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105.43 $c^2 = a^2 + bd$, a visual extension of the Pythagorean theorem

In [1] the author provides an interesting proof of the cosine rule, starting from a Pythagorean-like formula proved in [2] for obtuse triangles. We propose a visual proof which extends this formula, generalising to any triangle the typical rearrangement proof of the Pythagorean theorem.

The sides of the large quadrilateral in the right-hand diagram are all made up of two sides of a light grey triangle and the shortest side of a dark grey triangle. Therefore the two diagrams are of equal area. Moreover, the rectangle located in the upper part of the second diagram has the same dimensions as the dotted rectangle, then can be moved from the top and superimposed on it by rotating through a right angle.

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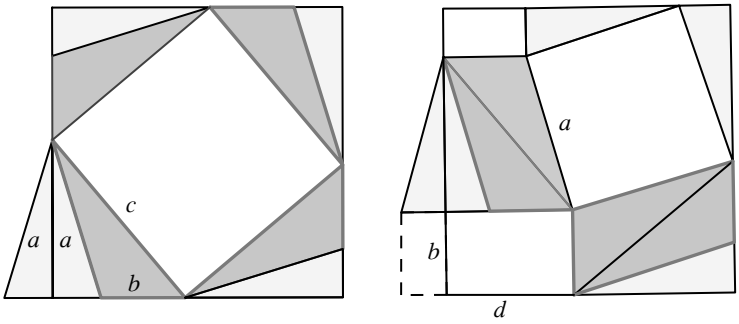


FIGURE 1

In conclusion the diagrams above show that, given an obtuse triangle having sides of lengths a, b, c with $c \geqslant a$, we have $c^2 = a^2 + bd$, where d is the projection of c onto b, obtained by the cevian congruent to a. The diagrams also show that, given an acute triangle having sides of lengths a, b, d with $c \geqslant a$, we have $c^2 = a^2 + bd$, where b is the projection of c onto d, obtained by the cevian congruent to a.

In particular, the angle opposite to the side c is right if, and only if, $b = d$, i.e. if, and only if, $c^2 = a^2 + b^2$.

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