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Partially transverse and partially longitudinal wave in non-uniform electron plasmas

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It is pointed out that in the slow time scale perturbations the displacement current is ignored but it does not imply that the electron density fluctuations vanish. The dispersion relation of the low-frequency electromagnetic wave described within the framework of electron magnetohydrodynamics (EMHD) is modified by taking into account the longitudinal effects. This wave can couple with plasma lower hybrid oscillations if ion dynamics is also considered. The low-frequency wave discussed here can have many applications in plasma transport and plasma opening switches.

It is well known that Maxwell's equation reduces to Ampere's law when the displacement current is neglected in the slow time scale phenomena. On the other hand, if ions are considered to be stationary, then the electron density fluctuations do not appear due to Ampere's law in previous investigations, both in unmagnetized (Stamper et al. 1971; Bol'shov et al. 1974; Brueckner and Jorna 1974; Raven et al. 1978; Stamper et al. 1978; Jones 1983) and magnetized (Huba 1991; Frushtman and Strauss 1992; Huba et al. 1994) plasmas.

A great deal of literature exists on the topic of magnetic field generation in laser-plasma (Stamper et al. 1971; Bol'shov et al. 1974; Brueckner and Jorna 1974; Raven et al. 1978; Stamper et al. 1978; Jones 1983) and cosmological (Lazarian 1992) scales assuming the system to be unmagnetized initially. On the other hand, efforts have also been made to find out some mechanism for the generation of magnetic fluctuations parallel to the external magnetic field, which can be very important in plasma opening switches (PoS) (Huba 1991; Huba et al. 1994). Thermomagnetic instability in laser plasmas has also been investigated assuming electron density to be fixed in the magnetized inhomogeneous plasma (Frushtman and Strauss 1992).

For the study of magnetic field generation and plasma switches, a simpler single fluid model called electron magnetohydrodynamics (EMHD) was presented (Kingssep et al. 1990). In this model, the electron inertia term in equation of motion is neglected and Ampere's law is used assuming the time scale $|\partial_t| \ll \omega_{pe}, ck$. Moreover, the ions are assumed to be static in the limit $\omega_{pi}, \Omega_i < |\partial_t|$, where $\omega_{pj} = (4\pi n_0 e^2/m_j)^{\frac{1}{2}}$ is the plasma oscillation frequency of jth species, k is the wave number, c is the speed of light and $\Omega_i = eB_0/m_i c$ is the ion gyrofrequency. The EMHD model is widely used to study the low-frequency electromagnetic phenomena in unmagnetized plasmas for the last many decades (Eliasson et al. 2009). A few years ago, the contribution of the longitudinal effects and the role of ion dynamics in such low-frequency perturbations

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have been highlighted (Saleem 2009). In magnetized plasmas, EMHD is supposed to be valid for $\Omega_i \ll |\partial_t| \ll \Omega_e$, where $\Omega_j = eB_0/m_j c$ is the gyrofrequency of the jth species.

In some works, the nonlinear whistler wave in pure electron plasmas have been studied. Since these waves are basically pure transverse in nature, therefore the electron density perturbations in nonlinear stage may be neglected (Eliasson and Shukla 2007). Our aim is to investigate a partially transverse and partially longitudinal mode in non-uniform pure electron and electron-ion plasmas with frequency $\omega < \Omega_e$.

Let us choose the wave geometry the same as was considered in the so called magnetic drift wave (MDW) in Huba (1991) and Huba et al. (1994). The constant external magnetic field is $\hat{\mathbf{B}}_0 = B_0\hat{\mathbf{z}}$, the density gradient is $\nabla n_{j0} = -\hat{\mathbf{x}}\frac{dn_0}{dx}$ and the perturbations are proportional to $e^{i(k_y y-\omega t)}$, while the subscript naught (0) denotes equilibrium quantities. The only difference is that we assume the density gradient along negative x-axis in resemblance with the well-known drift wave case while in the derivation of the dispersion relation for MDW it was assumed along positive x-axis.

In the limit $\omega_{pi}, \Omega_i \ll \omega$, the ions are assumed to be stationary and for $\omega \ll \omega_{pe}, ck, \Omega_e$ and $\frac{\omega}{\omega_{pe}} \ll \frac{\omega_{pe}}{\Omega_e}$ the displacement current is neglected in Maxwell's equation

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\partial_t \mathbf{E}, \tag{1}$$

which then becomes Ampere's law,

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J}. \tag{2}$$

In the case of pure electron plasma we have $\mathbf{J}_1 = -en_{e0}\mathbf{v}_{e1}$, where the subscript 1 denotes linearly perturbed quantities. Since Ampere's law implies $\nabla \cdot \mathbf{J}_1 = 0$, therefore in the derivation of dispersion relation of MDW it is deduced that $n_{e1} = 0$. Using the limit $|\partial_t| \ll \Omega_e$, the electron acceleration term is also neglected in the equation of motion, which then becomes

$$\mathbf{E}_1 = -\frac{c}{B_0}\mathbf{v}_{e1} \times \hat{\mathbf{z}}. \tag{3}$$

The magnetic fluctuations are assumed to be along z-axis and hence the Faraday's law

$$\nabla \times \mathbf{E}_1 = -\frac{1}{c}\partial_t \mathbf{B}_1 \tag{4}$$

requires $\mathbf{E}_1 = E_1\hat{\mathbf{x}}$, which implies $\nabla \cdot \mathbf{E}_1 = 0$ and therefore the wave seems to be purely transverse. Equation (2) gives

$$\nabla.\mathbf{v}_{e1} = \frac{c}{4\pi e n_0}\frac{\nabla n_0}{n_0}.(\nabla \times \mathbf{B}_1), \tag{5}$$

and hence $\nabla.\mathbf{v}_{e1} \neq 0$.

The final dispersion relation under local approximation turns out to be (Huba et al. 1994)

$$\omega = \lambda_e^2 k_y^2\left(\frac{\kappa_n}{k_y}\Omega_e\right) = k_y v_A\left(\frac{c}{\omega_{pi}L_n}\right), \tag{6}$$

where $v_A = B_0/\sqrt{4\pi n_0 m_e}$ is the electron Alfven speed, $\omega_{pi} = \left(4\pi n_0 e^2/m_i\right)^{\frac{1}{2}}$ is the ion plasma frequency, $L_n = 1/\kappa_n$, $\kappa_n = \left|\frac{1}{n_0}\frac{dn_0}{dx}\right|$ and $\lambda_e = c/\omega_{pe}$.

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The EMHD model was presented to reduce the time and spatial scales of two component electron-ion plasma to a single fluid, which could explain some important phenomena.

Now we modify the description of MDW by including longitudinal effects $\nabla.\mathbf{E} \neq 0$. Equation (5) implies $\nabla.\mathbf{v}_{e1} \neq 0$ and (2) requires

$$\nabla n_0.\mathbf{v}_{e1} + n_0\nabla.\mathbf{v}_{e1} = 0. \tag{7}$$

Thus the electron velocity has two non-zero components v_{ex1} and v_{ey1}. If it is so, then (3) yields $E_{1y} \neq 0$ and hence $\nabla.\mathbf{E}_1 \neq 0$.

It is important to mention here that when the longitudinal perturbations are taken into account, then (7) does not hold. If the wave has partially longitudinal and partially transverse character, then the electron density perturbations should be estimated using continuity equation although Ampere's law is used. It may be recalled here that a similar procedure is adopted for the study of Alfven waves in two fluid framework, which yields the linear dispersion relation in ideal electron-ion plasmas as

$$\omega^2 = \frac{v_A^2 k_\parallel^2 (1 + \rho_s^2 k_\perp^2)}{(1 + \lambda_e^2 k_\perp^2)}, \tag{8}$$

where $v_A = \left(\frac{B_0^2}{4\pi m_i n_0}\right)^{\frac{1}{2}}$ is the Alfven speed, $\rho_s = \frac{c_s}{\Omega_i}$ is the ion gyroradius at electron temperature, $c_s = (T_e/m_i)^{\frac{1}{2}}$ is the ion sound speed and $\lambda_e = (c/\omega_{pe})$ is the electron skin depth. For $\rho_s^2 k_\perp^2 \ll 1$ and $\lambda_e^2 k_\perp^2 \ll 1$, Alfven wave is a purely transverse mode and its dispersion relation becomes $\omega^2 = v_A^2 k_\parallel^2$, which can be obtained using magnetohydrodynamics (MHD). The effects of density fluctuations appear when compressibility ($\nabla.\mathbf{v} \neq 0$) is taken into account and we find $\rho_s^2 k_\perp^2 \neq 0$ or $\lambda_e^2 k_\perp^2 \neq 0$ or both the factors to be non-zero. In deriving (8), the electron and ion continuity equations are used along with Ampere's law. Two important points need attention:

(*a*) Ampere's Law does not necessarily imply that electron density fluctuations are zero as in case of Alfven waves we have noticed.

(*b*) Since $\kappa_n \ll k_y$ under the local approximation, ω can be near the lower hybrid frequency $(\Omega_e\Omega_i)^{\frac{1}{2}}$. Thus the ion dynamics should not be ignored particularly in the case of low-Z (like hydrogen) plasmas.

First we present a model for pure electron plasma, which is more useful for heavier ion plasmas where the approximation $\omega_{pi}, \Omega_i \ll \omega$ seems to be more logical and ions can be assumed to be static. Experiments have been performed to produce pure pair-ion fullerene plasmas (Oohara and Hatakeyama 2003; Oohara et al. 2005; Oohara et al. 2007) in Japan. However, a criterion for the pure pair-ion plasma has been defined (Saleem 2007) and it shows that the fullerene plasma produced in experiments was not pure pair-ion plasma. On the other hand, it has been learnt that there are plans to study electron shear flow effects in barium plasma in the United States (Koepke 2006). Since we are in the lower hybrid range of frequencies $\Omega_i \ll \omega \ll \Omega_e$, therefore we retain the electron inertia term in equation of motion, which for $\mathbf{E} = (E_{1x}, E_{1y}, 0)$ yields

$$v_{e1x} = \frac{e}{m_e\Omega_e^2}\{i\omega E_{1x} + \Omega_e E_{1y}\}, \tag{9}$$

$$v_{e1y} = \frac{e}{m_e\Omega_e^2}\{i\omega E_{1y} - \Omega_e E_{1x}\}. \tag{10}$$

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Note that $\nabla.\mathbf{v}_{e1} = \frac{\partial v_{e1x}}{\partial x} + ik_y v_{e1y} = ik_y v_{e1y}$ because $k_x = 0$. The electron density perturbations are estimated using continuity equations and Poisson's equation which yield

$$\Omega_e E_{1x} = \left\{ \left(1 + \frac{\Omega_e^2}{\omega_{pe}^2} \right) \omega + \Omega_e \frac{\kappa_n}{k_y} \right\} i E_{1y} - i \frac{\Omega_e}{k_y} \frac{\partial E_{1y}}{\partial x}; \tag{11}$$

Ampere's law yields

$$\Omega_e E_{1x} = \frac{i\omega}{\lambda_e^2 k_y^2} E_{1y} - i \frac{\Omega_e}{k_y} \frac{\partial E_{1y}}{\partial x}. \tag{12}$$

Then (11) and (12) give a new low (or hybrid) frequency electromagnetic wave in non-uniform electron plasmas as

$$\omega = -\frac{\lambda_e^2 k_y^2}{\left\{ 1 + \lambda_e^2 k_y^2 (1 + \frac{\Omega_e^2}{\omega_{pe}^2}) \right\}} \left(\frac{\kappa_n}{k_y} \Omega_e \right) \tag{13}$$

for $\frac{\omega}{c} \frac{\omega_{pe}}{\Omega_e} \ll k_y$.

Few comments are necessary at this stage. First, it is pointed out that a pure electrostatic low-frequency mode can also be obtained with dispersion relation

$$\omega = -\frac{1}{(1 + \Omega_e^2/\omega_{pe}^2)} \left(\frac{\kappa_n}{k_y} \Omega_e \right) \tag{14}$$

in an electron plasma using Poisson equation (11) for $E_{1x} = 0$ or using $1 \ll \lambda_e^2 k_y^2$ in (13). In lighter ions case, this mode will couple with lower hybrid oscillations, which will be presented later.

Second, the term $\mu = \lambda_e^2 k_y^2 \left(1 + \frac{\Omega_e^2}{\omega_{pe}^2} \right)$ is crucial in (13). The factor $\lambda_e^2 k_y^2$ appears due to electron inertia term in equation of motion and $\lambda_e^2 k_y^2 \left(\frac{\Omega_e^2}{\omega_{pe}^2} \right)$ appears due to electron density fluctuations through Poisson equation. In heavier ion plasmas, (13) will have many applications and even in hydrogen plasma it is important if $\kappa_n / k_y \sim (10^{-1})$, $\lambda_e^2 k_e^2 \sim (1)$ and $\Omega_e^2/\omega_{pe}^2 \lesssim 1$ so that $\Omega_i \ll \omega$ remains valid. In heavier ion plasmas, we may have $\kappa_n / k_y \ll 10^{-1}$ because m_e/m_i will be smaller and hence $\Omega_i \ll \omega < \Omega_e$ holds. However, in hydrogen plasmas, the frequency of this mode will be closer to $(\Omega_e \Omega_i)^{\frac{1}{2}}$ and therefore it is preferable to include ion dynamics. Ion momentum equation gives

$$v_{i1x} = \frac{e}{m_i} \frac{1}{(\omega^2 - \Omega_i^2)} (i\omega E_{1x} - \Omega_i E_{1y}) \tag{15}$$

and

$$v_{i1y} = \frac{e}{m_i} \frac{1}{(\omega^2 - \Omega_i^2)} (i\omega E_{1y} + \Omega_i E_{1x}). \tag{16}$$

Then the Poisson equation becomes

$$\left[\left(1 + \frac{\omega_{pe}^2}{\Omega_e^2} \right) \omega^3 + \left(\frac{\omega_{pe}^2}{\Omega_e^2} \frac{\kappa_n}{k_y} \right) \omega^2 - \omega_{pi}^2 \omega \right] i k_y E_{1y}$$
$$= \left[\omega_{pi}^2 (\Omega_i k_y - \omega \kappa_n) + \frac{\omega_{pe}^2}{\Omega_e^2} (\Omega_e k_y) \omega^2 \right] E_{1x}. \tag{17}$$

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In the electrostatic case $E_{1x} = 0$, it reduces to

$$\left(1 + \frac{\Omega_e^2}{\omega_{pe}^2}\right)\omega^2 + \frac{\kappa_n}{k_y}\Omega_e - \Omega_i\Omega_e = 0. \tag{18}$$

In the electromagnetic case we obtain

$$\left\{1 + \lambda_e^2 k_y^2\left(1 + \frac{\Omega_e^2}{\omega_{pe}^2}\right)\right\}\omega^2 + \lambda_e^2 k_y^2\left(\frac{\kappa_n}{k_y}\Omega_e\right)\omega - \lambda_e^2 k_y^2(\Omega_e\Omega_i) = 0. \tag{19}$$

The electromagnetic wave described in (13) couples with lower hybrid oscillations in (19) due to ions contribution in the limit $\omega^2 \simeq \Omega_e\Omega_i$, $\omega\kappa_n \simeq \Omega_i k_y$ and $\lambda_e^2 k_y^2 \sim (1)$.

To summarize, it has been pointed out that when the divergence of transverse current is zero, it does not mean that longitudinal current is zero. Therefore, the Ampere's law does not necessarily imply that there are no density perturbations in the charged particle system. Coupling of Ampere's law with Poisson equation gives a new partially longitudinal and partially transverse wave that exists in non-uniform magnetized electron plasmas.

The new electromagnetic wave described by dispersion relation (13) will have lot of applications in plasma dynamics, for example in plasma opening switches and in plasma transport. It is partially longitudinal and partially transverse.

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REFERENCES

Bol'shov, L. A., Dreizin, Yu. A. and Dykhne, A. M. 1974 *Pis'ma Zh. Edsp. Teor. Fiz.* **19**, 288; *JETP Lett.* **19**, 168.

Brueckner, K. A. and Jorna, S. 1974 *Rev. Mod. Phys.* **46**, 325.

Eliasson, E., Shukla, P. K. and Pavlenko, V. P. 2009 *Phys. Plasmas* **16**, 042306.

Eliasson, B. and Shukla, P. K. 2007 *Phys. Rev. Lett.* 99, 205005.

Frushtman, A. and Strauss, H. R. 1992 *Phys. Fluids* **B4**(6), 1397.

Huba, J. D. 1991 *Phys. Fluids* **B3**(12), 3217.

Huba, J. D., Grossmann, J. M. and Ottinger, P. F. 1994 *Phys. Plasmas* **1**(10), 3444.

Jones, R. D. 1983 *Phys. Rev. Lett.* **51**, 1269.

Kingssep, A. A., Chukbar, K. V. and Yan'Kov, V. V. 1990 In: *Reviews of Plasma Physics*, Vol. 16 (ed. B. B. Kadomtsev). New York: Consultants Bureau, p. 243.

Koepke, M. 2006 private communications. Laboratory experiments on the parallel electron velocity shear could potentially be done with the WVU Q Machine at West Virginia University USA.

Lazarian, A. 1992 *Astron. Astrophys.* **264**, 326.

Oohara, W., Data, D. and Hatakeyama, R. 2005 *Phys. Rev. Lett.* **95**, 175003.

Oohara, W. and Hatakeyama, R. 2003 *Phys. Rev. Lett.* **91**, 205005.

Oohara, W., Kuwabara, Y. and Hatakeyama, R. 2007 *Phys. Rev.* E **75**, 056403.

Raven, A., Willi, O. and Rumsby, R. T. 1978 *Phys. Rev. Lett.* **41**, 554.

Saleem, H. 2007 *Phys. Plasmas* **14**, 014505.

Saleem, H. 2009 In: *Proc. ICTP Summer College on Plasma Physics and Int. Symp. on Cutting Edge Plasma Physics, 10–28 August 2009* (ed. B. Eliasson and P. K. Shukla). Trieste, Italy, 233 pp.

Stamper, J. A., McLean, E. A. and Ripin, B. H. 1978 *Phys. Rev. Lett.* **40**, 1177.

Stamper, J. A., Papadapoulos, K., Sudan, R. N., Dean, S. O., McLean, E. A. and Dawson, J. M. 1971 *Phys. Rev. Lett.* **26**, 1012.

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