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A Method for Modular Robots Generating Dynamics Automatically

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SUMMARY

In this paper a method for the automatic generation of dynamics for modular robots is presented. A modular reconfigurable robot consists of link modules and joint modules of various specifications. We analyze the abstract architecture of modular robots. A total of nine types of connecting forms and three types of joint forms are identified with reference to their own systems. The geometric relationships are derived by the group theory. According to the modular idea, the formulations of the velocity, acceleration and other essential equations of the link module and the joint module are formed compensatively and recursively. Then the dynamic model is generated automatically by a compensative and recursive method.

KEYWORDS: Modular robot; Dynamics; Automatic generation

1. INTRODUCTION

Modular robots (MR) concepts and techniques have been of interest in the robotics area since the 1980s.[1–6] MR consist of link modular units and joint modular units on the building block principle. The joint modules are assumed to be self-actuated that can generate rotary (R) or translational (P) motions. The link modules are polyhedral objects with geometric symmetry; multiple connections of joint modules are allowed. Modular robots have been mostly developed to show that certain mechanical arm structures can be used for more than one application with minimal additions and/or modifications.

For an ordinary robot, the dynamic model depending on its geometry is derived manually. For a modular robot system, one can build numerous robots with distinct geometries out of an inventory of robot modules. Therefore, it's impossible and impractical to derive the dynamic model of every modular robot geometry case by case and store it as a library function in the computer.[7]

In this paper theoretical studies on modular joints and modular links are carried out. The modular joint consists of a rotary or translational joint and can be classified into three types: 1-DOF, 2-DOF and 3-DOF. Each transformation matrix T_J can be obtained. The link module is assumed to be a square prism, whose section is square. and 10 ports are considered; each transformation matrix T_L with its relatively dynamic feature expression can be obtained by the Lie algebra of the Euclidean group SE(3).[8,9] If modules are chosen, the related parameters are also chosen.

Now we present a methodology (compensative and recursive method) to automatically generate a dynamic model of a modular robot. The idea about the formulation of the dynamic model comes from the ordinary robot.[10,11] However, the dynamic model expression is simple and efficient for the modular robot. A further development is based on the modular robot. According to the modular idea and inertia properties, the velocity, acceleration, forces and other dynamic parameters of each module can be expressed compensatively. Then the dynamic model is generated automatically.

2. STRUCTURE CATALOGUE

In order to choose modules to construct MR or load and unload some modules to reconstruct a new MR on the basis of a known MR, we have to research the connecting configuration between joint modules and link modules. In this paper, we extract module features from real robots. After basic research, the joint module includes 1-DOF, 2-DOF and 3-DOF joints. The 1-DOF joint consists of a rotary or translational joint (Figure 1). Other multi-DOFs joints are composed of the 1-DOF joint. Considering the MR character, we design the 2-DOF joint whose axes are vertical. In order to obtain inverse kinematics, we design the 3-DOF joint whose axes intersect; their own transformation matrix T_J can be created. Joint modules are connected to the link through connecting ports. The link module is classified into a middle-link type and an end-link type. By the group theory the transformation matrix T_{Li-j} (port j relative to port i) of the link module and related dynamic feature expressions can be obtained.

2.1 Link module structure

According to the building block, MR mainly consist of some of the following 9 types: (**A**) *one link and one joint* (two kinds). (**B**) *one link and two joints* (seven kinds). Each kind has its relative transformation matrix. One link and one

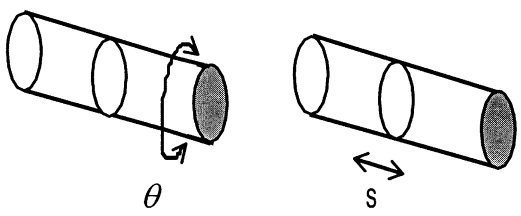


Fig. 1. Single DOF joint.

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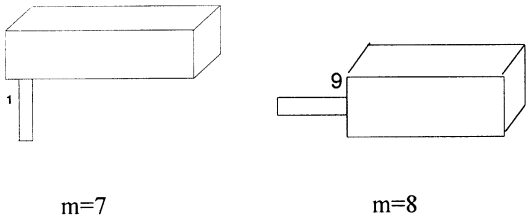


Fig. 2. One link and one joint.

joint (Figure 2) is mainly used as an end-link (for an open chain robot).

One link and two joints (Figure 3) system is mainly used as the middle-link of MR. With the bottom-up manner the link ports are numbered. The input ports are given odd numbers and the output ports even numbers (Figure 4). For the same character, only one kind is studied; m represents the module connecting type.

2.2 Mathematical description and the basic coordinate transformation matrix

Definition 1: Basic Group(G)*: In the basic group each set is a single cell base which forms other similar models with a relative twist cycle subgroup*, such as: $G = \{\{T_{L2\text{-}1}\}. \{T_{L10\text{-}1}\}, \ldots\}$.

In this paper the MR is based on a quadrivalent circular, whose twist angle is 2 $\pi/4$. Assuming the twist cycle subgroup of the transformation matrix is $\left(\sigma(\frac{2\pi}{4})^0 \ \sigma(\frac{2\pi}{4})^1 \ \sigma(\frac{2\pi}{4})^2 \ \sigma(\frac{2\pi}{4})^3 \right)$.

Then we can obtain:

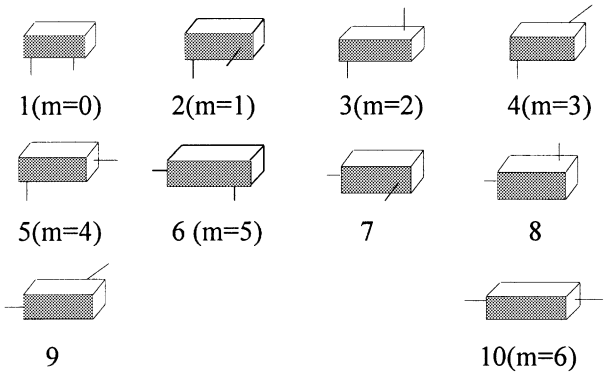
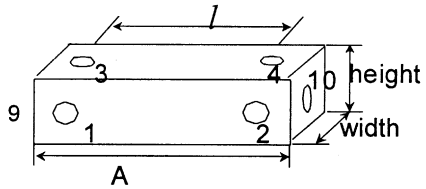


Fig. 3. One link and two joints.



(*l*-port separation, A-link length, *w*-link width)

Fig. 4. Link module.

$$\sigma\left(\frac{2\pi}{4}\right)^0 = I,$$

$$\sigma\left(\frac{2\pi}{4}\right)^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -\frac{\omega_{i-1}}{2} \\ 0 & 1 & 0 & -\frac{\omega_{i-1}}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \ldots,$$

Definition 2: Basic cycle subgroup (g)*: g is a twist cycle subgroup relative to the single cell base in the basic group. For example, $\{T_{L2\text{-}1}\}$ is a single cell base,

$$T_{L2-1} = \begin{bmatrix} 1 & 0 & 0 & -l_{i-1} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The basic cycle subgroup is:

$$g = T_{L2-1}\left(\sigma\left(\frac{2\pi}{4}\right)^0 \ \sigma\left(\frac{2\pi}{4}\right)^1 \ \sigma\left(\frac{2\pi}{4}\right)^2 \ \sigma\left(\frac{2\pi}{4}\right)^3 \right)$$

So $T_{L4\text{-}1} = T_{L2\text{-}1}.\sigma(\frac{2\pi}{4})^1$, $\sigma(\frac{2\pi}{4})^1$ is the transformation matrix and the twist angle is 2 $\pi/4$. According to the above definition and relation, the transformation matrix of other modules connecting type (Figure 3) can be obtained:

$$T_{L2\text{-}9} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & \frac{w_{i-1}}{2} \\ -1 & 0 & 0 & \frac{A_{i-1}+l_{i-1}}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$T_{L4\text{-}9} = T_{L2\text{-}9}.\sigma(\frac{\pi}{2})^1$ .... Like the above description, other parameters of each module can be analyzed.

Definition 3: twist equivalence*: After figure A_1 is transformed by f, A_2 can be obtained. $A_1 f = A_2$, so A_1 and A_2 are in twist equivalence.

For example:

$$T_{L2-1} \cdot \sigma\left(\frac{2\pi}{4}\right) = T_{L4-1},$$

$$T_{L2-1} \cdot \sigma\left(\frac{2\pi}{4}\right)^2 = T_{L6-1}, \ T_{L2-1} \cdot \sigma\left(\frac{2\pi}{4}\right)^3 = T_{L8-1},$$

So $T_{L2\text{-}1}$, $T_{L4\text{-}1}$, $T_{L6\text{-}1}$, $T_{L8\text{-}1}$ are also in twist equivalence. In the same way, $T_{L2\text{-}9}$, $T_{L4\text{-}9}$, $T_{L6\text{-}9}$, $T_{L8\text{-}9}$ are in twist equivalence, too. So we can only study the basic expressions about MR(m = 0, m = 4, m = 5, m = 6) (Figure 3). We take the connecting type (m = 0, Figure 5) as an example; then the

* Defined by author.

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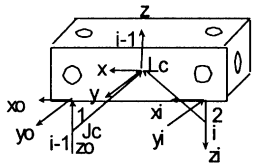


Fig. 5. One link and two joints (m = 0)

coordinate frame can be set up. Some formulations can be obtained, as follows:

$$T_{L2-1}=\begin{bmatrix}1&0&0&-l_{i-1}\\0&-1&0&0\\0&0&-1&0\\0&0&0&1\end{bmatrix} \tag{1}$$

$$\bar{e}_{i-1,i}=(0\ \ 0\ \ -1)^T \tag{2}$$

$$\bar{r}^{i-1}_{i-1,i-1}=\begin{bmatrix}-\frac{l_{i-1}}{2}&0&\frac{h_{i-1}+w_{i-1}}{2}\end{bmatrix}^T-q_{i-1}\cdot\bar{e}_{i-1,i-1}\cdot s_{i-1} \tag{3}$$

$$\bar{\rho}^{i-1}_{i-1}=\begin{pmatrix}-\frac{l_{i-1}}{2}&0&\frac{w_{i-1}}{2}\end{pmatrix}^T \tag{4}$$

$$\bar{r}^{i-1}_{i-1,i}=\begin{pmatrix}\frac{l_{i-1}}{2}&0&\frac{h_i+w_{i-1}}{2}\end{pmatrix}^T \tag{5}$$

$$\tilde{J}_{i-1}=\begin{pmatrix}\frac{m_{i-1}\cdot w_{i-1}^2}{6}&0&0\\0&\frac{m_{i-1}\cdot(l_{i-1}^2+w_{i-1}^2}{12}&0\\0&0&\frac{m_{i-1}\cdot(l_{i-1}^2+w_{i-1}^2)}{12}\end{pmatrix} \tag{6}$$

$$T_{Lc-1}=\begin{bmatrix}1&0&0&-\frac{l_{i-1}}{2}\\0&1&0&0\\0&0&1&\frac{w_{i-1}}{2}\\0&0&0&1\end{bmatrix}$$

where T_{L2-1} is the transformation matrix of port 2 relative to port 1; $\vec{e}_{i-1,i}$ is the vector of the i-th joint axis relative to the (i − 1)-th joint axis; $\vec{r}^{\,k}_{i,j}$ is the position of the center of the i-th link relative to the center of the j-th joint with reference to the k-th link input port coordinate system; $\vec{\rho}^{\,i-1}_{i-1}$ is the position of the center of the (i-1)-th link relative to the (i-1)-th link input port with reference to the input port coordinate system; $\tilde{J}_{i-1}$ is the inertia tensor of the (i-1)-th link module about its center of mass; T_{Lc-1} is the transformation matrix of the center of the link module relative to port 1. According to the basic expressions, other connecting types expressions can be obtained by the above group theory.

The end-link can be studied in the same way.

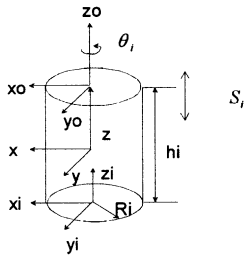


Fig. 6. Joint module coordinate system.

$$T_{Lm=7}=\begin{bmatrix}0&0&-1&-\frac{A_{i-1}+l_{i-1}}{2}\\1&0&0&0\\0&-1&0&\frac{w_{i-1}}{2}\\0&0&0&1\end{bmatrix},$$

$$T_{Lm=8}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&A_{i-1}\\0&0&0&1\end{bmatrix}$$

2.3 Joint module structure

In order to match the coordinate system of the link port, the coordinate system (Figure 6) is set up according to continuous multiplication. So the transformation matrix T_J of its own can be obtained (Suppose ξ is each joint axis relative to its own Plucker coordinate. $T_{JR}(\theta_i)$ is a matrix when the joint has an angle (θ_i), $T_{JP}(S_i)$ is a matrix when the joint has a translation (S_i).)

(a) 1-DOF joint. When it is a rotary joint $s_{i-1}=0$; when it is a translational joint, $s_{i-1}=1$. Assuming:

$$\pi_{i-1}=(1-s_{i-1})\cdot\theta_{i-1}+s_{i-1}\cdot S_{i-1},\ \ t_{i-1}=(1-s_{i-1})\cdot\theta_{i-1}$$

then:

$$T_{JR}(\pi_i)=e^{\hat{\xi}_i\cdot\pi_i}\cdot T_J(0)=\begin{bmatrix}\cos(t_i)&-\sin(t_i)&0&0\\\sin(t_i)&\cos(t_i)&0&0\\0&0&1&h_i+S_i\\0&0&0&1\end{bmatrix} \tag{7}$$

The 2-DOF joint or the 3-DOF joint module are based on the 1-DOF joint module. We adopt an *unreal-link* way. For example, the 2-DOF joint is made up of two 1-DOF joints and an *unreal-link* module. This matrix of the *unreal* 2-DOF joint can be generated. All of link parameters are supposed to be 0. Hence the matrices of the 2-DOF joint and the 3-DOF joint can be written. (Suppose the joint axes are vertical).

(b) 2-DOF joint. The chart and the unreal chart of the 2-DOF joint are shown in Figure 7 and the matrix can be obtained by:

$$\begin{aligned}T_{2dof}&=T_{J_{i-1}}\cdot T_{L_{i-1}}\cdot T_{J_i}\\&=e^{\hat{\xi}_{i-1}^{\,i-1}\cdot\theta_{i-1}}\cdot T_{J_{i-1}}(0)\cdot T_{L10-1}\cdot e^{\hat{\xi}_i^{\,i}\cdot\theta_i}\cdot T_{J_i}(0)\end{aligned} \tag{8}$$

Let $A_{i-1}=0$, $l_{i-1}=0$, $w_{i-1}=0$

suppose

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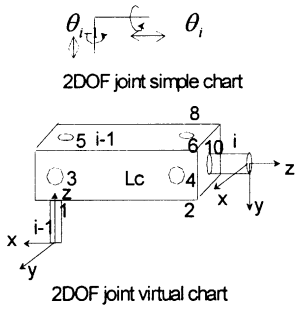


Fig. 7. 2-DOF joint.

$$T_{vir} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

is a virtual link (unreal-link) transformation matrix

$$T_{2df} = \begin{bmatrix} -\sin t_{i-1} \cdot \cos t_i & \sin t_{i-1} \cdot \sin t_i & -\cos t_{i-1} & -h_i \cdot \cos t_{i-1} - s_i \cdot \theta_i \cdot \cos t_{i-1} \\ \cos t_{i-1} \cdot \cos t_i & -\cos t_{i-1} \cdot \sin t_i & -\sin t_{i-1} & -h_i \cdot \sin t_{i-1} - s_i \cdot \theta_i \cdot \sin t_{i-1} \\ -\sin t_i & -\cos t_i & 0 & h_{i-1} + s_{i-1} \cdot \theta_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (9)$$

For example: A 2-DOF rotary joint which consists of two small segmentally spherical surfaces is studied. Inside there are two small motors which supply power and make the two small segmentally spherical surfaces touch. Thus we can obtain two rotary movements in vertical directions (Figure 8).

The analysis of the 3-DOF joint is the same as above. Its charts are shown in Figure 9.

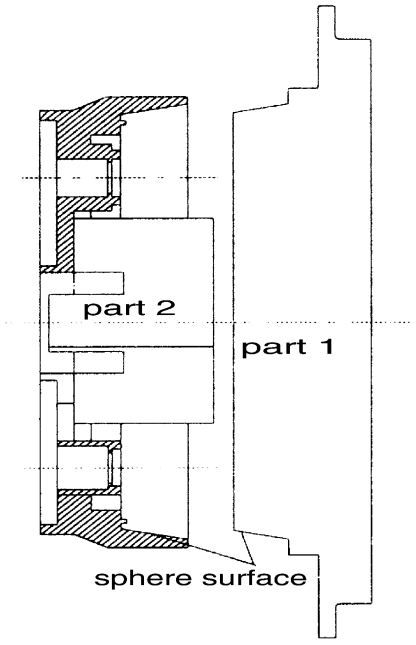


Fig. 8. The semblance of 2-DOF rotary joint.

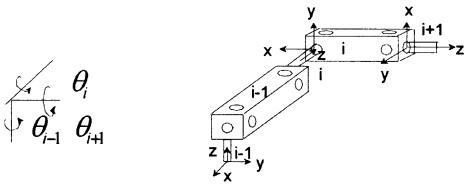


Fig. 9. 3-DOF joint.

Its transformation matrix is the function of θ_{i-1}, θ_i, θ_{i+1}, h_{i-1}, h_{i+1}.

If

$$A = \sin t_i \cdot \cos t_{i+1}, \quad B = \sin t_i \cdot \sin t_{i+1},$$

$$C = \sin t_{i-1} \cdot \cos t_i, \quad D = \cos t_{i-1} \cos t_i$$

then

$$T_{3dof} = \begin{bmatrix} A \cdot \sin t_{i-1} + \cos t_{i-1} \cdot \sin t_{i+1} & -B \cdot \sin t_{i-1} + \cos t_{i-1} \cdot \cos t_{i+1} & C & C(h_{i+1} + s_{i+1} \cdot \theta_{i+1}) - s_i \cdot \theta_i \cdot \cos t_{i-1} \\ -A \cdot \cos t_{i-1} + \sin t_{i-1} \cdot \sin t_{i+1} & B \cdot \cos t_{i-1} + \sin t_{i-1} \cdot \cos t_{i+1} & D & -D(h_{i+1} + s_{i+1} \cdot \theta_{i+1}) - s_i \cdot \theta_i \cdot \sin t_{i-1} \\ -\cos t_i \cdot \cos t_{i+1} & \cos t \cdot \sin t_{i+1} & \sin t_i & (h_{i+1} + s_{i+1} \cdot \theta_{i+1}) \cdot \sin t_i + h_{i-1} + s_{i-1} \cdot \theta_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (10)$$

3. AUTOMATIC GENERATION OF DYNAMICS

It can be assumed that $q = [q_1 \ldots q_n]$ represent generalized coordinate variables; P represents the torque. L represents the link module; J represents the joint module.

3.1 Representation of matrix A

Matrix A represents the pose of MR.[8]

$$A_j = R\left(e^{\hat{\xi}_1^1 \cdot \theta_1} \cdot T_{J1} \cdot T_{L1} \ldots e^{\hat{\xi}_j^j \cdot \theta_j} \cdot T_{Jj}\right)$$

$$= R\left(\prod_{i=1}^{j} e^{\hat{\xi}_{ig} \cdot \theta_i} \cdot T(0)\right), \quad (11)$$

$$A_{jL} = A_{jJ}$$

When it is a rotary joint, then

$$R\left(e^{\hat{\xi}_i^j \cdot \theta_i}\right) = \begin{bmatrix} c\theta_i & -s\theta_i & 0 \\ s\theta_i & c\theta_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (12)$$

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When it is a translational joint, then

$$R\left(e^{\hat{\xi}_i \cdot \theta_i}\right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{13}$$

3.2 Velocity and acceleration

In the recursive algorithm of a serial type robot, the forward iterations of the velocities and accelerations follow the natural order of links and joints. The general velocity and acceleration of the base will be given initially: $\vec{V}_0 = \vec{0}$, $\vec{W}_0 = \vec{0}$, $\vec{\omega}_0 = \vec{0}$, $\vec{\varepsilon}_0 = \vec{0}$. $\vec{V}_{iL}$ and $\vec{W}_{iL}$ represent the velocity and acceleration of the center of the i-th link module, $\vec{\omega}_{iL}$ and $\vec{\varepsilon}_{iL}$ represent its angular velocity and angular acceleration of the link module. $\vec{e}_i$ is the unit vector of the axis of joint i.

$$\vec{\omega}_{iL} = \vec{\omega}_{i-1L} + \dot{q}_i \cdot \vec{e}_i \cdot (1 - s_i) \tag{14}$$

$$\vec{V}_{iL} = \vec{V}_{i-1L} - \vec{\omega}_{i-1L} \times \vec{r}_{i-1,\,i} + \vec{\omega}_{iL} \times \vec{r}_{i,\,i} + \dot{q}_i \cdot \vec{e}_i \cdot s_i \tag{15}$$

$$\vec{\varepsilon}_{iL} = \dot{\vec{\omega}}_{iL} = \vec{\varepsilon}_{i-1L} + (\ddot{q}_i \cdot \vec{e}_i + \dot{q}_i \cdot \left(\dot{\vec{\omega}}_{i-1} \times \vec{e}_i\right)) \cdot \left(1 - s_i\right) \tag{16}$$

$$\begin{aligned} \vec{W}_{iL} = \dot{\vec{V}}_{iL} = \dot{\vec{V}}_{i-1L} - \Big(&\dot{\vec{\omega}}_{i-1L} \times \vec{r}_{i-i,\,i} + \vec{\omega}_{i-1L} \\ &\times \left(\vec{\omega}_{i-1L} \times \vec{r}_{i-i,\,i}\right)\Big) + \left(\dot{\vec{\omega}}_{iL} \times \vec{r}_{i,i} + \vec{\omega}_{iL} \times \left(\vec{\omega}_{iL} \times \vec{r}_{i,\,i}\right)\right) \\ &+ [\ddot{q}_i \cdot \vec{e}_i + 2\dot{q}_i \cdot \left(\vec{\omega}_{i-1} \times \vec{e}_i\right)] \cdot s_i \end{aligned} \tag{17}$$

In the same way we have the velocity and acceleration of the center of the modular joint.

$$\vec{\omega}_{iJ} = \vec{\omega}_{i-1L} + \dot{q}_i \cdot \vec{e}_i \cdot (1 - s_i) \tag{18}$$

$$\vec{\varepsilon}_{iJ} = \dot{\vec{\omega}}_{iJ} = \vec{\varepsilon}_{i-1L} + (\ddot{q}_i \cdot \vec{e}_i + \dot{q}_i \cdot \left(\vec{\omega}_{i-1L} \times \vec{e}_i\right)) \cdot \left(1 - s_i\right) \tag{19}$$

$$\vec{V}_{iJ} = \vec{V}_{i-1L} - \vec{\omega}_{i-1L} \times \vec{r}_{i-1,\,i} + \dot{q}_i \cdot \vec{e}_i \cdot s_i \tag{20}$$

$$\begin{aligned} \vec{W}_{iJ} = \vec{W}_{i-1L} &- \vec{\varepsilon}_{i-1L} \times \vec{r}_{i-1,\,i} - \vec{\omega}_{i-1L} \\ &\times \left(\vec{\omega}_{i-1L} \times \vec{r}_{i-1,\,i}\right) + (\ddot{q}_i \cdot \vec{e}_i + \vec{\omega}_{i-1L} \times \vec{e}_i \cdot \dot{q}_i) \cdot s_i \end{aligned} \tag{21}$$

So

$$W_i = [\beta_1^i \quad \ldots \quad \beta_i^i \quad 0 \quad \ldots \quad 0] \cdot \ddot{q} + [\delta^i] \tag{22}$$

$$\varepsilon_i = [\alpha_1^i \quad \ldots \quad \alpha_i^i \quad 0 \quad \ldots \quad 0] \cdot \ddot{q} + [\gamma^i] \tag{23}$$

β_j^i is $3\times\ 1$ vector coefficient of linear acceleration;

δ^i is the free item of linear accelertion;

α_j^i is 3×1 vector coefficient of angular acceleration;

γ^i is the free item of angular acceleration.

3.3 Representation of r

$\vec{r}_{i-1,\,i}$ is the position of the center of the $(i-1)$th link relative to the center of the i-th joint with reference to the fixed coordinate system; hence the vector $\vec{r}_{i-1,\,i} = A_{i-1} \cdot \vec{r}_{i-1,\,i}^{\,i-1}$. The connecting port between the joint k and the link k is k. $\vec{\rho}_k^{(k)}$ is the position relative to k. $\vec{r}_{iL}^{(k)}$ is the position of the center of the i-th link relative to port k with reference to the fixed coordinate system. $\vec{r}_{iJ}^{(k)}$ is the position of the center of the i-th joint relative to the port k with reference to the fixed coordinate system. Hence

$$\vec{r}_{k+iL}^{(k)} = A_k \cdot \vec{\rho}_k^{(k)} - \sum_{j=0}^{i-1} A_{k+j} \cdot \vec{r}_{k+j,\,k+1+j}^{(k+j)} + \sum_{l=0}^{i-1} A_{k+1+l} \cdot \vec{r}_{k+1+l,\,k+1+l}^{(k+1+l)} \tag{24}$$

$$\vec{r}_{k+iJ}^{(k)} = A_k \cdot \vec{\rho}_k^{(k)} - \sum_{j=0}^{i-1} A_{k+j} \cdot \vec{r}_{k+j,\,k+1+j}^{(k+j)} + \sum_{l=1}^{i-1} A_{k+l} \cdot \vec{r}_{k+l,\,k+l}^{(k+l)} \tag{25}$$

If

$$\vec{r}_i^{(j)} = \begin{bmatrix} r_{i1}^{(j)} & r_{i2}^{(j)} & r_{i3}^{(j)} \end{bmatrix},$$

assuming that

$$\underline{r_i^{(j)}} = \begin{bmatrix} 0 & -r_{i3}^{(j)} & r_{i2}^{(j)} \\ r_{i3}^{(j)} & 0 & -r_{i1}^{(j)} \\ -r_{i2}^{(j)} & r_{i1}^{(j)} & 0 \end{bmatrix} \tag{26}$$

3.4 Applications of the compensative and recursive method

From equations (14) to (23), we have compensative formulations of the modular link.

$$\varepsilon_i = [\alpha_1^i \ \ldots \ \alpha_i^i \ 0 \ \ldots \ 0] \cdot \ddot{q} + [\gamma^i]$$

$$\varepsilon_{i-1} = [\alpha_1^{i-1} \ \ldots \ \alpha_{i-1}^{i-1} \ 0 \ \ldots \ 0] \cdot \ddot{q} + [\gamma^{i-1}]$$

$$\begin{aligned} \vec{\varepsilon}_i = [&\alpha_1^{i-1} \ \ldots \ \alpha_{i-1}^{i-1} \ (1 - s_i) \cdot \vec{e}_i \quad 0 \ \ldots \ 0] \cdot \ddot{q} \\ &+ [\gamma^{i-1} + \dot{q}_i \cdot (\vec{\omega}_{i-1} \times \vec{e}_i) \cdot (1 - s_i)] \end{aligned}$$

$$W_i = [\beta_1^i \ \ldots \ \beta_i^i \ 0 \ \ldots \ 0] \cdot \ddot{q} + [\delta^i]$$

$$W_{i-1} = [\beta_1^{i-1} \ \ldots \ \beta_{i-1}^{i-1} \ 0 \ \ldots \ 0] \cdot \ddot{q} + [\delta^{i-1}]$$

So

$$\begin{cases} \alpha_j^i = \alpha_j^{i-1} \\ \alpha_i^i = (1 - s_i) \cdot \vec{e}_i \end{cases} j = 1, \ \ldots \ i - 1 \tag{27}$$

$$\gamma^i = \gamma^{i-1} + \dot{q}_i \cdot (\vec{\omega}_{i-1} \times \vec{e}_i) \cdot (1 - s_i) \tag{28}$$

$$\vec{\beta}_j^{\,i} = \vec{\beta}_j^{\,i-1} - \vec{\alpha}_j^{\,i-1} \times \vec{r}_{i-1,\,i} + \vec{\alpha}_j^{\,i} \times \vec{r}_{i,\,i} \tag{29}$$

$$\vec{\boldsymbol{\beta}}_i^{\,i} = \vec{\boldsymbol{\alpha}}_i^{\,i} \times \vec{\boldsymbol{r}}_{i,\,i} + \vec{\boldsymbol{e}}_i \cdot \boldsymbol{s}_i \tag{30}$$

$$\begin{aligned} \vec{\delta}^i = \vec{\delta}^{i-1} - \vec{\gamma}^{i-1} \times \vec{r}_{i-1,\,i} + \vec{\gamma}^i \times \vec{r}_{i,\,i} - \vec{\omega}_{i-1} \times (\vec{\omega}_{i-1} \\ \times \vec{r}_{i-1,\,i}) + \vec{\omega}_i \times (\vec{\omega}_i \times \vec{r}_{i,\,i}) + 2\vec{\omega}_{i-1} \times \vec{e}_i \cdot \dot{q}_i \cdot s_i \end{aligned} \tag{31}$$

We also have the compensative formulations of the modular joint as follows:

$$\begin{cases} \vec{\alpha}_j^{\,i} = \vec{\alpha}_j^{\,i-1} \\ \vec{\alpha}_i^{\,i} = (1 - s_i) \cdot \vec{e}_i \end{cases} \tag{32}$$

$$\vec{\gamma}^i = \vec{\gamma}^{i-1} + \dot{q}_i \cdot (\vec{\omega}_{i-1} \times \vec{e}_i) \cdot (1 - s_i) \tag{33}$$

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$$\vec{\delta}^i = \vec{\delta}^{i-1} - \vec{\gamma}^{i-1} \times \vec{r}_{i-1,i} - \vec{\omega}_{i-1} \times (\vec{\omega}_{i-1} \times \vec{r}_{i-1,i}) + \vec{\omega}_{i-1} \times \vec{e}_i \cdot s_i \cdot \dot{q}_i \quad (34)$$

$$\begin{cases} \vec{\beta}^i_j = \vec{\beta}^{i-1}_j - \vec{\alpha}^{i-1}_j \times \vec{r}_{i-1,i} \\ \vec{\beta}^i_i = \vec{e}_i \cdot s_i \end{cases} \quad (35)$$

3.5 Automatically generated dynamic equations

For a rotary joint Sk, it can be assumed that the port between the joint Sk and the link k is separated. Then we can consider the part of the robot from the joint Sk to the tip. The force $\vec{F}_{Rk}$ and the torque $\vec{M}_{Rk}$ are acting on the joint Sk. Then

$$\sum_{i=k}^{n} \vec{r}_i^{(k)} \times m_i \cdot \vec{W}_i + \sum_{i'=k}^{n-1} \vec{r}_{i'+1}^{(k)} \times m_{i'+1} \cdot \vec{W}_{i'+1}$$

$$+ \sum_{i=k}^{n} \vec{M}_i + \sum_{i'=k}^{n-1} \vec{M}_{i'+1}$$

$$= \vec{P}_k + \vec{M}_{Rk} + \sum_{i=k}^{n} \vec{r}_i^{(k)} \times m_i \vec{g}$$

$$+ \sum_{i'=k}^{n-1} \vec{r}_{i'+1}^{(k)} \times m_{i+1} \vec{g}$$

i is the order of the link, i′ is the order of the joint. After simplification, we have

$$\sum_{I=k}^{n} \vec{r}_I^{(k)} \times m_I \vec{W}_I + \sum_{I=k}^{n} \vec{M}_I = \vec{P}_k + \vec{M}_{Rk} + \sum_{I=k}^{n} \vec{r}_I^{(k)} \times m_I \vec{g}$$

$$(I = k_L, k+1_J, k+1_L, k+2_J, k+2_L, \ldots n_J, n_L) \quad (37)$$

Using compensative and recursive formulations, we have

$$A^* = \sum_{I=j}^{n} (\underline{r}_I^{(j)} \cdot m_I \cdot [\beta_1^I \ldots \beta_I^I 0 \ldots 0] + A_I \cdot \tilde{J}_I \cdot A_I^{-1} \cdot [\alpha_1^I \ldots \alpha_1^I 0 \ldots 0]) \quad (38)$$

$$B^* = \sum_{I=j}^{n} e_j^T \{\underline{r}_I^{(j)} \cdot m_I \cdot [\delta^I] - \underline{r}_I^{(j)} \cdot m_I \cdot \vec{g} + A_I \cdot \tilde{J}_I \cdot A_I^{-1} \cdot [\gamma^I] - A_I \cdot \left(\tilde{J}_I \cdot \vec{\tilde{\omega}}\right) \times \vec{\tilde{\omega}}_I\} \quad (39)$$

So we have

$$A^* \cdot \ddot{q} + B^* = P^M \quad (40)$$

For a translational joint Sk, we also have

$$\sum_{i=k}^{n} m_i \cdot \vec{W}_i + \sum_{i'=k}^{n-1} m_{i'+1} \cdot \vec{W}_{i'+1} = \vec{P}_k + \sum_{i=k}^{n} m_i \vec{g}$$

$$+ \sum_{i'=k}^{n-1} m_{i'+1} \vec{g} + \vec{F}_{Rk}$$

namely:

$$\sum_{I=k}^{n} m_I \cdot \vec{W}_I = \vec{P}_k + \sum_{I=k}^{n} m_I \vec{g} + \vec{F}_{Rk} \quad (41)$$

$$A^* = \sum_{I=j}^{n} e_j^T \cdot m_I \cdot [\beta_1^I \ldots \beta_I^I 0 \ldots 0] \quad (42)$$

$$B^* = \sum_{I=j}^{n} e_j^T \cdot m_I \cdot [\delta^I] - e_j^T \cdot m_I \cdot \vec{g} \quad (43)$$

So we have

$$A^* \cdot \ddot{q} + B^* = P^F \quad (44)$$

According to equations (40 and (44), we have:

$$A^* \cdot \ddot{q} + B^* = P$$

When the kinematic geometry of MR is known, the link module and the joint module can be determined. According to the above mathematical description (section 2.2), the relatively dynamic parameters (such as $\vec{r}_{ii}$, $\vec{\rho}$, J, $\vec{e}$ . . .) may be known. Then A*, B* can be automatically and iteratively computed by the compensative and recursive method. Hence dynamic equations can be generated automatically by the above method.

Since the method is recursive, it can be easily realized on the computer. The procedure to obtain an automatic generation of dynamics can be summarized as follows:

Input: the structure of MR, and the joint angles
Compute: link module parameters
Choose: 1) rotary joint or 2) translation joint
compute: joint module parameters
Compute: A, B**
Output: P.

We can set up a coordinate system of a 2-DOF MR (Figure 10). By the above compensative and recursive method we can get some equations about this MR:

$$A_{1J} = R(e^{\hat{\xi}_1 \cdot \theta_1} \cdot T_{J1}(0)) = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_{1L} = R(e^{\hat{\xi}_1 \cdot \theta_1} \cdot T_{J1}(0) \cdot T_{L1C}(0)) = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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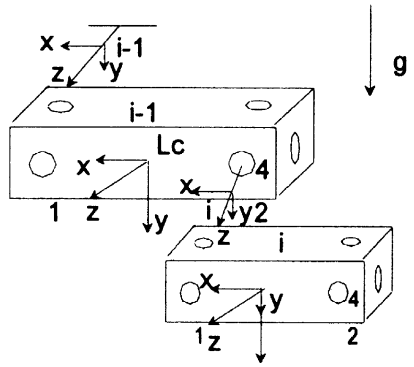


Fig. 10. Setting up a coordinate system of a 2-DOF MR

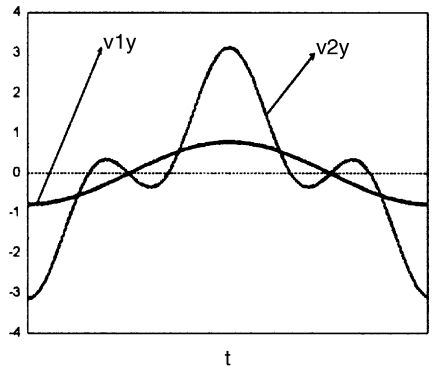


Fig. 11. The velocity of the center of each link: v1y, v2y.

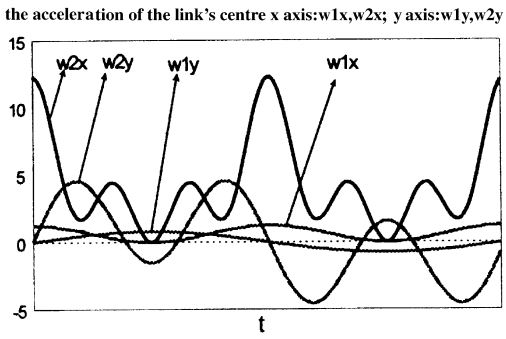


Fig. 12. The acceleration of the center of each link.

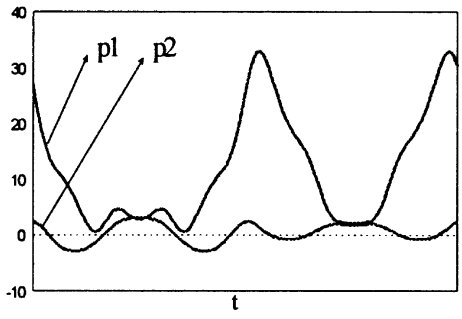


Fig. 13. The moment of forces of joints.

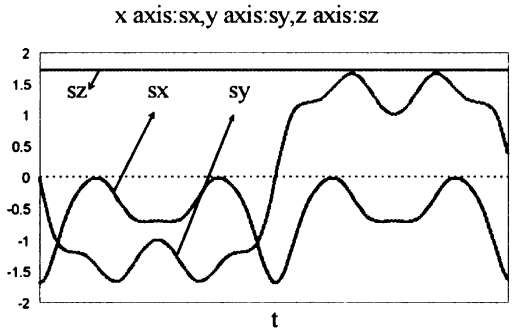


Fig. 14. The trajectory of links end points.

$$A_{2J}=\begin{bmatrix} c\theta_{12} & -s\theta_{12} & 0 \\ s\theta_{12} & c\theta_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_{2L}=A_{2J}\cdot T_{L2C}(0)=\begin{bmatrix} c\theta_{12} & -s\theta_{12} & 0 \\ s\theta_{12} & c\theta_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underline{r_{1L}^{(1)}}=\begin{pmatrix} 0 & -\frac{w_1}{2} & -\frac{l_1}{2}\cdot s\theta_1 \\ \frac{w_1}{2} & 0 & \frac{l_1}{2}\cdot c\theta_1 \\ \frac{l_1}{2}\cdot s\theta_1 & -\frac{l_1}{2}\cdot c\theta_1 & 0 \end{pmatrix}$$

$$\underline{r_{2J}^{(1)}}=\begin{pmatrix} 0 & -\left(w_1+\frac{h_2}{2}\right) & -l_1\cdot s\theta_1 \\ w_1+\frac{h_2}{2} & 0 & l_1\cdot c\theta_1 \\ l_1\cdot s\theta_1 & -l_1\cdot c\theta_1 & 0 \end{pmatrix}$$

$$\underline{r_{2L}^{(1)}}=\begin{pmatrix} 0 & -\left(w_1+h_2+\frac{w_2}{2}\right) & -l_1\cdot s\theta_1-\frac{l_2}{2}\cdot s\theta_{12} \\ w_1+h_2+\frac{w_2}{2} & 0 & l_1\cdot c\theta_1+\frac{l_2}{2}\cdot c\theta_{12} \\ l_1\cdot s\theta_1+\frac{l_2}{2}\cdot s\theta_{12} & -l_1\cdot c\theta_1-\frac{l_2}{2}\cdot c\theta_{12} & 0 \end{pmatrix}$$

$$\underline{r_{2L}^{(2)}}=\begin{pmatrix} 0 & -\frac{w_2}{2} & -\frac{l_2}{2}\cdot s\theta_{12} \\ \frac{w_2}{2} & 0 & \frac{l_2}{2}\cdot c\theta_{12} \\ \frac{l_2}{2}\cdot s\theta_{12} & -\frac{l_2}{2}\cdot c\theta_{12} & 0 \end{pmatrix}$$

$$\beta_1^{1L}=\left(l_1\cdot s\theta_{1/2} \quad -l_1\cdot c\theta_{1/2} \quad 0\right)^T$$

$$\beta_1^{2J}=(l_1\cdot s\theta_1 \quad -l_1\cdot c\theta_1 \quad 0)^T$$

$$\beta_2^{2J}=0$$

$$\beta_1^{2L}=\left(l_1\cdot s\theta_1+\frac{l_2}{2}\cdot s\theta_{12} \quad -l_1\cdot c\theta_1-\frac{l_2}{2}\cdot c\theta_{12} \quad 0\right)^T$$

$$\beta_2^{2L}=\left(\frac{l_2}{2}\cdot s\theta_{12} \quad -\frac{l_2}{2}\cdot c\theta_{12} \quad 0\right)^T$$

$$\delta^{1L}=\left(\frac{l_1}{2}\cdot c\theta_1\cdot\dot{\theta}_1^2 \quad \frac{l_1}{2}\cdot s\theta_1\cdot\dot{\theta}_1^2 \quad 0\right)^T$$

$$\delta^{2J}=\left(l_1\cdot c\theta_1\cdot\dot{\theta}_1^2 \quad l_1\cdot s\theta_1\cdot\dot{\theta}_1^2 \quad 0\right)^T$$

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$$\delta^{2L} = \begin{pmatrix} l_1 \cdot c\theta_1 \cdot \dot{\theta}_1^2 + \frac{l_2}{2} \cdot c\theta_{12} \cdot (\dot{\theta}_1 + \dot{\theta}_2)^2 \\ l_1 \cdot s\theta_1 \cdot \dot{\theta}_1^2 + \frac{l_2}{2} \cdot s\theta_{12} \cdot (\dot{\theta}_1 + \dot{\theta}_2)^2 \\ 0 \end{pmatrix}$$

$$(\alpha_1^1) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \ (\alpha_1^{2J}, \alpha_2^{2J}) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{pmatrix},$$

$$(\alpha_1^{2L}, \alpha_2^{2L}) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{pmatrix}$$

$$\left(\gamma^{1L}\right) = 0, \left(\gamma^{2J}\right) = 0, \left(\gamma^{2L}\right) = 0$$

so we can get P_1^M, P_2^M:

$$P_1^M = \sum_{I=1}^{2} (\vec{r}_1^{(1)} \times m_1 \cdot \vec{W}_1 - \vec{r}_1^{(1)} \times m_1 \vec{g}) \cdot \vec{e}_1 + \sum_{I=1}^{2} \vec{M}_1 \cdot \vec{e}_1$$

$$P_2^M = \sum_{I=2}^{2} (\vec{r}_1^{(2)} \times m_1 \cdot \vec{W}_1 - \vec{r}_1^{(2)} \times m_1 \vec{g}) \cdot \vec{e}_2 + \sum_{I=2}^{2} \vec{M}_1 \cdot \vec{e}_2$$

Assuming $\theta_1 = \frac{\pi}{2} \cdot \sin(t)$, $\theta_2 = \frac{\pi}{2} \cdot \sin(3 \cdot t)$, and that the two angles are changed in a certain range, we obtain some figures: Figure 11 shows the velocity of the center of each link; Figure 12 shows the acceleration of the center of each link; Figure 13 shows the moment of force of each joint, Figure 14 shows the trajectory of link end point. From Figure 14 we can see that sz is a constant, i.e. the MR moves on a plane.

According to the above analysis and the procedure of the compensative and recursive method, we can automatically generate dynamics and obtain some dynamic figures. Figure 15 shows the accelerations of the centers of links of a 3-DOF MR (m1 = 3, m2 = 1, m3 = 7)in x, y, z directions.

4. CONCLUSIONS

There are several methods to formulate the dynamics of an ordinary robot. However, for a modular robot it has its specific features. In this paper we analyze the abstract structure of the modular reconfigurable robot by the group theory. According to modular ideas, we can automatically generate dynamic expressions of each module by the compensative and recursive method. Various codes have been developed under the *Windows* environment for the dynamic simulation. When a modular robot is considered, we can generate its dynamic model automatically and quickly with this method. Some simple examples of 2-DOF, 3-DOF MR indicate that the above mentioned method is feasible.

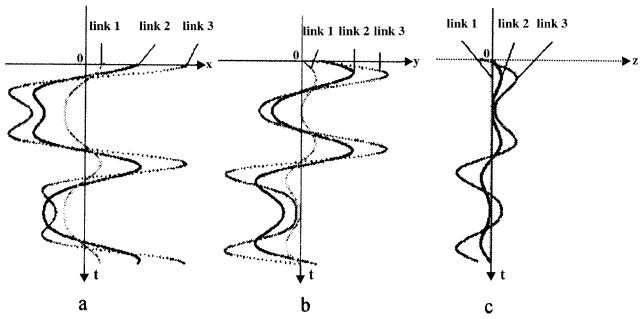


Fig. 15. The accelerations of the centers of links of the 3-DOF MR.

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