Science in Context **20**(4), 571–600 (2007). Copyright © Cambridge University Press
doi:10.1017/S0269889707001457 Printed in the United Kingdom

Force, Mathematics, and Physics in Newton's *Principia*: A New Approach to Enduring Issues

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Argument

This paper investigates the conceptual treatment and mathematical modeling of force in Newton's *Principia*. It argues that, contrary to currently dominant views, Newton's concept of force is best understood as a physico-mathematical construct with theoretical underpinnings rather than a "mathematical construct" or an ontologically "neutral" concept. It uses various philosophical and historical frameworks to clarify interdisciplinary issues in the history of science and draws upon the distinction between axiomatic systems in mathematics and physics, as well as discovery patterns in science. It also dwells on Newton's "philosophy" of mathematics, described here in terms of mathematical naturalism. This philosophy considers mathematical quantities to be physically significant quantities whose motions are best mapped by geometry. It then shows that to understand the epistemic status of force in the *Principia*, it is important to scrutinize both Newton's mathematical justificatory strategies and his background assumptions about force – without constructing, however, an overarching metaphysical framework for his science. Finally, the paper studies scientific attempts to redefine or eliminate force from science during the period between Newton and Laplace. From a philosophical standpoint, the paper implicitly suggests that questions about the reality of force be distinguished from questions about the validity of force, and that both sets of questions be distinguished from questions about the utility of the concept of force in science.

> Thus the historian seems to find in Newton himself firm injunctions to heed only his positive achievements in science and to disregard any "hypotheses" of a secondary order upon which they might appear to depend; yet the same historian might be uncomfortably aware that such positive achievements of Newton remain incomplete and intellectually indigestible if they are taken in isolation, divorced from his deeper views about the structure of things and about the causes of those phenomena which are susceptible of positive discussion. Moreover, as the historian extends his knowledge of the Newtonian *oeuvre*, he finds that Newton himself was so far incapable of remaining on the plane of icy positivism to which he found it convenient to retreat from time to time, that he openly showed how there were multiple leads that might be pursued at the fascinating level of hypothesis. (Hall and Hall 1970, 55-6)

https://doi.org/10.1017/S0269889707001457 Published online by Cambridge University Press

Introduction

What is the epistemic status of force in Newton's *Principia*? There is profound disagreement among scholars on this issue. Two conflicting interpretations that we may, roughly speaking, call mathematical reconstructivism and holistic finalism seem to dominate the debate. In this paper, I primarily, but not exclusively, associate mathematical reconstructivism with I. B. Cohen and holistic finalism with Betty Jo Dobbs. Mathematical reconstructivism holds that Newton invented a scientific style consisting of treating force mathematically in a way that weaned the concept from physical and ontological assumptions, particularly in Book I of the *Principia*. According to this view, force is a "mathematical construct" rather than a physico-mathematical construct with theoretical underpinnings.

Mathematical reconstructivism has already been revised in late studies of Book II showing physical assumptions in Newton's concepts and justificatory strategies (Smith 2001 and 2002a). In recent years, however, a new interpretation, derived in part from mathematical reconstructivism, has arisen which argues that force is ontologically a neutral concept in the *Principia* and that Newton established a "fairly rigid demarcation" between his various fields of research. Proponents of this interpretation urge that the history of these different fields be reconstructed, each within its own rigid boundaries, as if they had never been tangential or overlapping at any point in Newton's work (Iliffe 2004). Let's call this new trend asymptotic reconstructivism.

Asymptotic reconstructivism is primarily a reaction against holistic finalism, which holds that in his inquiries in multiple areas Newton was motivated by a belief in an ultimate secret about the "divine principles beyond the veil of nature" and "received revelation" (Dobbs 1991, 7). According to this view, all of Newton's studies converge toward the discovery of one single and final truth, "the unified wisdom of Creation" (McGuire and Rattansi 1966, 138). What is interesting is that while Cohen, *mutatis mutandis*, attempted to reconcile mathematical reconstructivism with holistic narratives by conceding that non-scientific disciplines played a key role in motivating Newton to undertake a scientific study of gravity as an attractive force (Newton [1687] 1999, 58, 60), asymptotic reconstructivists contend that Newton created an "unbridgeable gap" between diverse concepts deriving from his intellectual pursuits in "incompatible fields" (Iliffe 2004).

Throughout this paper, I maintain that holistic finalists overstate their case. We know that what prompted the project of the *Principia* was in part an attempt to answer Halley's mathematical question about force. This by itself makes it doubtful that Newton's motivation during the years 1684–87 reflected an overarching motivational goal of a theological nature. But I also maintain that it is useful to take Cohen's concession as a starting point rather than as a limiting case. Accordingly, I show that the claims of the asymptotic reconstructivists are untenable. First, I focus on the "interdisciplinary" issue in order to argue that the irreducibility of scientific disciplines is not synonymous with field insularity. Second, dwelling on the conceptualization and the mathematical

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modeling of force in the *Principia*, I demonstrate that Newton's later philosophy of mathematics does not seem to support the current understanding of Newton's concept of force as a mathematical construct. Third, I place the "*Principia*-type forces" in the broader context of Newton's "culture of force" and discuss Newton's mathematical justificatory strategies in Book I. Fourth, I suggest that the reception of the *Principia* up until Laplace seems to indicate that the contention about the neutrality of force may be relatively more appropriate to the eighteenth-century reworking of Newton's science than to the science in the *Principia*.

All in all, my thesis is that the concept of force in the *Principia* is best understood as a physico-mathematical construct embodying epistemic and ontological assumptions. In this regard, Newton's later "philosophy" of mathematics and the development of science between Newton and Laplace are, perhaps, evidentiary blind-spots that threaten the viability of mathematical reconstructivism and its derived research programs. These types of counterevidence cannot be parsimoniously explained in their frameworks. But my goal is less to provide a conclusive refutation of the current views than to furnish epistemic reasons that can operate as rational exit signals to indicate which orientations one can abandon in this debate without great loss. Methodologically speaking, I suggest a critical naturalistic approach to Newtonian mechanics. At a time when positivism has been beaten to death and when social construction reads more and more like social distraction, this approach has the advantage of grounding scientific methods and aims in actual scientific practice and writings without sacrificing anything from or to the logical structure of theories.

I. In Newton's Eyes

There is a long tradition of interpretation dating back at least to Auguste Comte, Ernst Mach, and the Vienna Circle according to which Newton set the stage for positive sciences by forfeiting a metaphysical inquiry into the cause of gravity in order to focus on laws that phenomenally interconnect natural events. But when Newton's manuscripts emerged in the 1940s, some scholars began to challenge this interpretation by arguing that disciplines such as *prisca sapientia*, "Neo-Platonist" metaphysics, theology and alchemy, hereafter referred to as "non-canonic disciplines," influenced the mechanics and the cosmology of the *Principia* (McGuire and Rattansi 1966; Westfall 1971 and 1980a; Dobbs 1975 and 1991; Manuel 1968 and 1994; Force 2000). Critics counterattacked that Newton operated with "disciplinary compartmentalization" and created a sharp dichotomy between his "private" metaphysical views and his "public" scientific ideas. At the heart of this objection against "holistic" interpretations is a legitimate concern about the necessary disciplinary division that guarantees "discipline-specific discourses and epistemological demands" (Iliffe 2004, 429–30).

However, holistic finalists are not reductionists. They do not argue, for example, that, for Newton, higher-level questions (in this instance metaphysical ones) are reducible to

https://doi.org/10.1017/S0269889707001457 Published online by Cambridge University Press

low-level mathematical and experimental questions, as scientific reductionists usually contend. Nor do they suggest that, for Newton, low-level physical problems can be simply assimilated to higher-level metaphysical problems. What they maintain is that it is possible to subsume Newton's various disciplinary-specific goals under an allegedly broader metaphysical goal. Some also believe that Newton's scientific methods can be subsumed under a broader "avenue to Truth" wrought out by Newton (Dobbs 1991, 7). Furthermore, it is not only holistic finalists who linked non-canonic disciplines to Newton's science. Richard Westfall, for one, argued against holistic finalism that Newton used non-canonic disciplines to create a secular science in which our own secular world is rooted (Westfall 1980a and 2000).

From the perspective taken in this paper then, claims about the domain of autonomy of Newton's various disciplinary undertakings are unproblematic. Take for example contemporary biology. Two molecular geneticists working, say, one on prokaryotes and the other on eukaryotes or, to put it even more simply, one on Mendelian disorders and the other on complex diseases, both face different epistemic requirements. Nonetheless, this does not entail a "compartmentalization" of these areas of research. Evolutionary biology and psychology are separate disciplines, but a breakthrough in one can guide research in the other. Because interdisciplinarity and the irreducibility of the sciences are not opposing epistemic notions, the most relevant issue at stake in the debate over the relationship between Newton's multiple areas of research is, to use modern terminology, the issue of "interdisciplinary" insight and the transfer of conceptual and problem-solving tools.

Nonetheless, holistic finalism contains a potential danger. It risks overestimating motivational factors or goals that, in a specific context, may have been rather secondary. Take for example the writing of *De Motu* and its dramatically expanded version, the *Principia*. Events starting from Edmond Halley's visit in 1684 and culminating in the priority controversy over the inverse square law with Robert Hooke seem to indicate that Newton's more pressing goals in elaborating his sophisticated "rational mechanics" at the close of the seventeenth century may have been just ordinary, i.e., scientific and perhaps polemical. Halley's question about the trajectory described by a body subjected to an inverse square law was the major driving issue behind *De Motu*.[1] In the preface

[1] Since Johann Bernoulli there has been a long controversy over the so-called "inverse problem" (see Lohne 1960). To make a long story short, Halley's mathematical inquiry seems to have been about the trajectory described by a body under the $1/r^2$ law. Newton's reported spontaneous and verbal answer to Halley was that the trajectory is an ellipse. But *De Motu*, the draft or the version of it that was sent to Halley three months later, and the *Principia* did not derive the trajectory from the law of force, but assumed the trajectory to be indeed a conic section and derived the law of force from the trajectory instead. About two decades later, Johann Bernoulli criticized Newton for bypassing the mathematically more difficult problem, the inverse problem of central forces, and solved it by use of the differential equation. It was thus Johann Bernoulli who first solved the direct problem apparently asked by Halley. For the purpose of this paper, however, it is enough to realize that the historical issue at the origin of the writing of the *Principia* is a mathematical quest, whether about the trajectory or the law of force of planetary motion.

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to the first edition of the *Principia*, Newton himself acknowledged his debt to Halley's initiative. But the structure of the *Principia* itself indicates that it is in the main an answer to Halley's inquiry about orbital motion following the physico-mathematical pathways of Galileo and Huygens. Although constructed within an anti-Cartesian mechanistic mindset, the *Principia* is an obviously direct dialogue neither with Henry More and the Cambridge Platonists nor with Michael Maier and the alchemists. That is, there are multiple motivational factors and the theological explanation, by itself, is insufficient and misleading. But do the weaknesses of holistic finalism justify asymptotic reconstructivism?

Whatever the dominant factors in the elaboration of his rational mechanics between 1684 and 1687 might have been, when asked about its theological implications, Newton unequivocally responded that while writing the *Principia* he "had an eye upon such principles as might work with considering men for the belief in a Deity [sic]" (Newton 1961, 233). In his letters to Bentley, Newton emphatically maintained that the motion of the planets and the harmony in the system of the world could not derive from a simple natural cause or from chance, but required a "Counsel" or an agent "very well skilled in mechanics and geometry." In brief, "divine power," Newton asserted, is the true explanation for natural phenomena.[2] He even alleged that, in addition to the specific points of the *Principia* Bentley referred to in the first letter, he himself had a stronger proof for the existence of God. Newton, however, argued that until the principles on which that proof rested were "better received," he found it "more advisable to let it sleep" (ibid., 236).

It is reasonable to raise a red flag with respect to these enthusiastic Newtonian speculations about the metaphysical implications of the *Principia*. The skeptic is bound to object that they were written for a clergyman who, eager himself to debunk atheism, pressured Newton for ammunition. Yet, earlier in *De Gravitatione*, a treatise related to the *Principia* (Dobbs 1991, 138–46; Newton 1999, 47, 58),[3] Newton expressed fierce criticism of Cartesian physics and maintained that our conception of body is not neutral vis-à-vis metaphysical issues, but is relevant to the problem of the existence of God: "For we cannot postulate bodies of this kind without at the same time supposing that God exists. . . . If we say with Descartes that extension is body, do we not manifestly offer a path to atheism?" (Newton [1962] 1978, 142–3). According to Newton, there is no better case for atheism than the notion of bodies conceived as autonomous realities. It is an important goal of the treatise to counter the alleged atheism implicit in Descartes' physics by arguing that, whether viewed from the perspective of *potentia*

[2] See Newton's three successive letters to Bentley on December 10, 1692; January 17, 1693; and February 11, 1693.

[3] The Halls, who first published *De Gravitatione*, were uncertain about its date, but suggested that it might reflect the metaphysical thought of young Newton. Howard Stein on his part cautions about the Halls' dating while questioning Dobbs' (Stein 2002, notes 27 and 39). Cohen, for one, seems to side with Dobbs on this chronological issue. For a new attempt to date this manuscript, see Feingold 2004.

https://doi.org/10.1017/S0269889707001457 Published online by Cambridge University Press

Dei ordinata or *absoluta*, matter and its motion depend on God. Also, in MS 3965, just as in the letters to Bentley and in the *Principia*, Newton linked the regularity of mathematical laws of motion to the omnipresence of an absolute God. In a debate with Thomas Burnet over the validity of the creationist story in *Genesis*, Newton, instead of rigidly confining himself to theological arguments, advanced scientific arguments to support the biblical account (Westfall 2000, 50; Force 2004). But it is in the famous General Scholium of the *Principia* that we learn that "to treat of God from phenomena is certainly a part of natural philosophy" (Newton 1999).[4]

Similar stories can be told about various other aspects of Newton's scientific achievements and his private metaphysical views. For instance, in *De Gravitatione*, Newton maintained that space and time are effects emanating from an eternal and immutable being and that as such, space and time are eternal and immutable (Newton [1962] 1978, 136). In the Definitions of the *Principia*, Newton seems to be as much metaphysician as mathematician and experimentalist with respect to space and time. But it is the 2nd edition of the *Principia* that explained that attributes such as "eternity" and "infinity" have God as referent, and by enduring always and being everywhere God "constitutes duration and space" (Newton 1999, 941), while Query 31 of the *Optics* simply referred to space as a *Sensorium Dei*.

Three conclusions can be drawn from the above considerations. First, Newton himself many times hinted at the fact that at least some of his scientific concepts had metaphysical underpinnings. Second, his "private" and "public" documents sometimes converged. Third, from an evidential standpoint, there is no indication that Newton built a sort of theoretical Berlin Wall between his various areas of interest. Textual evidence seems rather to suggest that while solving the equations of motion Newton had in fact an "eye," but only "one" and not "two," on issues that for us do not fall under rational mechanics. Clearly, while the case for strong holism (holistic finalism) seems weak and easy to rebut, weak holism seems strong and hard to refute.

Newton's epistemic behavior does not appear to lack philosophical justification. According to Phillip Kitcher, "the aim of the sciences is to address the issues that are significant for people at a particular stage in the evolution of human culture." For Kitcher, the goal of scientific inquiry is not simply to find truths but to find "significant truths." Drawing on the cloning of Dolly, the Scottish sheep, he argues for a "significance graph" made of various lines and levels of significance deriving from "healthy natural curiosity." Scientists, like cartographers, Kitcher maintains, depict reality according to our evolving capacities and interests (Kitcher 2001, 59, 63–82).

There is no need to accept Kitcher's view as a whole, but his notion of a "significance graph" is useful for our discussion. Just as for genomics in our time, in the seventeenth century the resolution of mathematical equations pertaining to the system of the world was embedded in a web of various lines of significance. If it is no sign of intellectual

[4] Notice that one can fairly articulate a reductionist thesis on these grounds.

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disorder or psychological melodrama on the part of a contemporary genomic scientist to have an eye, for example, on the ethical, legal, and social implications (ELSI) of his or her research, Newton's interest in physics, mathematics, and non-canonic disciplines is a sign of nothing at all, except perhaps of a seventeenth-century "healthy natural curiosity." The "significance graph" allows us to recognize multiple motivational factors distributed along social, scientific, and metaphysical lines of significance. In the seventeenth century, however, scientific questions very often had philosophical and theological implications (Funkenstein 1986; Dobbs 1991; Manuel 1968 and 1994; Force 2000; McGuire 2000). Indeed, for Newton, or more exactly for Clarke, "natural and supernatural are nothing at all different with regard to God, but distinctions merely in our conception of things." Causing the sun to move regularly or stopping it for a day are both effects of the same "Power." The divisions into natural and supernatural – and consequently into secular and religious – are not essential divisions in things themselves (Leibniz 1965, vol. 4, 362). We cannot, however, take this to mean that Newton's world is dominated by religion or is totally secular. Perhaps we should simply accept that Newton provides no remedy to pacify or exorcise the "metaphysical" phantoms that agitate our own secular sleep in this post-positivist era.

Actually, at the time of its publication, the *Principia* had already galvanized scholars with at least four different points of view: a) those who saw in it only the birth of a rational mechanics or an extraordinary potential to develop such a mechanics in a way that definitively set the stage for the modern scientific approach; b) those like Leibniz who perceived in it excellent mathematical achievements unfortunately coupled with a retreat from reason due to allegedly uncritical metaphysical assumptions (Newton 1961, vol. 3, 258; Leibniz 1965, vol. 7, 417; Leibniz 1973); c) those like William Law for whom Newton's perspective was perfectly consistent with mystical traditions (Dobbs 1975, 9–10); and d) those like Newton himself who, while defending (a), claimed also a connection between gravitational (as well as heliocentric) theory and *prisca sapientia* (Newton 1744, vol. 1; Newton 1961, 384–9). Newton devoted time and energy to the so-called "classical scholia" with the intention of inserting them in the second edition of the *Principia* (Casini 1984). The scholia were "publicly" announced in David Gregory's *Memorandum* in 1694 and published in his *Astronomia* in 1702.

But the manuscript *De Gravitatione*, mentioned earlier, has its own fascinating story. This document was a "private" document par excellence not only because Newton did not publish it but also because, at Newton's death, Thomas Pellet marked it "not fit to be printed" along with his other manuscripts. Interestingly, the metaphysics-based theory of matter that Newton opposed to Descartes' in this "private" document had already been briefly presented "publicly" by Pierre Coste in his French translation of John Locke's *Essay* after a visit to Newton (Locke [1689] 1735; De Gandt 1985, 108). Furthermore, Mme du Châtelet, the famous French translator of Newton's *Principia*, also discussed Newton's theory of matter in 1740 in her *Institutions de physique*. The text of her discussion was reproduced *verbatim* in one of the most publicized works in the eighteenth century, the *Encyclopédie* of Denis Diderot and Jean d'Alembert (see

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Maglo, forthcoming). In fact, glimpses of Newton's "private" views about gravity and its cause, space, body, etc., can be found in his own public writings, as well as his correspondence, the published works of his disciples and public accounts by friends and visitors of conversations with him.

This signals how we may rationally exit this debate even though we cannot pretend to see through Newton's eyes. The creation of a radical dichotomy between "private" and "public" in the Newtonian studies is not always illuminating. Newton's theory of fluxions, until the priority dispute with Leibniz, remained a "private" mathematics, discussed only with a relatively few people, just as alchemy was discussed only with selected people such as Boyle.[5] In the *Principia*, this "private" mathematics appeared in a very limited number of pages as a problem-solving tool, just as alchemical concepts appeared in Hypothesis 3 of the first edition of the *Principia*. Furthermore, Newton justified the predominance of geometry or synthetic methods in the *Principia* by appealing to the method of the ancients who, he claimed, investigated truths analytically, but presented them geometrically (Newton 1967–1981, vol. 8, 453–5; Newton [1687] 1999, 49–54). If Newton's belief in *prisca sapientia* influenced his mathematical choices in the *Principia*, then it is useful to see Newton as a rational and pragmatic scientist capable of cleverly and efficiently transferring, when need be, problem-solving tools from one field to another and of reworking them according to the internal epistemic constraints of each field.

To be sure, it is the emergence of Newton's manuscripts in the twentieth century that gave momentum to a deeper interdisciplinary scrutiny of the theoretical assumptions beneath his scientific concepts. It is in this context that Newton's insight into alchemy,[6] for example, has been evoked as a powerful tool that allowed him to overcome the limits of Cartesian mechanistic philosophy. But even today scholars remain divided about whether Newton's alchemy itself should be reconstructed in an interdisciplinary framework as exhibiting metaphysical, experimental, and quantitative features or whether it should be squarely defined as an experimental practice only (Figala 2002; Newman 2002; Principe 2000 and 2004).[7] Actually, reactions to Newton's alchemy were from the beginning strongly negative in part because alchemy was perceived as a

[5] Although a manuscript on the theory of fluxions did reach Leibniz, Newton's research in this area did not circulate widely nor was it published. The point I am making is that arguments based on the fact that Newton did not publish his non-canonic research are at best speculative.

[6] My aim in the next two pages is not to privilege alchemy over the other non-canonic disciplines but to treat it simply as a case study to make a point. The discussion sheds light also on the problematic character of reductionist approaches to Newton's non-canonic scrutiny.

[7] In fact, after the nineteenth and early twentieth centuries' biased reduction of alchemy to a spiritual and psychological endeavor and following the recent philosophical reevaluation of the role of experiment in science, there seems to be a temptation to promote laboratory practice as the defining moment of Newton's alchemy and to downplay its metaphysical components. If this new trend triumphs, it might help some to accept Newton's alchemy and, perhaps, its contribution to his science. However, the problem with the new trend is that as no unbalanced approach in human intellectual history has ever brought about a stable comprehensive view, the backlash is easy to predict.

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mysticism-driven discipline with the potential to undermine the belief that Newton's science was based on mechanistic philosophy. Opponents of alchemy first tried to link Newton to Boyle, second to claim for Newton the title of a mechanistic philosopher "in the spirit of his age," and third to argue that Newton stripped his key scientific concepts such as attraction of any alchemical connotations (Hall and Hall 1958 and 1962; Hall 1958). But by the early 1990s, the opposition to Newton's alchemy partly collapsed when Rupert Hall open-mindedly declared that it was a mistake to contend, as his wife and he himself did 30 years earlier, that "Newton was not in any admissible sense of the word an alchemist" (Hall 1992, 199–200, 382).

Students of Newton's alchemy used various, sometimes conflicting, strategies in the controversy. Westfall seems to have studied Newton's manuscripts in light of the conflict mounted by Newton over the Cartesian mechanistic philosophy of nature (Westfall 1971, 1972, 1980a, 1980b, and 2000). By putting Newton's experimental papers in perspective, Dobbs suggested in her earlier work that not only was Newton truly an alchemist but also that Newton was after the philosopher's stone (Dobbs 1975, 145–6).[8] While Spargo (1993) and Newman (1994 and 2002) carry on the experimental line of research, Frank Manuel and Dobbs (in her later work) reconstructed a religious framework for the study of Newton's alchemy (Dobbs 1991 and [1994] 2000; Manuel 1963, 1968, and 1994). Hunter (1990 and 1994) and Principle (1998 and 2004) showed in the meantime that even Boyle was more anchored in alchemy than previously thought. Newman and Principle (1998 and 2001) showed that there is more to alchemy than mysticism. Newman (1994 and 2002) explored in particular the link between George Starkey and Newton. For Figala (1977, 1984, and 1992), the manuscripts suggest that they were informed by Newton's mathematical rationalism. But she still argues that Starkey as well as Michael Maier influenced Newton to the extent that "alchemy cannot be seen solely in connection with his chemical experiments only but was also a link between his religious beliefs and his scientific aims" (Figala 2002, 375).

At the turn of the twenty-first century, a second sign of the collapse of the resistance to Newton's alchemy came from another key player. In the "Guide" to the new English translation of Newton's *Principia*, Cohen open-mindedly wrote: "There can no longer be any doubt that Newton's thinking in the area of mechanics or cosmic physics was closely related to his very deep concern with alchemy and the structure and interactions of matter." After decades of resisting alchemy, Cohen bluntly stated that perhaps without alchemy Newton might not have developed gravitational theory as a science of force (Newton [1687] 1999, 58, 62). This restatement of what we may call Newton's "interdisciplinary" insight enables us to tackle the fundamental question about how Newton conceptualized and mathematically treated force in the *Principia*.

[8] Not everyone agrees, however, on this characterization of Newton's alchemical goals (Hall 1992; Spargo 1993).

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II. Newton's Later "Philosophy" of Mathematics: Evidentiary Blind-Spot #1

> Therefore, let the reader beware of thinking that by words of this kind [attraction and impulse] I am anywhere defining a species or mode of action or a physical cause or reason, or that I am attributing forces in a true and physical sense to centers (which are mathematical points) if I happen to say that centers attract or that centers have forces. (Newton 1999, 408)

I shall call this Newton's methodological disclaimer. This disclaimer occurred twice in Definition 8 alone and also in the introduction and the scholium of Section 11 of Book I. But Corollary 7 of Proposition 4 and the scholium of Section 5 of Book II, for example, distinguish between questions for physics and questions for mathematics. Moreover, the preface to the first edition of the *Principia* explains that Books I and II aim at establishing the mathematical propositions underlying the physics of Book III. Newton seems *prima facie* to argue that his approach in the *Principia* is that of physical inference from what we may call physics-blind mathematics or, more precisely, physics-blind geometry. But equally, the preface explains that rational mechanics as set forth in the *Principia* is a research program of the ancients amended on two key points. The opening sentence of the *Principia* is actually very telling and reads as follows:

> SINCE THE ANCIENTS (according to Pappus) considered *mechanics* to be of the greatest importance in the investigation of nature and science and since the moderns – rejecting substantial forms and occult qualities – have undertaken to reduce the phenomena of nature to mathematical laws, it has seemed best in this treatise to concentrate on *mathematics* as it relates to natural philosophy. (Newton [1687] 1999, 381)

The ancients, Newton maintains, studied rational mechanics by focusing on "manual powers" or "the *five powers*" but for the most part neglected natural powers. Newton therefore defines his goal in the *Principia* as an investigation of "natural rather than manual powers."

Thus from the research program of the "ancients," Newton seems to adopt the conceptual basis of his science as a science of force, arguing that he suspects that "all phenomena may depend on certain forces by which particles of bodies" relate to each other and that the whole business of science amounts to the discovery of "the forces of nature from the phenomena of motions and then to demonstrate the other phenomena from these forces" (ibid., 382). But because the exact nature of these forces are unknown and because it has led many astray in the past, Newton appeals to the approach of the moderns, which consists of reducing natural phenomena to mathematical laws. Clearly, the *Principia* unsealed the contours of Newton's project, the inquiry about the mathematical laws of the forces underlying the motion of bodies, as a combined borrowing from the best of both "the ancients" and "the moderns."

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However, even the mathematical methods of the *Principia* were deliberately anchored in a *prisca sapientia*'s framework. In Book I, Section 5, Lemma 19, Corollary 2, Newton explicitly downgraded calculus, boasting that he had solved a conic problem with geometrical methods as required by ancients such as Euclid and Apollonius rather than with the modern analytic tools of Descartes. The "*non calculus, sed compositio geometrica*" of the *Principia* is a conscious effort on Newton's part to align his mathematical choices with a putative ancient methodological practice. Newton seems to harbor at least three criticisms against differential calculus: 1) unlike geometrical representation, it does not allow for visualization; 2) it was not the preferred method of the ancients; 3) in its Leibnizian version, it is unnatural or artificial because it is based on indivisibles or differential quantities which have "no being either in geometry or in nature" and "we have no ideas of infinitely little quantities."

If one tentatively accepts calling the above and similar Newtonian views of and assumptions about mathematics "Newton's philosophy of mathematics" – even if Newton is not a classical philosopher in the sense we take for example Plato, Descartes, Leibniz, or Russell to be – one can see that there is in this "philosophy" something we may term mathematical naturalism or simply mathematical "realism."[9] Compared with this mathematical naturalism, differential calculus appears to Newton as ontologically unsound, while the theory of fluxions, which strives to "proceed by finite quantities," appears "more natural and geometrical." For Newton, quantities have ontological content or referents in nature and this explains why ancient mathematicians never admitted indivisibles in geometry (Newton 1967–1981, vol. 8, 597–8; Guicciardini 1999, 35).[10] Otherwise put, nature gives meaning to quantities and geometry allows the mapping of this natural ontology. These Newtonian views of mathematics drive home what I have been suggesting apropos of the ontological content of force in the *Principia*.

Scholars who deny that Newton's concept of force has ontological content, i.e., assumptions about a referent in nature, frequently cite Newton's methodological disclaimer as proof. This allows them to argue that Newton treated force only as a mathematical quantity and a neutral term devoid of any specific ontological dimension. Now, Newton's thoughts and assumptions about mathematics or, simply, Newton's "philosophy" of mathematics indicates that he treated mathematical quantities as

[9] In using the term "mathematical realism," I do not refer to any treatise by Newton on the philosophy of mathematics but rather to views he held and assumptions he made in the course of his mathematical practice. Although terms like this, as applied to Newton, might appear excessive to some, they nevertheless have in this debate the advantage of helping the reader to intuitively and sharply grasp what is at stake. But they also invite comparative studies with the views of traditional philosophers such as Plato. In any event, the controversy between Newton and Leibniz over the foundations of calculus, Pemberton's report on Newton's preferences in terms of mathematical style, *View of Sir Isaac Newton's Philosophy* (1728), and Berkeley's *Analyst* (1734), etc., all indicate the necessity of spelling out Newton's philosophical understanding of mathematics.

[10] For more details on the controversy over the foundations of calculus, see Guicciardini 1999. On the priority dispute, see Hall 1980 and Bertoloni Meli 1993.

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physically significant "units" because their "geneses take place in the reality of physical nature and are daily witnessed in the motion of bodies" (Newton 1967–1981, vol. 8, 122–3). It is the progression of these physically significant units that Newton, the mathematical "realist," wants geometry to show. According to this philosophy of mathematics, neither the theory of fluxions nor geometry are physically blind or, more precisely, *rerum natura* blind. He wrote for instance: "fluxions are finite quantities and real and ought to have their own symbols; and each time it can be conveniently so be done, it is preferable to express them by finite lines visible to the eye rather than by infinitely small ones" (ibid., 113–5). In *De Gravitatione*, Newton argues that geometrical figures are innate in nature "for the material delineation of any figure is not a new production of that figure with respect to space, but only a corporeal representation of it, so that what was formally insensible in space now appears to the senses to exist" (Newton [1962] 1978, 133).

In short, Newton's later philosophical understanding of mathematics appears to nullify his methodological disclaimer when construed as suggesting drawing physical inferences from physics-blind mathematics because his kind of mathematical "realism" does not grant a radical formalism, wherein mathematical quantities are entirely devoid of any physical meaning. Actually, the *Principia* itself opens up with this philosophy. In the preface to the first edition, Newton articulates the relationship between geometry and mechanics by arguing that geometry presupposes mechanics and is founded on some mechanical principles. Geometry belongs to "universal mechanics which reduces the art of measuring to exact proportions and demonstrations" (Newton [1687] 1999, 382). Thus, Newton, rather than reducing mechanics to a simple branch of mathematics, saw mathematics itself as part of mechanics and as founded on mechanical practice.

This falsifies characterizations of Newton's scientific method that tend simply to restate his disclaimer. The emphasis that is usually placed on the disclaimer stems from a naturalistic attitude that aims at rooting the description of Newton's scientific methods and goals in his actual scientific practice and writings, in this instance, the *Principia*. But this naturalistic attitude frequently tends also to rule out interdisciplinary approaches to the positive content of Newton's science. Indeed, if Newton's method in the *Principia* is, as he allegedly claimed it was, a physical inference from physics-blind mathematics, and if Newton treated force purely as a neutral term, then how could non-canonic disciplines have possibly contributed to the *Principia*? Asymptotic reconstructivists seem to deny any such contribution. But this radical position will automatically fall if we can prove that the mathematical constructivist claim on which it apparently depends is untenable. For example, although admitting a role for alchemy in Newton's science, Cohen seems to construe this role in the spirit of demarcationist philosophies of science (Reichenbach 1938; Popper [1959] 1968). For him, alchemy explains "the mindset or basic philosophy of Newton the individual" but "not the features of the *Principia*" (Newton [1687] 1999, 62) because Newton has successfully eliminated from the *Principia* the traces of his alchemical findings in such a way that "it

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has taken us three centuries of intellectual archeology to get behind the façade of the *Principia* to reveal the full content of his thought" (ibid., 60).

Otherwise put, non-canonic disciplines constituted a powerful but private tool in Newton's hands in the context of discovery, but had no role in the context of justification. According to this interpretation, Newton's approach consists of three steps: 1) stylize the physical entities in such a way as to make them mathematically tractable; 2) apply mathematical rules and propositions deduced from experimental and observational data; and 3) compare the mathematical substitute for nature to the external world. Based on this typical physico-mathematical approach, Cohen curiously, however, described the so-called "Newtonian style" in terms of "mathematical construct." This style, it is said, was forged between 1679 and 1680 (Cohen 1980 and 1982; Newton [1687] 1999).

There is no question about Newton's attempts at "masking" his background assumptions. The problem with mathematical reconstructivists lies, to put it in Leibnizian terms, not so much in what they advance but in what they deny. The term "mathematical construct" seems to force a conclusion that is hard to maintain both philosophically and historically. From the fact that Newton relied on mathematics to postpone a theoretical justification of gravitational attraction it does not follow that Newton lacked any working physical assumptions. Subjecting "force" and "attraction" to rigorous mathematical treatment does not entail that "force" and "attraction" are transformed into pure mathematical concepts without any physical meaning. It is logically a *non sequitur*. As Carnap already pointed out, there is "an essential difference between an axiomatic system in mathematics and an axiomatic system in physics." In theoretical physics, entities such as force, field, electron, temperature, mass, inertia, etc., cannot be defined by logical terms only, as is the case for line, point, plane, etc., in pure mathematics. That is, they cannot have "a splendid isolation from the world" (Carnap 1966, 236–7). In other words, valid theoretical terms, as opposed to fictional ones, are already physically interpreted to a significant degree.

In fact, in this debate, Newton's "philosophy" of mathematics is a kind of rational exit signal that forbids "naïve" and "dogmatic" naturalistic approaches, so to speak, to Newton's scientific methods. Rather, "sophisticated" (Lakatos 1970) or, better yet, critical naturalism is needed which is informed not only by Newton's "philosophy" of mathematics according to which physics is already built into his mathematics but also by an awareness of the Carnapian essential distinction between an axiomatic system of physics and an axiomatic system of mathematics. As with any physical term, the concept of force has meaning not in pure mathematics but in physics. Huygens, for one, seems to have been aware of this. Huygens showed a great admiration for the *Principia* and its "beautiful discoveries" (Huygens 1888–1950, vol. 9, 305). He was particularly impressed that "the celebrated Mr. Newton" explained the elliptical path of the planets by gravity alone, disposing thus of Cartesian vortices (ibid., vol. 21, 143). Yet, the *Principia* revealed to Huygens his own intellectual misfortune. He affirmed that, long before Newton, he had himself thought about generalizing the effect of

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gravity but that the Cartesian vortex theory, which he had accepted, prevented him from doing so (ibid., vol. 9, 472–4).

Thus, apparently erroneous physical intuitions rather than mathematical ability prevented Huygens from anticipating the key achievement of the *Principia*. Although prior to reading the book, he had warned against making "suppositions like that of attraction" (ibid., vol. 9, 190), he had become, while reading it, prepared to accept the concept in so far as it did not imply action at a distance (ibid., vol. 21, 472–4). But Huygens was soon to realize that, despite the methodological disclaimer, Newton's calculations were like the Trojan horse of theoretical intuitions he could not accept, that is, the assumption that particles of bodies interact with each other at a distance (Newton [1687] 1999, Preface and Book I, Proposition 69 and Book I, Section 11, Scholium). Indeed, Huygens wrote: "I am not in agreement with a principle he assumes in this calculation and elsewhere, which is that all the parts one could imagine in two or more different bodies mutually attract and tend to approach each other"[11] (Huygens 1888–1950, vol. 21, 471). Because Huygens seems to realize that in an axiomatic system of physics theoretical assumptions impact mathematical inference, he wondered how difficult calculations such as Newton's could have been guided by such "incoherent" theoretical intuitions alone (ibid., vol. 9, 538).

What does Huygens' reading of the *Principia* reveal? It shows that three centuries before McGuire, Westfall, Dobbs, etc., began to suggest that non-canonic disciplines supplied Newton with concepts he could not find in the Cartesian mechanistic philosophy, Huygens had already cogently disclosed the theory-laden structure of Newton's calculations and described the principles underlying gravitational theory as "absurd" (ibid., vol. 9, 538).[12] But as we shall see later, Huygens was not the only one to notice and criticize the *Principia*'s background assumptions. The lesson is that

[11] Of course, today, we know experimentally that Newton was right that any two particles of matter interact with each other. But at the same time we know that a field medium is required for such an interaction. Also, I have already shown that, for Newton in *De Gravitatione*, metaphysical assumptions underlie our conception of bodies. Furthermore, the conception of bodies in the passages of the *Principia* incriminated by Huygens is to be found not only in the preface to the first edition of the *Principia* and the general scholium but also in the unpublished conclusion to the *Principia* and the queries of the *Optics*, for example. Scholars generally agree that alchemy is relevant to the understanding of Newton's theory of matter in the *Optics* and in the *Conclusio* (Newman 2002; Figala 2002). It is not unreasonable to extend this finding to Newton's *Magnum Opus*. In any event, my point is that on the specific issue of attraction, action at a distance, and theory of matter, both Newton and his critics very often display astounding insights but historical accounts do not always do justice to both camps. The same point is valid for the controversy over absolute space and time.

[12] Notice that claims about theory-ladenness are not claims about falsity. Actually, Leibniz even boldly compared "attractive force" to the so-called mystery of the Eucharist (Leibniz 1973, 37–40). His problem was not so much that Newton's *Principia* argues that "a grain of sand on this earth exerts an attractive force on the sun," but that it suggests that this happens without physical contact of any sort. In the seventeenth and eighteenth centuries, or more precisely in this pre-Faraday/Maxwell/Einstein period, this kind of interaction can indeed only be understood as physically unmediated. So Leibniz wondered: "how could the gentlemen" who espouse these kinds of explanations in science "deny that, by the omnipotence of God, we could have a participation in the body and blood of Jesus Christ . . .?" What Leibniz was arguing is that Newton's "attractive force" seems

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Newton's justification strategies are not completely disconnected from the context of discovery of the *Principia* that, by itself, tells the story of its own theoretical background, although Newton's manuscripts are needed to understand the details of that story.

III. Mathematics and Newton's Culture of Force

Force is the conceptual backbone of Newton's *Principia* and Euler's algebraic formulation of the second law of motion has become "the soul of classical mechanics" (Wilczek 2004). F = ma is, in the Kuhnian sense, the paradigm or exemplar par excellence for students training in classical mechanics. But the scientific reality and necessity of the concept of force remain dubious. The concept had no prominent role in pre-Newtonian physics starting from Galileo, Descartes, and Huygens.[13] After Newton, it has been assigned a lesser function by d'Alembert's principle and Lagrange's Action equation. Moreover, it does not appear in the fundamental equations of the theory of general relativity and quantum mechanics. But the concept still remains popular in science for pragmatic reasons and "a big part of the explanation of its continued use is no doubt (intellectual) inertia" (ibid., 12). In fact, what Wilczek, winner of the Nobel Prize for physics, terms "the culture of force," i.e., the "tacit culture of working assumptions" embedded in the interpretation of the algorithmic function f = ma that students acquire through the learning process, is rooted in a tacit culture of Newton's own background assumptions.

Mathematical reconstructivists deny the existence or downplay the relevance of such a culture of background assumptions to Newton's *Principia*, while holistic finalists locate it essentially in Newton's religious framework. For example, Dobbs maintains that "Newton's methodology was much broader than that implied by the creation of mathematical models" because "mathematics was just one avenue to Truth" and Newton's search as such was directed to the secrets behind the divine creation (Dobbs 1991, 7). Although critical naturalism resists naïve and dogmatic naturalism, it is not committed to holistic finalism. Because it conceptualizes Newton's religious interests in terms of "significance graphs," but not in terms of an overarching theological goal, and because it takes for granted that the historical starter of the science of *De Motu* and the *Principia* is at least in part Halley's mathematical quest, critical naturalism reads the *Principia* as primarily written for people like Halley, Huygens, and Johan Bernoulli, rather than for the followers of Michael Maier, George Starkey, or Henry

to have theological underpinnings and, as such, it provides a good way to embarrass those who, like Newton's countrymen, believe that the "reality of Eucharist is absurd."

[13] The concept of force does appear in Descartes' and Huygens' work, which might have, to some extent, influenced Newton. But Descartes and Huygens held matter and motion as the two essential explanatory principles in science (see Maglo 2003).

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More. But because Newton's late "philosophy" of mathematics and the scientific developments of the eighteenth century (see below) indicate that Newton did have in the *Principia* ontological commitments deriving from his culture of force, critical naturalism interprets the *Principia* in terms of "heuristic models" transfer.

According to Thomas Kuhn, heuristic and ontological models provide researchers with "permissible analogies and metaphors" that guide problem-solving apparatuses in determining and evaluating a given scientific explanation (Kuhn 1962, 184). Taken this way, heuristic models have something in common with what Gerard Holton calls *themata*, which in their methodological function operate like guiding principles for mathematical modeling and experimental justification (Holton 1973, 1978). Divergence in heuristic models or thematic guiding principles may create disagreement and/or explain scientific discovery as they orient researchers differently. For instance, while the thematic heuristics of the vortex apparently prevented Huygens from generalizing gravity, the thematic ideas embedded in Newton's culture of force very likely provided him with models having surplus explanatory power.

Newton's culture of force was not a spontaneous philosophy of nature. In the 1660s, Newton accepted the vortex heuristics in explaining gravity. It is also during these years that he discovered calculus, or more precisely his method of fluxions. Although he by no means discarded this tool, he failed to realize its full importance for reasons I discussed earlier. Indeed, the decade of the 1670s marked a turning point in Newton's intellectual life and the beginning of his intense alchemical activities.[14] During a trip to London in 1669, Newton procured the six volumes of the *Theatricum chemicum* and bought two kilns, some chemicals, and laboratory glassware. This allowed him to begin his first forays into experiments in alchemy. Through Isaac Barrow and Henry More, both critics of Descartes, Newton joined a clandestine English society for alchemy (Westfall 1980a, 286). This intellectual environment, combined with his own research, perhaps led Newton to realize the limits of the vortex heuristics of physical impact and to incorporate non-canonic explanatory models based on active principles and subtle spirits into his philosophy of nature (Dobbs 1975, 194–232; Westfall 1980a, 281–334). But it is in the 1680s that Newton's philosophy of nature underwent the radical change Huygens' did not. Perhaps because his research in non-canonic disciplines had gradually provided him with competing attractionist heuristic models, by the mid-1680s, Newton called into question the vortex theory. While his calculations in *De Motu* were confirming the attractionist heuristic models, his pendulum experiments, related to *De Gravitatione* and the *Principia*, presented the vortex heuristic models with some significant difficulties (Dobbs 1991, 138–46).

Newton was not the first modern scientist to use the term attraction. Roberval, Fermat, Kepler, Gilbert, and the magnetists did as well. But here we need to distinguish

[14] This paragraph summarizes findings by scholars such as Westfall and Dobbs. But, as suggested above and below, works of scholars such as McGuire also indicate that Henry More and the Cambridge Platonists constitute another significant non-canonic source of Newton's culture of force.

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the "signified" from the "signifier." Newton's predecessors seem to defend views that we may, roughly speaking, call inherentism. In this view, attraction is inherent in bodies which are endowed with certain animistic (*anima*) or magnetic properties capable of explaining gravitational phenomena. Newton rejected this view at least during the period of the rise of the *Principia*. First, although for Newton, magnetic forces are centripetal forces like gravitational attraction, gravity is not a magnetic force (Newton [1687] 1999, Definition 5, 8).[15] Second, in *De Gravitatione*, Newton was skeptical about the role of an *anima mundi* as a cause of bodies and their motion (Newton [1962] 1978). Third, Rule 3 of the *Principia* clearly maintains that gravity, unlike inertia, is not inherent in bodies. Fourth, Newton demarcated himself from the inherentist tradition in his famous 1692 letter to Bentley. He wrote:

> That gravity should be innate, inherent, and essential to matter, so that one body may act upon another at a distance through a vacuum, without the mediation of anything else, by and through which their action and force may be conveyed from one to another, is to me so great an absurdity, that I believe no man who has in philosophical matters a competent faculty of thinking, can ever fall into it. (Newton 1961, 254, 244, 240)

Thus in the 1680s, Newton held a non-inherentist view of attraction although in 1713 he allowed Cote's preface to introduce in the *Principia* the inherentist view he previously derided as "a great absurdity" unworthy of a sound and healthy philosophical mind. In short, Newton accepted attraction neither because of sympathy for the ancients or moderns nor because the term was available in some kind of surrounding intellectual environment. He accepted it perhaps because the specific heuristic models he had begun to attribute to the concept of force privately, but also publicly at least since the "Hypothesis" of 1675, had more explanatory power than the rival modern vortex theory. Works by McGuire (1968 and 1977), Dobbs (1975 and 1991), Westfall (1971, 1972, 1975, 1980a, and 1980b), and McMullin (1978), to name but a few, have deepened our understanding of the process through which Newton came to incorporate active principles, alongside the passive principle of matter, as explanatory factors in his philosophy of nature. Interdisciplinary investigations into his canonic and non-canonic areas of research may allow us to put in perspective some of his background assumptions about gravity, force as a primary entity, and the goal of science as a search for forces.

Take for example *The Emerald Tablet*, which Newton studied for many years. Not only does this document assert the reality of varieties of forces in the system of the world but it also explains that everything in this system derives from one unique procedure. It affirms the existence of a unique and general principle captured as "the force of forces" which permeates both micro and macro entities and which is the explanatory

[15] In an unpublished draft intended to be a conclusion to the *Principia*, Newton drew a vivid and sharp contrast between gravitational, electric, and magnetic attractions (Newton [1687] 1999, 287).

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principle of the world. Force is presented in this document as a fundamental reality and the discovery of its modalities of operating in the world system is considered to be the essential aim of the human epistemic endeavor (Hermes Trismegistus 1995).[16]

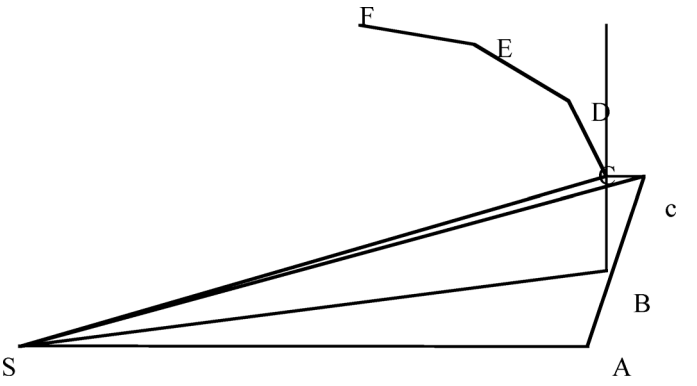
Interestingly, Hypothesis 3 of the first edition of the *Principia* asserts the transmutation of bodies. This may indicate that while writing the *Principia*, Newton did not lose sight of the findings of his alchemical and hermetical research. Hypothesis 3 was, however, suppressed from subsequent editions of the *Principia* but not its use against views such as the Aristotelian and the Cartesian in Book III, Proposition 6, Corollary 2. In fact, as Cotes explained in his preface to the second edition of the *Principia*, there are three kinds of natural scientists. These are the Aristotelians who endow "the individual species of things with specific occult qualities," the Cartesians who feign "certain occult fluids," and the Newtonians who base their science on experiment and do not "contrive" or "admit" hypotheses "into the natural science otherwise than as questions whose truth may be discussed" (Newton [1687] 1999, 385–6). However, it is a mistake to dig out one line of evidence as a monolithic source of the *Principia*. Newton's mathematical modeling of force in the *Principia* was influenced by Huygens' work and Robert Hooke's 1679 suggestion to Newton to decompose orbital motion into tangential and central components (ibid.; Guicciardini 1999). Accordingly, the contribution of non-canonic disciplines to Newton's science should be neither dogmatically ignored nor vindictively overdone. Be that as it may, I shall now dwell on the actual mathematical treatment of force by Newton.

[16] I use this example simply to invite more close and open-minded readings of some of Newton's non-canonic manuscripts. It is obvious that it is not crucial to the central argument of this paper. Nonetheless, it should not be misconstrued. To suggest that vivid descriptions of nature in some non-canonic disciplines such as alchemy and (Cambridge Neo-Platonist) metaphysics might have contributed something, how little this contribution may be, to Newton's natural philosophy is not to suggest extrapolating from these disciplines to findings in the *Principia*.

Actually, what Hermes Trismegistus teaches is that everything derives from a unique procedure and that "the sun is its father, and the moon is its mother. The wind has borne it in its belly, and the earth has nourished it with its milk." What the wind carries in its belly is a starred *regulus* with its center of force. *Regulus*, rather than designating antimony with respect to its relationship to gold, the king of metals, identifies it instead, according to Dobbs, because of its capacity to produce a star (Dobbs 1975). In the center of the crystalline structure of the *regulus*, the alchemist's star is found. At the experimental level, Newton confirms having isolated the *regulus* of Mars in a letter to Oldenburg dating from January 18, 1672. Alchemists seem traditionally to interpret the crystal lines in a stelliform *regulus* as converging to a central point. The *regulus* is also believed to embody the philosopher's stone and to have both medical powers and attractive properties. For Basil Valentine, antimony is endowed with spirit that, like its force, permeates it invisibly in the same manner that magnetic properties permeate a magnet (ibid., 148–50, 146–66; Westfall 1980a, 295). But that, adds Hermes, "is the force of forces which surmounts all subtle things" and "penetrates all dense things" (Hermes Trismegistus 1995, 16). Ultimately, Hermes, later identified by Newton as Christ, is himself "the principle of the world and he who leads it" (ibid., 11). For Michael Maier, however, what "the wind bore in its belly" is chemically the sulfur, physically the fetus, arithmetically the cube root, musically the double octave, geometrically the point and astronomically the center of the planets Saturn, Jupiter, and Mars (ibid., 59–74).

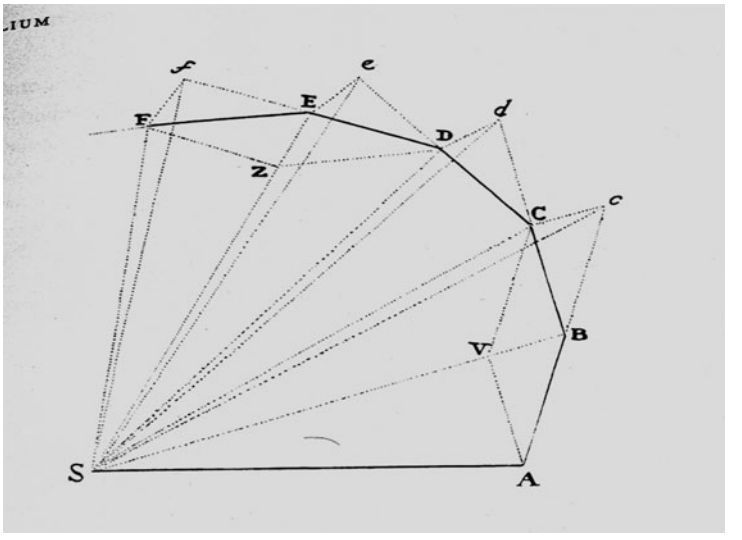
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By the time of Halley's visit in 1684, Newton's "tacit culture of working assumptions" and the heuristic models generating "permissible analogies and metaphors" seem to have been in place, generally speaking. Theorem 1 of *De Motu* seems to illustrate these various contributions. Newton envisioned a force acting from an immobile center, an attractive force identified, however, only by its geometrical "stigma" at this central point. This force does not confront perturbing effects from any corporeal medium and its role is to communicate a deflecting acceleration toward the center to a body moving uniformly along a straight line according to its inertial force. Geometry allows materializing and visualizing the action of this force as, for example, in this diagram from Theorem 1 of *De Motu*.

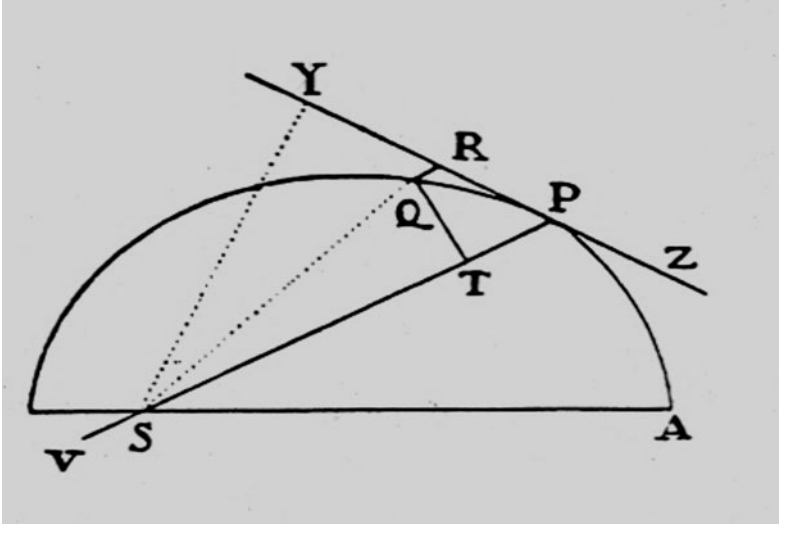


Let us, however, focus on Proposition 1 of the *Principia*. It states that the "areas which bodies made to move in orbits describe by radii drawn to an unmoving center of forces lie in unmoving planes and are proportional to the times." The proof runs as follows: first let the time be divided into equal parts. Then suppose that a body is moving with a given initial velocity during the first interval of time in the direction AB. Left to its "inherent force" only, the body will go straight to *c* during the second part of the time. In this case AB = B*c* and the area ASB = BS*c*. Now, imagine instead that a force is acting on the body from the unmoving center S. With a single strong impulse, the centripetal force will cause the body moving by the inertial force along AB to deviate, at point B, from the line B*c* to BC toward the focus S during the second time interval. If the centripetal force acts successively, the position of the body at different intervals of time can be represented by the polygonal trajectory A B C D E F. The segment *c*C materializes the effect of the force and is parallel to BS. SBC and SAB are equal and lie in the same plane. So are the triangles SCD, SDE and SEF. But in the limit, when the time interval approaches zero, the action of the force can be conceived of as continuous and as describing a smooth curvilinear progression of the motion of the body around S.

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However, it is in Proposition 6, section II of Book I of the *Principia* that Newton's dynamical measure of force appears (see also Theorem III of *De Motu*). Following the same principle of the decomposition of orbital motion into tangential and radial components, suppose that a body P orbits around the unmoving center S describing the curved path APQ. Let the straight line ZY be the tangent at P. Draw the segment QR, parallel to SP, intersecting the curve APQ at Q and the tangent ZY at R. If QT is dropped perpendicular to SP, then the centripetal force will be inversely proportional to $SP^2 \cdot QT^2/QR$, when Q approaches P. The displacement QR is proportional to the square of the time Δt. It materializes the effect of the force and is also proportional to the force.



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Thus while Proposition 1 shows that if there is a centripetal force then Kepler's areas law holds, Proposition 6 provides a quantitative evaluation of this force, which is as $QR/SP^2 \cdot QT^2$. As by the areas rule $SP^2 \cdot QT^2$ is proportional to Δt^2, then $F = <QR/\Delta t^2$. Interestingly, some scholars suggest that Newton might have preferred the already by then anachronistic formula $SP^2 \cdot QT^2/QR$ of dimension 3 to the dimension -3 formula $QR/SP^2 \cdot QT^2$ because of a concern about "dimensional realism" as the latter lacks "intuitive significance" (De Gandt 1995, 32). Whatever the reason might be, Newton's mathematical achievement in *De Motu* and in Book I of the *Principia* presupposes a conceptual break with the vortex thematic heuristics, an important key unavailable to Huygens.[17]

IV. Force between Newton and Laplace: Evidentiary Blind-Spot #2

What is the epistemic status of the "*Principia*-type forces" in the period following the publication of the *Principia* when young mathematicians and physicists came to accept both Newtonian and Leibnizian achievements in order to reorganize science? Did scientists simply follow Newton in treating force as a primary entity? This is a complex issue that is far from being fully understood, but it has the potential to provide us with additional rational exit signals by shedding new light on the debate. How Newton conceptually constructed force and mathematically treated it can also be grasped in part by interrogating the actual scientific pathways of the triumphant rise in the eighteenth century of Newtonian mechanics. In fact, the science set in motion by the *Principia* did not reach its "completion" until Laplace, and the reworking process of that science provides a context for assessing Newton's achievement.

As Cohen put it about Newton's concept of inertia:

> Although the use of *vis* or "force" in the context of inertia seems outlandish to a twenty-first century reader, this was not the case for Newton's successors in developing the science of dynamics. For example, Jean d'Alembert, in his *Traité de dynamique* (1743), wrote: "I follow Newton in using the name 'force of inertia' for the properties which bodies have of remaining in the state in which they are." (Cohen 2002, 61–2)

Beyond the issue of the historical accuracy of his interpretation, Cohen correctly assumed that our attempt to understand Newton's achievement in its original context must also confront the eighteenth-century revision of Newton's science in order to avoid anachronisms. Furthermore, the possibility of conflating the views Newton laid out in the *Principia* and in related documents and the views standardized in the Newtonian mechanics is real. It is thus important if not to disentangle entirely both sets of ideas at least to sketch out the complex scientific history of the "*Principia*-type forces"

[17] Of course, we know that Leibniz, *ex post facto*, attempted to reconcile the mathematics of the *Principia* with the vortex theory but we shall not be concerned here with that story.

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in the eighteenth century. This history also corroborates, I believe, the suggestion that force is in the *Principia* a physico-mathematical construct with ontological and philosophical underpinnings.

Newton's *Principia* was a gift to human intellectual history, to say the least. But it is to be expected that eighteenth-century scientists who stood on Newton's "shoulders" saw in many respects further than he did, just as he himself saw further than the giants such as Descartes, Huygens, and Hooke who cleared the way for him. Newton's followers such as Euler, d'Alembert, Clairaut, Lagrange, and Laplace were at the same time admirers, critics, and innovators. Some of them even provided, incidentally, a genuine standpoint for the study of scientific growth in the period. D'Alembert, for one, emphasized Newton's debt to Huygens for the theory of central forces and to Isaac Barrow for the theory of fluxions, but also the fact that scientists of his own generation were departing significantly from Newton's science. Nonetheless, d'Alembert maintained that the depth of science in the *Principia*, even when Newton was mistaken, is such that scientists of his own time should still pay tribute to Newton as their master (D'Alembert 1967, 355–6). Actually, the history of Newtonian mechanics is not a history of a straightforward scientific development either at a conceptual or methodological level. In genealogical terms, it is a "reticulate network" (Sober 1988) of multiple ancestral originations with striking Newtonian features. Let's consider, in this respect, the concept of inertia mentioned by Cohen.

McMullin has already observed that Newton's use of the concept of *vis* for inertia indicates that Newton's philosophy of nature accumulated serious tensions since inertia is supposed to belong to matter which is in Newton's own system inert (McMullin 1978). Figala, for her part, pointed to conceptual links between Newton's alchemical concept of "immutable seeds" and his concept of inertia understood as an "unchangeable" inherent force or *vis insita* (Figala 2002). Actually, documentary evidence indicates that Newton's concept of force of inertia is very complex. For instance, in MS 3970, a draft connected to the *Principia*, Newton himself attributed the law of inertia to Lucretius, Aristotle, and Anaxagoras.[18] But because in the *Principia*, Newton chose instead to credit the law to Galileo, we can ignore for the moment alchemical seeds, *clinamen*, and so forth. In fact, in this discussion about "inertia" one needs to distinguish between the principle, the term, and the concept. In its scientific content, the law of inertia is a Cartesian principle baptized by Newton with the Keplerian term "inertia." Newton's originality in the formulation of this law seems to be his conceptualization of inertia in "Definition 3" and "Rule 3" of the *Principia* as an inherent force essential to bodies. In the eighteenth century, however, both Euler and d'Alembert launched serious attacks on Newton's concept of *vis inertiae*. Euler maintained that "force of inertia" is a misnomer because force is what changes the state of a body and conceptually is the opposite of the quality of a body remaining

[18] Notice that Huygens opposed the intrusion of all these traditions in physics. But Newton does not seem to have conceived of gravity, unlike inertia, in light of these traditions (see letter to Bentley, February 25, 1692).

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in a state of rest or motion. For Euler, force is neither an action at a distance nor an entity inherent to bodies, but results from impenetrability. Thus, he described attractive force as "unintelligible" (Euler [1768] 1843). Euler did not, however, seem to forgo completely force as a primary concept in physics (Ravi 2005; Hankins 1970).

D'Alembert also took issue with Newton's conceptual framework. Although he used the term "force of inertia," he took great care to argue that inertia, just like velocity, is by no means inherent in bodies. In a word, contrary to Cohen's opinion, d'Alembert's concept of inertia, just like the modern one, is not Newton's (Westfall 1980a, 302). D'Alembert, for one, was an eliminativist vis-à-vis force, which he characterized as an "obscure" concept but still employed for conventional reasons. Using the principle of virtual work to recast dynamics in the framework of statics, d'Alembert devised a clear conceptual apparatus to "neutralize," if not eliminate, force from physics. He achieved this by creating a kinetic equilibrium thanks to virtual movements opposite to the acceleration of bodies in motion (D'Alembert [1743] 1758). For Lagrange, the actual formulation of the so-called d'Alembert's principle is "awkward and difficult" but the principle itself is powerful. He saluted d'Alembert's *Traité de dynamique* as a crucial turning point in science because it was the first successful attempt to provide "a general and direct method to solve or at least to put into the form of equations all the problems of dynamics that can be imagined" by reducing "all the laws of motions of bodies to those of their equilibrium" (Lagrange [1788] 1997).

The principle itself has deep roots and goes at least as far back as Huygens. Versions of it are to be found in the works of Jacob Herman and Euler. D'Alembert generalized the principle and tied it closely to an eliminativist research program vis-à-vis force. His diagnosis of the fundamental problem of the science of his time is simple: rational mechanics has no rational foundations. His target is clear: the concept of "force," whether conceived in the Newtonian sense as "motive cause" or "inherent forces" or in the Leibnizian sense as *vis viva*. For him, these concepts refer to nothing less than "metaphysical and obscure beings" that plunge science "into darkness" (D'Alembert [1743] 1758). His goal is likewise unambiguous: an epistemic reform of science by reducing the principles of mechanics to just a few and by providing science with a general method for solving problems. It is in this respect that statics became a lightening path to dynamics. D'Alembert's principle was for Lagrange, who also strived to reduce "all the principles of statics to one general formula," a point of departure (Lagrange [1788] 1997, 180). Both Lagrange's Action integral and d'Alembert's principle turned force into a secondary concept.[19] In Newton's *Principia*, mechanics is primarily a science of force conceived of as an inherent entity and motive cause. Newton's followers did not see these concepts as simple mathematical constructs and neutral

[19] Of course, force did not disappear from the work of Lagrange, Laplace, and Hamilton. Moreover, the concept is still in use today in science and we even speak of weak and strong nuclear forces. But the concept does not seem to be indispensable to science.

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terms. It is interesting that contemporary scholars seem unable to see in the *Principia* what Newton's followers saw and labored to transform, sometimes with extraordinary success.

Newton's gravitational theory, rather than generating on the Continent what is today called "normal science," inaugurated a period of long crisis instead. Eighteenth-century Continental revolutionaries found themselves with three tasks: a) reorganizing science on more rational bases; b) extending science beyond the point mass dynamics of the *Principia*; and c) determining the most progressive mathematical methods of solving problems. It is in this context that, like d'Alembert's treatise, Euler's *Mechanica*, the first great treatise of analytical mechanics, inaugurated a new scientific era by divorcing science from Newton's synthetic methods. As one scholar recently put it, "After Euler the *Principia*'s mathematical methods belong definitely to what is past and obsolete" (Guicciardini 1999, 6; Guicciardini 2004). The "Eulerian mathematical style" which dominated analytic mechanics is in fact the completion of a research program initiated by scholars such as Pierre Varignon, Jacob Herman, and Johan Bernoulli. It consists of recasting, sometimes with increasing content, Newton's *Principia* into Leibnizian mathematical methods (Blay 1992; Maglo 2003).

An awareness of the specific challenges coming out of Newton's science in the eighteenth century as perceived by the scientific players of the time has the potential to shed light on some tough issues in Newtonian studies. For example, d'Alembert's characterization of inherent forces as obscure metaphysical entities does not detract from Newton's own attribution of the law of inertia to Aristotle, Lucretius, and Anaxagoras in MS 3970. In addition, in light of the eighteenth-century struggle with Newton's concept of force, Figala's suggestion that Newton's *vis insita* may have corresponded to alchemical principles no longer appears unreasonable. Non-canonic disciplines are good candidates in the explanation of Newton's appropriation of the expression *vis insita* in the context of defining inertia.

What I am suggesting is that although Newton subjected "force" to a rigorous mathematical treatment in the *Principia*, the metaphysical and ontological underpinnings of this concept do not appear to have been significantly "neutralized" until the period of Euler, d'Alembert, Lagrange, and Laplace. The "*Principia*-type forces" sparked heated debate even among Newton's followers about whether force as such should be considered a primary or a derived concept or be simply eliminated from science. Despite the lack of resolution of this issue in the eighteenth century, the debate it engendered seems to indicate that Newton's concept of force was, even for the key players of the period, more than a simple mathematical construct. Moreover, d'Alembert and Laplace were aware that Newton's preference for geometry or, simply put, his "*non calculus, sed compositio geometrica*" was influenced by *prisca sapentia* (D'Alembert 1770, vol. 4, 176; idem 1965, vol. 2, 320; Laplace 1878–1912, vol. 6, 464). But the development of science particularly from Euler and d'Alembert up to Langrage and Laplace aimed, among other things, to "neutralize" and eliminate some of Newton's "puzzling" theoretical assumptions. Accordingly, the fact that after

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Laplace most scholars apparently became incapable of perceiving these assumptions in the *Principia* needs historical explanation.

Conclusion

Clifford Truesdell once noted that Newton is not "alive in the barren speculations of the Queries" of the *Optics* "but in the solid permanent mathematical pages of the *Principia*" (Truesdell 1970, 226). I have been arguing that appraisals such as Truesdell's of Newton's scientific achievement are accurate. Nonetheless, we can gain a greater understanding of some of the features of Newton's positive accomplishments by placing them in the interdisciplinary context of his interests in diverse areas. In this respect, Newton's concept of force and what I tentatively called his "philosophy" of mathematics are good candidates.

Three important points emerge, however, from the early reception of Newton's *Principia*. First, both d'Alembert's principle and Lagrange's Action equation reassigned a different epistemic status to Newton's concept of force in rational mechanics. Later on, both the theory of relativity and quantum mechanics reinforced the view that force is at best a derived notion rather than a primary concept. Second, from Johann Bernoulli, Pierre Varignon, and Jakob Hermann to Euler, d'Alembert, and Alexis Claude Clairaut, there was a progressive but effective dissociation from the *Principia*'s *prisca sapientia*-driven geometrical methods, which prompted, in the standardized works of Lagrange and Laplace, the incorporation of Leibnizian calculus into Newtonian mechanics. In fact, it is even likely that there was an "invisible revolution" in the eighteenth century that brought the "Scientific Revolution" to its completion (ibid.; Blay 1992; Greenberg 1995; Guicciardini 1999; Smith 2002b; Maglo 2003). Third, Huygens and Leibniz's criticisms of attraction and absolute space were later echoed in the development of science from Faraday and Maxwell to Einstein. Einstein remarkably took on the issue of the cause of gravity, whose forfeiture in the *Principia* is often interpreted as indicative of the Newtonian style. We can now see that this style, at least as currently interpreted, has not been generally followed in the history of science. We can also see that it is not the case that "with certain notable exceptions, the whole history of dynamics from Newton to Einstein can be thought of as an exploitation, albeit infinitely ingenious and resourceful, of the definitions, principles and propositions in the *Principia*" (Herivel 1965).

How then are we to understand the sharp contrast frequently drawn between Newton, allegedly phenomenalistic, agnostic, etc., in his approach to science, and Leibniz and Huygens, allegedly metaphysics driven "philosophers"? This contrast often seems aimed at arguing that Newton invented the current scientific method which, thanks to its putative "agnostic" structure, leaves no room for non-canonic disciplines. But it is the way of drawing the contrast itself that is problematic. First, the heuristic value of the cleavage between Newton, allegedly a symbol of the "new"

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way of doing science, and Leibniz and Huygens, symbols of the "old" way of doing science, simply founders when one brings into the debate scientists such as Johann Bernoulli, Euler and d'Alembert, whose criticisms of Newton sometimes converge with Leibniz's and Huygens'. Second, it is significant that Leibniz and Huygens, although left with philosophical arguments only, forcefully anticipated two of the most important criticisms of Newton's science, action at a distance by Michael Faraday and absolute space by Albert Einstein. It is telling that in his defense of the incompleteness of quantum mechanics, Einstein (2005) compared himself to Leibniz. On the 15th of September 1950, he wrote to Max Born: "For the time being, I am alone in my views – as Leibniz was with respect to the absolute space of Newton's theory."[20]

Acknowledgments

This paper is based partly on my Ph.D. dissertation and partly on a talk I gave at the Dibner Institute in April 2005 while a Visiting Assistant Professor at MIT. Special thanks to Niccolò Guicciardini, François De Gandt, Gorge Smith, Emily Grosholz, Roger Ariew, Alexandre Métraux, and John McEvoy for their comments on an earlier version. I am also grateful to many of my students at MIT with whom I had very stimulating discussions.

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[20] Despite Leibniz's alleged "inconsistencies" in his criticisms of absolute space or Newton's "background dependent physics," it is fair to grant Leibniz a conceptual point about the possibility of a "background independent physics." If one accepts this minimalist claim, then what is striking is that both the Leibnizian and the Newtonian intuitions, so to speak, still conflict in contemporary science, particularly in string theory and loop quantum theory of gravity. To some extent, we live amid intuitions of the past and sometimes prove some with more appropriate tools and reject others.

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