On the Relation between Dualities and Gauge Symmetries

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We make two points about dualities in string theory. The first point is that the conception of duality, which we will discuss, meshes with two dual theories being 'gauge related' in the general philosophical sense of being physically equivalent. The second point is a result about gauge/gravity duality that shows its relation to gauge symmetries to be subtler than one might expect: each of a certain class of gauge symmetries in the gravity theory, that is, diffeomorphisms, is related to a position-dependent symmetry of the gauge theory.

1. Introduction. In this article, we make two main points about how duality and gauge symmetry are connected. Both points are about dualities in string theory, and both have the 'flavor' that two dual theories are 'closer in content' than you might think. For both points, we adopt a simple conception of a duality as an 'isomorphism' between theories. In section 3, we take a theory to be given by a triple comprising a set of states, a set of quantities, and a dynamics, so that a duality is an appropriate 'structure-preserving' map between such triples. This discussion will be enough to establish our first point, namely, dual theories can indeed 'say the same thing in different words'—which is reminiscent of gauge symmetries.

Our second point (secs. 4 and 5) is much more specific. We give a result about a specific (complex and fascinating) duality in string theory, gauge/gravity duality, which we introduce in section 4, using section 3's conception of duality. We state this result in section 5. (More details are in De Haro, Teh, and Butterfield [2016], and the proof is in De Haro [2016b].) It says, roughly speaking, that each of an important class of gauge symmetries in one of the dual theories (a gravity theory defined on a bulk volume) is mapped

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by the duality to a gauge symmetry of the other theory (a conformal field theory defined on the boundary of the bulk volume). This is worth stressing since some discussions suggest that all the gauge symmetries in the bulk theory will not map across to the boundary theory but instead be 'invisible' to it. To set the stage for these points, section 2 describes the basic similarity between the ideas of duality and gauge symmetry: that they both concern 'saying the same thing in different words'.

2. Saying the Same Thing in Different Words. We use the general term 'duality' to denote the *formal equivalence* between two theories, that is, a bijection between the sets of mathematical 'states' and 'quantities' of a formal theory, without further specifying the physical interpretation of those states and quantities, such as unitary equivalence for quantum theories. At the other extreme, one might be using the duality to describe the "universe" (as in quantum gravity): on an 'internal interpretation' (cf. Dieks, van Dongen, and de Haro 2015) the equivalence is then not merely formal or schematic but also empirical and physical. As to *gauge symmetry*, it has (i) a general philosophical meaning and (ii) a specific physical meaning, as do cognate terms like 'gauge-dependent', and so on, as follows:

i) (**Redundant**) If a physical theory's formulation is redundant (i.e., roughly, it uses more variables than the number of degrees of freedom of the system being described), one can often think of this in terms of an equivalence relation, 'physical equivalence', on its states, so that gauge symmetries are maps leaving each class (called a 'gauge orbit') invariant. Leibniz's criticism of Newtonian mechanics provides a putative example: he believed that shifting the entire material contents of the universe by one meter must be regarded as changing only its description and not its physical state.
ii) (**Local**) If a physical theory has a symmetry (i.e., roughly, a transformation of its variables that preserves its Lagrangian) that transforms some variables in a way dependent on space-time position (and is thus 'local'), then this symmetry is called 'gauge'. In the context of Yang-Mills theory, these variables are 'internal', whereas in the context of general relativity, they are space-time variables—both types of examples will occur in sections 4 and 5. Although Local is often a special case of Redundant, it will be important to us that this is not always so. For we will be concerned with Local gauge symmetries (specifically diffeomorphisms) that do not tend to the identity at space-like infinity and that can thus change the state of a system relative to its environment.[1]

1. Compare the discussion in Greaves and Wallace (2014) and Teh (2016).

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These sketches are enough to suggest that duality and gauge symmetry are likely to be related. The obvious suggestion to make is that the differences between two dual theories will be like the differences between two formulations of a gauge theory: they 'say the same thing, despite their differences'. Indeed, for several notable dualities—including some string theory dualities (and our case of gauge/gravity duality) in which the dual theories are strikingly disparate—this is the consensus among physicists. Besides, several philosophical commentators endorse this consensus (e.g., Rickles 2011, secs. 2.3, 5.3; 2015; Dieks et al. 2015, sec. 3.3.2; De Haro 2016a, sec. 2.4; Huggett 2016, secs. 2.1, 2.2). But beware of a *false* impression: roughly, that dualities between gauge theories must ignore Local gauge structure. To see how this impression arises, assume for the moment (falsely) that all Local gauge symmetries exemplify Redundant. Then given a duality between two theories' gauge-invariant structures, it would be surprising if the duality also mapped their gauge-dependent structures (gauge symmetries and gauge-dependent quantities) into each other. For, think of the everyday analogy in which (i) a duality is like a translation scheme between languages, and so (ii) gauge structure, a theory allowing several gauges, is like a language having several synonyms for one concept. (Indeed, this analogy is entrenched in physics: physicists call the definition of the duality transformation the 'dictionary', etc.) One would not expect a translation scheme to match the languages' synonym structures, that is, to translate each of a set of synonyms in language L_1 by just one of the corresponding (synonymous) set of synonyms in L_2 and vice versa. Analogously, it seems that for a duality between gauge theories, the gauge-dependent structures on the two sides will not be related by the duality. Each such structure, on one side, will be 'invisible' to the other side—at least, 'invisible' if you are using just the duality map 'to look through'.

Thus, the impression is tempting. Indeed we think the impression is widespread because of this line of thought: as we will see, no lesser authors than Horowitz and Polchinski seem to endorse it.

It is this impression that we will rebut. We of course admit that in general, Local gauge-dependent structures may be 'invisible from the other side'. But surprisingly, for the case of gauge/gravity duality, some Local gauge structure is visible. This is our result in section 5: roughly, that each of a certain class of Local gauge symmetries of the gravity/bulk theory is mapped by the duality to Redundant gauge symmetries of its dual, that is, the position-dependent conformal symmetries of the boundary conformal field theory.

3. Duality as a Symmetry between Theories. In this section, we propose schematic definitions of a physical theory and of a duality. The definitions will be general enough to apply to both classical and quantum physics, although we of course have quantum physics, especially quantum field theory and string theory, mostly in mind.

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The idea of the definitions is that duality is a symmetry between theories. But while symmetry is a matter of sameness between distinct situations, according to a theory, duality is a matter of sameness between theories. So a duality is, roughly, an 'isomorphism' between two theories or from a theory to itself. To make these ideas more precise, we define, first, theories (sec. 3.1) and, then, duality maps (sec. 3.2).

3.1. The Definition of '*Theory*'. We take a *theory* as a triple $T = \langle \mathcal{S}, \mathcal{Q}, D \rangle$ of state-space $\mathcal{S}$, set of quantities $\mathcal{Q}$, and dynamics D. States and quantities are assignments of values to each other. So there is a natural pairing, and we write $\langle Q; s \rangle$ for the value of Q in s. In classical physics, we think of this as the system's intrinsic possessed value for Q, when in s; in quantum physics, we think of it as the (orthodox, Born-rule) expectation value of Q, for the system in s. As to the dynamics, D, this can here be taken as the deterministic time evolution of states.

Our construal of theory allows us to recognize that usually there are many token systems of the type treated by a theory in our sense. This prompts an important point: we of course recognize (as everyone must) that there can be cases of two disjoint parts of reality—in Hume's phrase, two 'distinct existences'—whose formal or schematic structure matches exactly, 'are isomorphic', in the taxonomy used by some theory but are otherwise different, that is, distinct and known to be distinct. This point has an obvious corollary for what in section 1 we announced as our first main point: that dual theories 'can say the same thing'. For it is natural to take 'saying the same thing' to mean saying (i) the same assertions about (ii) the same objects. Then we must beware that a definition of dual theories as 'isomorphic' (as in sec. 3.2 below) will cue in only to i, and the possibility of 'distinct but isomorphic existences' implies that this does not secure ii. This leads in to a final point.

There is one scientific context in which the idea of distinct but isomorphic existences falls by the wayside, namely, when we aim to write down a cosmology, that is, a theory of the whole universe. For such a theory, there will be, ex hypothesi, only one token of its type of system, that is, the universe. Agreed, this scientific context is very special and very ambitious: we rarely aim to write down a cosmology. But of course, it is the context of much work in string theory and quantum gravity more generally. So it will apply when we turn, in section 4, to string theory and gauge/gravity duality.[2]

2. In this context, one might go further than setting aside the possibility of distinct but isomorphic existences. One might also hold that the interpretation of our words, i.e., of the symbols in the cosmological theory, must be fixed from within the theory: this view is endorsed, under the label 'internal point of view', in Dieks et al. (2015, sec. 3.3.2) and De Haro (2016a, sec. 2.4). We should note, however, that a lot of work on gauge/gravity duality concerns systems that are much smaller than the universe. For gauge/gravity ideas

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3.2. The Definition of 'Duality'. We now define a duality as a 'meshing' map between two theories: $T_1 = \langle \mathcal{S}_1, \mathcal{Q}_1, D_1 \rangle$ is *dual* to $T_2 = \langle \mathcal{S}_2, \mathcal{Q}_2, D_2 \rangle$ if and only if there are bijections $d_s : \mathcal{S}_1 \to \mathcal{S}_2, d_q : \mathcal{Q}_1 \to \mathcal{Q}_2$ (d for 'duality') that give matching values of quantities on states in the following sense:[3]

$$\langle Q_1; s_1 \rangle_1 = \langle d_q(Q_1); d_s(s_1) \rangle_2, \quad \forall\, Q_1 \in \mathcal{Q}_1, \forall\, s_1 \in \mathcal{S}_1. \tag{1}$$

This definition of 'duality' is obvious and simple, given our conception of 'theory'. One could strengthen the definition in various ways, for example, to require that d_s be a symplectomorphism for Hamiltonian theories, unitary for quantum theories. And there is a whole tradition of results relating the requirement of matching values (eq. [1]) to such strengthenings. But we do not need to pursue such strengthenings. It is not just that a simple definition is clearer and can be weakened and qualified, as needed. Also, with this definition, we immediately establish our first main point: that two dual theories can be gauge related, in section 2's general philosophical sense of being physically equivalent. More precisely, the point follows immediately, when we bear in mind our preceding discussion and the "can be" in 'can be gauge related' (i.e., sec. 3.1's allowance of distinct but isomorphic 'existences') and how this allowance falls by the wayside for a cosmology, that is, theory of the whole universe. And more important, we will see in section 4 that this definition of duality is indeed instantiated, albeit formally, by gauge/gravity duality and using maps d_s, d_Q that are (formally) unitary.

4. Gauge/Gravity Duality. We first (sec. 4.1) give a brief introduction to the original and most studied case of gauge/gravity duality: AdS/CFT, the duality between a gravity theory on anti–de Sitter space-time (AdS) and a conformal field theory (CFT) on its boundary. Then in section 4.2, we argue that section 3.2's simple definition of duality is indeed instantiated, albeit formally, by AdS/CFT. For a philosophically informed introduction to gauge/gravity dualities, see De Haro, Mayerson, and Butterfield (2016).

4.1. Introducing AdS/CFT. The general idea of gauge/gravity duality is that some gauge quantum field theories (QFTs) in d space-time dimensions are dual to some quantum theories of gravity in a $(d + 1)$-dimensional space-

turn out to be very useful for understanding strongly coupled microscopic systems, like a quark-gluon plasma as explored by instruments like the Relativistic Heavy Ion Collider; cf. McGreevy (2010).

3. We also require the dynamics to mesh in the obvious sense: i.e., d_s commutes with (is equivariant for) the two theories' dynamics. In sec. 4.2 we add a stronger condition, i.e., that the values of the quantities on any pair of states match: $\langle Q_1; s_1, s_2 \rangle_1 = \langle d_q(Q_1); d_s(s_1), d_s(s_2) \rangle_2$, $\forall\, s_1, s_2 \in \mathcal{S}_1$. For quantum theories, this strengthening of this section's simple approach is natural because it amounts to unitary equivalence.

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time that has the d-dimensional manifold of the QFT as its conformal boundary. Pictorially, AdS_{d+1} is a family of copies of a d-dimensional Minkowski space-time of varying sizes. The family is parameterized by the coordinate r. The boundary is the locus $r \to 0$.

The original case (Maldacena 1999) takes the QFT to be a strong coupling regime of a supersymmetric gauge theory (SYM, for 'super Yang-Mills') in four space-time dimensions ($d = 4$), with gauge group $SU(N)$. To display the AdS/CFT correspondence in a simple case, take the path integral for the scalar field, in Euclidean signature, as a function of the boundary conditions:[4]

$$Z[\phi_{(0)}] \coloneqq \int_{\phi(r,x)|_{\text{bdy}}=\phi_{(0)}(x)} \mathcal{D}\phi \exp(-S_{\text{bulk}}[\phi]). \tag{2}$$

The AdS/CFT correspondence now states that this is the generating functional in the CFT:

$$Z[\phi_{(0)}] = \langle \exp\left(-\int d^4x \phi_{(0)}(x)\mathcal{O}(x)\right)\rangle, \tag{3}$$

where $\phi_{(0)}(x)$ is a 'source' that couples to a certain gauge-invariant operator $\mathcal{O}(x)$, whose scaling dimension Δ is determined by the mass of the bulk scalar field (in a CFT, the scaling dimension uniquely determines a gauge-invariant, scalar operator). The exponential is evaluated in the vacuum state of the theory.

In the CFT, the vacuum correlation functions $\langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_n)\rangle$ are calculated from the generating functional $Z[\phi_{(0)}]$: one takes functional derivatives of the right-hand side of (3) with respect to the source and sets the source to zero.

But according to the correspondence (eqq. [2] and [3]), the correlation functions can also be calculated using the bulk theory. For instance, to calculate the two-point function, one can use the leading classical approximation to (2), which is just the classical action evaluated on solutions that satisfy the prescribed boundary conditions. Up to normalization, the result is

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle = \frac{1}{|x-y|^{2\Delta}}, \tag{4}$$

where Δ is the scaling dimension of the operator, and $|x-y|$ the distance between the two boundary points. This result matches the CFT result precisely.

4. For simplicity, we are now taking the metric to be fixed, and we are suppressing it in the notation.

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4.2. AdS/CFT Exemplifies the Definition of Duality. We now turn to how (3) instantiates our definition of duality (i.e., eq. [1]). We begin with the boundary, CFT, side.

In a CFT, the quantities one is interested in are the correlation functions:

$$\langle s|\mathcal{O}(x_1)\cdots\mathcal{O}(x_n)|s'\rangle, \tag{5}$$

of products of gauge-invariant operators, in some states s, s'. In section 4.1 we showed how to calculate the two-point function in the vacuum state, leading to (4). Thus, the gauge theory side of the duality is, by construction, framed in the language of states and quantities that was used in section 3.1 to define a theory. We admit that in the absence of proper nonperturbative methods for rigorously defining higher-dimensional theories such as SYM, one should be wary of expressions such as (5). To assume that SYM makes sense nonperturbatively is to assume that expressions such as (5) make sense. Similar remarks hold for the path-integral expressions (2) used in the bulk. In the present state of knowledge, one simply assumes that these formal structures will some day be defined with rigorous mathematics: defining these expressions nonperturbatively would amount to proving AdS/CFT. This is why the AdS/CFT correspondence still has the status of a conjecture.

So in order to substantiate our claim—that section 3.2's simple definition of duality is instantiated by AdS/CFT—we need to argue that the bulk side in (2) can be written in the language of states and operators. Accepting the comments in the previous paragraph, the point follows readily from the correspondence between path integral quantization and the Hilbert space description of states and operators, provided one can adapt that correspondence to take into account the fact that (2) contains boundary, rather than bulk, sources. This can in fact be done: for details, compare De Haro, Mayerson, and Butterfield (2016, secs. 5, 6.1) and De Haro, Teh, and Butterfield (2016, sec. 4.2).

So to sum up, the bulk side of the duality gives rise to the theory $T_{\text{bulk}} = \langle \mathcal{H}_1, \mathcal{Q}_1\rangle$, whereas the boundary dual gives rise to $T_{\text{bdy}} = \langle \mathcal{H}_2, \mathcal{Q}_2\rangle$, where $\mathcal{H}_i$ is a Hilbert space, $\mathcal{Q}_i$ is some algebra of operators, and unitary dynamics is implicit. The above remarks show that (3) yields the duality maps d_s and d_q, which are isomorphisms of Hilbert spaces and operators, respectively, satisfying (1).

5. A Pandora's Box for Gauge Invariance

5.1. Gauge Invariance and Duality in AdS/CFT. Let us now turn to the topic of gauge invariance in AdS/CFT. One typically identifies Local gauge symmetry at the level of the classical Lagrangian, which enters into the path integral formulas of (3). In the bulk theory, the main Local gauge

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symmetry is diffeomorphism symmetries. Furthermore, as mentioned at the end of section 4.1, one can add $U(1)$ (Yang-Mills type) gauge symmetry by coupling a $U(1)$ gauge field to the bulk metric. In the boundary CFT, however, one has an $SU(N)$ gauge symmetry acting on the gauge fields.

What about gauge invariance at the quantum level? Were we dealing (albeit perturbatively) with a simple case of gauge theory (e.g., QED), we would obtain the quantum state space by constructing a nilpotent Hermitian (BRST) operator Q and defining the physical (gauge-invariant) states as those annihilated by Q: cohomology then ensures that each state represents only a gauge orbit.[5] Thus, for two such quantum theories T_1 and T_2, there can be no question about whether a duality map relates their classical gauge symmetries: these symmetries are simply not represented to begin with.

In contrast, one cannot proceed in this way for the gauge theories involved in AdS/CFT (and many other dualities) because each side of the duality typically contains a nonperturbative sector, for which we cannot directly construct the quantum state space. It is precisely here that the duality relation (3) is strikingly useful; for example, by exchanging the nonperturbative sector of T_{bdy} with the perturbative sector of T_{bulk} (i.e., semiclassical supergravity), it allows us to indirectly construct the quantum state space of T_{bdy} by means of T_{bulk}.

But this also opens up a Pandora's box for gauge invariance. For, prima facie, it allows that the Local gauge symmetries of T_{bulk} might be related to the symmetries of T_{bdy}. As we will now explain, Horowitz and Polchinski have argued against this possibility, that is, for what section 2 called 'invisibility'.

Recall that (naively) a duality is a bijection between the gauge-invariant content (states and quantities) of two theories T_1 and T_2. Now suppose one is skeptical that a duality would match elements of a gauge orbit $G_1 (G_1 \subset \mathcal{S}_1)$ in theory T_1 with elements of the dual gauge orbit $G_2 \coloneqq d_s(G_1)$ in T_2. That is, in terms of section 2's everyday analogy with translations between languages, one doubts that a translation will match each synonym in a set of synonyms in T_1 with a synonymous member of an equinumerous set of synonyms in T_2. Then one would naturally expect that the duality mapping (e.g., as given for AdS by eq. [3]) secures

> (**Invisibility**) Each gauge symmetry of T_1, that is, permutation of $\mathcal{S}_1$ that leaves each gauge orbit invariant (in the analogy, permutation of T_1's words that leaves each synonymy equivalence class invariant) carries over to only the identity permutation on $\mathcal{S}_2$ and vice versa. In this sense, the duality only relates the gauge orbits of T_1 and T_2.

5. Failing to perform this move, i.e., naively quantizing the gauge fields of the theory, would lead to an indefinite Fock space (i.e., a space containing states with negative norm).

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This is indeed the definition of 'invisibility' that Horowitz and Polchinski (2006, sec. 1.3.2) use. Polchinski (2016, secs. 2.3 and 2.4) demonstrates this property for the duality between *p*-form gauge theories, and Horowitz and Polchinski (2006, sec. 1.3.2) claim that in AdS/CFT, even the Local gauge symmetry of bulk, that is, diffeomorphism symmetry, displays such 'invisibility', thus showing that diffeomorphism symmetry (and relatedly, spacetime) is an 'emergent' property: "the gauge variables of AdS/CFT are trivially invariant under the bulk diffeomorphisms, which are entirely invisible in the gauge theory" (12).

Is this latter claim correct? To be sure, there exist simple examples of 'invisibility' in AdS/CFT, from both the bulk and the boundary perspective. For example, from the bulk: Since all correlation functions defined by (3) are invariant under boundary Local gauge symmetry, this symmetry is not seen in the bulk.

Thus, one might expect that the fundamental Local gauge symmetry of the bulk theory, that is, diffeomorphism symmetry, is also invisible in the boundary theory. In the next section, we argue that not all the diffeomorphisms of the bulk theory are invisible, and we sketch criteria to distinguish visible from invisible diffeomorphisms. Furthermore, there is a sense in which the duality map (3) maps Local gauge symmetries in the bulk to position-dependent Redundant symmetries in the boundary.

5.2. AdS/CFT's Visible and Invisible Diffeomorphisms. As mentioned in section 5.1, we will scrutinize the statement that all the bulk diffeomorphisms are invisible to the boundary theory. We construe the notion of 'visible' diffeomorphism, along the lines of Horowitz and Polchinski's own examples, as one that does not restrict to the identity map on the boundary, whereas 'invisible' means that it does restrict to the identity. First, consider the class of diffeomorphisms that satisfy the following three conditions:

i) (**Fixed**) Leave the form of the bulk metric fixed;
ii) (**Invisibility**) Are equal to the identity, at the boundary;
iii) (**Existence**) Are nontrivial (i.e., not the identity map) in the bulk.

One can show that if the boundary dimension *d* is odd, then there are no such diffeomorphisms.[6] More precisely, there are no diffeomorphisms that are equal to the identity at infinity and extend nontrivially to the bulk. The desiderata i–iii cannot all be met.

6. In the case of even *d*, diffeomorphism invariance is broken by the presence of the conformal anomaly. For a discussion, see De Haro, Mayerson, and Butterfield (2016, sec. 6.2) and De Haro, Teh, and Butterfield (2016, secs. 4.2.1 and 5.2.2).

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We sketch the argument for this in a moment. But to do so, we need to consider relaxing the above conditions because this will allow us to identify the relevant visible diffeomorphisms. And this relaxation will also lead in to our final point about Local diffeomorphisms that are Redundant.

We thus replace Invisibility by the weaker

ii′) (**Invariance**) Leave all the boundary quantities (in particular, the metric) invariant; that is, the bulk diffeomorphisms can be nonvanishing (hence, visible) on the boundary but must leave all the CFT quantities invariant.

A sketch of the argument that shows the incompatibility of i–iii is as follows. Let the bulk metric be that of an Einstein space (i.e., a solution of general relativity's field equations with a negative cosmological constant), so that an arbitrary metric is induced at the boundary. Requiring that the infinitesimal diffeomorphisms leave the asymptotic form of the metric fixed, one finds the following condition for them:

$$\nabla_i \xi_j(x) + \nabla_j \xi_i(x) - \frac{2}{d} g_{ij}(x) \nabla^k \xi_k(x) = 0. \tag{6}$$

Here, ξ_i is a reparameterization of the boundary coordinates x (a d-dimensional vector). There are two important points about this equation, which is the mathematical representation of Invariance:

a) This equation is the condition for an infinitesimal boundary coordinate transformation ξ_i to give a local scale transformation: thus, the ξ_i generate the *boundary conformal group*.
b) As a consequence of *a*, when one requires in addition to Invariance also Invisibility, that is, that the boundary coordinate transformations vanish (i.e., $\xi_i = 0$), then one can subsequently show that the diffeomorphism in fact is the identity map throughout the bulk. Requiring Invisibility of the diffeomorphism thus deprives it of Existence.

But, Invariance is compatible with Existence, that is, allowing ξ_i to be nonzero. In this case, the bulk diffeomorphism has two parts: (1) a nontrivial boundary coordinate transformation (of a special kind: a conformal transformation) and (2) a compensating reparameterization of the bulk coordinate r, so that the overall boundary metric is invariant, yet the diffeomorphism nontrivial.

The above is exactly what one expects: we have identified the 'residual' diffeomorphisms, that is, the ones that preserve the bulk form of the metric (as per Fixed) and preserve the boundary metric (as per Invariance) as the boundary conformal group. Thus, the CFT's group of invariances arises in this way explicitly from bulk diffeomorphisms.

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This technical discussion returns us to our distinction between Local and Redundant and the 'internal point of view' mentioned in footnote 2. Thus, the boundary coordinate transformations ξ_i considered here are Local: they are position-dependent coordinate transformations, but they represent overall transformations of the correlation functions (5) derived from either (2) or (3). The correlation functions are indeed covariant under such coordinate transformations.[7] Therefore, on the 'internal point of view' mentioned in footnote 2, two states of the universe related by such transformations are physically equivalent: the transformations count as Redundant gauge symmetries. But, the contrary 'external point of view' interprets (3) as coupled to an already interpreted physical background, represented by $\phi_{(0)}(x)$, and in that case, the diffeomorphisms would not be Redundant but would become physical. When we describe Galileo's ship relative to the quay, its state of motion is indeed physically meaningful.

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7. Of course, they are invariant under the full bulk diffeomorphisms considered, i.e., the boundary coordinate transformations and the compensating rescaling of r.

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