Macroeconomic Dynamics, **24**, 2020, 1574–1594. Printed in the United States of America.
doi:10.1017/S1365100518000883

A NOTE ON ROBUST MONETARY POLICY AND NON-ZERO TREND INFLATION

KOHEI HASUI
Matsuyama University

This paper studies how model uncertainty influences economic fluctuation when trend inflation is high. We introduce Hansen and Sargent's [(2008) *Robustness*, Princeton University Press] robust control techniques into a New Keynesian model with non-zero trend inflation. We reveal the following three points. First, we find that robust monetary policy responds more aggressively. This aggressiveness increases with trend inflation. Second, as the trend inflation rises, the response of macroeconomic variables is larger under robust policy. Third, stronger robustness tends to lead to indeterminate equilibrium as trend inflation increases. Consequently, the economy might be volatile when trend inflation is high due to robustness from the view of both variance and determinacy. We interpret the results as indicating that the model uncertainty might be the one of the factors causing large macroeconomic fluctuations when trend inflation is high.

Keywords: Trend Inflation, Robust Control, Discretionary Monetary Policy

1. INTRODUCTION

The relation between inflation and monetary policy is one of the most important topics. In a standard New Keynesian model, a more than 1% increase in policy rate in response to a 1% increase in inflation rate leads to a determinate equilibrium. This is called the *Taylor Principle*. However, once the trend of inflation is positive, the Taylor Principle does not hold. Ascari and Ropele (2009) show that the standard Taylor rule fails to stabilize and therefore the economy falls into indeterminate equilibria. The relation between trend inflation and monetary policy has been explored by determinacy analyses. Moreover, some papers analyze this relationship in a New Keynesian model from an empirical perspective. Coibion and Gorodnichenko (2011) argue that the US economy experienced high volatility because high trend inflation leads to economy indeterminacy of the rational expectation equilibrium. Kobayashi and Muto (2013) also explain

The author thanks the editor William Barnet, two anonymous referees, Masakatsu Nakamura, Kengo Nutahara, and any other participants at the 52nd KMSG workshop in Meiji Gakuin University, at the Japan Society of Monetary Economics Spring Meeting in Senshu University, and at the International Finance and Development Economics Workshop in Kurume University for valuable suggestions and comments. The author acknowledges financial support from JSPS KAKENHI Grants Nos. 16K03920 and 17K13768. Address correspondence to: Kohei Hasui, Department of Economics, Matsuyama University, Japan; e-mail: khasui@g.matsuyama-u.ac.jp. Phone: +81-89-926-7764.

© 2018 Cambridge University Press 1469-8056/18

https://doi.org/10.1017/S1365100518000883 Published online by Cambridge University Press

this macroeconomic phenomenon by introducing adaptive learning to the New Keynesian model with non-zero trend inflation and show that high trend inflation leads to the rational expectation equilibrium becoming expectational unstable.

In the present paper, we take an alternative approach to analyzing the relation between monetary policy and trend inflation. We introduce Hansen and Sargent's (2008) robust control technique as the imperfect knowledge about the economic structure. Under the robust control problem, the true economic structure deviates from the so-called reference macroeconomic structure by the misspecification terms. The central bank cannot know this true economic structure and cannot formulate the probability for the misspecification terms. This uncertainty is called *Knightian* uncertainty. When the central bank designs a monetary policy supposing the worst-case outcome, it is called *robust policy*. Thus, robust policy deviates from policy without Knightian uncertainty (henceforth, the *reference model*).

We introduce a robust control approach developed by Giordani and Söderlind (2004) and Hansen and Sargent (2008) to a New Keynesian model with non-zero trend inflation as imperfect knowledge. First, we analyze how changes in trend inflation affect the deviation between robust policy and the reference model. Moreover, we analyze how this deviation affects volatility in macroeconomic variables. We derive closed-form analytical solutions of the model under optimal discretionary policy. We find that deviations between robust policy and the reference model become larger as trend inflation rises. Moreover, we find this increase in deviation leads to higher economic volatility with trend inflation.

The key mechanism for the result is a slope of the Phillips curve. The value of the slope becomes small as trend inflation increases. In response to positive cost-push shock, the central bank lowers the output gap by raising the policy rate to mitigate the impact on the inflation rate. However, the central bank needs to decrease the output gap more, because the slope is small and the impact on inflation is also small when trend inflation is high. The central bank knows this mechanism, and robust policy responds more aggressively as trend inflation increases. This leads to the aggressive response of the inflation rate.

Second, along the lines of determinacy analysis in the literature of trend inflation, we investigate the determinacy region for degree of trend inflation and model uncertainty. We find that stronger robustness leads to indeterminate equilibria as the trend inflation increases. This result is consistent with Kobayashi and Muto (2013). A higher trend inflation makes the model more forward-looking. On the other hand, model uncertainty increases the volatility of the economy. Combining these two factors, the model deviates from equilibrium substantially and tends to be indeterminate.

Our paper is related to two strands in the literature. First, our paper is related to robust monetary policy. The seminal work of Brainard (1967) first showed that a policy maker will become more cautious (called the "Brainard's conservative mechanism") when multiplicative uncertainty exists. Giordani and Söderlind (2004) and Leitemo and Söderström (2008a,b) analyze the robust policy by

https://doi.org/10.1017/S1365100518000883 Published online by Cambridge University Press

introducing Hansen and Sargent's (2008) robust control technique. Tillmann (2009a) examines the robust monetary policy in a New Keynesian model with a cost channel, and Gerke and Hammermann (2016) investigate a robust monetary policy in a New Keynesian model with incomplete interest rate pass-through.[1] In addition to Hansen and Sargent's (2008) technique, some papers analyze monetary policy under imperfect knowledge in alternative ways. Aoki (2003) explores the optimal monetary policy when measurement errors exist in data and shows the policy cautiousness.[2] Giannoni (2002, 2007) analyzes the robust monetary policy when the parameter uncertainty exists. Cogley et al. (2008) consider US robust monetary policy for Bayesian parametric uncertainty. Orphanides and Wieland (2013) examine the robust policy rules by evaluating alternative policies toward the various models for the Euro area.

Second, our paper is related to the literature of trend inflation. Ascari (2004), Ascari and Ropele (2007, 2009), Sbordone (2007), and Cogley and Sbordone (2008) introduce positive trend inflation into the sticky price model. Ascari (2004) and Ascari and Ropele (2007, 2009) analyze how a positive trend inflation affects economic dynamics, firms' pricing behavior, and the stabilization problem in monetary policy. Ascari and Ropele (2007) show that optimal discretionary policy does not guarantee a determinate equilibrium with positive trend inflation, and that positive trend inflation deteriorates social welfare under the optimal commitment policy. Ascari and Ropele (2009) find that the standard Taylor rule fails to stabilize the economy, which falls into indeterminate equilibria since positive trend inflation makes pricing behavior more forward-looking.[3] Sbordone (2007) and Cogley and Sbordone (2008) investigate the inflation dynamics under a positive trend inflation rate. They show that a positive long-run trend inflation rate can explain the persistent dynamics of the inflation rate without introducing intrinsic inflation persistence. Masolo and Monti (2017) explain the motion of US trend inflation by introducing Knightian uncertainty to the central bank's monetary policy conduct. While our analysis is based on the optimal discretionary policy, they present a numerical analysis of monetary policy under a Taylor rule.

The remainder of this paper is organized as follows: Section 2 explains the model and robust control problem. Sections 3 and 4 analyze the effects of trend inflation on robust monetary policy. First, we solve the model analytically and confirm the fundamental property of robust policy. Second, we analyze the effects of trend inflation on robust monetary policy numerically. Section 5 presents our conclusions.

2. THE MODEL

2.1. The Macroeconomic Structure

The model is a New Keynesian model with trend inflation, proposed by Sbordone (2007) and Cogley and Sbordone (2008).

https://doi.org/10.1017/S1365100518000883 Published online by Cambridge University Press

TABLE 1. Coefficients on forward terms for degree of trend inflation

$\bar{\pi}$ (%)	$\beta\xi(\Pi)$	$\beta\chi(\Pi)$	$\beta\zeta(\Pi)$	$\lambda(\Pi)$
0	0.99	0	0	0.0062903
1	1.0117	0.00013398	0.0087346	0.0057769
2	1.033	0.0002519	0.016975	0.0052884
3	1.054	0.00035389	0.024695	0.0048242

TABLE 2. Parameter values

Parameters	Values	Explanation
β	0.99	Discount factor
σ	1.5	Intertemporal substitution of consumption
ω	0.429	Sensitivity of real marginal cost to output
α	0.588	Price stickiness
ϵ	9.8	Elasticity of substitution for individual goods
σ_r	1.0160	Standard deviation of natural rate shock
σ_u	0.154	Standard deviation of cost-push shock

$$x_t = \mathbb{E}_t x_{t+1} - \sigma(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n), \tag{1}$$

$$\pi_t = \left[\kappa(\Pi) - (1-\phi)^{-1}\chi(\Pi)\right] x_t + \beta\xi(\Pi)\mathbb{E}_t\pi_{t+1} + \beta\chi(\Pi)\mathbb{E}_t x_{t+1} + \beta\chi(\Pi)\mathbb{E}_t \sum_{j=2}^{\infty} \phi^{j-1} x_{t+j} + \beta\zeta(\Pi)\mathbb{E}_t \sum_{j=2}^{\infty} \phi^{j-1}\pi_{t+j} + \gamma(\Pi) r_t^n + u_t, \tag{2}$$

where x_t, i_t, r_t^n, and u_t denote output gap, nominal interest rate, natural rate of interest, and cost-push shock, respectively. π_t denotes the percent deviation of the inflation rate from trend inflation. Π denotes the gross rate of trend inflation. We define $\bar{\pi}$ as the annual rate of trend inflation.[4] Parameters ϕ, $\kappa(\Pi)$, $\chi(\Pi)$, $\xi(\Pi)$, $\zeta(\Pi)$, and $\gamma(\Pi)$ denote structural parameters. We give definitions of these structural parameters in Appendix A.1 in the Supplementary Material. r_t^n and u_t are *i.i.d* disturbance terms with standard deviation σ_r and σ_u, respectively. The explanations and values of deep parameters are listed in Table 2.

As Ascari and Ropele (2007) point out, trend inflation influences firms' pricing behavior, and the model becomes more forward-looking. We check how values of coefficients on forward-looking terms $\beta\xi(\Pi)$, $\beta\chi(\Pi)$, and $\beta\zeta(\Pi)$ vary as trend inflation increases by setting annualized trend inflation as 0%, 1%, 2%, and 3%. Table 1 shows that $\beta\xi(\Pi)$, $\beta\chi(\Pi)$, and $\beta\zeta(\Pi)$ increase as trend inflation rises. This indicates that a positive trend inflation makes the model more forward-looking.

To compute the model, we change the form of the New Keynesian Phillips curve (NKPC) simply by introducing the auxiliary variable.

$$\pi_t = \mathcal{K}(\Pi)x_t + \mathcal{B}(\Pi)\mathbb{E}_t\pi_{t+1} + h_t + \gamma(\Pi)r_t^n + u_t,$$
$$h_t = \beta\chi(\Pi)\mathbb{E}_t x_{t+1} + \beta\zeta(\Pi)\mathbb{E}_t\pi_{t+1} + \phi\mathbb{E}_t h_{t+1},$$

https://doi.org/10.1017/S1365100518000883 Published online by Cambridge University Press

where h_t denotes the auxiliary variable. We define $\mathcal{K}(\Pi) \equiv \kappa(\Pi) - (1-\phi)^{-1}\chi(\Pi)$ and $\mathcal{B}(\Pi) \equiv \beta\,[\xi(\Pi) - \zeta(\Pi)]$.

2.2. The Central Bank's Objective Function

The welfare loss function of the economy is derived by second-order approximation of the household's utility function. Under non-zero trend inflation, the welfare loss function includes trend inflation, since it affects price dispersion. Alves (2012, 2014) derives the trend inflation welfare-based loss function by approximating the second-order utility function around the steady state with trend inflation. The central bank's intertemporal problem is given as follows:

$$\mathcal{L}_t = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j L_{t+j},$$

where the period loss function is

$$L_t = \pi_t^2 + \lambda(\Pi) x_t^2, \quad \lambda(\Pi) = \frac{1-\bar{\alpha}}{1-\bar{\alpha}\vartheta}\frac{\kappa(\Pi)}{\epsilon}.$$

The weight on the output gap depends on the level of trend inflation. As indicated in Table 1, $\lambda(\Pi)$ takes 0.0062903, 0.0057769, 0.0052884, and 0.0048242 for annualized trend inflation 0%, 1%, 2%, and 3%, respectively. The weights on output gap decrease and relative weights on inflation increase as trend inflation increases. One can interpret this as follows. The central bank weights on inflation rate increase with trend inflation because the fluctuation in inflation might become larger as trend inflation becomes higher.[5]

2.3. Robust Control Problem

Up to the previous section, we assumed that the central bank and economic agent know the model exactly. The model satisfies the certainty equivalence, because terms of shocks are additively separable. We call the model up to this point the "reference model".

Now, we relax the assumption. In the actual world, we cannot capture the economy exactly. We describe this situation by introducing Hansen and Sargent's (2008) robust control technique. The central bank knows the reference model but does not know the true model. The true model lies in the neighborhood of the reference model. The robust control approach introduces specification error terms, denoted v_t^x, v_t^π, and v_t^h to describe the deviation between the true model and reference model. The misspecified economic model is given by the following equations[6]:

$$\begin{aligned}
x_t &= \mathbb{E}_t x_{t+1} - \sigma[i_t - \mathbb{E}_t\pi_{t+1} - (r_t^n + v_t^x)],\\
\pi_t &= \mathcal{K}(\Pi)x_t + \mathcal{B}(\Pi)\mathbb{E}_t\pi_{t+1} + h_t + \gamma(\Pi)(r_t^n + v_t^x) + u_t + v_t^\pi,\\
h_t &= \beta\chi(\Pi)\mathbb{E}_t x_{t+1} + \beta\zeta(\Pi)\mathbb{E}_t\pi_{t+1} + \phi\mathbb{E}_t h_{t+1} + v_t^h.
\end{aligned}$$

https://doi.org/10.1017/S1365100518000883 Published online by Cambridge University Press

Moreover, the central bank cannot formulate the probability to v_t^x, v_t^π, and v_t^h. Thus, the robust control approach assumes that the central bank designs a "robust" monetary policy supposing the worst possible outcome is realized. We refer to this model as "worst-case". However, even though monetary policy adopts a robust policy supposing the worst-case, misspecification does not exist in practice. We refer to this model as the "approximating model". The specification error terms are set by an "evil agent" to maximize the loss in the central bank's objective function. The evil agent sets specification error terms subject to the following constraint:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j v_{t+j}' v_{t+j} \leq v_0,$$

where $v_t = [v_t^x; v_t^\pi; v_t^h]$. According to Giordani and Söderlind (2004) and Hansen and Sargent (2008), the robust control problem can be recast as

$$\begin{aligned} &\min_{x_t,\pi_t,i_t} \max_{v_t} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j [L_{t+j} - \theta v_{t+j}' v_{t+j}], \\ &\text{s.t} \\ &x_t = \mathbb{E}_t x_{t+1} - \sigma[i_t - \mathbb{E}_t \pi_{t+1} - (r_t^n + v_t^x)], \\ &\pi_t = \mathcal{K}(\Pi) x_t + \mathcal{B}(\Pi)\mathbb{E}_t \pi_{t+1} + h_t + \gamma(\Pi)(r_t^n + v_t^x) + (u_t + v_t^\pi), \\ &h_t = \beta\chi(\Pi)\mathbb{E}_t x_t + \beta\zeta(\Pi)\mathbb{E}_t \pi_{t+1} + \phi\mathbb{E}_t h_{t+1} + v_t^h, \end{aligned} \tag{3}$$

where θ denotes the central bank's preference for robustness, that is, the degree of robustness. $\lim_{v\to 0} \theta = \infty$ denotes the case that no misspecification exists in the model, that is, the reference model in the present paper. On the other hand, a low value of θ denotes the case where strong misspecification exists.

We solve problem (3) and obtain solutions for forward variables x_t, π_t, and h_t. In addition to the solution of forward variables, misspecification error terms can be solved as a function of r_t^n and u_t.

$$v_t = F_v \begin{bmatrix} r_t^n \\ u_t \end{bmatrix}.$$

F_v denotes a 3×2 matrix. The approximating model is obtained by setting $F_v = \mathbf{0}_{3\times 2}$.[7]

To evaluate θ, we introduce the detection error probability. Given the data from the model, the probability is calculated as the rate of wrong choices between the worst-case model and approximating model. Thus, the detection error probability indicates the difficulty of distinguishing between models with and without misspecification. Following Giordani and Söderlind (2004), we set θ so that the detection error probability becomes 20% for numerical analysis in Section 4.

https://doi.org/10.1017/S1365100518000883 Published online by Cambridge University Press

3. THE EFFECTS OF ROBUST POLICY

This section derives the analytic policy function of i_t, π_t, and x_t under a robust control problem. The first-order conditions under discretion are given as follows[8,9]:

$$x_t = -\frac{\mathcal{K}(\Pi)}{\lambda(\Pi)}\pi_t, \tag{4}$$

$$-\sigma\phi_{1t} = 0, \tag{5}$$

$$\phi_{2t} - \phi_{3t} = 0, \tag{6}$$

$$-\theta v_t^x + \sigma\phi_{1t} + \gamma(\Pi)\phi_{2t} = 0, \tag{7}$$

$$-\theta v_t^\pi + \phi_{2t} = 0, \tag{8}$$

$$-\theta v_t^h + \phi_{3t} = 0. \tag{9}$$

Equation (4) denotes a *targeting rule*. The parameter of robustness θ does not enter into the coefficient of the targeting rule. This shows that the targeting rule is not affected by the robust control approach, which is consistent with Leitemo and Söderström (2008b).

Now, we derive policy functions for π, x_t, and i_t by the undetermined coefficient method using structural equations and first-order conditions. We guess the policy functions as follows[10]:

$$\pi_t = a_r r_t^n + a_u u_t,$$
$$x_t = b_r r_t^n + b_u u_t,$$
$$i_t = c_r r_t^n + c_u u_t.$$

The value of coefficients of policy function depends on preference of robustness θ. This leads to a complication in analyzing the effects of robustness. We pose Assumption 1 following Leitemo and Söderström (2008b).

ASSUMPTION 1. *We analyze the effects of small decreases in θ starting from $\theta = \infty$.*

Assumption 1 shows that we analyze the effects of small increases in robustness starting from the rational expectation.

Supposing reasonable values of deep parameters, we pose Assumption 2 for some coefficients that depend on a value of trend inflation.

ASSUMPTION 2. *We assume $\sigma \geq 1$, $0 < \alpha < 1$, $0 < \omega < 1$, $\epsilon > 1$, and $\mathcal{K}(\Pi) > 0$.*

The solution of policy function for inflation rate is given as follows:

$$\pi_t = a_r(\Pi, \theta) r_t^n + a_u(\Pi, \theta) u_t$$
$$= \frac{\gamma(\Pi)}{A(\Pi, \theta)} r_t^n + \frac{1}{A(\Pi, \theta)} u_t,$$

https://doi.org/10.1017/S1365100518000883 Published online by Cambridge University Press

where

$$A(\Pi,\theta)\equiv\frac{[\mathcal{K}(\Pi)]^2}{\lambda(\Pi)}-\frac{1}{\theta}\{[\gamma(\Pi)]^2+2\}+1>0.$$

Combining the policy function of inflation with the targeting rule, we obtain the policy function of the output gap as follows:

$$\begin{aligned}x_t&=b_r(\Pi,\theta)r_t^n+b_u(\Pi,\theta)u_t\\&=-\frac{\mathcal{K}(\Pi)}{\lambda(\Pi)}[a_r(\Pi,\theta)r_t^n+a_u(\Pi,\theta)u_t],\end{aligned}$$

where $b_r<0$ and $b_u<0$. Finally, we obtain the policy function of the nominal interest rate as follows:

$$\begin{aligned}i_t&=c_r(\Pi,\theta)r_t^n+c_u(\Pi,\theta)u_t\\&=\left[\left(\frac{1}{\theta}+\frac{\mathcal{K}(\Pi)}{\sigma\lambda(\Pi)}\right)a_r(\Pi,\theta)+1\right]r_t^n+\left(\frac{1}{\theta}+\frac{\mathcal{K}(\Pi)}{\sigma\lambda(\Pi)}\right)a_u(\Pi,\theta)u_t^n,\end{aligned}$$

where $c_r>0$ and $c_u>0$.

Now, we analyze the effects of robustness on policy function. We evaluate the effect following Leitemo and Söderström (2008a).

$$-\frac{\partial|z_j(\Pi,\theta)|}{\partial\theta},\ z=a,b,c,\ j=r,u. \tag{10}$$

This partial derivative shows changes in the coefficient in response to a small decrease in θ. Since $\theta=\infty$ corresponds to the case of rational equilibrium, a decrease in θ shows the effect of robustness.

First, we analyze the effect on coefficients of inflation, $a_r(\Pi,\theta)$ and $a_u(\Pi,\theta)$. The effects of the central bank's robustness to $a_u(\Pi,\theta)$ are given by

$$\begin{aligned}-\frac{\partial|a_u(\Pi,\theta)|}{\partial\theta}&=\frac{1}{A(\Pi,\theta)^2}\frac{\partial A(\Pi,\theta)}{\partial\theta}\\&=\frac{[\gamma(\Pi)]^2+2}{[\theta A(\Pi,\theta)]^2}>0.\end{aligned} \tag{11}$$

Similarly for $a_r(\Pi,\theta)$,

$$-\frac{\partial|a_r(\Pi,\theta)|}{\partial\theta}=-\frac{\partial|a_u(\Pi,\theta)|}{\partial\theta}\gamma(\Pi)>0. \tag{12}$$

Equations (11) and (12) show that inflation rate responds aggressively to cost-push shock and natural rate shock under robust monetary policy.

Next we analyze the effects of robustness on coefficients of the output gap, $b_j(\Pi,\theta), j=r,u$. The absolute value $|b_j|$ is $-b_j$, because b_j is negative. The effects of robustness to $b_u(\Pi,\theta)$ are

https://doi.org/10.1017/S1365100518000883 Published online by Cambridge University Press

$$-\frac{\partial|b_u(\Pi,\theta)|}{\partial\theta} = \frac{\mathcal{K}(\Pi)}{\lambda(\Pi)}\frac{\partial a_u(\Pi,\theta)}{\partial\theta} = \frac{\mathcal{K}(\Pi)}{\lambda(\Pi)}\frac{[\gamma(\Pi)^2+2]}{[\theta A(\Pi,\theta)]^2} > 0. \tag{13}$$

Similarly, the effects of robustness to $b_r(\Pi,\theta)$ are given as follows:

$$-\frac{\partial|b_r(\Pi,\theta)|}{\partial\theta} = -\frac{\partial|b_u(\Pi,\theta)|}{\partial\theta}\gamma(\Pi) > 0. \tag{14}$$

Equations (13) and (14) show that the output gap responds aggressively to cost-push shock and natural rate shock under robust policy.

Finally, we derive the effects of robustness on the coefficients of the nominal interest rate, $c_r(\Pi,\theta)$ and $c_u(\Pi,\theta)$. The effects of robustness to $c_u(\Pi,\theta)$ are given as follows:

$$-\frac{\partial|c_u(\Pi,\theta)|}{\partial\theta} = \frac{a_u(\Pi,\theta)}{\theta^2} - \left(\frac{1}{\theta}+\frac{\mathcal{K}(\Pi)}{\sigma\lambda(\Pi)}\right)\frac{\partial a_u(\Pi,\theta)}{\partial\theta} = \frac{1}{\theta^2 A(\Pi,\theta)} + \left(\frac{1}{\theta}+\frac{\mathcal{K}(\Pi)}{\sigma\lambda(\Pi)}\right)\frac{[\gamma(\Pi)]^2+2}{[\theta A(\Pi,\theta)]^2} > 0. \tag{15}$$

The effects on the coefficient of the natural rate $c_r(\Pi,\theta)$ are

$$-\frac{\partial|c_r(\Pi,\theta)|}{\partial\theta} = -\frac{\partial|c_u(\Pi,\theta)|}{\partial\theta}\gamma(\Pi) > 0. \tag{16}$$

Equations (15) and (16) show that monetary policy responds aggressively to cost-push shock and natural rate shock under robust policy.

From the results of equations (11)–(16), we obtain the following proposition.

PROPOSITION 1. *In the worst-case scenario, equations (11)–(16) show that stronger policy robustness increases sensitivity in inflation, output gap, and policy rate in response to cost-push shock and natural rate shock under Assumption 2.*

In terms of the inflation rate, Proposition 1 is consistent with Giordani and Söderlind (2004), but opposite to the Leitemo and Söderström's (2008b) result, which found that the inflation rate responds cautiously to cost-push shock.[11] However, as Giordani and Söderlind (2004) pointed, robustness leads to more persistent response in inflation rate, output gap, and nominal interest rate in a standard New Keynesian model. Our findings are consistent with this result.

Moreover, our result shows responses to natural rate shock under robust policy. In a standard New Keynesian model, robust policy does not deviate from the rational expectation, because the monetary policy does not face a trade-off. However, in our model, natural rate shock appears in a Phillips curve. This means that the central bank faces a trade-off. The robust policy deviates from the rational expectation. In our analytic solution, the robust policy responds to the natural rate more aggressively.

In addition to Proposition 1, equations (11)–(16) show that sensitivity in response to the robustness possibly depends on trend inflation. We analyze

https://doi.org/10.1017/S1365100518000883 Published online by Cambridge University Press

how the amount of change in sensitivity depends on trend inflation by calculating the cross derivatives of $\partial^2|a_u(\Pi,\theta)|/\partial\theta\partial\Pi$, $\partial^2|b_u(\Pi,\theta)|/\partial\theta\partial\Pi$, and $\partial^2|c_u(\Pi,\theta)|/\partial\theta\partial\Pi$ for cost-push shock.[12] These cross derivatives depend on the derivative of $\mathcal{K}(\Pi)$, $\lambda(\Pi)$, $\gamma(\Pi,\theta)$, and $A(\Pi,\theta)$ with respect to Π. We calculate these derivatives for $\sigma=1$ and $\sigma>1$ separately. The case where $\sigma=1$ is simpler than that where $\sigma>1$, because some structural parameters are reduced to zero.

Under $\sigma>1$, the sign of $\mathcal{K}(\Pi)$ and $\lambda(\Pi)$ is as follows[13]:

$$\frac{\partial\mathcal{K}(\Pi)}{\partial\Pi}<0,\ \frac{\partial\lambda(\Pi)}{\partial\Pi}<0\ \text{ if }\ \frac{\alpha(\epsilon-1)\Pi^{2\omega-\epsilon(\omega+1)-2}(\Pi^{1+\omega\epsilon}-1)}{1-\alpha\beta\Pi^{\epsilon(1+\omega)}}\leq 1+\omega\epsilon. \quad \textbf{(17)}$$

Under $\sigma=1$, the signs of $\mathcal{K}(\Pi)$, $\lambda(\Pi)$, $\gamma(\Pi,\theta)$, and $A(\Pi,\theta)$ are determined as follows:

$$\frac{\partial\kappa(\Pi)}{\partial\Pi}<0,\ \frac{\partial\lambda(\Pi)}{\partial\Pi}<0,\ \frac{\partial A(\Pi,\theta)}{\partial\Pi}<0,\ \frac{\partial\gamma(\Pi)}{\partial\Pi}=0,$$

where $\mathcal{K}(\Pi)=\kappa(\Pi)$ if $\sigma=1$. The directions of $\partial A(\Pi,\theta)/\partial\Pi$ and $\partial\lambda(\Pi)/\partial\Pi$ are mainly determined by the direction of $\partial\kappa(\Pi)/\partial\Pi$.

First, we begin with the policy function of inflation rate. The cross derivative of $a_u(\Pi,\theta)$ is given as follows:

$$-\frac{\partial^2|a_u(\Pi,\theta)|}{\partial\theta\partial\Pi}=\frac{2\gamma(\Pi)}{[\theta A(\Pi,\theta)]^2}\frac{\partial\gamma(\Pi)}{\partial\Pi}-2\frac{[\gamma(\Pi)]^2+2}{\theta^2[A(\Pi,\theta)]^3}\frac{\partial A(\Pi,\theta)}{\partial\Pi}. \quad \textbf{(18)}$$

The sign of equation (18) can be positive and negative. If $\partial\gamma(\Pi)/\partial\Pi>0$ and equation (17) are satisfied, the sign of equation (18) can be non-negative. Moreover, if $\sigma=1$, the cross derivative of $a_u(\Pi,\theta)$ is reduced to a simpler form as follows:

$$-\frac{\partial^2|a_u(\Pi,\theta)|}{\partial\theta\partial\Pi}=-\frac{4}{\theta^2[A(\Pi,\theta)]^3}\frac{\partial A(\Pi,\theta)}{\partial\Pi}\geq 0. \quad \textbf{(19)}$$

For inflation rate in response to cost-push shock, equation (19) shows that an increase in trend inflation amplifies the effects of robustness in the case where $\sigma=1$.

Second, we derive the cross derivative of the output gap. The cross derivative of $b_u(\Pi,\theta)$ is given as follows:

$$-\frac{\partial^2|b_u(\Pi,\theta)|}{\partial\theta\partial\Pi}=\frac{\mathcal{K}(\Pi)}{\lambda(\Pi)}\frac{[\gamma(\Pi)]^2+2}{[\theta A(\Pi,\theta)]^2}\times\left[\left(2-\frac{\kappa(\Pi)}{\mathcal{K}(\Pi)}\right)\frac{\alpha(\epsilon-1)\Pi^{\epsilon-2}}{1-\alpha\Pi^{\epsilon-1}}+\left(1-\frac{\kappa(\Pi)}{\mathcal{K}(\Pi)}\right)\frac{\epsilon-1}{\Pi}\right.$$

https://doi.org/10.1017/S1365100518000883 Published online by Cambridge University Press

$$-\frac{\kappa(\Pi)}{\mathcal{K}(\Pi)}\frac{\alpha\beta\epsilon(1+\omega)\Pi^{\epsilon(1+\omega)-1}}{1-\alpha\beta\Pi^{\epsilon(1+\omega)}}+\frac{1-\sigma^{-1}}{\sigma^{-1}+\omega}\left(1-\frac{\kappa(\Pi)}{\mathcal{K}(\Pi)}\right)\left(\frac{\alpha(1+\omega\epsilon)\Pi^{\epsilon(1+\omega)-1}}{1-\alpha\Pi^{\epsilon-1}}-\frac{\alpha^2(\epsilon-1)(\Pi^{1+\omega\epsilon-1})\Pi^{2\omega-3}}{(1-\alpha\Pi^{\epsilon-1})(1-\alpha\beta\Pi^{\epsilon(1+\omega)})}\right)\Bigg]-\frac{\mathcal{K}(\Pi)}{\lambda(\Pi)}\frac{\partial^2|a_u(\Pi,\theta)|}{\partial\theta\partial\Pi}. \tag{20}$$

It is different from $a_u(\Pi,\theta)$. The sign of the cross derivative of $b_u(\Pi,\theta)$ is ambiguous even though the derivative of $\partial\gamma(\Pi)/\partial\Pi\geq 0$ and condition (17) is satisfied. Under $\sigma=1$, the derivative is reduced as follows:

$$-\frac{\partial^2|b_u(\Pi,\theta)|}{\partial\theta\partial\Pi}=\frac{\kappa(\Pi)}{\lambda(\Pi)}\frac{[\gamma(\Pi)]^2+2}{[\theta A(\Pi,\theta)]^2}\times\left[\frac{\alpha(\epsilon-1)\Pi^{\epsilon-2}}{1-\alpha\Pi^{\epsilon-1}}-\frac{\alpha\beta\epsilon(1+\omega)\Pi^{\epsilon(1+\omega)-1}}{1-\alpha\beta\Pi^{\epsilon(1+\omega)}}\right]-\frac{\kappa(\Pi)}{\lambda(\Pi)}\frac{4}{\theta^2[A(\Pi,\theta)]^3}\frac{\partial A(\Pi,\theta)}{\partial\Pi}.$$

It is unlikely in the case of $a_u(\Pi,\theta)$ that the cross derivative of $b_u(\Pi,\theta)$ can be both positive and negative, even though $\sigma=1$.

Finally, we derive the cross derivative for coefficients on nominal interest rate. The cross derivative of $c_u(\Pi,\theta)$ is given as follows:

$$-\frac{\partial^2|c_u(\Pi,\theta)|}{\partial\theta\partial\Pi}=-\frac{1}{[\theta A(\Pi,\theta)]^2}\frac{\partial A(\Pi,\theta)}{\partial\Pi}-\frac{1}{\sigma}\frac{\partial^2|b_u(\Pi,\theta)|}{\partial\theta\partial\Pi}. \tag{21}$$

The cross derivative of $c_u(\Pi,\theta)$ for $\sigma=1$ is also ambiguous, because the sign of the cross derivative of $b_u(\Pi,\theta)$ is ambiguous even though $\sigma=1$. From the results of equations (18)–(21), we obtain the following proposition.

PROPOSITION 2. *In the worst-case scenario, equations (18)–(21) show that the amount of change in sensitivity in inflation, output gap, and policy rate depends on trend inflation.*

How trend inflation affects the sensitivity in inflation, output gap, and policy rate is complicated. We tackle this question numerically in Section 4.

4. NUMERICAL RESULTS

This section analyzes numerically how trend inflation affects the change in sensitivity in inflation, output gap, and policy rate. Our numerical simulation is derived using Giordani and Söderlind's (2004) MATLAB codes.[14]

4.1. Main Result

Table 2 presents the values of parameters. We set parameters α, β, ω, and ϵ following Cogley and Sbordone (2008), and set standard deviations of shocks

https://doi.org/10.1017/S1365100518000883 Published online by Cambridge University Press

TABLE 3. Policy function for i_t under discretion

(a) Coefficient on cost-push shock (u_t)				
Trend inflation (%)	0	1	2	3
i_t ($\theta = \infty$)	4.0729	4.1696	4.2485	4.3033
i_t (worst-case)	4.1481	4.2845	4.4089	4.5051
Change in percent	1.8471	2.7559	3.7771	4.6915
(b) Coefficient on natural rate shock (r_t^n)				
Trend inflation (%)	0	1	2	3
i_t ($\theta = \infty$)	1	1.0028	1.0062	1.0106
i_t (worst-case)	1	1.0031	1.0105	1.0346
Change in percent	0	0.030722	0.42327	2.3739

TABLE 4. Policy function for π_t under discretion

(a) Coefficient on cost-push shock (u_t)				
Trend inflation (%)	0	1	2	3
π_t ($\theta = \infty$)	0.6234	0.65499	0.68771	0.72137
π_t (worst-case)	0.72214	0.76588	0.81325	0.85645
Change in percent	15.8379	16.9303	18.2538	18.7261
(b) Coefficient on natural rate shock (r_t^n)				
Trend inflation (%)	0	1	2	3
π_t ($\theta = \infty$)	0	0.0013414	0.0030664	0.0053243
π_t (worst-case)	0	0.0040958	0.013599	0.041227
Change in percent	0	205.3302	343.4835	674.3166

following Adam and Billi (2006). We set $\sigma = 1.5$ for a numerical reason.[15] Tables 3–5 show the policy functions of i_t, π_t, and x_t. Panel (a) in each table shows the coefficients on u_t when trend inflation is 0%, 1%, 2%, and 3%; panel (b) shows the coefficients on r_t^n.

To capture the fundamental effect of the robust control, we begin with the case where the trend inflation is zero. Panel (a) in each table shows that the coefficients on cost-push shock in the worst-case are amplified by coefficients of the reference model ($\theta = \infty$). Monetary policy responds to cost-push shock more aggressively than policy under the reference model. This is because the central bank fears a large impact on the inflation rate by cost-push shock [c.f. Leitemo and Söderström (2008b)]. π_t and x_t respond aggressively to cost-push shock in the worst-case. These results are consistent with Giordani and Söderlind (2004),

https://doi.org/10.1017/S1365100518000883 Published online by Cambridge University Press

TABLE 5. Policy function for x_t under discretion

(a) Coefficient on cost-push shock (u_t)				
Trend inflation (%)	0	1	2	3
x_t ($\theta = \infty$)	−6.1093	−6.2545	−6.3728	−6.455
x_t (worst-case)	−7.0769	−7.3134	−7.5361	−7.6637
Change in percent	15.8379	16.9303	18.2538	18.7261
(b) Coefficient on natural rate shock (r_t^n)				
Trend inflation (%)	0	1	2	3
x_t ($\theta = \infty$)	0	−0.012809	−0.028415	−0.047643
x_t (worst-case)	0	−0.03911	−0.12601	−0.36891
Change in percent	0	205.3302	343.4835	674.3166

but not with Leitemo and Söderström (2008b). In the Giordani and Söderlind's (2004) analysis, macroeconomic variables response is more persistent under robust policy than in the reference model.

Panel (b) in the tables shows the coefficients on the natural rate. However, results under the worst-case do not change from the reference model, because monetary policy does not face a trade-off in response to natural rate. This result is consistent with Leitemo and Söderström (2008a) and Leitemo and Söderström (2008a,b).

We now turn to the analysis of how trend inflation affects changes in coefficients between the worst-case and reference models. The coefficients on cost-push shock in the worst-case are amplified by the coefficients of the reference model. However, the amount of amplification increases with trend inflation. Changes in coefficients of the nominal interest rate are particularly large relative to inflation and output gap. Changes in percent of nominal interest rate increase about 1% as trend inflation increases 1%.

Changes in coefficients on natural rate also increase as trend inflation increases. However, the amount of change is large, since values of coefficients are very low.

Though it is a little complicated to disentangle why high trend inflation leads to an aggressive policy response to cost-push shock under robust policy, we attempt to interpret the mechanism. To obtain insight, it is useful to recall the model for rational expectation equilibrium. The key reason is a main part of the slope of the Phillips curve $\kappa(\Pi)$. $\kappa(\Pi)$ is a decreasing function with respect to trend inflation.[16]

$$\kappa(\Pi) = \frac{(1-\bar{\alpha}(\Pi))(1-\phi\vartheta(\Pi))}{\bar{\alpha}(\Pi)} \frac{\sigma^{-1}+\omega}{1+\omega\epsilon}, \quad \bar{\alpha}(\Pi) = \alpha\Pi^{\epsilon-1}. \tag{22}$$

In response to a positive cost-push shock, the central bank lowers the output gap by increasing the policy rate to mitigate the impact of the shock on the inflation

https://doi.org/10.1017/S1365100518000883 Published online by Cambridge University Press

rate. However, the central bank needs to decrease the output gap aggressively when trend inflation is high, because the $\kappa(\Pi)$ becomes small. The policy maker is aware of this mechanism. In our model, robust control leads to a more aggressive policy response. The decrease in the slope $\kappa(\Pi)$ increases this mechanism through the trend inflation. The policy maker adjusts the output gap largely to cost-push shock when the model uncertainty is taken into account. This leads to a more aggressive response of the nominal interest rate and output gap under robust policy when trend inflation is higher.[17]

On the other hand, the output gap is related to the inflation rate directly through the targeting rule (4). The aggressive response of the output gap leads to an aggressive response of inflation rate. However, the targeting rule also includes trend inflation. As we will demonstrate in Section 4.2, an increase in trend inflation decreases the coefficient in the targeting rule, and this decrease mitigates the aggressive response of output gap and inflation rate. However, the effect of trend inflation in the structural equations is dominant in our model, and inflation and the output gap respond more aggressively.[18] Our inflation rate response is starkly different from that of Leitemo and Söderström (2008b). Under the standard New Keynesian model without trend inflation, robust discretionary policy responds to the cost-push shock aggressively, and the inflation response is smaller. This is because the central bank fears a large impact on the inflation rate from the cost-push shock. However, our result shows that a positive trend inflation could dominate the central bank's preference of robustness to tighten the inflation rate through the flatter NKPC slope.

To summarize, the model uncertainty from robust control amplifies the responses of macroeconomic variables as trend inflation increases. The fluctuation of the economy might be larger when trend inflation is higher.[19]

To confirm the fluctuation in the present model, we derive the changes in variance between the worst-case and reference models. Figure 1 shows the result.

Solid lines in the figure illustrate percent change in variance of inflation, output gap, and nominal interest rate between the worst-case and reference models for the level of trend inflation. Dashed lines in Figure 1 indicate percent changes in variance between the approximating model and reference model. As may be seen, the increase in both the worst-case and approximating models rises with an increase in trend inflation. Thus, macroeconomic fluctuation rises large when trend inflation increases.

4.2. Trend Inflation in Loss Function versus Structural Equation

Given the result in the previous subsection, it is natural to ask whether the trend inflation in the loss function or structural equation affects the increases in the effects of the robust control. To answer this question, we set trend inflation in the structural equation $\Pi_{str} = 1$ explicitly.[20] Tables 6–8 show the results.

The fundamental effect of the robust control approach is the same as in the previous subsection. Robust control leads to aggressive responses of interest

https://doi.org/10.1017/S1365100518000883 Published online by Cambridge University Press

TABLE 6. Policy function for i_t under discretion ($\Pi_{str} = 1$)

(a) Coefficient on cost-push shock (u_t)

Trend inflation (%)	0	1	2	3
i_t ($\theta = \infty$)	4.0729	4.2912	4.5218	4.7652
i_t (worst-case)	4.1481	4.3491	4.5602	4.7814
Change in percent	1.8471	1.3495	0.84766	0.33947

(b) Coefficient on natural rate shock (r_t^n)

Trend inflation (%)	0	1	2	3
i_t ($\theta = \infty$)	1	1	1	1
i_t (worst-case)	1	1	1	1
Change in percent	0	0	0	0

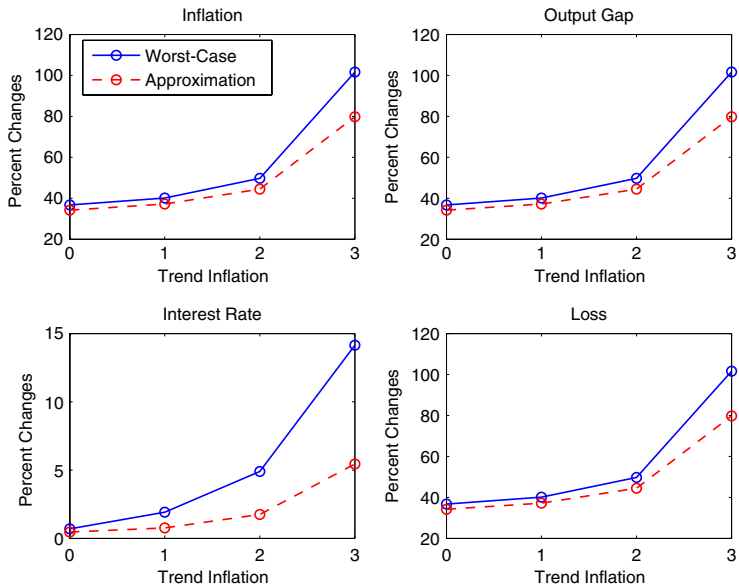


FIGURE 1. Percent changes in variances and losses.

rate, inflation, and output gap. However, changes in coefficients between the worst-case and reference models are different when the trend inflation increases. Changes in percent between the worst-case and reference models decrease as trend inflation increases. Under our baseline calibration, the coefficient in the targeting rule $\mathcal{K}(\Pi)/\lambda(\Pi)$ decreases with trend inflation.[21] The impact on the inflation rate of a fluctuation in the output gap becomes small in the targeting rule. This makes the fluctuation of inflation small. This mechanism therefore mitigates the aggressive responses from the effect of robustness.

https://doi.org/10.1017/S1365100518000883 Published online by Cambridge University Press

TABLE 7. Policy function for π_t under discretion ($\Pi_{str} = 1$)

(a) Coefficient on cost-push shock (u_t)				
Trend inflation (%)	0	1	2	3
π_t ($\theta = \infty$)	0.6234	0.60322	0.58188	0.55938
π_t (worst-case)	0.72214	0.69697	0.67077	0.64235
Change in percent	15.8379	15.5419	15.2747	14.8319
(b) Coefficient on natural rate shock (r_t^n)				
Trend inflation (%)	0	1	2	3
π_t ($\theta = \infty$)	0	0	0	0
π_t (worst-case)	0	0	0	0
Change in percent	0	0	0	0

TABLE 8. Policy function for x_t under discretion ($\Pi_{str} = 1$)

(a) Coefficient on cost-push shock (u_t)				
Trend inflation (%)	0	1	2	3
x_t ($\theta = \infty$)	−6.1093	−6.4368	−6.7828	−7.1478
x_t (worst-case)	−7.0769	−7.4372	−7.8189	−8.208
Change in percent	15.8379	15.5419	15.2747	14.8319
(b) Coefficient on natural rate shock (r_t^n)				
Trend inflation (%)	0	1	2	3
x_t ($\theta = \infty$)	0	0	0	0
x_t (worst-case)	0	0	0	0
Change in percent	0	0	0	0

Now, we analyze how trend inflation in the structural equation affects the effect of robustness. We set trend inflation in the central bank's objective function $\Pi_{loss} = 1$ explicitly. Tables 9–11 show the results.

The robust control amplifies the responses of interest rate, inflation, and output gap. This result is consistent with the main result in Section 4. Changes in percent between the worst-case and reference models increase as trend inflation increases. This shows that the increases in deviation between the worst-case and the reference models are caused by trend inflation in the structural equations.

4.3. Determinacy Analysis

Thus far, we have derived policy responses under a robust policy. However, Coibion and Gorodnichenko (2011) and Kobayashi and Muto (2013) derive the determinate region for trend inflation. Along the lines of their analysis, we also

https://doi.org/10.1017/S1365100518000883 Published online by Cambridge University Press

TABLE 9. Policy function for i_t under discretion ($\Pi_{\text{loss}} = 1$)

(a) Coefficient on cost shock (u_t)				
Trend inflation (%)	0	1	2	3
i_t ($\theta = \infty$)	4.0689	3.9362	3.7546	3.5253
i_t (worst-case)	4.1446	4.066	3.9376	3.7588
Change in percent	1.8589	3.2988	4.8757	6.6224
(b) Coefficient on natural rate shock (r_t^n)				
Trend inflation (%)	0	1	2	3
i_t ($\theta = \infty$)	1	1.002	1.0037	1.005
i_t (worst-case)	1	1.002	1.0038	1.0053
Change in percent	0	0.0064964	0.017769	0.032925

TABLE 10. Policy function for π_t under discretion ($\Pi_{\text{loss}} = 1$)

(a) Coefficient on cost shock (u_t)				
Trend inflation (%)	0	1	2	3
π_t ($\theta = \infty$)	0.62376	0.67431	0.72402	0.77174
π_t (worst-case)	0.72278	0.7909	0.85984	0.92898
Change in percent	15.8735	17.291	18.7584	20.3741
(b) Coefficient on natural rate shock (r_t^n)				
Trend inflation (%)	0	1	2	3
π_t ($\theta = \infty$)	0	0.00033803	0.00070537	0.0010938
π_t (worst-case)	0	0.00039648	0.00083768	0.0013167
Change in percent	0	17.291	18.7584	20.3741

analyze the determinacy. We derive the determinate region for trend inflation and robustness.[22] Figure 2 plots the region of determinacy.[23] The horizontal axis denotes degrees of robustness θ, and the vertical axis denotes the annualized trend inflation $\bar{\pi}$.

Figure 2 shows that stronger robustness (i.e., low θ) leads to indeterminate equilibrium as trend inflation increases. This is mainly because of coefficients on the forward term. To obtain an insight, we substitute the first-order conditions into a Phillips curve and iterate it forward.

$$\pi_t = \lim_{T\to\infty} \mathbb{E}_t\{\delta(\Pi,\theta)^T \pi_{t+T}\} + \Gamma(\Pi,\theta)\mathbb{E}_t \sum_{T=0}^{\infty} \delta(\Pi,\theta)^T \left(h_{t+T} + \gamma(\Pi) r_{t+T}^n + u_{t+T}\right), \tag{23}$$

https://doi.org/10.1017/S1365100518000883 Published online by Cambridge University Press

TABLE 11. Policy function for x_t under discretion ($\Pi_{\text{loss}} = 1$)

(a) Coefficient on cost shock (u_t)				
Trend inflation (%)	0	1	2	3
x_t ($\theta = \infty$)	−6.1035	−5.9044	−5.6319	−5.2881
x_t (worst-case)	−7.0723	−6.9253	−6.6884	−6.3655
Change in percent	15.8735	17.291	18.7584	20.3741
(b) Coefficient on natural rate shock (r_t^n)				
Trend inflation (%)	0	1	2	3
x_t ($\theta = \infty$)	0	−0.0029599	−0.0054868	−0.0074949
x_t (worst-case)	0	−0.0034716	−0.006516	−0.0090219
Change in percent	0	17.291	18.7584	20.3741

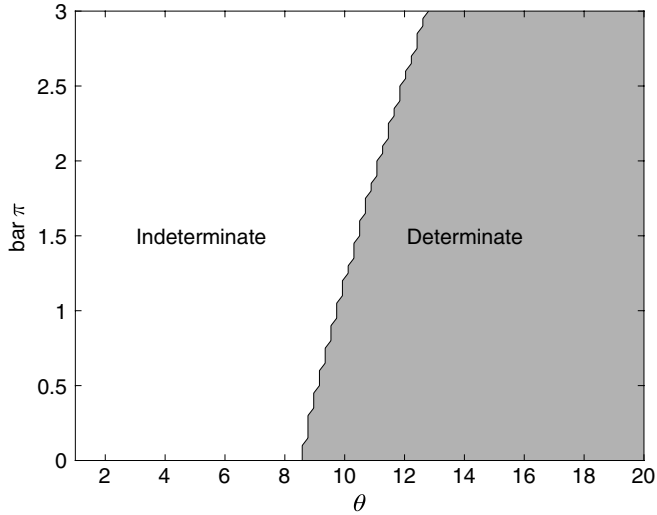


FIGURE 2. Determinate region for $\bar{\pi}$ and θ. Grid intervals for $\bar{\pi}$ and θ are 0.01 and 0.1, respectively.

where

$$\delta(\Pi, \theta) = \Gamma(\Pi)\mathcal{B}(\Pi), \quad \Gamma(\Pi, \theta) = \frac{1}{1 + \mathcal{K}(\Pi)^2/\lambda(\Pi) - (\gamma(\Pi)^2 + 1)/\theta}.$$

If $\delta(\Pi, \theta)$ is outside an unit circle, equation (23) diverges. Under our baseline calibration, $\mathcal{B}(\Pi)$ increase together with trend inflation. This leads to higher values in $\delta(\Pi, \theta)$. On the other hand, increases in θ lower the value of $\Gamma(\Pi, \theta)$. Therefore, higher trend inflation needs weaker robustness to lower the $\delta(\Pi, \theta)$. This leads a

https://doi.org/10.1017/S1365100518000883 Published online by Cambridge University Press

positively sloped border line in Figure 2.[24] Under our baseline calibration, robustness makes the economy more volatile. When this meets higher trend inflation, the model deviates from equilibrium substantially and tends to be indeterminate.

5. CONCLUDING REMARKS

This paper investigated how model uncertainty influences economic fluctuation when trend inflation is high. We introduced Hansen and Sargent's (2008) robust control techniques into a New Keynesian model with non-zero trend inflation. We revealed the following two points. First, we found that robust policy monetary responds more aggressively, and macroeconomic variables have a larger response to cost-push shock as trend inflation rises. Second, stronger robustness tends to lead to indeterminate equilibrium with higher trend inflation. Consequently, the economy might become volatile when trend inflation is high through determinacy and robustness. We interpret that model uncertainty might be the one of the factors causing large macroeconomic fluctuations when trend inflation is high.

SUPPLEMENTARY MATERIAL

To view supplementary material for this article, please visit http://doi.org/S1365100518000883.

NOTES

1. Tillmann (2009b) analyzes the robust monetary policy in the New Keynesian model with a cost channel under Onatski and Williams's (2003) structured uncertainty.

2. Aoki (2006) analyzes the advantage of optimal commitment policy when inflation and output gap are available with noisy data.

3. Kobayashi and Muto (2013) and Kurozumi (2014) analyze the stabilization problem in monetary policy under non-zero trend inflation. They introduce adaptive learning and analyze the relation between trend inflation and E-stability, as well as determinacy.

4. The relation between gross rate of trend inflation and annual rate of trend inflation is

$$\Pi = 1 + \frac{\bar{\pi}}{400}.$$

5. Conversely, Alves (2012, 2014) explains that the weight on the output gap increases as trend inflation rises in Coibion et al.'s (2012) model. This study derives the loss function in the model with trend inflation around the zero inflation steady-state equilibrium, different from Alves (2012). However, our overall result does not change much if both loss functions are applied.

6. We introduce the specification error term v_t^h even though there are no exogenous shocks in the equation of the auxiliary variable. However, we suppose the situation that the central bank does not capture all of the structural equations. In the numerical part, we plot the results of introducing exogenous shock to the equation of the auxiliary variable.

7. $F_v = \mathbf{0}_{3\times 2}$ means that no misspecification exists. However, solutions for forward variables include θ.

8. We derive the solutions under discretion but commitment. Under the commitment policy, it is too complicated to solve the model in our hands because of the history dependence.

https://doi.org/10.1017/S1365100518000883 Published online by Cambridge University Press

9. The first-order conditions in the present paper correspond to the Nash solution in Leitemo and Söderström (2008b).

10. Appendix A.2 in the Supplementary Material gives a detailed derivation of policy function.

11. The difference from Leitemo and Söderström (2008b) is explained in Section 4.

12. We explain the derivation of cross derivative and associated conditions in Appendix A.3 in the Supplementary Material. The cross derivative of coefficients on natural rate is small. We report the results in numerical analysis.

13. See Appendix A.3 in the Supplementary Material for detailed calculation for derivation.

14. Giordani and Söderlind's (2004) code solves the robust control model under discretion using Backus and Driffill's (1986) iterative algorithm. This algorithm reoptimizes the problem every period by taking the private sector's expectation formation as given.

15. Cogley and Sbordone (2008) set $\sigma = 6.25$. However, under values greater than 1.5 in σ, detection error probabilities do not converge when trend inflation is high.

16. Appendix A.1 in the Supplementary Material of this paper presents definitions of structural parameters.

17. This aggressive response is related to Giordani and Söderlind's (2004) warning that the robust policy maker looks like a foolish catastrophist. They suggest a detection error probability as one defense to this warning. Our interpretation of aggressive policy response comes from the central bank's fundamental stabilization policy to inflation fundamentally. Therefore our interpretation is reasonable to policy makers, and we consider that it does not conflict with this warning.

18. We also investigate in detail the effect of trend inflation in the central bank's objective function in Section 4.2.

19. This model property seems consistent with the US economy in the 1970s, but its mechanism might not be consistent.

20. Henceforth, we express trend inflation in the structural equations as Π_{str}, where $\Pi_{str} = 1$ corresponds to zero annual trend inflation. On the other hand, we express trend inflation in the central bank's objective function as Π_{loss}.

21. Under our baseline calibration, values of $\mathcal{K}(\Pi)/\lambda(\Pi)$ take 9.8, 9.549, 9.2666, and 8.9482 for 0%, 1%, 2%, and 3%, respectively, in trend inflation.

22. It is difficult to derive the determinacy region for parameters except θ, because we cannot derive the detection error probability if the model is indeterminate.

23. We derive the region of determinacy in relying on Giordani and Söderlind's (2004) MATLAB code.

24. Under our baseline calibration, values of $\mathcal{B}(\Pi)$ take 0.99, 1.0029, 1.016, and 1.0293 for 0%, 1%, 2%, and 3%, respectively, in trend inflation.

REFERENCES

Adam, K. and R. M. Billi (2006) Optimal monetary policy under commitment with a zero bound on nominal interest rates. *Journal of Money, Credit and Banking* 38, 1877–1905.

Alves, S. A. L. (2012) Optimal Policy When the Inflation Target is not Optimal. Research Department, Central Bank of Brazil, Working Papers Series 271.

Alves, S. A. L. (2014) Lack of divine coincidence in New Keynesian models. *Journal of Monetary Economics* 67, 33–46.

Aoki, K. (2003) On the optimal monetary policy response to noisy indicators. *Journal of Monetary Economics* 50, 501–523.

Aoki, K. (2006) Optimal commitment policy under noisy information. *Journal of Economic Dynamics and Control* 30, 81–109.

Ascari, G. (2004) Staggered prices and trend inflation: Some nuisances. *Review of Economic Dynamics* 7, 642–667.

Ascari, G. and T. Ropele (2007) Optimal monetary policy under low trend inflation. *Journal of Monetary Economics* 54, 2568–2583.

https://doi.org/10.1017/S1365100518000883 Published online by Cambridge University Press

Ascari, G. and T. Ropele (2009) Trend inflation, Taylor principle, and indeterminacy. *Journal of Money, Credit and Banking* 41, 1557–1584.
Backus, D. and J. Driffill (1986) The Consistency of Optimal Policy in Stochastic Rational Expectations Models. CEPR discussion papers: No. 124, CEPR.
Bils, M. and P. J. Klenow (2004) Some evidence on the importance of sticky prices. *Journal of Political Economy* 112, 947–985.
Brainard, W. C. (1967) Uncertainty and the effectiveness of policy. *American Economic Review* 57, 411–425.
Cogley, T., R. Colacito, L. P. Hansen and T. J. Sargent (2008) Robustness and U.S. monetary policy experimentation. *Journal of Money, Credit and Banking* 40, 1599–1623.
Cogley, T. and A. M. Sbordone (2008) Trend inflation, indexation, and inflation persistence in the New Keynesian Phillips curve. *American Economic Review* 98, 2101–26.
Coibion, O. and Y. Gorodnichenko (2011) Monetary policy, trend inflation, and the great moderation: An alternative interpretation. *American Economic Review* 101, 341–370.
Coibion, O., Y. Gorodnichenko and J. Wieland (2012) The optimal inflation rate in New Keynesian models: Should central banks raise their inflation targets in light of the zero lower bound? *The Review of Economic Studies* 79, 1371.
Gerke, R. and F. Hammermann (2016) Robust monetary policy in a New Keynesian model with imperfect interest rate pass-through. *Macroeconomic Dynamics* 20, 1504–1526.
Giannoni, M. P. (2002) Does model uncertainty justify caution? Robust optimal monetary policy in a forward-looking model. *Macroeconomic Dynamics* 6, 111–144.
Giannoni, M. P. (2007) Robust optimal monetary policy in a forward-looking model with parameter and shock uncertainty. *Journal of Applied Econometrics* 22, 179–213.
Giordani, P. and P. Söderlind (2004) Solution of macromodels with HansenSargent robust policies: Some extensions. *Journal of Economic Dynamics and Control* 28, 2367–2397.
Hansen, L. P. and T. J. Sargent (2008) *Robustness*. Princeton, NJ: Princeton University Press.
Kobayashi, T. and I. Muto (2013) A note on expectational stability under nonzero trend inflation. *Macroeconomic Dynamics* 17, 681–693.
Kurozumi, T. (2014) Trend inflation, sticky prices, and expectational stability. *Journal of Economic Dynamics and Control* 42, 175–187.
Leitemo, K. and U. Söderström (2008a): Robust monetary policy in a small open economy. *Journal of Economic Dynamics and Control* 32, 3218–3252.
Leitemo, K. and U. Söderström (2008b) Robust monetary policy in the New Keynesian framework. *Macroeconomic Dynamics* 12, 126–135.
Masolo, R. M. and F. Monti (2017) Ambiguity, Monetary Policy and Trend Inflation. Discussion papers: No. 1709, Centre for Macroeconomics (CFM).
Nakamura, E. and J. Steinsson (2008) Five facts about prices: A reevaluation of menu cost models. *The Quarterly Journal of Economics* 123, 1415–1464.
Onatski, A. and N. Williams (2003) Modeling model uncertainty. *Journal of the European Economic Association* 1, 1087–1122.
Orphanides, A. and V. Wieland (2013) Complexity and monetary policy. *International Journal of Central Banking* 9, 167–204.
Sbordone, A. M. (2007) Inflation persistence: Alternative interpretations and policy implications. *Journal of Monetary Economics* 54, 1311–1339.
Tillmann, P. (2009a) Optimal monetary policy with an uncertain cost channel. *Journal of Money, Credit and Banking* 41, 885–906.
Tillmann, P. (2009b) Robust monetary policy with the cost channel. *Economica* 76, 486–504.

https://doi.org/10.1017/S1365100518000883 Published online by Cambridge University Press