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RESEARCH ARTICLE

A maximum problem of stopping time for correlated random walks

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Let $\{X_n\}_{n\geq 1}$ be a Markov chain taking values in $\{1,-1\}$ with transition probabilities

$$p = P(X_{n+1} = 1 \mid X_n = 1)\ ,\ q = P(X_{n+1} = -1 \mid X_n = -1).$$

Define a correlated random walk $(S_n)_{n\geq 0}, S_0 = 0$ and $S_n = \sum_{i=1}^{n} X_i, n \geq 1$. Let A and B be positive integers, and define a stopping time

$$\tau = \inf\{n \geq 1 : S_n = A \text{ or } S_n = -B\}.$$

Using the martingale approach, Pozdnyakov [4] (Martingale approach to gambler's ruin problem for correlated random walks. *Journal of Applied Probability* 62(4): 1594–1606.) derived the expected stopping time $E(\tau)$.

This study investigates the monotonicity of $E(\tau)$ with respect to the parameters p and q under the condition of symmetric boundaries $A = B$. We prove that when $p + q = t$ is fixed, $E(\tau)$ increases as p increases from 0 to $\frac{t}{2}$, which implies that $E(\tau)$ attains its maximum at $p = q = \frac{t}{2}$.

1. Introduction

For the classical gambler's ruin problem, Peköz and Ross [3] used a smart coupling method to prove that the duration of the game is stochastically maximized when the game is fair. Subsequently, Hsiau and Yao [1, 2] established stochastic ordering and likelihood ratio ordering results for the duration of the gambler's ruin game. Their results revealed a close connection between symmetry of the underlying random walk and the persistence of the game.

A natural extension of the simple random walk is the correlated random walk, in which successive increments form a two-state Markov chain. Pozdnyakov [4] derived explicit expressions for the ruin probabilities and the expected stopping time of a correlated random walk by creating several clever martingales. These formulas provide a starting point for studying how the duration of the game depends on the transition parameters.

Above results naturally raise the question of whether the maximal-duration phenomenon observed in the classical gambler's ruin problem persists in the correlated setting. As a first step toward understanding possible stochastic ordering properties in the correlated setting, we study the behavior of the expected stopping time. More precisely, consider a two-state Markov chain $\{X_k\}_{k\geq 1}$ with state space

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$\{-1, 1\}$ and transition probabilities

$$p = P(X_{k+1} = 1 \mid X_k = 1), \qquad q = P(X_{k+1} = -1 \mid X_k = -1).$$

Throughout the paper, we assume that the initial distribution is stationary:

$$P(X_1 = 1) = \frac{1-q}{2-p-q}, \qquad P(X_1 = -1) = \frac{1-p}{2-p-q}.$$

Let A and B be positive integers. Define $S_0 = 0,\ S_k = \sum_{i=1}^{k} X_i,\ k \geq 1$, and let

$$\tau = \inf\{k \geq 1 : S_k = A \text{ or } S_k = -B\}.$$

The corresponding ruin probabilities are

$$\alpha = P(S_\tau = A), \qquad \beta = P(S_\tau = -B),$$

with

$$\alpha + \beta = 1.$$

Recently, Pozdnyakov [4] obtained explicit expressions for both the ruin probabilities and the expected stopping time of a correlated random walk by means of martingale techniques. When $p = q$,

$$\alpha = \frac{B - 1 + \dfrac{1}{2(1-p)}}{A + B - 2 + \dfrac{1}{1-p}},$$
$$E(\tau) = AB\,\frac{1-p}{p} + (A+B)\,\frac{2p-1}{2p}.$$

When $p \neq q$ (after correcting a typographical error in the original formula),

$$\alpha = \frac{\left(\frac{q}{p}\right)^{-B}\sqrt{\frac{q(1-q)}{p(1-p)}} - \frac{\sqrt{(1-p)(1-q)}}{2-p-q}\left(\sqrt{\frac{q}{p}} + \sqrt{\frac{p}{q}}\right)}{\left(\frac{q}{p}\right)^{-B}\sqrt{\frac{q(1-q)}{p(1-p)}} - \left(\frac{q}{p}\right)^{A}\sqrt{\frac{p(1-p)}{q(1-q)}}},$$
$$E(\tau) = \frac{2-p-q}{p-q}(A\alpha - B\beta) + \frac{p+q-1}{p-q}(\alpha - \beta) + \frac{1-p-q}{2-p-q}.$$

In this paper, we restrict attention to the symmetric boundary case $A = B = n, n \geq 2$, and investigate the behavior of $E(\tau)$ along the line $p + q = t,\ 0 < t < 2$. For $p \neq t/2$, we rewrite the expression for α by multiplying its numerator and denominator by
$(2-p-q)(pq)^{n-1}\sqrt{pq(1-p)(1-q)}$, and then the preceding formulas reduce to

$$\alpha = \begin{cases} \dfrac{1}{2}, & p = \dfrac{t}{2}, \\ \dfrac{(2-t)(1-q)p^{2n-1} + t(1-p)(1-q)(pq)^{n-1}}{(2-t)\left[p^{2n-1}(1-q) - q^{2n-1}(1-p)\right]}, & p \neq \dfrac{t}{2}. \end{cases} \tag{1}$$

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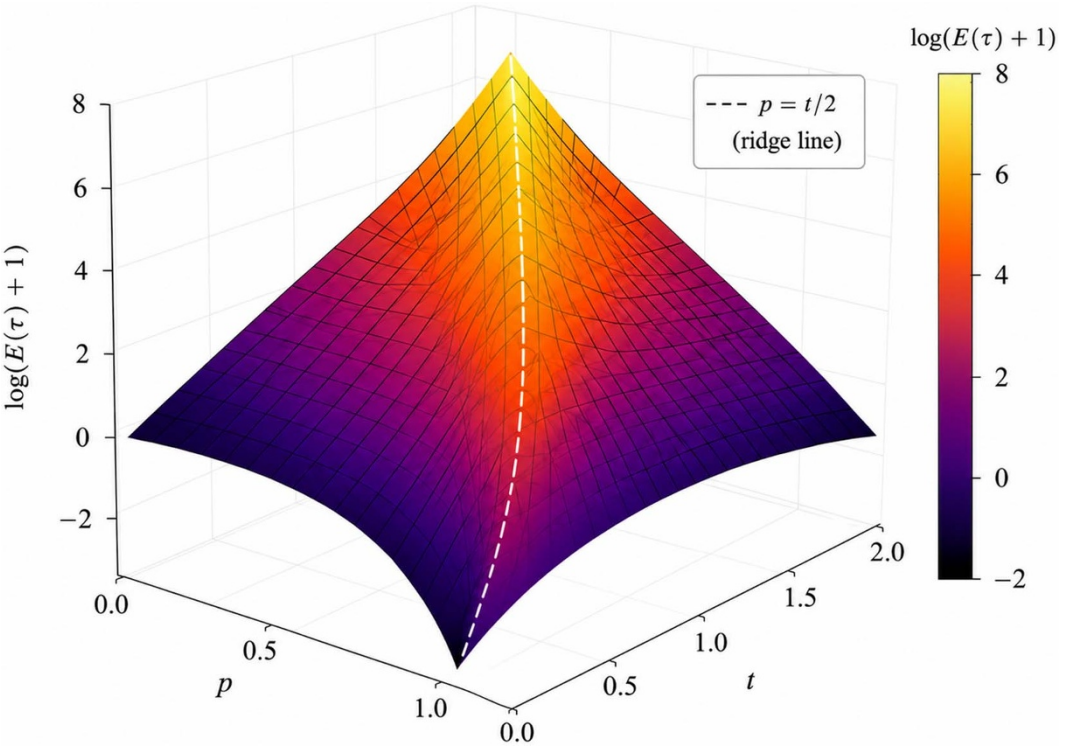


Figure 1. *The surface of* $\log(E(\tau)+1)$ *for* $n = 3$.

$$E(\tau) = \begin{cases} \dfrac{n}{t}\Big[n(2-t) + 2(t-1)\Big], & p = \dfrac{t}{2}, \\ \dfrac{2\alpha - 1}{p - q}\Big[n(2-t) + (t-1)\Big] + \dfrac{1-t}{2-t}, & p \neq \dfrac{t}{2}. \end{cases} \tag{2}$$

The following Figure 1 shows the behavior of $E(\tau)$ respect to p and t for $n = 3$.
The main result of this paper is the following theorem.

Theorem 1. *Let* $n \geq 2,\ 0 < t < 2$ *be fixed. As* $A = B = n,\ p + q = t$, *the expected stopping time* $E(\tau)$ *is an increasing function of* p *on the interval* $0 \leq p \leq \frac{t}{2}$. *Consequently,* $E(\tau)$ *attains its maximum at* $p = q = \frac{t}{2}$.

To establish this result, we require the following lemma.

Lemma 1. *For* $p \neq q$,

$$\frac{2\alpha - 1}{p - q} = \frac{1 + 2f}{2 - t},$$

where

$$f = \frac{1}{p - q} \cdot \frac{(1-p)(1-q)\left(p^n - q^n\right)\left(p^{n-1} - q^{n-1}\right)}{p^{2n-1}(1-q) - q^{2n-1}(1-p)}.$$

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The proof of Lemma 1 is postponed to Section 3. Lemma 1 and (2) immediately imply that $E(\tau)$ is continuous at $p = t/2$. Indeed,

$$\lim_{p\to t/2} E(\tau) = \frac{n}{t}\Big[n(2-t) + 2(t-1)\Big].$$

Therefore, to prove Theorem 1, it suffices to show that $\frac{dE(\tau)}{dp} > 0$ for $0 < p < \frac{t}{2}$. By (2) and Lemma 1, $\frac{dE(\tau)}{dp} > 0$ is equivalent to $\frac{df}{dp} > 0$. Let $g = \frac{1}{f}$. Then $\frac{df}{dp} > 0$ is equivalent to $g' = \frac{dg}{dp} < 0$. Therefore, it remains to establish that $g'(p) < 0$ for $0 < p < \frac{t}{2}$. To this end, we decompose g into three components.

$$g := \frac{1}{f} = \frac{q-p}{(1-p)(1-q)}(g_1 + g_2 + g_3),$$

where

$$g_1 = \frac{1}{2}\frac{q^{n-1}+p^{n-1}}{q^{n-1}-p^{n-1}}, \qquad g_2 = \frac{1}{2}\frac{q^n+p^n}{q^n-p^n}, \qquad g_3 = -\frac{q^{n-1}+p^{n-1}}{q^n-p^n}pq.$$

Define

$$h_i = \frac{d}{dp}\left(\frac{q-p}{(1-p)(1-q)}\,g_i\right), \qquad i = 1, 2, 3.$$

A direct calculation yields $g' = h_1 + h_2 + h_3$, where

$$\begin{aligned} h_1 = {} & \frac{q-p}{(1-p)(1-q)} \cdot \frac{(n-1)(pq)^{n-2}(p+q)}{(q^{n-1}-p^{n-1})^2} \\ & - \frac{1}{2}\left[\frac{1}{(1-p)^2} + \frac{1}{(1-q)^2}\right]\frac{q^{n-1}+p^{n-1}}{q^{n-1}-p^{n-1}} \end{aligned} \tag{3}$$

$$h_2 = \frac{q-p}{(1-p)(1-q)} \cdot \frac{n(pq)^{n-1}(p+q)}{(q^n-p^n)^2} - \frac{1}{2}\left[\frac{1}{(1-p)^2} + \frac{1}{(1-q)^2}\right]\frac{q^n+p^n}{q^n-p^n} \tag{4}$$

$$h_3 = \left[\frac{1}{(1-p)^2} + \frac{1}{(1-q)^2}\right]\frac{q^{n-1}+p^{n-1}}{q^n-p^n}pq - \frac{q-p}{(1-p)(1-q)}\left[\frac{n(pq)^{n-1}(p+q)^2}{(q^n-p^n)^2} + 1\right] \tag{5}$$

2. Proof of Theorem 1

To complete the proof of Theorem 1, it suffices to show that

$$h_1 + h_2 + h_3 < 0.$$

For convenience, define

$$\begin{aligned} F &= \frac{q-p}{(1-p)(1-q)} \cdot \frac{(n-1)(pq)^{n-2}(p+q)}{(q^{n-1}-p^{n-1})^2}, \\ G &= \frac{1}{2}\left[\frac{1}{(1-p)^2} + \frac{1}{(1-q)^2}\right]\frac{q^{n-1}+p^{n-1}}{q^{n-1}-p^{n-1}}, \\ H_1 &= \frac{npq(1-p-q)(q^{n-1}-p^{n-1})^2}{(n-1)(q^n-p^n)^2}, \end{aligned} \tag{6}$$

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$$H_2 = \frac{(q^{n-1} - p^{n-1})\left[q^n + p^n - 2pq(q^{n-1} + p^{n-1})\right]}{(q^n - p^n)(q^{n-1} + p^{n-1})}. \tag{7}$$

By (3), we have $h_1 = F - G$. Furthermore, combining (4) and (5) yields

$$\begin{aligned} h_2 + h_3 &= \frac{q-p}{(1-p)(1-q)}\left[\frac{n(pq)^{n-1}(p+q)}{(q^n-p^n)^2} - \frac{n(pq)^{n-1}(p+q)^2}{(q^n-p^n)^2}\right] \\ &\quad - \left[\frac{1}{(1-p)^2} + \frac{1}{(1-q)^2}\right]\left[\frac{1}{2}\frac{q^n+p^n}{q^n-p^n} - pq\frac{q^{n-1}+p^{n-1}}{q^n-p^n}\right] \\ &\quad - \frac{q-p}{(1-p)(1-q)} \\ &= \frac{q-p}{(1-p)(1-q)}\left[\frac{n(pq)^{n-1}(p+q)(1-p-q)}{(q^n-p^n)^2}\right] \\ &\quad - \left[\frac{1}{(1-p)^2} + \frac{1}{(1-q)^2}\right]\left[\frac{q^n+p^n-2pq(q^{n-1}+p^{n-1})}{2(q^n-p^n)}\right] \\ &\quad - \frac{q-p}{(1-p)(1-q)} \\ &= FH_1 - GH_2 - \frac{q-p}{(1-p)(1-q)}. \end{aligned}$$

Consequently,

$$h_1 + h_2 + h_3 = F(1+H_1) - G(1+H_2) - \frac{q-p}{(1-p)(1-q)}.$$

The following two lemmas will be established in Section 3.

Lemma 2. *For all* $0 < p < q$, *we have*

$$0 < F < G.$$

Lemma 3. *For all* $0 < p < q$, *we have*

$$0 < 1 + H_1 < 1 + H_2.$$

Combining Lemmas 2 and 3, we obtain $F(1+H_1) - G(1+H_2) < 0$. Therefore,

$$h_1 + h_2 + h_3 = F(1+H_1) - G(1+H_2) - \frac{q-p}{(1-p)(1-q)} < 0.$$

This proves Theorem 1.

3. Proofs of Lemmas 1–3

Proof of Lemma 1. Using the expression for α given in (1),

$$\frac{2\alpha - 1}{p-q} = \frac{[(1-q)+(1-p)]\left[p^{2n-1}(1-q) + q^{2n-1}(1-p)\right] - 2(1-p)(1-q)(p+q)(pq)^{n-1}}{(2-p-q)(p-q)\left[p^{2n-1}(1-q) - q^{2n-1}(1-p)\right]}$$

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$$
\begin{aligned}
&= \frac{[(1-q)-(1-p)]\left[p^{2n-1}(1-q)-q^{2n-1}(1-p)\right]}{(2-p-q)(p-q)\left[p^{2n-1}(1-q)-q^{2n-1}(1-p)\right]} \\
&\quad + \frac{2(1-p)(1-q)\left(p^{2n-1}+q^{2n-1}\right)-2(1-p)(1-q)(p+q)(pq)^{n-1}}{(2-p-q)(p-q)\left[p^{2n-1}(1-q)-q^{2n-1}(1-p)\right]} \\
&= \frac{(p-q)\left[p^{2n-1}(1-q)-q^{2n-1}(1-p)\right]+2(1-p)(1-q)(p^n-q^n)(p^{n-1}-q^{n-1})}{(2-p-q)(p-q)\left[p^{2n-1}(1-q)-q^{2n-1}(1-p)\right]} \\
&= \frac{1}{2-p-q}\left[1+2\,\frac{1}{p-q}\,\frac{(1-p)(1-q)(p^n-q^n)(p^{n-1}-q^{n-1})}{p^{2n-1}(1-q)-q^{2n-1}(1-p)}\right] \\
&= \frac{1+2f}{2-t}.
\end{aligned}
$$

□

Proof of Lemma 2. For all $p < q$, it is immediate that

$$F = \frac{q-p}{(1-p)(1-q)} \cdot \frac{(n-1)(pq)^{n-2}(p+q)}{(q^{n-1}-p^{n-1})^2} > 0.$$

By the arithmetic–geometric mean inequality,

$$\frac{1}{2}\left[\frac{1}{(1-p)^2}+\frac{1}{(1-q)^2}\right] \geq \frac{1}{(1-p)(1-q)}.$$

Moreover,

$$
\begin{aligned}
\frac{q^{2n-2}-p^{2n-2}}{q^2-p^2} &= \sum_{i=0}^{n-2} q^{2(n-2-i)}p^{2i} \\
&\geq (n-1)\left(\prod_{i=0}^{n-2} q^{2(n-2-i)}p^{2i}\right)^{\frac{1}{n-1}} \\
&= (n-1)\left(q^{(n-1)(n-2)}p^{(n-1)(n-2)}\right)^{\frac{1}{n-1}} \\
&= (n-1)(pq)^{n-2}.
\end{aligned}
$$

Therefore,

$$\frac{1}{2}\left[\frac{1}{(1-p)^2}+\frac{1}{(1-q)^2}\right]\frac{q^{2n-2}-p^{2n-2}}{q^2-p^2} \geq \frac{(n-1)(pq)^{n-2}}{(1-p)(1-q)}.$$

Since $q > p$, it follows that

$$\frac{1}{2}\left[\frac{1}{(1-p)^2}+\frac{1}{(1-q)^2}\right]\frac{q^{n-1}+p^{n-1}}{q^{n-1}-p^{n-1}} > \frac{q-p}{(1-p)(1-q)} \cdot \frac{(n-1)(pq)^{n-2}(p+q)}{(q^{n-1}-p^{n-1})^2}.$$

That is,

$$G > F.$$

□

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Proof of Lemma 3. From (6),

$$1 + H_1 = \frac{(n-1)(q^n - p^n)^2 + npq(1-p-q)(q^{n-1} - p^{n-1})^2}{(n-1)(q^n - p^n)^2}.$$

Its numerator is

$$\begin{aligned}
&(n-1)(q^n - p^n)^2 + npq(1-p-q)(q^{n-1} - p^{n-1})^2 \\
&\quad \geq (n-1)(q^n - p^n)^2 - np^2q(q^{n-1} - p^{n-1})^2 \\
&\quad \geq (n-1)(q^n - p^n)^2 - n(pq^{n-1} - p^n)^2 \\
&\quad \geq (n-1)(q^n - p^n)(pq^{n-1} - p^n) - n(pq^{n-1} - p^n)^2 \\
&\quad = (pq^{n-1} - p^n)\left[(n-1)(q^n - p^n) - n(pq^{n-1} - p^n)\right] \\
&\quad = (pq^{n-1} - p^n)\left[(n-1)(q^n - pq^{n-1}) - p(q^{n-1} - p^{n-1})\right] \\
&\quad = (pq^{n-1} - p^n)(q-p)\left[(n-1)q^{n-1} - p\sum_{i=0}^{n-2} q^{n-2-i}p^i\right] > 0.
\end{aligned}$$

Hence, $1 + H_1 > 0$. It remains to show that $1 + H_1 < 1 + H_2$.
By (6) and (7),

$$\begin{aligned}
&H_2 - H_1 \\
&= \frac{(q^{n-1} - p^{n-1})\left[q^n + p^n - 2pq(q^{n-1} + p^{n-1})\right]}{(q^n - p^n)(q^{n-1} + p^{n-1})} - \frac{npq(1-p-q)(p^{n-1} - q^{n-1})^2}{(n-1)(q^n - p^n)^2} \\
&= \frac{q^{n-1} - p^{n-1}}{(n-1)(q^n - p^n)^2\left(q^{n-1} + p^{n-1}\right)} \cdot \\
&\quad \left\{(n-1)(q^n - p^n)\left[(q^n + p^n) - 2pq\left(q^{n-1} + p^{n-1}\right)\right] - npq(1-p-q)\left(q^{2n-2} - p^{2n-2}\right)\right\} \\
&= \frac{q^{n-1} - p^{n-1}}{(n-1)(q^n - p^n)^2\left(q^{n-1} + p^{n-1}\right)} \cdot \\
&\quad \left\{\left[(n-1)(q^{2n} - p^{2n}) - npq(q^{2n-2} - p^{2n-2})\right]\right. \\
&\quad \left. - pq\left[(n-2)(q^{2n-1} - p^{2n-1}) + 2(n-1)(pq)^{n-1}(q-p) - npq(q^{2n-3} - p^{2n-3})\right]\right\}.
\end{aligned}$$

We first rewrite the first term as

$$\begin{aligned}
&(n-1)(q^{2n} - p^{2n}) - npq(q^{2n-2} - p^{2n-2}) \\
&\quad = (q-p)\left[(n-1)\sum_{i=0}^{2n-1} q^{2n-1-i}p^i - npq\sum_{i=0}^{2n-3} q^{2n-3-i}p^i\right] \\
&\quad = (q-p)\left[(n-1)\left(q^{2n-1} + p^{2n-1}\right) - \sum_{i=1}^{2n-2} q^{2n-1-i}p^i\right] \\
&\quad = (q-p)\left[\sum_{i=1}^{n-1}\left(q^{2n-1} + p^{2n-1} - q^{2n-1-i}p^i - q^ip^{2n-1-i}\right)\right]
\end{aligned}$$

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$$= (q-p)\left[\sum_{i=1}^{n-1}\left(q^{2n-1-i}-p^{2n-1-i}\right)\left(q^i-p^i\right)\right].$$

For the second term, we have

$$\begin{aligned}
&pq\Big[(n-2)(q^{2n-1}-p^{2n-1})+2(n-1)(pq)^{n-1}(q-p)-npq(q^{2n-3}-p^{2n-3})\Big]\\
&\quad=(q-p)pq\left[(n-2)\sum_{i=0}^{2n-2}q^{2n-2-i}p^i+2(n-1)(pq)^{n-1}-npq\sum_{i=0}^{2n-4}q^{2n-4-i}p^i\right]\\
&\quad=(q-p)pq\left[(n-2)\left(q^{2n-2}+p^{2n-2}\right)-2\sum_{i=1}^{2n-3}q^{2n-2-i}p^i+2(n-1)(pq)^{n-1}\right]\\
&\quad=(q-p)pq\left[(n-2)\left(q^{2n-2}+p^{2n-2}\right)-2\sum_{i=1}^{n-2}\left(q^{2n-2-i}p^i+q^ip^{2n-2-i}\right)+2(n-2)(pq)^{n-1}\right]\\
&\quad=(q-p)pq\left[\sum_{i=1}^{n-2}\left(q^{2n-2}+p^{2n-2}-q^{2n-2-i}p^i-q^ip^{2n-2-i}\right)\right.\\
&\qquad\qquad\left.-\sum_{i=1}^{n-2}\left(q^{2n-2-i}p^i-q^ip^{2n-2-i}-2(pq)^{n-1}\right)\right]\\
&\quad=(q-p)pq\left[\sum_{i=1}^{n-2}\left(q^{2n-2-i}-p^{2n-2-i}\right)\left(q^i-p^i\right)-\sum_{i=1}^{n-2}\left(q^{n-1-\frac{i}{2}}p^{\frac{i}{2}}-q^{\frac{i}{2}}p^{n-1-\frac{i}{2}}\right)^2\right]\\
&\quad<(q-p)pq\left[\sum_{i=1}^{n-2}\left(q^{2n-2-i}-p^{2n-2-i}\right)\left(q^i-p^i\right)\right]\\
&\quad<(q-p)\left[\sum_{i=1}^{n-2}\left(q^{2n-1-i}-p^{2n-2-i}q\right)\left(q^i-p^i\right)\right].
\end{aligned}$$

Therefore,

$$\begin{aligned}
&H_2-H_1\\
&\quad>\frac{(q^{n-1}-p^{n-1})(q-p)}{(n-1)(q^n-p^n)^2\left(q^{n-1}+p^{n-1}\right)}\\
&\qquad\left[\sum_{i=1}^{n-1}\left(q^{2n-1-i}-p^{2n-1-i}\right)\left(q^i-p^i\right)-\sum_{i=1}^{n-2}\left(q^{2n-1-i}-p^{2n-2-i}q\right)\left(q^i-p^i\right)\right]>0.
\end{aligned}$$

This completes the proof. □

Competing interest. The authors declare no competing interests.

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