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Some infinite integrals involving Whittaker functions and generalized hypergeometric polynomials, with their applications

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1. *Introduction.* We have defined the generalized hypergeometric polynomial ((6), eqn. (2·1), p. 79) by means of

$$F_n(x) = x^{(\delta-1)n}\, {}_{p+\delta}F_q\left[\begin{matrix}\Delta(\delta, -n), a_1, a_2, \ldots, a_p \\ b_1, b_2, \ldots, b_q\end{matrix}; \lambda x^c\right], \tag{1·1}$$

where δ and n are positive integers and the symbol $\Delta(\delta, -n)$ represents the set of δ-parameters

$$\frac{-n}{\delta}, \quad \frac{-n+1}{\delta}, \ldots, \frac{-n+\delta-1}{\delta}.$$

The polynomial is in a generalized form which yields many known polynomials with proper choice of parameters and therefore the results are of general character.

For ease in writing, we shall follow the contracted notation

$${}_pF_q(x) = {}_pF_q\left(\begin{matrix}a_p \\ b_q\end{matrix}\middle| x\right) = \sum_{r=0}^{\infty} \frac{(a_p)_r x^r}{(b_q)_r r!}.$$

Thus $(a_p)_r$ is to be interpreted as $\prod_{j=1}^{p} (a_j)_r$ and similarly $(b_q)_r$.

The object of this paper is to evaluate some infinite integrals involving the product of Whittaker functions and generalized hypergeometric polynomials. A number of particular cases have been obtained for different polynomials on specializing the parameters.

2. *The integrals.* The integrals to be evaluated are

$$\int_0^\infty x^{l-1} e^{-\frac{1}{2}x} W_{\kappa,m}(x) \left\{x^{(\delta-1)n}\, {}_{p+\delta}F_q\left[\begin{matrix}\Delta(\delta, -n), a_p \\ b_q\end{matrix}; \lambda x^c\right]\right\} dx$$

$$= \frac{\Gamma[l+(\delta-1)n+m+\frac{1}{2}]\,\Gamma[l+(\delta-1)n-m+\frac{1}{2}]}{\Gamma[l+(\delta-1)n-\kappa+1]}$$

$$\times {}_{p+\delta+2c}F_{q+c}\left[\begin{matrix}\Delta(\delta, -n), a_p, \Delta(c, l+(\delta-1)n+m+\frac{1}{2}), \Delta(c, l+(\delta-1)n-m+\frac{1}{2}) \\ b_q, \Delta(c, l+(\delta-1)n-\kappa+1)\end{matrix}; \lambda c^c\right], \tag{2·1}$$

where $R(l+(\delta-1)n \pm m+\frac{1}{2}) > 0$, δ, n and c are positive integers.

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$$\int_0^\infty x^{l-1}e^{-\frac{1}{2}x}W_{\kappa,m}(x)\left\{x^{(\delta-1)n}\,{}_{p+\delta}F_q\left[\begin{matrix}\Delta(\delta,-n),a_p\\ b_q\end{matrix};\lambda x^{-c}\right]\right\}dx$$

$$=\frac{\Gamma[l+(\delta-1)n+m+\frac{1}{2}]\,\Gamma[l+(\delta-1)n-m+\frac{1}{2}]}{\Gamma[l+(\delta-1)n-\kappa+1]}$$

$$\times\,{}_{p+\delta+c}F_{q+2c}\left[\begin{matrix}\Delta(\delta,-n),a_p,\Delta(c,-l-(\delta-1)n+\kappa)\\ b_q,\Delta(c,\frac{1}{2}-l-(\delta-1)n-m),\Delta(c,\frac{1}{2}-l-(\delta-1)n+m)\end{matrix};\frac{\lambda(-1)^c}{c^c}\right] \quad (2\cdot2)$$

where δ, n, c are positive integers and $R(l+(\delta-1)n\pm m+\frac{1}{2})>0$.

Proofs. (A) To establish the integral (2·1), writing down the series for the generalized hypergeometric polynomial in the integrand and changing the order of integration and summation which is easily seen to be justified, we obtain

$$\sum_{r=0}^{\infty}\frac{\prod_{i=0}^{\delta-1}\left(\frac{-n+i}{\delta}\right)_r(a_p)_r\lambda^r}{r!\,(b_q)_r}\int_0^\infty x^{l+(\delta-1)n+cr-1}e^{-\frac{1}{2}x}W_{\kappa,m}(x)\,dx.$$

Now evaluating the integral with the help of Goldstein's integral

$$\int_0^\infty x^{l-1}e^{-\frac{1}{2}x}W_{\kappa,m}(x)\,dx=\frac{\Gamma(l+m+\frac{1}{2})\,\Gamma(l-m+\frac{1}{2})}{\Gamma(l-\kappa+1)}, \quad (2\cdot3)$$

where $R(l\pm m+\frac{1}{2})>0$ and using the following relations

$$(\alpha)_n=\frac{\Gamma(\alpha+n)}{\Gamma(\alpha)},\quad (\alpha)_{n\kappa}=\kappa^{n\kappa}\prod_{i=0}^{\kappa-1}\left(\frac{\alpha+i}{\kappa}\right)_n,$$

we get

$$\frac{\Gamma[l+(\delta-1)n+m+\frac{1}{2}]\,\Gamma[l+(\delta-1)n-m+\frac{1}{2}]}{\Gamma[l+(\delta-1)n-\kappa+1]}$$

$$\times\sum_{r=0}^{\infty}\frac{\prod_{i=0}^{\delta-1}\left(\frac{-n+i}{\delta}\right)_r(a_p)_r\prod_{i=0}^{c-1}\left(\frac{l+(\delta-1)n+m+\frac{1}{2}+i}{c}\right)_r\prod_{i=0}^{c-1}\left(\frac{l+(\delta-1)n-m+\frac{1}{2}+i}{c}\right)_r\lambda^r c^{cr}}{r!\,(b_q)_r\prod_{i=0}^{c-1}\left(\frac{l+(\delta-1)n-\kappa+1+i}{c}\right)_r}$$

which is the expression on the right of (2·1).

(B) The integral (2·2) is similarly established on applying the same procedure as above and after using (2·3) and the following relations

$$(\alpha)_n=\frac{\Gamma(\alpha+n)}{\Gamma(\alpha)},\quad \frac{\Gamma(1-\alpha-n)}{\Gamma(1-\alpha)}=\frac{(-1)^n}{(\alpha)_n},\quad (\alpha)_{n\kappa}=\kappa^{n\kappa}\prod_{i=0}^{\kappa-1}\left(\frac{\alpha+i}{\kappa}\right)_n,$$

we have

$$\frac{\Gamma[l+(\delta-1)n+m+\frac{1}{2}]\,\Gamma[l+(\delta-1)n-m+\frac{1}{2}]}{\Gamma[l+(\delta-1)n-\kappa+1]}$$

$$\times\sum_{r=0}^{\infty}\frac{\prod_{i=0}^{\delta-1}\left(\frac{-n+i}{\delta}\right)_r(a_p)_r\prod_{i=0}^{c-1}\left(\frac{-l-(\delta-1)n+\kappa+i}{c}\right)_r\lambda^r(-1)^{cr}}{r!\,(b_q)_r\prod_{i=0}^{c-1}\left(\frac{\frac{1}{2}-l-(\delta-1)n-m+i}{c}\right)_r\prod_{i=0}^{c-1}\left(\frac{\frac{1}{2}-l-(\delta-1)n+m+i}{c}\right)_r c^{cr}}$$

which is the value of the integral (2·2).

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3. *Applications.* (A) When $\delta = c = \lambda = 1$ in (2·1):

(*a*) With $a_1 = n+\alpha+\beta+1$, $b_1 = 1+\alpha$, $b_2 = \frac{1}{2}$ and multiplying both sides by $\{(1+\alpha)_n\}/n!$, we obtain

$$\int_0^\infty x^{l-1} e^{-\frac{1}{2}x} W_{\kappa,m}(x) \oint_n^{(\alpha,\beta)} \left(\begin{matrix} a_2, \ldots, a_p \\ b_3, \ldots, b_q \end{matrix}; x\right) dx$$

$$= \frac{\Gamma(l+m+\frac{1}{2})\,\Gamma(l-m+\frac{1}{2})}{\Gamma(l-\kappa+1)} \oint_n^{(\alpha,\beta)} \left(\begin{matrix} a_2, \ldots, a_p, l+m+\frac{1}{2}, l-m+\frac{1}{2} \\ b_3, \ldots, b_q, l-\kappa+1 \end{matrix}; 1\right), \qquad (3\cdot1)$$

where $R(l \pm m + \frac{1}{2}) > 0$ and

$$\oint_n^{(\alpha,\beta)} \left(\begin{matrix} a_2, \ldots, a_p \\ b_3, \ldots, b_q \end{matrix}; x\right) = \frac{(1+\alpha)_n}{n!} {}_{p+1}F_q\left[\begin{matrix} -n, n+\alpha+\beta+1, a_2, \ldots, a_p \\ 1+\alpha, \frac{1}{2}, b_3, \ldots, b_q \end{matrix}; x\right]$$

is a generalized Sister Celine's polynomial ((6), eqn. (2·2), p. 80) which reduces to Sister Celine's polynomial ((2), p. 806) when $\alpha = \beta = 0$.

In (3·1), setting $p = q = 3$, $a_2 = \rho$, $a_3 = \frac{1}{2}$ and $b_3 = \sigma$, we have

$$\int_0^\infty x^{l-1} e^{-\frac{1}{2}x} W_{\kappa,m}(x) H_n^{(\alpha,\beta)}(\rho, \sigma, x)\, dx$$

$$= \frac{\Gamma(l+m+\frac{1}{2})\,\Gamma(l-m+\frac{1}{2})\,(1+\alpha)_n}{\Gamma(l-\kappa+1)\,n!} {}_5F_3\left[\begin{matrix} -n, n+\alpha+\beta+1, \rho, l+m+\frac{1}{2}, l-m+\frac{1}{2} \\ 1+\alpha, \sigma, l-\kappa+1 \end{matrix}; 1\right], \qquad (3\cdot2)$$

where $R(l \pm m + \frac{1}{2}) > 0$ and

$$H_n^{(\alpha,\beta)}(\rho, \sigma, x) = \frac{(1+\alpha)_n}{n!} {}_3F_2\left[\begin{matrix} -n, n+\alpha+\beta+1, \rho \\ 1+\alpha, \sigma \end{matrix}; x\right]$$

is a generalized Rice's polynomial ((3), eqn. (2·3), p. 158) which reduces to Rice's polynomial ((4), p. 108) when $\alpha = \beta = 0$.

Further substituting $\rho = \sigma$ in (3·2), we obtain

$$\int_0^\infty x^{l-1} e^{-\frac{1}{2}x} W_{\kappa,m}(x)\, P_n^{(\alpha,\beta)}(1-2x)\, dx$$

$$= \frac{\Gamma(l+m+\frac{1}{2})\,\Gamma(l-m+\frac{1}{2})\,(1+\alpha)_n}{\Gamma(l-\kappa+1)\,n!} {}_4F_2\left[\begin{matrix} -n, n+\alpha+\beta+1, l+m+\frac{1}{2}, l-m+\frac{1}{2} \\ 1+\alpha, l-\kappa+1 \end{matrix}; 1\right], \qquad (3\cdot3)$$

where $R(l \pm m + \frac{1}{2}) > 0$ and $P_n^{(\alpha,\beta)}(x)$ is the Jacobi polynomial.

(*b*) With $p = 0$, $q = 1$, $b_1 = 1+\alpha$ and multiplying both sides by $\{(1+\alpha)_n\}/n!$, we obtain

$$\int_0^\infty x^{l-1} e^{-\frac{1}{2}x} W_{\kappa,m}(x)\, L_n^{(\alpha)}(x)\, dx$$

$$= \frac{\Gamma(l+m+\frac{1}{2})\,\Gamma(l-m+\frac{1}{2})\,(1+\alpha)_n}{\Gamma(l-\kappa+1)\,n!} {}_3F_2\left[\begin{matrix} -n, l+m+\frac{1}{2}, l-m+\frac{1}{2} \\ 1+\alpha, l-\kappa+1 \end{matrix}; 1\right], \qquad (3\cdot4)$$

where $R(l \pm m + \frac{1}{2}) > 0$ and $L_n^{(\alpha)}(x)$ is the generalized Laguerre polynomial.

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(B) When $\delta = c = 2$ in (2·2):

(i) Substituting $p = 1$, $q = 2$, $a_1 = \gamma - \beta$, $b_1 = \gamma$, $b_2 = 1-\beta-n$, $\lambda = 1$ and multiplying both sides by $\{2^n(\beta)_n\}/n!$, we obtain

$$\int_0^\infty x^{l-1} e^{-\frac{1}{2}x} W_{\kappa,m}(x) R_n(\beta,\gamma;x)\,dx$$
$$= \frac{\Gamma[l+n+m+\frac{1}{2}]\,\Gamma[l+n-m+\frac{1}{2}]\,2^n(\beta)_n}{\Gamma[l+n-\kappa+1]\,n!}$$
$$\times {}_5F_6\left[\begin{matrix}\Delta(2,-n), \gamma-\beta, \Delta(2,-l-n+\kappa) \\ \gamma,\ 1-\beta-n,\ \Delta(2,\tfrac{1}{2}-l-n-m),\ \Delta(2,\tfrac{1}{2}-l-n+m)\end{matrix}; \frac{1}{2^2}\right] \qquad (3\cdot5)$$

where $R(l+n\pm m+\frac{1}{2}) > 0$ and

$$R_n(\beta,\gamma;x) = \frac{(2x)^n(\beta)_n}{n!}\,{}_3F_2\left[\begin{matrix}\Delta(2,-n), \gamma-\beta \\ \gamma,\ 1-\beta-n\end{matrix}; x^{-2}\right]$$

is the Bedient's polynomial ((5); eqn. (1), p. 297) which reduces to the Gegenbauer polynomial $C_n^\beta(x)$ when $\lim_{\gamma\to\infty} R_n(\beta,\gamma;x)$.

(ii) Setting $p = q = 0$, $\lambda = -1$ and multiplying both sides by 2^n, we have

$$\int_0^\infty x^{l-1} e^{-\frac{1}{2}x} W_{\kappa,m}(x) H_n(x)\,dx$$
$$= \frac{\Gamma(l+n+m+\frac{1}{2})\,\Gamma(l+n-m+\frac{1}{2})\,2^n}{\Gamma(l+n-\kappa+1)}\,{}_4F_4\left[\begin{matrix}\Delta(2,-n), \Delta(2,-l-n+\kappa) \\ \Delta(2,\tfrac{1}{2}-l-n-m), \Delta(2,\tfrac{1}{2}-l-n+m)\end{matrix}; -\frac{1}{2^2}\right], \qquad (3\cdot6)$$

where $R(l+n\pm m+\frac{1}{2}) > 0$ and

$$H_n(x) = (2x)^n\,{}_2F_0\left[\begin{matrix}\Delta(2,-n) \\ -\end{matrix}; -x^{-2}\right]$$

is the Hermite polynomial.

(iii) With $p = 0$, $q = 1$, $b_1 = \frac{1}{2}-n$, $\lambda = 1$ and multiplying both sides by $\{(\frac{1}{2})_n 2^n\}/n!$ we obtain

$$\int_0^\infty x^{l-1} e^{-\frac{1}{2}x} W_{\kappa,m}(x) P_n(x)\,dx = \frac{\Gamma(l+n+m+\frac{1}{2})\,\Gamma(l+n-m+\frac{1}{2})\,(\frac{1}{2})_n\,2^n}{\Gamma(l+n-\kappa+1)\,n!}$$
$$\times {}_4F_5\left[\begin{matrix}\Delta(2,-n), \Delta(2,-l-n+\kappa) \\ \frac{1}{2}-n,\ \Delta(2,\tfrac{1}{2}-l-n-m),\ \Delta(2,\tfrac{1}{2}-l-n+m)\end{matrix}; \frac{1}{2^2}\right], \qquad (3\cdot7)$$

where $R(l+n\pm m+\frac{1}{2}) > 0$ and

$$P_n(x) = \frac{(2x)^n(\frac{1}{2})_n}{n!}\,{}_2F_1\left[\begin{matrix}\Delta(2,-n) \\ \frac{1}{2}-n\end{matrix}; x^{-2}\right]$$

is the Legendre polynomial.

4. *Particular cases of* (2·1). (i) With $k = 0$: using the known result ((1), eqn. (14), p. 265)

$$W_{0,m}(x) = \sqrt{\left(\frac{x}{\pi}\right)}\,k_m(\tfrac{1}{2}x),$$

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we obtain

$$\int_0^\infty x^{l+(\delta-1)n-\frac{1}{2}} e^{-\frac{1}{2}x} k_m(\tfrac{1}{2}x)\, {}_{p+\delta}F_q\left[\begin{matrix}\Delta(\delta,-n), a_p \\ b_q\end{matrix}; \lambda x^c\right] dx$$

$$= \frac{\sqrt{\pi}\,\Gamma(l+(\delta-1)n+m+\frac{1}{2})\,\Gamma(l+(\delta-1)n-m+\frac{1}{2})}{\Gamma(l+(\delta-1)n+1)}$$

$$\times\, {}_{p+\delta+2c}F_{q+c}\left[\begin{matrix}\Delta(\delta,-n), a_p, \Delta(c, l+(\delta-1)n+m+\frac{1}{2}), \Delta(c, l+(\delta-1)n-m+\frac{1}{2}) \\ b_q, \Delta(c, l+(\delta-1)n+1)\end{matrix}; \lambda c^c\right] \tag{4·1}$$

where $R(l+(\delta-1)n \pm m+\frac{1}{2}) > 0$, δ, n, c are positive integers and $k_m(\frac{1}{2}x)$ is the Bateman's function ((1), p. 267).

(ii) With the application of the relation ((1), eqn. (70), p. 221)

$$e^{-\frac{1}{2}x} W_{\kappa,m}(x) = \sqrt{\left(\frac{x}{2\pi}\right)}\, 2^\kappa G^{40}_{24}\left(\frac{x^2}{2^2}\middle|\begin{matrix}\frac{1}{4}-\frac{1}{2}\kappa, \frac{3}{4}-\frac{1}{2}\kappa \\ \frac{1}{2}+\frac{1}{2}m, \frac{1}{2}-\frac{1}{2}m, \frac{1}{2}m, -\frac{1}{2}m\end{matrix}\right),$$

we have

$$\int_0^\infty x^{l+(\delta-1)n-\frac{1}{2}} G^{40}_{24}\left(\frac{x^2}{2^2}\middle|\begin{matrix}\frac{1}{4}-\frac{1}{2}\kappa, \frac{3}{4}-\frac{1}{2}\kappa \\ \frac{1}{2}+\frac{1}{2}m, \frac{1}{2}-\frac{1}{2}m, \frac{1}{2}m, -\frac{1}{2}m\end{matrix}\right) {}_{p+\delta}F_q\left[\begin{matrix}\Delta(\delta,-n), a_p \\ b_q\end{matrix}; \lambda x^c\right] dx$$

$$= \frac{\sqrt{\pi}\,\Gamma[l+(\delta-1)n+m+\frac{1}{2}]\,\Gamma[l+(\delta-1)n-m+\frac{1}{2}]}{2^{\kappa-\frac{1}{2}}\Gamma[l+(\delta-1)n-\kappa+1]}$$

$$\times\, {}_{p+\delta+2c}F_{q+c}\left[\begin{matrix}\Delta(\delta,-n), a_p, \Delta(c, l+(\delta-1)n+m+\frac{1}{2}), \Delta(c, l+(\delta-1)n-m+\frac{1}{2}) \\ b_q, \Delta(c, l+(\delta-1)n-\kappa+1)\end{matrix}; \lambda c^c\right], \tag{4·2}$$

where $R(l+(\delta-1)n \pm m+\frac{1}{2}) > 0$ and δ, n, c are positive integers and $G^{mn}_{pq}(x)$ is the Meijer's G-function ((1), p. 207).

(iii) With $m = \frac{1}{4}$. Using the formula

$$W_{\kappa,\pm\frac{1}{4}}(x) = 2^{-\kappa}(2x)^{\frac{1}{4}} D_{2\kappa-\frac{1}{2}}[(2x)^{\frac{1}{2}}],$$

we get

$$\int_0^\infty x^{l+(\delta-1)n-\frac{3}{4}} e^{-\frac{1}{2}x} D_{2\kappa-\frac{1}{2}}[\sqrt{(2x)}]\, {}_{p+\delta}F_q\left[\begin{matrix}\Delta(\delta,-n), a_p \\ b_q\end{matrix}; \lambda x^c\right] dx$$

$$= \frac{2^{\kappa-\frac{1}{4}}\Gamma[l+(\delta-1)n+\frac{3}{4}]\,\Gamma[l+(\delta-1)n+\frac{1}{4}]}{\Gamma[l+(\delta-1)n-\kappa+1]}$$

$$\times\, {}_{p+\delta+2c}F_{q+c}\left[\begin{matrix}\Delta(\delta,-n), a_p, \Delta(c, l+(\delta-1)n+\frac{3}{4}), \Delta(c, l+(\delta-1)n+\frac{1}{4}) \\ b_q, \Delta(c, l+(\delta-1)n-\kappa+1)\end{matrix}; \lambda c^c\right], \tag{4·3}$$

where $R(l+(\delta-1)n \pm \frac{1}{4}+\frac{1}{2}) > 0$, δ, n and c are positive integers and $D_\nu(x)$ is the Parabolic cylinder function.

(iv) With $\kappa = \frac{1}{2}\nu+\frac{1}{4}$, $m = \frac{1}{4}$.

Using the formula

$$W_{\frac{1}{2}\nu+\frac{1}{4},\pm\frac{1}{4}}(x) = 2^{-\frac{1}{2}\nu}x^{\frac{1}{4}} e^{-\frac{1}{2}x} H_\nu[(2x)^{\frac{1}{2}}],$$

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where $H_\nu(x)$ is the Hermite polynomial, we obtain

$$\int_0^\infty x^{l+(\delta-1)n-\frac{3}{4}} e^{-x} H_\nu[\sqrt{(2x)}]\, {}_{p+\delta}F_q\left[\begin{matrix}\Delta(\delta,-n), a_p \\ b_q\end{matrix}; \lambda x^c\right] dx$$

$$= \frac{2^{\frac{1}{2}\nu}\Gamma[l+(\delta-1)n+\frac{3}{4}]\,\Gamma[l+(\delta-1)n+\frac{1}{4}]}{\Gamma[l+(\delta-1)n-\frac{1}{2}\nu+\frac{3}{4}]}$$

$$\times {}_{p+\delta+2c}F_{q+c}\left[\begin{matrix}\Delta(\delta,-n), a_p, \Delta(c, l+(\delta-1)n+\frac{3}{4}), \Delta(c, l+(\delta-1)n+\frac{1}{4}) \\ b_q, \Delta(c, l+(\delta-1)n-\frac{1}{2}\nu+\frac{3}{4})\end{matrix}; \lambda c^c\right], \tag{4·4}$$

where $R(l+(\delta-1)n\pm\frac{1}{4}+\frac{1}{2})>0$, δ, n and c are positive integers.

(v) With $\kappa = m+\nu+\frac{1}{2}$. Using the formula

$$W_{m+\nu+\frac{1}{2},\pm m}(x) = (-1)^\nu \nu!\, x^{m+\frac{1}{2}} e^{-\frac{1}{2}x} L_\nu^{(2m)}(x),$$

where $L_\nu^{(\alpha)}(x)$ is the Laguerre polynomial, we have

$$\int_0^\infty x^{l+(\delta-1)n+m-\frac{1}{2}} e^{-x} L_\nu^{(2m)}(x)\, {}_{p+\delta}F_q\left[\begin{matrix}\Delta(\delta,-n), a_p \\ b_q\end{matrix}; \lambda x^c\right] dx$$

$$= \frac{(-1)^\nu \Gamma[l+(\delta-1)n+m+\frac{1}{2}]\,\Gamma[l+(\delta-1)n-m+\frac{1}{2}]}{\nu!\,\Gamma[l+(\delta-1)n-m-\nu+\frac{1}{2}]}$$

$$\times {}_{p+\delta+2c}F_{q+c}\left[\begin{matrix}\Delta(\delta,-n), a_p, \Delta(c, l+(\delta-1)n+m+\frac{1}{2}), \Delta(c, l+(\delta-1)n-m+\frac{1}{2}) \\ b_q, \Delta(c, l+(\delta-1)n-m-\nu+\frac{1}{2})\end{matrix}; \lambda c^c\right], \tag{4·5}$$

where $R(l+(\delta-1)n\pm m+\frac{1}{2})>0$, δ, n and c are positive integers.

On specializing the parameters in (4·1)–(4·5), we may obtain results for the known polynomials.

Proceeding as above we may obtain similar results for other polynomials with proper choice of parameters in (2·2).

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