J. Plasma Physics (2006), *vol.* 72, *part* 5, *pp.* 605–608. © 2006 Cambridge University Press
doi:10.1017/S0022377806004582 First published online 23 May 2006 Printed in the United Kingdom

Letter to the Editor

Shear electromagnetic waves in strongly magnetized quantum electron–positron plasmas

P. K. SHUKLA[1]† and L. STENFLO[2]

[1]Institut für Theoretische Physik IV and Centre of Excellence for Plasma Science and Astrophysics, Fakultät für Physik und Astronomie, Ruhr-Universität Bochum, D-44780 Bochum, Germany
(ps@tp4.rub.de)

[2]Centre for Nonlinear Physics, Department of Physics, Umeå University, SE-90187 Umeå, Sweden
(lennart.stenflo@physics.umu.se)

(Received 17 March 2006)

Abstract. We study the dispersion properties of low-frequency (in comparison with the electron/positron gyrofrequency) shear electromagnetic waves in an ultra-cold quantum magnetoplasma composed of electrons and positrons. By using the electron and positron fluid velocities deduced from the quantum momentum equations, together with the continuity equations and Ampère's law, we derive the governing equation for the shear waves. This equation is then Fourier transformed to obtain the dispersion relation, and to show the modification of the electromagnetic wave dispersions due to the electron and positron quantum effects. The latter can play a significant role in magnetic reconnection and in the formation of localized electromagnetic wave structures at quantum scales in ultra-cold quantum magnetoplasmas.

It is well known that magnetized electron–positron plasmas, which are composed of particles of the same mass and opposite charge, exist in the early universe [1, 2], in the bipolar outflows (jets) in active galactic nuclei [3, 4], in the polar regions of neutron stars [5], in the inner regions of the accretion disks surrounding black holes [6], at the centre of our own galaxy [7], as well as in plasmas in intense laser fields [8] and in tokamaks [9]. In such plasmas, there are various new linear and nonlinear wave modes and structures [10–17]. However, when the magnetized pair plasma is extremely dense, the dispersion caused by density correlations due to quantum fluctuations can also play an important role [18–22]. Several authors

† Also at the Centre for Nonlinear Physics, Department of Physics, Umeå University, SE-90187 Umeå, Sweden; Department of Physics, University of Strathclyde, Glasgow G4ONG, UK; Centro de Física dos Plasmas, Departamento de Fisica, Instituto Superior Técnico, Universidade Técnica de Lisboa, 1049-001 Lisbon, Portugal; and Centre for Fundamental Physics, Rutherford Appleton Laboratory, Chilton, Didcot, Oxfordshire 0X11 OQX, UK

https://doi.org/10.1017/S0022377806004582 Published online by Cambridge University Press

[23–31] have thus investigated different aspects of collective interactions in quantum electron–ion plasmas.

In this paper, we consider the linear propagation of low-frequency (in comparison with the electron gyrofrequency) shear electromagnetic waves in a homogeneous ultra-cold magnetoplasma, composed of cold electrons and positrons, in an external magnetic field $\hat{\mathbf{z}}B_0$, where $\hat{\mathbf{z}}$ is a unit vector along the z-axis and B_0 is the strength of the magnetic field. At equilibrium, we have $n_{e0} = n_{p0} \equiv n_0$, where $n_{e0}(n_{p0})$ is the unperturbed electron (positron) number density. In the presence of a low-frequency (in comparison with the electron/positron gyrofrequency $\omega_c = eB_0/mc$, where e is the magnitude of the electron charge, m is the electron/positron mass, and c is the speed of light in vacuum) electromagnetic wave with the electric and magnetic fields $\mathbf{E} = -\nabla\phi - c^{-1}\partial A_z/\partial t$ and $\mathbf{B}_\perp = \nabla_\perp A_z \times \hat{\mathbf{z}}$, where ϕ is the electrostatic potential and A_z is the component of the vector potential along $\hat{\mathbf{z}}$, the perpendicular (to $\hat{\mathbf{z}}$) components of the electron and positron fluid velocities are the sum of the $\mathbf{E}_\perp \times \hat{\mathbf{z}}$ and polarization drifts, namely,

$$\mathbf{u}_{j\perp} \approx \frac{c}{B_0}\hat{\mathbf{z}} \times \nabla\varphi_j + \alpha_j \frac{c}{B_0\omega_c}\frac{\partial \nabla_\perp \varphi_j}{\partial t}, \tag{1}$$

where $\alpha_e = 1$, $\alpha_i = -1$, $\varphi_j = \phi + (\alpha_j\hbar^2/4n_0em)\nabla^2 n_{j1}$, $n_{j1}(\ll n_0)$ is the density perturbation of particle species j (j equals e for electrons and p for positrons), and $\hbar$ is the Planck constant divided by 2π. We note that (1) has been obtained in the drift approximation ($|\partial/\partial t| \ll \omega_c$) from the momentum equations that include the linearized form of the quantum correction [30]. The electron diamagnetic drift has not been included in (1), since we are considering the case where $T_{jF}n_{j1} \ll (\hbar^2/m)\nabla^2 n_{j1}$, where T_{jF} is the Fermi temperature. The density perturbations n_{j1} are obtained from the continuity equation

$$\frac{\partial n_{j1}}{\partial t} + n_0 \nabla_\perp \cdot \mathbf{u}_{j\perp} + n_0 \frac{\partial u_{jz}}{\partial z} = 0, \tag{2}$$

where the parallel component of the fluid velocity u_{jz} is determined from

$$m\frac{\partial u_{jz}}{\partial t} = \alpha_j e\left[\frac{\partial \varphi_j}{\partial z} + \frac{1}{c}\frac{\partial A_z}{\partial t}\right]. \tag{3}$$

The system of equations is closed by the Poisson equation

$$\nabla^2\phi = 4\pi e(n_{e1} - n_{p1}) \tag{4}$$

and Ampère's law

$$\nabla_\perp^2 A_z = \frac{4\pi e n_0}{c}(u_{ez} - u_{pz}). \tag{5}$$

We now substitute the electron and positron fluid velocities from (1) into the corresponding continuity equations (2), eliminate u_{jz} by using (3), and then use (4) and (5) to obtain

$$\frac{\partial}{\partial t}\left[\nabla^2\phi + \frac{\omega_p^2}{\omega_c^2}(2 + \lambda_q^4\nabla^4)\nabla_\perp^2\phi\right] + c\frac{\partial \nabla_\perp^2 A_z}{\partial z} = 0, \tag{6}$$

where $\omega_p = (4\pi n_0 e^2/m)^{1/2}$ is the electron plasma frequency and $\lambda_q = (\hbar^2/4m^2\omega_p^2)^{1/4}$

https://doi.org/10.1017/S0022377806004582 Published online by Cambridge University Press

is the quantum wavelength. Furthermore, from (3)–(5) we find

$$(1-\lambda_e^2\nabla_\perp^2)\frac{\partial A_z}{\partial t}+c\frac{\partial}{\partial z}\left(1+\frac{1}{2}\lambda_q^4\nabla^4\right)\phi=0, \tag{7}$$

where $\lambda_e=c/\sqrt{2}\omega_p$ is the skin depth.

Combining (6) and (7) we obtain

$$(1-\lambda_e^2\nabla_\perp^2)\left[\nabla^2+\frac{\omega_p^2}{\omega_c^2}(2+\lambda_q^4\nabla^4)\nabla_\perp^2\right]\frac{\partial^2\phi}{\partial t^2}-c^2\left(1+\frac{1}{2}\lambda_q^4\nabla^4\right)\frac{\partial^2\nabla_\perp^2\phi}{\partial z^2}=0. \tag{8}$$

Assuming that ϕ is proportional to $\exp(-i\omega t+i\mathbf{k}\cdot\mathbf{r})$, where ω and $\mathbf{k}$ are the frequency and wavevector, respectively, we then Fourier transform (8) to obtain

$$\omega=\frac{ck_zk_\perp(1+b_q/2)^{1/2}}{K(1+b_e)^{1/2}}, \tag{9}$$

where $b_q=k^4\lambda_q^4$, $b_e=k_\perp^2\lambda_e^2$, and $K^2=k^2+k_\perp^2(2+b_q)\omega_p^2/\omega_c^2$. Here $k^2=k_z^2+k_\perp^2$, where k_z and $k_\perp$ are the components of the wavevector along and across $\hat{\mathbf{z}}$, respectively. For $b_q\gg1$ and $b_e\ll1$, we have $\omega=ck_zk_\perp(b_q/2)^{1/2}/(k^2+k_\perp^2b_q\omega_p^2/\omega_c^2)^{1/2}$, which indicates a strong dependence of the electron quantum effect on the wave frequency. Furthermore, for $k_z\ll k_\perp$, $b_q\ll1$, $b_e\ll1$, and $\omega_p^2\ll\omega_c^2$, we have from (9) that

$$\omega\approx k_zc(1+k_\perp^4\lambda_q^4/4), \tag{10}$$

which exhibits the dispersion due to the electron–positron density correlations at the quantum scale.

In summary, we have presented the dispersion relation for a low-frequency shear electromagnetic wave in an ultra-cold quantum pair plasma in an external magnetic field. It is found that the dispersions caused by the electron and positron density correlations can play an important role in modifying the electromagnetic wave propagation characteristics at the quantum scale. In closing, we stress that the present results are a necessary prequisite for studying magnetic field reconnection and localized electromagnetic waves (viz. solitons and vortices) in dense quantum magnetoplasmas whose constituents are electrons and positrons.

Acknowledgements

This work was partially supported by the Deutsche Forschungsgemeinschaft through the Sonderforschungsbereich 591, and by the Swedish Research Council.

References

[1] Misner, W., Thorne, K. and Wheeler, J. A. 1973 *Gravitation*. San Francisco: Freeman, p. 763.

[2] Rees, M. J. 1983 *The Very Early Universe* (ed. G. W. Gibbons, S. W. Hawking and S. Siklas). Cambridge: Cambridge University Press.

[3] Miller, H. R. and Witta, P. 1987 *Active Galactic Nuclei*. Berlin: Springer, p. 202.

[4] Begelman, M. C., Blandford, R. D. and Rees, M. J. 1984 *Rev. Mod. Phys.* **56**, 255.

[5] Michel, F. C. 1982 *Rev. Mod. Phys.* **54**, 1.
Michel, F. C. 1991 *Theory of Neutron Star Magnetosphere*. Chicago: Chicago University Press.

[6] Rees, M. J. 1971 *Nature (London)* **229**, 312.

https://doi.org/10.1017/S0022377806004582 Published online by Cambridge University Press

[7] Burns, M. L. 1983 *Positron–Electron Pairs in Astrophysics* (ed. M. L. Burns, A. K. Harding and R. Ramaty). New York: American Institute of Physics.
[8] Berezhiani, V., Tskhakaya, D. D. and Shukla, P. K. 1992 *Phys. Rev.* A **46**, 6608.
Liang, E. P., Wilks, S. C. and Tabak, M. 1998 *Phys. Rev. Lett.* **81**, 4887.
[9] Helander, P. and Ward, D. J. 2003 *Phys. Rev. Lett.* **90**, 135004.
[10] Shukla, P. K., Rao, N. N., Yu, M. Y. and Tsintsadze, N. L. 1986 *Phys. Rep.* **138**, 1.
[11] Yu, M. Y., Shukla, P. K. and Stenflo, L. 1986 *Astrophys. J. Lett.* **309**, L63.
[12] Stenflo, L., Shukla, P. K. and Yu, M. Y. 1985 *Astrophys. Space Sci.* **117**, 303.
[13] Gedalin, M. E., Lominadze, J. G., Stenflo, L. and Tsytovich, V. N. 1985 *Astrophys. Space Sci.* **108**, 393.
[14] Iwamoto, N. 1993 *Phys. Rev.* E **47**, 604.
[15] Gangadhara, R. T., Krishan, V. and Shukla, P. K. 1993 *Mon. Not. R. Astron. Soc.* **262**, 151.
[16] Popel, S. I., Vladimirov, S. V. and Shukla, P. K. 1995 *Phys. Plasmas* **1**, 716.
[17] Eliasson, B. and Shukla, P. K. 2006 *Phys. Rep.* **422**, 225.
[18] Kremp, D., Bornath, Th., Bonitz, M. and Schlanges, M. 1999 *Phys. Rev.* E **60**, 4725.
[19] Andreev, A. V. 2000 *JETP Lett.* **72**, 238.
[20] Chabier, G., Douchin, F. and Potekhin, Y. 2002 *J. Phys.: Conden. Matter* **14**, 9133.
[21] Jung, Y. F. 2001 *Phys. Plasmas* **8**, 3842.
[22] Opher, M., Silva, L. O., Danger, D. E., Decyk, V. K. and Dawson, J. M. 2001 *Phys. Plasmas* **8**, 2454.
[23] Haas, F., Manfredi, G. and Feix, M. 2000 *Phys. Rev.* E **62**, 2763.
[24] Manfredi, G. and Haas, F. 2001 *Phys. Rev.* B **64**, 075316.
[25] Anderson, D., Hall, B., Lisak, M. and Marklund, M. 2002 *Phys. Rev.* E **65**, 046417.
[26] Haas, F., Garcia, L. G., Goedert, J. and Manfredi, G. 2003 *Phys. Plasmas* **10**, 3858.
[27] Garcia, L. G., Haas, F., de Oliveira, L. P. and Goedert, J. 2005 *Phys. Plasmas* **12**, 012302.
[28] Marklund, M. 2005 *Phys. Plasmas* **12**, 082110.
[29] Luque, A., Schamel, H. and Fedele, R. 2004 *Phys. Lett.* A **324**, 185.
[30] Haas, F. 2005 *Phys. Plasmas* **12**, 062117.
[31] Shukla, P. K. 2006 *Phys. Lett.* A **352**, 242.

https://doi.org/10.1017/S0022377806004582 Published online by Cambridge University Press