NURBS-BASED PROFILE RECONSTRUCTION USING CONSTRAINED FITTING TECHNIQUES

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ABSTRACT

Ordered data point cloud obtained by laser-scanning, image processing or contact digitizing techniques is widely used for reconstructing a cross section profile in the field of reverse engineering. In this research, a comprehensive algorithm for reconstruction of 2D profile is developed based on NURBS parametric curve theory. In this regard, the line, arc and B-spline segments are firstly extracted from the set of ordered data points using a developed segment detection algorithm. Then, the final profile is reconstructed using the newly proposed algorithm of constrained local fitting which approximates these segments by NURBS curves with appropriate geometric continuity conditions. The validity, capability and functionality of the developed method are investigated by some practical case studies. Results show that developed algorithm can be integrated and improve the functionality of exiting reverse engineering systems.

Keywords: NURBS curves, Reverse engineering, Constrained local fitting.

1. INTRODUCTION

Reconstructing profiles of an existing object from a set of data points is a promising field in the area of reverse engineering [1]. Representing a profile with one freeform curve cannot be always a proper and efficient method in data fitting and may lead to loss of natural features of the profile such as straightness of lines and fillets. As a result, data fitting procedure usually includes more types of curve definition which are locally approximated with respect to geometrical constraints. Constrained data fitting techniques are firstly taken into account by Porrill [2]. Other researchers such as Werghi *et al*. [3] and Benkő *et al*. [4] also achieve useful results using constrained optimization techniques. Ke *et al*. [5] extend above mentioned researches by considering B-spline curves in the local fitting procedure. In the previously proposed constrained data fitting techniques, input data points are assumed to be segmented. Consequently, these techniques are limited use in reverse engineering applications.

Data segmentation is always decisive to reconstruct a reasonable profile. Existing data are usually in the form of data sets or point clouds where may be obtained by laser-scan, image processing or digitizing. In order for data points to be locally fitted, input data must be segmented at first. So that line, circular arc and B-spline data become separate inputs for the constrained local fitting algorithm. One of the well-known methods for detecting lines and circles, especially concerning image processing problems, is Hough transform [6]. For the circle recognition in a set of data points, some other methods such as least squares [7] and Newtonian-Gaussian optimization [8] were also presented. Further on, for the line detection, clustering [9] and split-and-merge [10] algorithms have been presented. Ke *et al*. [11] extract line, arc and B-spline data from a sectional profiles by defining the concepts of discrete derivative and discrete curvature. In addition to the above mentioned approaches, Xavier *et al*. [12] also propose the inscribed angle variance (IAV) method for circle detection which has been chosen for implementation in this research.

The main objective of this paper is to present a new NURBS-based method for reconstructing 2D object profiles. In this approach, data points are first segmented using the developed segment detection algorithm and data correspond to line, arc and B-spline are

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extracted. Hence, separated data are approximated by respective parametric curves of the NURBS family using the proposed constrained local fitting algorithm. In this regard, line segments will be reconstructed by B-spline curves of degree one and arc segments by quadratic NURBS curves. Although each segment is locally approximated, the fitting process must be globally applied for the entire profile to satisfy all geometric constraints at segment joints. The constrained local fitting algorithm will be discussed in Section 2. In Section 3, the segment detection algorithm will be presented, followed by some case studies in Section 4. Finally, Section 5 concludes the results.

2. CONSTRAINED LOCAL FITTING ALGORITHM

A planar profile usually represented by a composite curve consists of line, arc and B-spline segments which are the most common features in engineering applications. To represent these features, conventionally, a line and a circle are implicitly expressed as $L(x, y) = l_0x + l_1y + l_2 = 0$ and $C(x, y) = c_0(x^2 + y^2) + c_1x + c_2y + c_3 = 0$, respectively. Three unknowns $\{l_0\ l_1\ l_2\}$ of the line and four unknowns $\{c_0\ c_1\ c_2\ c_3\}$ of the circle should be determined using optimization techniques subjected to geometric constraints [4]. In this paper, to have a unified representation, the expression of line, arc and B-spline features are in the common parametric form of NURBS definition noting that a circular arc can be parametrically modeled by a NURBS curve of degree two and a line by a B-spline curve of degree one. Therefore, the optimization parameters are the respective unknown control points of each segment. There are many advantages to use parametric representation instead of implicit forms which are deeply studied in [13]. For instance, it is difficult to represent bounded curve segments with the implicit form; however, by parametric definition, bounded curves can be represented in a convenient manner. Furthermore, a parametric form such as B-spline definition is more common for computer design applications. The coefficients of parametric functions possess considerable geometric significance which results in intuitive design methods and numerically stable algorithms. The other advantage of the proposed approach, in comparison to previous researches in [4,5] and regardless of the curve types, is that the geometric continuity constraints of the optimization procedure will be easily expressed in terms of control points.

Starting with curve definition, a B-spline curve of degree p with $n + 1$ control points $\mathbf{P}_i$ is expressed as [13]:

$$\mathbf{C}(u) = \sum_{i=0}^{n} N_{i,p}(u)\ \mathbf{P}_i \tag{1}$$

where $\mathbf{C}(u)$ is a vector-valued function whose components are represented separately as an explicit function of parameter u and $N_{i,p}(u)$ is the i-th B-spline basis function of degree p. As an example, Fig. 1 shows a cubic B-spline curve with 5 control points. In the fitting algorithm, $\mathbf{D}$ is considered as a set of $r + 1$ input data points and in order to fit a B-spline curve through these points, it is required to find the unknown control points as the output.

Therefore, according to least-squares minimization, one can write:

$$F_B = \sum_{k=0}^{r} \left\| \mathbf{D}_k - \mathbf{C}(u_k) \right\|^2 = \sum_{k=0}^{r} \left\| \mathbf{D}_k - \sum_{i=0}^{n} N_{i,p}(u_k)\ \mathbf{P}_i \right\|^2 \tag{2}$$

where F_B is the objective function which must be minimized in order to find unknown control points as optimization parameters. Since all parametric NURBS curves have a representation similar to that was shown in Eq. (1), the objective function stated by Eq. (2) can be generalized for all types of curve definition.

A line, actually, is a simplified form of a B-spline curve of degree one with two control points (Fig. 2) where the basis functions converted into $N_{0,1}(u) = 1 - u$ and $N_{1,1}(u) = u$. The line parametric definition will be represented by:

$$\mathbf{C}(u) = (1-u)\,\mathbf{P}_0 + u\,\mathbf{P}_1 \tag{3}$$

Due to the fact that a line has a definition similar to a B-spline curve, the objective function of a line segment, F_L, has a same form as that was written in Eq. (2).

A circular arc can be represented by a quadratic NURBS curve. Generally, a NURBS curve of degree p with $n + 1$ control points is defined by [13]:

$$\mathbf{C}(u) = \frac{\sum_{i=0}^{n} N_{i,p}(u)\, w_i\, \mathbf{P}_i}{\sum_{j=0}^{n} N_{i,p}\ (u)\, w_j} = \sum_{i=0}^{n} R_{i,p}\ (u)\, \mathbf{P}_i \tag{4}$$

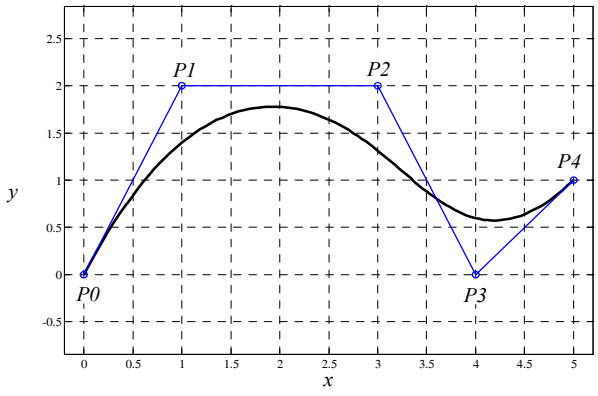


Fig. 1 A cubic B-spline with 5 control points



Fig. 2 Representing a line with two control points

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where $R_{i,p}(u)$ is the i-th basis function of a NURBS curve. A circular arc needs three control points $\mathbf{P}_0$, $\mathbf{P}_1$ and $\mathbf{P}_2$ and corresponding weights $w_0 = 1$, $w_1 = w$ and $w_2 = 1$ where w is a positive scalar. Under this condition, the B-spline basis functions are $N_{0,2}(u) = (1-u)^2$, $N_{1,2}(u) = 2u(1-u)$ and $N_{2,2}(u) = (u)^2$. Comparing Eq. (4) with Eq. (1), the objective function of a circular arc, F_C, is also similar to Eq. (2). It should be noted that in addition to control points, the unknown weight w must also be calculated as an optimization parameter in the fitting process. Therefore, the objective function becomes nonlinear due to multiplication of w in $\mathbf{P}_2$.

$$F_C = \sum_{k=0}^{r} \| \mathbf{D}_k - \mathbf{C}(u_k) \|^2 = \sum_{k=0}^{r} \left\| \mathbf{D}_k - \sum_{i=0}^{2} R_{i,p}(u_k)\, \mathbf{P}_i \right\|^2 \quad (5)$$

Referring to Fig. 3, the control polygon of the arc must be an isosceles triangle while the value of w is equal to $\sin\theta$ [14]. These two conditions are intrinsic constraints of a circle fitting process.

Once the objective function for local fitting of each curve feature is generated, the global optimization procedure can be applied to find all unknown control points and weights for reconstructing the final object profile. In this regard, the overall objective function of the fitting process should be defined as the summation of all individual objective functions of curve segments:

$$F = \sum F_L + \sum F_C + \sum F_B \quad (6)$$

The overall objective function will be minimized taking into account the geometric continuity conditions at each joint. Hence, the final reconstructed profile not only locally approximates each curve segment, but also globally satisfies the geometric constraints.

Two major continuity conditions usually applied at segment joints are positional C^0 and tangent G^1 conditions which have been extensively investigated in curve fitting processes [5,11,15]. Using the parametric forms, the continuity conditions are represented in terms of control points and regardless of the curve segment type. Considering two arbitrary NURBS curves (Fig. 4), the C^0 continuity condition enforces the curves to have equal control points at the joint. Furthermore, in order for two curves to be G^1 continuous, the adjacent legs of the corresponding control polygons must be collinear. These geometric constraints can be simply formulated as follows [15]:

$$\mathbf{P}_n^L = \mathbf{P}_0^R = \frac{\beta}{\alpha+\beta}\mathbf{P}_{n-1}^L + \frac{\alpha}{\alpha+\beta}\mathbf{P}_1^R \quad (7)$$

where $\alpha = \| \mathbf{P}_n^L \mathbf{P}_{n-1}^L \|$ and $\beta = \| \mathbf{P}_0^R \mathbf{P}_1^R \|$ are the length of the adjacent legs and superscripts L and R are referred to the left and right curves, respectively.

Having formulated the continuity conditions for two adjacent curves, they can be applied for all types of curve segments throughout the profile reconstruction process. Referring to Eq. (7), it should be noted that α and β are unknown variables for the optimization process. However, they can be pre-assigned using the initial values of involving control points. It would be a reasonable assumption to decrease the nonlinearity and also increase the speed and convergence of the global optimization. Initial values have a decisive role in iterative optimization procedures. For the proposed data fitting process, the initial values of unknown control points for the global optimization can be determined from the local fitting of individual curve segments regardless of continuity constraints.

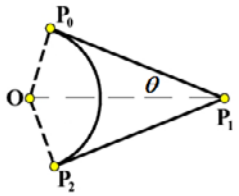


Fig. 3 Representing of an arc by a quadratic NURBS curve [14]

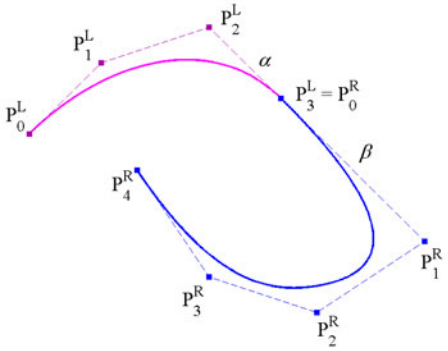


Fig. 4 Two arbitrary NURBS curves with G^1 continuity at the joint

In order to illustrate the applicability of the constrained fitting algorithm, Fig. 5 demonstrates a data set consist of separated lines, arcs and B-spline segments. The global iterative optimization procedure has been implemented in the optimization toolbox of MATLAB® software [16].

3. SEGMENT DETECTION ALGORITHM

In the current work, the concept of relative angles is introduced to detect line segments in a set of data. Assuming an ordered data points depicted in Fig. 6, the position of point $\mathbf{D}_i$ with respect to $\mathbf{D}_{i-1}$ can be represented by vector $\mathbf{s}_i$. The relative angle between vectors $\mathbf{s}_i$ and $\mathbf{s}_1$ can be expressed by θ_i as:

$$\theta_i = \cos^{-1} \frac{\mathbf{s}_1 . \mathbf{s}_i}{\| \mathbf{s}_1 \| \| \mathbf{s}_i \|} \qquad i = 2, 3, \ldots \quad (8)$$

In a set of data points with the index j to k (where $j < k$), when the value of relative angle θ remains invariant with respect to indices, or for a noisy data remains in the prescribed variation limit, the data set $\{\mathbf{D}_j\ \mathbf{D}_{j+1} \ldots \mathbf{D}_{k-1}\ \mathbf{D}_k\}$ will represent a line. For example, assume a set of data points illustrated in Fig. 7, the variation of relative angles θ is demonstrated in Fig. 8.

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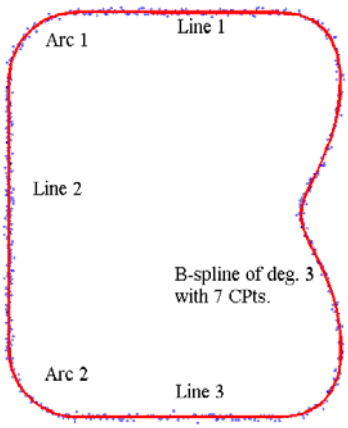


Fig. 5 Constrained local fitting consisting three lines, two arcs and one B-spline with G^1 continuity at joints

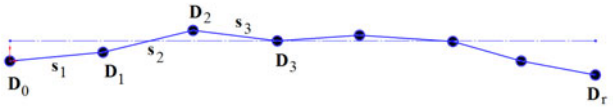


Fig. 6 Relative positions of data points



Fig. 7 Example data set for segment detection algorithm

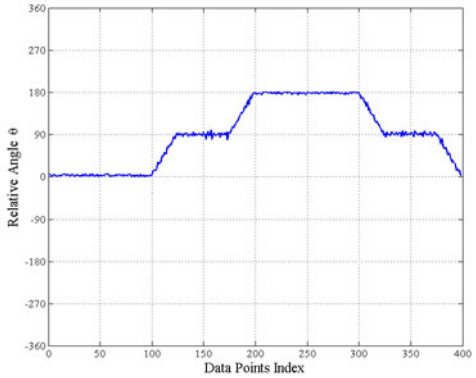


Fig. 8 Variation of relative angle θ vs. data points index

In Fig. 8, the horizontal portions refer to the invariant domains of θ which represent line segments. By prescribing an allowable standard deviation of 3.5° for θ, the start and end points of each line segment can be extracted (Fig. 9)

Once data points for line segments are extracted, the remaining data points either belong to arcs or B-splines. For detecting arc segments, the algorithm proposed in [12] can now be applied. In other words, all points lying on one arc have the same inscribed angle with respect to the extreme points. In Fig. 10, the relative positions of the point $\mathbf{D}_i$ with regard to extreme points $\mathbf{D}_0$ and $\mathbf{D}_r$ are expressed as $\mathbf{s}_{0i}$ and $\mathbf{s}_{ri}$, respectively. Hence, the inscribed angle ψ_i between vectors $\mathbf{s}_{0i}$ and $\mathbf{s}_{ri}$ can be determined as follows:

$$\psi_i = \cos^{-1} \frac{\mathbf{s}_{0i} . \mathbf{s}_{ri}}{\| \mathbf{s}_{0i} \| \| \mathbf{s}_{ri} \|} \qquad i = 1,\ 2,\ 3,... \tag{9}$$

Using Eq. (9), all remaining segments in Fig. 9 are reported as arcs with the maximum variation of 6° for the inscribed angles in each segment. For this case, there is no reported B-spline segment.

Detected data points of lines and arcs will then be used for the constrained local fitting algorithm which was described in Section 2. The final reconstructed profile of the example data is presented in Fig. 11.

4. CASE STUDIES

In this section, the proposed algorithm for segment detection and constraint local fitting will be implemented on two case studies. The first one is an airfoil profile (Fig. 12) which is manufactured by a CNC machining center. Data points measured by RENISHAW® 3D contact digitizing machine are depicted in Fig. 13. The line, arc and B-spline segments of the measured data points are recognized using the proposed segment detection algorithm. Figure 14 illustrates the computed relative angles using Eq. (8) while detected curve segments are demonstrated in Fig. 15. Once segment detection algorithm is implemented, data points are classified into line, arc and B-spline features; thus they can now be treated as separated inputs for the constrained local fitting algorithm. The unknown control points and weights of all segments are calculated by minimizing Eq. (6) with respect to intrinsic and continuity constraints. As stated in Section 2, each line segment has two and each arc segment has three control points while the number of control points for a B-spline segment depends on the complexity of the profile. However, in order to find the proper number of control points, a compromise between fitting error and convergence speed of the solution should be made. Figure 16 shows the final reconstructed airfoil profile that consists of one line, two circular arcs and one cubic B-spline curve with 18 control points. As a matter of fact, the degree of B-spline is set to 3 which is the most widely accepted value in CAD applications [13]. The maximum fitting error in each segment as well as G^1

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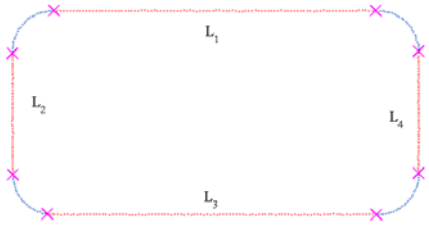


Fig. 9 Line detection for example assembly

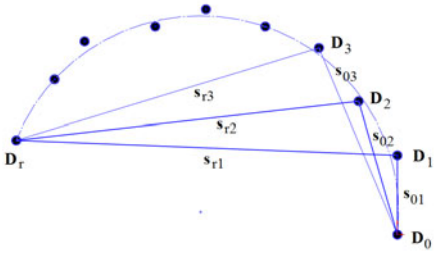


Fig. 10 Relative positions of points lying on an arc with respect to extreme points

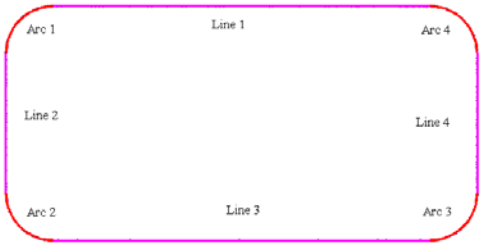


Fig. 11 Final reconstructed profile of example data set

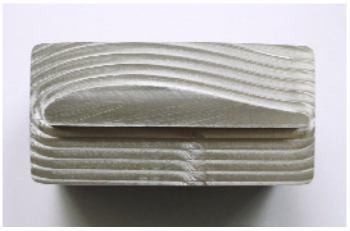


Fig. 12 Airfoil profile as an experimental case study

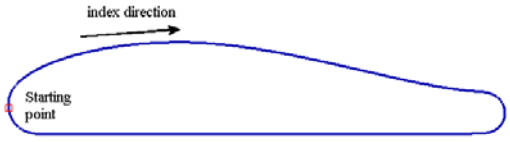


Fig. 13 Measured data points of airfoil profile

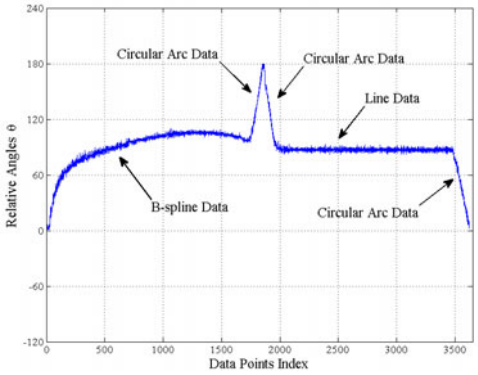


Fig. 14 Relative angles θ vs. data points index

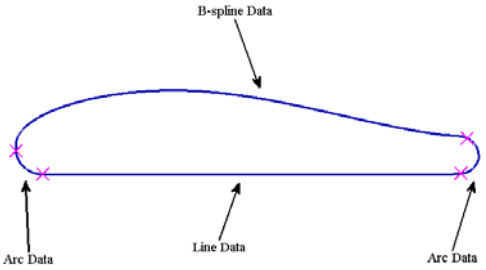


Fig. 15 Segment detection for airfoil data

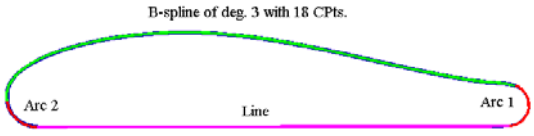


Fig. 16 Final reconstruction of the airfoil profile

continuity error at the end of each segment is also tabulated in Table 1. The G^1 continuity error is computed as the angular difference between the tangent vectors of two adjacent segments.

The base frame of the seat belt fastener mechanism (Fig. 17) is selected as the second case study for testing the functionality of proposed approach. Figure 18 depicts measured data points of the L-shape profile. After applying the segment detection algorithm for this profile, it is inferred that the data set includes only line and arc segments and there is not any B-spline segment. The reconstructed profile is depicted in Fig. 19. The maximum fitting and continuity errors are also reported in Table 2.

5. CONCLUSIONS

In this paper, an approach for 2D profile reconstruction of ordered data points is presented. This approach incorporates two main modules: 1- the segment detection algorithm by which line, arc and B-spline features are extracted from the set of ordered data points and 2- the constrained local fitting algorithm which reconstructs the final profile using the NURBS theory. The fitting algorithm is an iterative nonlinear optimization procedure with desired continuity conditions at joints and intrinsic conditions for circular segments as the constraints.

In comparison to previous research works, the main contributions of the current work can be categorized as follows. Similar to B-spline curves, lines and arcs are also explicitly represented in the parametric formulation of NURBS curves. There are many advantages to use parametric representation instead of implicit forms. For example the objective functions and optimization constraints have a unified formulation defined in terms of control points regardless of the curve types. Also, an automatic optimization procedure in MATLAB environment is developed using the proposed algorithm. It should be noted that the feature or segment detection algorithm is implemented in an appropriate manner. Finally, the developed algorithm is successfully implemented for two profiles selected from practical case studies.

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Table 1 Maximum errors for airfoil profile

Curve segment	Maximum fitting error (mm)	G^1 continuity error (degree) at the end of the segment
Arc 1	0.04	0.020
Line	0.03	0.163
Arc 2	0.02	0.082
B-spline	0.02	0.288

Table 2 Maximum errors for L-shape profile

Curve segment	Maximum fitting error (mm)	G^1 continuity error (degree) at the end of the segment
C_1	0.03	0.657
L_1	0.02	0.515
C_2	0.05	0.268
L_2	0.08	0.130
C_3	0.01	0.238
L_3	0.05	0.046
C_4	0.02	0.265
L_4	0.02	0.234
C_5	0.03	0.008
L_5	0.05	0.081
C_6	0.03	0.258
L_6	0.03	0.111
C_7	0.04	0.002
L_7	0.02	0.006



Fig. 17 Base frame of seat belt fastener before bending

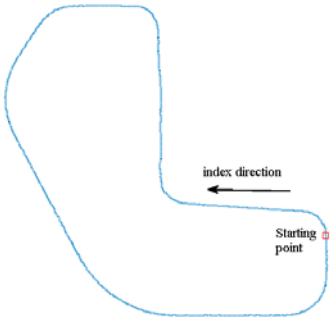


Fig. 18 Measured data points of L-shape profile

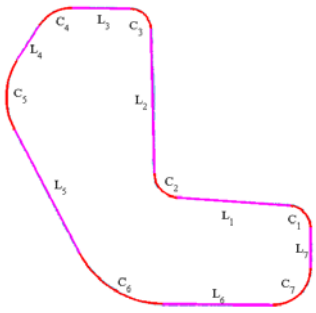


Fig. 19 Final reconstruction of L-shape profile

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