqrt{2}/2 \quad 0]^{T}$$

$$\mathbf{u}_3 = {}^{M}\mathbf{v}_3 = [-1/2 \quad -\sqrt{2}/2 \quad 0]^{T}$$

$$\mathbf{h} = [0 \quad 0 \quad 1]^{T}$$

Therefore, the unit vector $\mathbf{w}_i$ is computed as

$$\mathbf{w}_1 = [-0.4819 \quad 0 \quad 0.8762]^{T},$$

$$\mathbf{w}_2 = [0.2410 \quad -0.4174 \quad 0.8762]^{T},$$

$$\mathbf{w}_3 = [0.2410 \quad 0.4174 \quad 0.8762]^{T}$$

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(b) Numerical Example

For numerical example, assume that the stroke of the actuators is defined as

$$l_1 = 1.55\,, l_2 = 1.2\,, l_3 = 1.75$$

Now, we can easily calculate the angle β_i with regard to the triangle A_iSB_i.

$$\beta_1 = 92.5110°\,, \beta_2 = 67.7983°\,, \beta_3 = 109.4487°$$

We substitute these values from the previous steps. Therefore, we can compute the coefficients of equation (31) as shown in Table 1. Therefore, values of φ and θ can be calculated. The results are listed in Table 2. Since the polynomial of degree eight results eight real solutions, the degree of the polynomial in Eq. (31) is minimal and optimal. According to these results, the solutions (assembly modes) are shown graphically in Figs. 4 ~ 11.

As we know, a type of singularity in parallel manipulators occurs when a point or a set of points whereby different branches of the forward displacement problem meet [29]. In this type, the manipulator gains one or more uncontrollable degrees of freedom or, equivalently, can not resist moments in one or more directions, even if all the actuators are locked. Therefore, we can avoid the problems of the singularities based on the calculation of the forward displacement problem. Furthermore, the performance indices (such as global conditioning index, global stiffness index and *etc*.), for all assembly modes are shown in Figs. 4 ~ 11, can be calculated to select one of assembly modes as a home posture for the manipulator.

Table 1 Values of coefficients N_i.

N_8	N_7	N_6	N_5	N_4	N_3	N_2	N_1	N_0
1.2853	6.5956	9.9172	–0.3594	–11.3438	–6.2256	0.6588	0.7295	0.0004

Table 2 Solutions of the numerical example.

Solution (assembly mode)	φ	θ	Roll(φ_x)	Pitch(φ_y)	Yaw(φ_z)
1	–89.3470°	88.4131°	–152.4645°	8.0243°	85.9523°
2	–91.3868°	–0.0573°	119.0284°	–8.9617°	84.1771°
3	87.6855°	34.9676°	95.9626°	–7.2558°	–87.4008°
4	6.0504°	31.7703°	24.0653°	22.6259°	–172.4059°
5	171.7785°	–64.2859°	25.2014°	–33.0504°	–8.5785°
6	66.9215°	–119.7023°	–55.0278°	2.5196°	–105.9383°
7	92.0743°	–123.3807°	–62.6777°	–9.2761°	–83.5793°
8	116.2188°	–117.7199°	–56.3738°	–19.5719°	–62.4574°

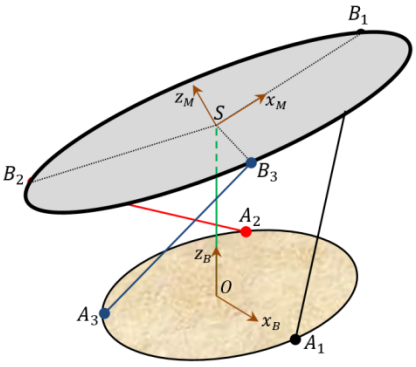


Fig. 4 Assembly mode 1.

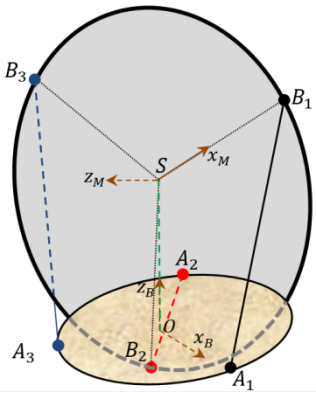


Fig. 5 Assembly mode 2.

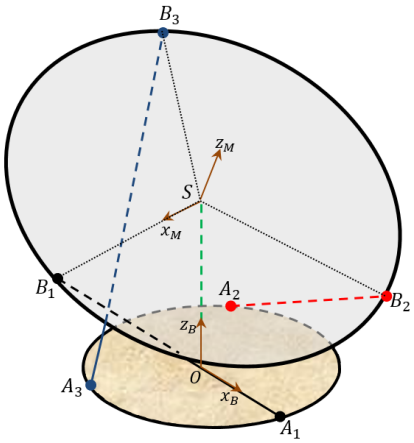


Fig. 6 Assembly mode 3.

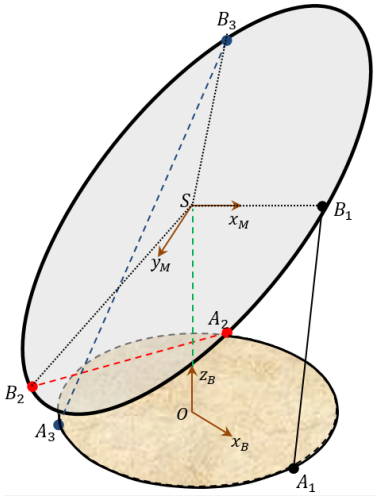


Fig. 7 Assembly mode 4.

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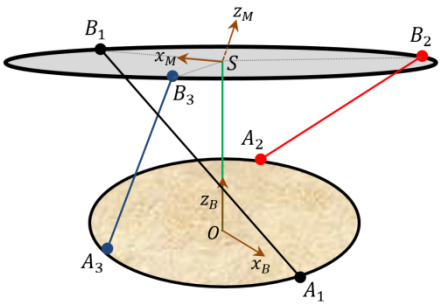


Fig. 8 Assembly mode 5.

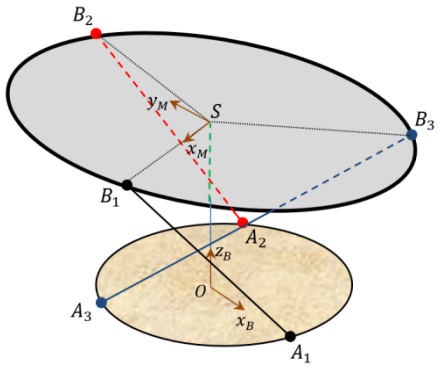


Fig. 9 Assembly mode 6.

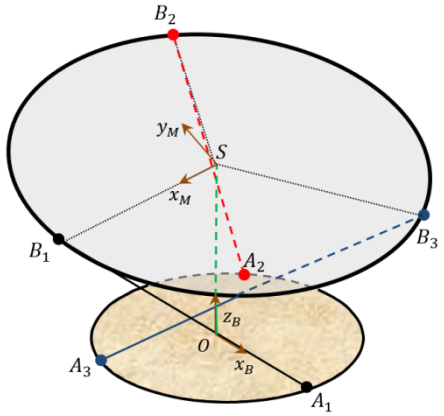


Fig. 10 Assembly mode 7.

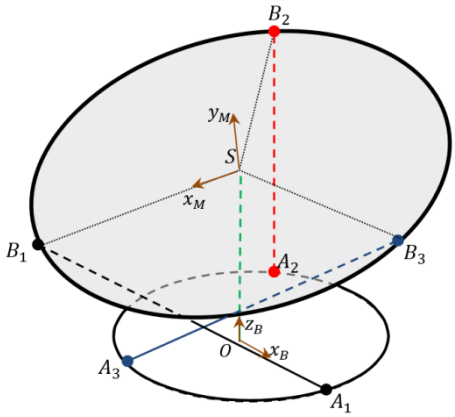


Fig. 11 Assembly mode 8.

4. CONCLUSIONS

A novel approach modeling for the forward displacement analysis of a 3UPS-S fully spherical parallel manipulator was proposed. First, the geometry of the manipulator was introduced as well as coordinate frames. Analysis of the forward displacement problem was performed using mathematical tools, Rodrigues' rotation formula and Bezout's elimination method. Next, two coupled trigonometric equations were obtained. The two coupled equations were transformed to a polynomial of degree eight. This is an advantage compared to the proposed methods by Parenti-Castelli [9] and Ji and Wu [28]. Furthermore, with the proposed method in this paper, the forward displacement analysis equations are derived with compact coefficients, which leads to a robust procedure. Therefore, the proposed method can be easily used to avoid singularity configuration, selecting one of assembly modes as a home posture according to the performance indices, real-time path generation using selecting the admissible solution of the forward displacement analysis [30], real-time control and dynamics simulation. Finally, for an example was shown that the polynomial has eight real solutions. Therefore, the polynomial being minimal and the innovative method is optimal. Furthermore, the proposed method can be extended to the other spherical parallel manipulators (*e.g.*, 3(RRS)-S, 3(RPSP)-S and 3(PSS)-S).

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