J. Plasma Physics (2013), *vol.* 79, *part* 1, *pp.* 33–35. © Cambridge University Press 2012
doi:10.1017/S0022377812000633

Drift wave instability in a radially bounded dusty magnetoplasma with parallel ion velocity shear

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(Received 13 March 2012; revised 7 June 2012; accepted 8 June 2012; first published online 17 July 2012)

Abstract. Properties of the coupled dust ion-acoustic drift wave instability in a radially bounded dusty magnetoplasma with an equilibrium sheared parallel ion (SPI) flow are investigated. By using the two-fluid model for the electrons and ions, a wave equation for the low-frequency coupled dust ion-acoustic drift waves in a bounded plasma with stationary charged dust grains is derived. The wave equation admits a linear dispersion relation, which exhibits that the radial boundary affects the growth rate of the coupled ion-acoustic drift wave instability which is excited by the SPI flow. The results should be relevant to dusty magnetoplasma experiments with an SPI flow.

In his classic paper, D'Angelo (1965) demonstrated that the parallel sheared ion (SPI) flow can excite coupled ion-acoustic drift waves (Kadomtsev 1965; Shukla et al. 1981, 1984, 1995, 2002; Sorasio et al. 2004) that propagate obliquely to a homogeneous magnetic field, $\hat{\mathbf{z}}B_0$. These waves have low frequency (in comparison with the ion gyrofrequency ω_{ci}), low magnetic field-aligned phase speed (in comparison with the electron thermal speed), and long wavelength (in comparison with the thermal ion gyroradius). The model plasma for the above-cited studies is a low-$\beta(=8\pi n_0 k_B T_e/B_0^2)$ electron–ion plasma in which $\hat{\mathbf{z}}B_0$ is directed transverse to the equilibrium density and parallel ion velocity gradients. Here $\hat{\mathbf{z}}$ is the unit vector along the z-axis of a Cartesian coordinate system, B_0 the strength of the magnetic field, n_0 the unperturbed plasma number density, k_B the Boltzmann constant, and T_e the electron temperature. In the presence of the three-dimensional (3D) coupled ion-acoustic drift wave potential ϕ, the electrons follow a helical orbit with the $\mathbf{V}_E = (c/B_0)\hat{\mathbf{z}} \times \nabla\phi$ and $\mathbf{V}_D = -(ck_B T_e/eB_0 n_e)\hat{\mathbf{z}}\times\nabla n_e$ drifts, as well as a rapid magnetic field-aligned motion, where c is the speed of light in vacuum, and e is the magnitude of the electron charge. Due to the predominant motion along $\hat{\mathbf{z}}$, electrons may rapidly thermalize along $\hat{\mathbf{z}}$, establishing the Boltzmann law for the density distribution $n_e = n_{e0}\exp(e\phi/k_B T_e)$, where n_{e0} is the unperturbed electron number density. The Boltzmann relationship between the electron density n_e and the wave potential comes about since the parallel (to $\hat{\mathbf{z}}$) electric force $e\partial\phi/\partial z$ is in balance with the pressure gradient $k_B T_e \partial \ln n_e/\partial z$. Furthermore, in the coupled ion-acoustic drift wave electric fields, 3D ions execute the $\mathbf{V}_E$ and the polarization ($\mathbf{V}_P = -(c/B_0\omega_{ci})\partial\nabla_\perp\phi/\partial t$) drifts, in addition to the magnetic field-aligned acceleration of the ions due to the parallel electric field. Since the parallel ion velocity gradient is convected by the $\mathbf{V}_E$ drift, the parallel ion velocity perturbation cannot keep in phase with the wave potential. This leads to an instability of the coupled ion-acoustic drift waves on account of the free energy stored in the equilibrium parallel ion velocity gradient. The parallel ion velocity shear (PIVS) instability has been observed in laboratory experiments (Willig et al. 1997a, b), and its importance has also been recognized in auroral plasmas (Migliuolo 1984; Basu and Coppi 1988, 1989; Ganguli 1996; Gavrishchaka et al. 1999) that are magnetized and non-uniform.

The presence of charged dust impurities, which are common in laboratory (Shukla and Mamun 2002; Shukla and Eliasson 2009) and fusion (Krasheninnikov et al. 2011) plasmas, can affect the properties of the ion-acoustic (Shukla and Silin 1992) and drift waves (Shukla et al. 1991) in a dusty plasma composed of electrons, ions, and charged dust. Charged dust impurities modify the equilibrium quasi-neutrality condition

$$n_{i0} = n_{e0} + \epsilon Z_d n_{d0}, \tag{1}$$

where ϵ equals 1 for negative dust and -1 for positive dust, Z_d is the number of charges residing on dust grain, and n_{i0} and n_{d0} are the unperturbed ion and dust number densities, respectively. The effect of stationary charged dust grains on PIVS instability in an infinite plasma has been studied both theoretically (D'Angelo and Song 1990; Bharuthram and Shukla 1992) and experimentally (Luo et al. 2001).

https://doi.org/10.1017/S0022377812000633 Published online by Cambridge University Press

In this paper, we present an investigation of PIVS instability involving the coupled dust ion-acoustic drift waves in a radially bounded non-uniform dusty magnetoplasma with the equilibrium density ($n_{j0}(r)$) and parallel ion velocity ($\hat{\mathbf{z}}u_0(r)$) gradients. Our plasma is composed of magnetized warm electrons and cold ions, as well as stationary charged dust impurities. In our cylindrical plasma column, the coupled dust ion-acoustic collisional drift waves propagate obliquely to the direction of the axial (the z-axis) magnetic field in a cylindrical system. The equilibrium density and parallel ion velocity gradients are $\partial(n_{j0})/\partial r$ and $\partial u_0/\partial r$, where j equals e for electrons, i for ions, and d for charged dust impurities.

The electron and ion number density perturbations, $n_{e1}(\ll n_{e0})$ and $n_{i1}(\ll n_{i0})$, associated with the low-frequency [viz. $|(\partial/\partial t)+u_0(\partial/\partial z)+\nu_i|\ll\omega_{ci}=eB_0/m_ic$ and $|\partial/\partial t|\ll\nu_e\ll\omega_{ce}=eB_0/m_ec$), where $m_i(m_e)$ is the ion (electron) mass and ν_e the electron collision frequency], long wavelength (in comparison with the ion gyroradius and $(v_e/\omega_{ce})\hat{\mathbf{z}}\times\nabla\ln n_{e0}\cdot\nabla\phi\ll\partial^2\phi/\partial z^2$), and low magnetic field-aligned phase speed (viz. $|\partial n_{e1}/\partial t|\ll(V_{Te}^2/\nu_e)\partial^2 n_{e1}/\partial z^2$, where V_{Te} is the electron thermal speed) electrostatic waves in our dusty magnetoplasma are given by, respectively,

$$\frac{n_{e1}}{n_{e0}}\approx\frac{e\phi}{k_BT_e}, \tag{2}$$

and

$$\left(\frac{\partial}{\partial t}+u_0\frac{\partial}{\partial z}\right)n_{i1}-c\hat{\mathbf{z}}\times\nabla n_{i0}\cdot\nabla\phi$$

$$-\frac{cn_{i0}}{B_0\omega_{ci}}\left(\frac{\partial}{\partial t}+u_0\frac{\partial}{\partial z}+\nu_i\right)\nabla_\perp^2\phi+n_{i0}\frac{\partial u_z}{\partial z}=0, \tag{3}$$

where the magnetic field-aligned ion fluid velocity perturbation u_z is determined from

$$\left(\frac{\partial}{\partial t}+u_0\frac{\partial}{\partial z}+\nu_i\right)u_z-\frac{c}{B_0}\hat{\mathbf{z}}\times\nabla u_0\cdot\nabla\phi+\frac{e}{m_i}\frac{\partial\phi}{\partial z}=0, \tag{4}$$

where ν_i is the ion-neutral collision frequency. Several comments are in order. The second and third terms in (3) represent the contributions of the divergence of $n_{i0}\mathbf{V}_E$ and $n_{i0}\mathbf{V}_P$, respectively, while the fourth term in (3) comes from the parallel component $n_{i0}\partial u_z/\partial z$. Furthermore, the second term on the left-hand side in (4) comes from the $\mathbf{V}_E$ advection of the equilibrium parallel ion velocity flow. Here the subscript $\perp$ denotes the perpendicular (to $\hat{\mathbf{z}}$) component of a physical quantity. Since in typical laboratory dusty plasmas, the ion temperature is much smaller than the electron temperature, we have neglected the ion pressure gradient, assuming that the ions are cold.

Substituting (1) into (4) from (3), and eliminating u_z from the resultant equation by using (4), one can derive the wave equation in our plasma with $n_{e1}=n_{i1}$; the latter holds for long wavelength (in comparison with the electron Debye radius in a dense dusty magnetoplasma with $\omega_{pi}\gg\omega_{ci}$, where ω_{pi} is the ion plasma frequency). The result is

$$\left(\frac{d}{dt}+\nu_i\right)\left[\frac{d\phi}{dt}-\alpha\rho_s^2\left(\frac{d}{dt}+\nu_i\right)\nabla_\perp^2\phi\right]$$

$$-\alpha\rho_s^2\hat{\mathbf{z}}\times\nabla\ln n_{i0}\cdot\nabla\left(\frac{d}{dt}+\nu_i\right)\phi$$

$$-\alpha C_s^2\frac{\partial^2\phi}{\partial z^2}+\alpha\rho_s^2\omega_{ci}(\hat{\mathbf{z}}\times\nabla u_0)\cdot\nabla\frac{\partial\phi}{\partial z}=0, \tag{5}$$

where $\alpha=n_{i0}/n_{e0}$, $C_s=(k_BT_e/m_i)^{1/2}$ is the ion-acoustic speed, and $\rho_s=C_s/\omega_{ci}$ is the ion-acoustic gyroradius. We have denoted the operator $d/dt=(\partial/\partial t)+u_0\partial/\partial z$.

Assuming that $\phi(r,\theta,z)=\varphi(r)\exp(-\omega t+im\theta+ik_zz)$, where ω is the wave frequency, m the azimuthal mode number, and k_z the component of the wave vector $\mathbf{k}$ along the axial direction, we Fourier transform (6) to obtain

$$\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}+D\varphi=0, \tag{6}$$

where we have denoted

$$D=-\frac{(\Omega-\Omega_*)}{(\Omega+i\nu_i)\alpha\rho_s^2}-k_\theta^2+\frac{\omega_{ci}^2k_z^2}{(\Omega+i\nu_i)^2}-\frac{k_zk_\theta\Omega_s\omega_{ci}}{(\Omega+i\nu_i)^2}, \tag{7}$$

with $\Omega=\omega-k_zu_0$, $\Omega_*=\alpha k_\theta C_s^2K_i/\omega_{ci}$, $K_i=-\partial\ln n_{i0}/\partial r>0$, and $\Omega_s=\partial u_0/\partial r$. Here k_θ $(=m/r)$ is the azimuthal component of the wave vector.

A possible solution of the zero-order Bessel Equation (6), which is finite at the origin, is $\varphi(r)=J_0(\sqrt{D}r)$, where J_0 is the Bessel function of zero order. The requirement of $\varphi=0$ at $r=R$, where R is the radius of our cylindrical dusty plasma, implies that $J_0(\sqrt{D}R)=0$. Thus,

$$\sqrt{D}R=\gamma_n, \tag{8}$$

which yields the dispersion relation

$$(\Omega-\Omega_*)(\Omega+i\nu_i)-\alpha k_z^2C_s^2+\frac{\alpha k_zk_\theta C_s^2\Omega_s}{\omega_{ci}}$$

$$+\alpha K^2\rho_s^2(\Omega+i\nu_i)^2=0, \tag{9}$$

where $K^2=k_\theta^2+\gamma_n^2/R^2\equiv k_\theta^2+k_r^2$, γ_n are the roots of J_0 (the first few zeros of J_0 are (Abramowitz and Stegun 1964; Shukla and Rosenberg 1999) $\gamma_1=2.4,\gamma_2=5.5,\gamma_3=8.7,\gamma_4=11.8$).

Letting $\Omega=\Omega_r+i\Omega_i$ in (9), and assuming that $\nu_i,\Omega_i\ll k_zu_0+\Omega_r$, we obtain

$$\Omega_r=\frac{1}{2}\Omega_{e*}\pm\frac{1}{2}\left(\Omega_{e*}^2+4\Omega_A^2-4\Omega_A^2\frac{k_\theta\Omega_s}{k_z\omega_{ci}}\right)^{1/2}, \tag{10}$$

which predicts an oscillatory instability with the real frequency $\omega_r=k_zu_0+\Omega_{e*}/2$ if

$$\Omega_s>\omega_{ci}\frac{k_z}{k_\theta}\left(1+\frac{\Omega_{e*}^2}{4\Omega_A^2}\right), \tag{11}$$

where $\Omega_{e*}=\Omega_*/(1+\alpha K^2\rho_s^2)$, and $\Omega_A=\sqrt{\alpha}k_zC_s/(1+\alpha K^2\rho_s^2)^{1/2}$. The growth rate above the threshold is $\Omega_A\left(k_\theta\Omega_s/k_z\omega_{ci}\right)^{1/2}$ for $k_\theta/k_z>0$. It emerges that both the real part of the wave frequency and the growth rate of

https://doi.org/10.1017/S0022377812000633 Published online by Cambridge University Press

the parallel ion velocity gradient instability are modified by the plasma boundary effect in a non-uniform collisionless dusty magnetoplasma. The critical shear can be affected by the presence of the stationary charged dust grains, through the parameter $\alpha = n_{i0}/n_{e0}$, which is >1 for negatively charged grains (see D'Angelo and Song 1990; Bharuthram and Shukla 1992; Luo et al. 2001).

In a collisional dusty magnetoplasma with $|\Omega| \ll \nu_i$, taking $\Omega = \Omega_r + i\gamma$, with $|\gamma| \ll |\Omega_r|$, we obtain from (9) the real part of the wave frequency

$$\Omega_r \approx \frac{\Omega_*}{1 + 2\alpha K^2 \rho_s^2}. \tag{12a}$$

The growth rate is obtained as

$$\gamma(1 + 2\alpha K^2 \rho_s^2) = \frac{\alpha k_z k_\theta C_s^2 \Omega_s}{\nu_i \omega_{ci}} - \frac{\alpha k_z^2 C_s^2}{\nu_i}\left(1 + \frac{K^2 \nu_i^2}{k_z^2 \omega_{ci}^2}\right), \tag{12b}$$

provided that $k_z/k_\theta > 0$ and

$$\Omega_s > \omega_{ci}\frac{k_z}{k_\theta}\left(1 + \frac{K^2 \nu_i^2}{k_z^2 \omega_{ci}^2}\right). \tag{13}$$

It follows that the boundary effect modifies the growth rate (12b) and the threshold condition (13) of PIVS instability in a collisional dusty magnetoplasma.

To summarize, we have studied the plasma boundary effect on PIVS-driven low-frequency (in comparison with the ion gyrofrequency) electrostatic modes in a non-uniform dusty magnetoplasma composed of 3D electrons, 3D ions, and stationary charged dust impurities which modify the equilibrium charge neutrality condition. The two fluid plasma model with the guiding center drifts for the electrons and ions is used to derive the wave equation in a cylindrical plasma column. The wave equation admits a linear dispersion relation, which exhibits that the plasma boundary effect modifies the real part of the frequency and the growth rate of the collisionless PIVS instability. The modification is through the introduction of the radial wavenumber $k_r = \gamma_n/R$ in the dispersion of the coupled dust ion-acoustic and drift waves that are propagating obliquely to the external magnetic field direction. Furthermore, the plasma boundary effect does not affect the real part of the wave frequency, but the growth rate and threshold of PIVS-driven drift modes are affected by the radial boundary effect in a highly collisional non-uniform dusty magnetoplasma. Both in collisionless and collisional dusty magnetoplasmas, there arise phase differences between the magnetic field-aligned ion fluid velocity perturbations and the wave potential due to the $(c/B_0)\hat{\mathbf{z}} \times \nabla\phi$ advection of the equilibrium parallel ion velocity gradient. As a result, the free energy stored in the latter is coupled to the low-frequency electrostatic modes in our bounded dusty magnetoplasma that is non-uniform. The presence of the stationary charged dust grains can affect the threshold shear condition for the onset of instability (D'Angelo and Song 1990; Bharuthram and Shukla 1992; Luo et al. 2001). The present results should be useful for understanding the properties of the low-frequency non-thermal drift wave fluctuations, as well as associated anomalous cross-field parallel ion momentum transport (Coppi 2002) and reduction in the magnetic field-aligned ion flow in low-temperature non-uniform laboratory dusty plasma devices with magnetic fields.

Acknowledgements

This research was supported by NSF PHY0903808. P. K. Shukla enjoys the hospitality of his colleague Marlene Rosenberg at the Department of Electrical and Computer Engineering, University of California San Diego, La Jolla, California.

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