Thermal rockets 5

The general principle of a rocket is to expel propellant in one direction, thereby forcing the rocket to move in the opposite via conservation of momentum. Most rockets built so far are propelled by burning a fuel to produce a gas jet. In this chapter we will see how a convergent–divergent nozzle is used to accelerate the gas to a large velocity by maximizing the conversion of its thermal energy (random motion) into kinetic energy (linear motion). We will examine the characteristics of liquid and solid fuels, and look at a few examples of real rockets that use them.

5.1 Rocket thermodynamics

The basic idea of a thermal rocket is to heat a gas to a high temperature and pressure in a combustion chamber that has a hole at one end (figure 5.1). Beyond the hole is either the near-vacuum of space or the atmosphere of a planet. Provided the pressure of the gas in the combustion chamber is higher than the ambient pressure, the gas escapes through the hole. This gas has momentum relative to the rocket, so by conservation of momentum the rocket acquires equal and opposite momentum. The microscopic mechanism for this momentum transfer is the collisions of the gas particles with the walls of the combustion chamber. For an axisymmetric combustion chamber, the collisions perpendicular to the axis impart no net momentum on the rocket. Because the hole is only at one end of the rocket, there is a net momentum exchange along the axis.

Although this approach imparts momentum on the rocket, we will see that passing the gas through a specially shaped nozzle can accelerate the gas to a larger velocity. It does this by extracting more thermal energy from the gas.

Let us first consider how the properties of a gas change as it moves along a nozzle or pipe without yet specifying its cross-sectional shape. Inside the combustion chamber, the hot gas is at high pressure, whereas at the end of the nozzle the gas expands into a lower pressure environment. If this change takes place rapidly, we can approximate it as an adiabatic process, one in which there is no heat exchange with the environment (in particular the walls of the nozzle). We also assume there is no energy dissipation, so it is also a reversible process, and thus isentropic.

As the gas moves along the nozzle, then depending on the nozzle's shape, the gas will be compressed or expanded, resulting in a change in its pressure p, temperature T, and density ρ (a compressible flow). Assuming the gas moves steadily along the nozzle without turbulence, we can also consider its bulk velocity u at any point along the nozzle. To determine how these quantities relate, we consider the total specific energy E (i.e. per unit mass) of the gas. This is the sum of all the different specific energies involved. These are the internal energy U, the

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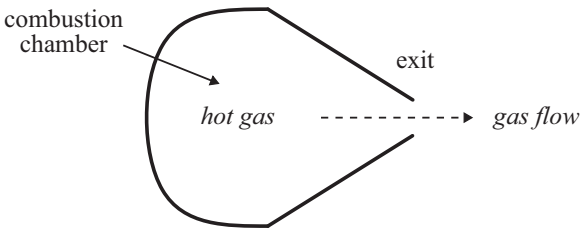


Fig. 5.1 A simple thermal rocket consists of a combustion chamber in which a gas is heated and can escape from a hole at one end.

kinetic energy $K = \frac{1}{2}u^2$, and the ability to do work (arising from the finite entropy of the gas) $\int pdV$, which we write as pV, where V is the specific volume (inverse density). Thus

$$E = U + pV + K. \tag{5.1}$$

As we are considering an adiabatic process and assuming the gas is not doing work on any external bodies (the sides of the nozzle are rigid), then E is conserved along the flow. At this point it is useful to introduce the specific enthalpy, which is defined as

$$H = U + pV. \tag{5.2}$$

The change in energy between any two points along the nozzle is then

$$\Delta E = \Delta H + \Delta K = 0. \tag{5.3}$$

The reason for introducing the enthalpy is that it is directly related to a quantity we can measure. If the gas changes its temperature by ΔT then

$$\Delta H = c_p \Delta T, \tag{5.4}$$

where c_p is the specific heat capacity of the gas at constant pressure.

The gas in the combustion chamber is well mixed with a single temperature T_c and is on average stationary relative to the rocket, that is, $u_c = 0$. As the pressure in the combustion chamber is higher than the ambient pressure at the exit, the gas will flow along the nozzle. Let the temperature and velocity of the gas at some arbitrary point be T and u, respectively. It follows from equations 5.3 and 5.4 that

$$c_p(T_c - T) + \frac{1}{2}u_c^2 - \frac{1}{2}u^2 = 0 \tag{5.5}$$

so

$$u^2 = 2c_p(T_c - T). \tag{5.6}$$

This tells us that if the gas temperature drops along the nozzle, the gas will accelerate. The gas gains kinetic energy (K) at the expense of its internal energy (U) through conservation of energy. The hotter the gas in the combustion chamber, and the more we can get the gas to cool by the time it leaves the nozzle, the faster the gas, and thus the larger the change in momentum of the rocket. Indeed, the velocity at the exit of the nozzle is, in an ideal world, the effective exhaust velocity of the rocket that appears in the rocket equation (equation 3.4). In reality, the exhaust velocity will be lower due to losses in the nozzle arising from heat transfer, viscosity, and imperfect collimation of the exiting gas flow.

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It is neither immediately obvious what the temperature T is, nor whether it has to drop along the nozzle, so it is more insightful to write equation 5.6 in terms of other properties of the gas. To do this we assume an ideal gas, the equation of state for which is $p = \rho RT/\mathcal{M}$, where $\mathcal{M}$ is the molar mass ($\mathrm{kg\,mol^{-1}}$) of the gas and R is the universal gas constant ($\mathrm{J\,mol^{-1}\,K^{-1}}$). For an ideal gas undergoing reversible, adiabatic changes we also have the relation

$$Tp^{(1-\gamma)/\gamma} = \text{constant}, \tag{5.7}$$

where γ is the ratio of the specific heat capacity at constant pressure to that at constant volume, which is always larger than one. This relation tells us that if the gas pressure decreases from combustion chamber to the outside, so does the gas temperature. To avoid having both γ and c_p in our equations, we can write the latter as

$$c_p = \frac{\gamma}{\gamma - 1}\frac{R}{\mathcal{M}}. \tag{5.8}$$

Substituting equations 5.7 and 5.8 into equation 5.6 we can now write the latter as

$$u^2 = \frac{2\gamma}{\gamma - 1}\frac{RT_c}{\mathcal{M}}\left[1 - \left(\frac{p}{p_c}\right)^{(\gamma-1)/\gamma}\right]. \tag{5.9}$$

This gives the velocity of an ideal gas under the assumption of a steady isentropic flow along a nozzle when the pressure has decreased from p_c to p. $\mathcal{M}$ and γ are constant along the flow: the gas is assumed not to undergo any chemical changes after the combustion (and we assume γ does not vary with temperature). Figure 5.2 plots how this gas velocity varies as a function of the pressure ratio for a given gas. With all other quantities held constant, this shows that the exit velocity is maximized by reducing the pressure at the exit to the lowest value possible. This can be achieved by operating the rocket in a vacuum.

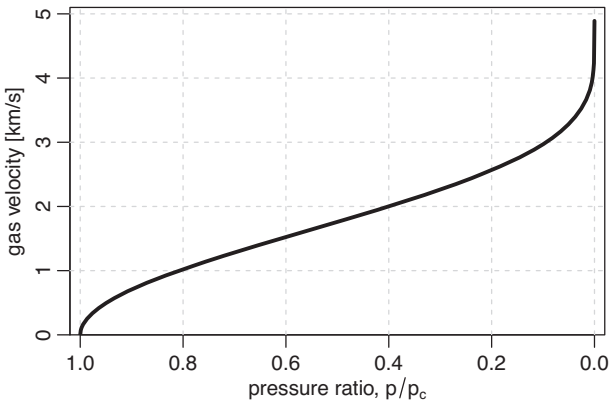


Fig. 5.2 The variation of gas velocity in a nozzle as a function of the ratio of the pressure in the gas to the pressure in the combustion chamber (equation 5.9) for $\gamma = 1.25$, $\mathcal{M} = 0.0113\ \mathrm{kg\,mol^{-1}}$, $T_c = 3250$ K (applicable for the combustion of hydrogen in oxygen). Note that the horizontal axis is flipped. The pressure p could represent either the pressure at the exit of the nozzle or the pressure at some point along the nozzle.

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When the ambient pressure is essentially zero and the gas pressure drops to this value at the exit of the nozzle, then the exit velocity is

$$u_{\max} = \sqrt{\frac{2\gamma}{\gamma - 1}\frac{RT_c}{\mathcal{M}}}. \tag{5.10}$$

This expression shows that there are various ways to maximize the exit velocity:

- raise the temperature in the combustion chamber (T_c);
- use a lighter propellant ($\mathcal{M}$);
- use a propellant with a smaller value of γ, because $\gamma/(\gamma - 1)$ increases with decreasing γ and $\gamma > 1$. The quantity γ is determined by the number of degrees of freedom f of the gas via $\gamma = 1 + 2/f$. For an ideal monatomic gas $f = 3, \gamma = 5/3$ $(= 1.67)$ and for a diatomic gas $f = 5, \gamma = 7/5$ $(= 1.4)$. More complicated molecules have higher values of f (lower values of γ). To maximize u we therefore want to have a gas with lots of degrees of freedom. Rocket fuel exhausts are often polyatomic with an average value of $f = 8, \gamma = 5/4$ $(= 1.25)$, although for a real gas γ varies with temperature.

Note that γ and $\mathcal{M}$ refer to properties of the exhaust gases, the products of combustion, rather than the fuel that is burned to make them.

An ideal gas has two independent properties, for example p and T, which then determine ρ. Under the additional assumption that the gas behaves adiabatically, these two properties are no longer independent. We see from equation 5.7 how they vary, although this is more explicit if we differentiate:

$$\frac{dT}{T} = \left(\frac{\gamma - 1}{\gamma}\right)\frac{dp}{p}. \tag{5.11}$$

As $\gamma > 1$, this means that changes in p and T for an adiabatic process are positively correlated. We can also write the adiabatic relationship in terms of pressure and density as

$$\rho p^{-1/\gamma} = \text{constant} \tag{5.12}$$

and differentiating this gives

$$\frac{dp}{p} = \gamma\frac{d\rho}{\rho}, \tag{5.13}$$

showing that pressure and density are also positively correlated. Thus p, T, and ρ all vary in a common direction.

5.2 The convergent–divergent nozzle

Equation 5.9 shows that the lower the gas pressure at the nozzle exit, the larger the gas velocity. How can we achieve this low gas pressure? Just because the ambient pressure is low (or even zero), this does not mean the gas pressure at the exit equals this. The exiting gas could have a larger pressure, in which case its velocity would not be as large.

To arrange for a low exit gas pressure, we need to consider the geometry of the nozzle. To do this we examine the mass flow rate ($\mathrm{kg\,s^{-1}}$) of the gas at any point along the nozzle,

$$\dot{m} = \rho A u, \tag{5.14}$$

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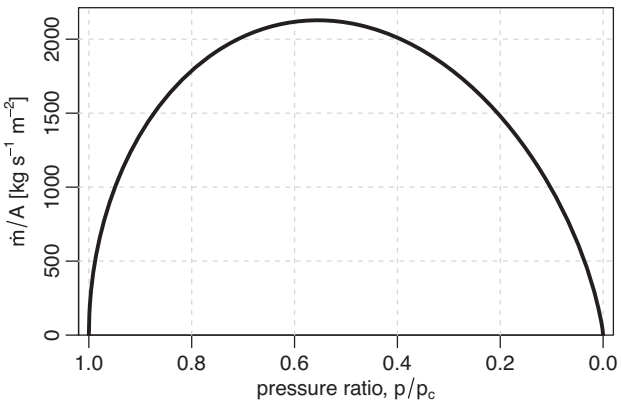


Fig. 5.3 The variation of the mass flow rate per unit cross-sectional area in a nozzle as a function of the gas pressure expressed as a fraction of the combustion chamber pressure (equation 5.17) for $\gamma = 1.25$, $\mathcal{M} = 0.0113$ kg mol^{-1}, $T_c = 3250$ K, and $p_c = 5$ MPa. Note that the horizontal axis is flipped.

where A is the cross-sectional area of the nozzle. We can write the density in terms of the pressure using the adiabatic gas relation

$$\frac{\rho}{\rho_c} = \left(\frac{p}{p_c}\right)^{1/\gamma}. \tag{5.15}$$

Using the ideal gas law, the density in the combustion chamber is

$$\rho_c = \frac{\mathcal{M}}{R}\frac{p_c}{T_c}. \tag{5.16}$$

Putting these and the expression for u from equation 5.9 into equation 5.14 allows us to write the mass flow rate per unit cross-sectional area as

$$\frac{\dot{m}}{A} = p_c\sqrt{\frac{2\gamma}{\gamma-1}\frac{\mathcal{M}}{RT_c}\left(\frac{p}{p_c}\right)^{2/\gamma}\left[1-\left(\frac{p}{p_c}\right)^{(\gamma-1)/\gamma}\right]}. \tag{5.17}$$

This is plotted as a function of the pressure ratio in figure 5.3. We see that as the pressure ratio drops, $\dot{m}/A$ initially increases, hits a maximum, and then decreases. The pressure ratio on the horizontal axis could be interpreted as the exit to chamber ratio or as the ratio at any point along the nozzle. In either case the mass of the propellant is conserved, so $\dot{m}$ must be constant along the nozzle. Thus equation 5.17 tells us how the cross-sectional area of the nozzle must vary in order to achieve a low gas pressure – hence large velocity – at the exit.

Moving away from the combustion chamber, the cross-sectional area of the nozzle must initially decrease (a converging nozzle). As it does so, the gas pressure decreases and the gas velocity increases (figure 5.2). The smallest area corresponds to the maximum in figure 5.3. If this was the exit of this nozzle (a purely converging nozzle), then the corresponding pressure ratio determines the velocity of the gas at the exit. Even if the ambient pressure were lower than this gas pressure, the gas velocity would be no higher. (The gas would then expand further outside the rocket nozzle, but this would not transfer any momentum to the spacecraft.)

To achieve a lower pressure ratio, figure 5.3 shows that the area of the nozzle must now increase. Continuing along this now diverging nozzle, shown in figure 5.4, the pressure of

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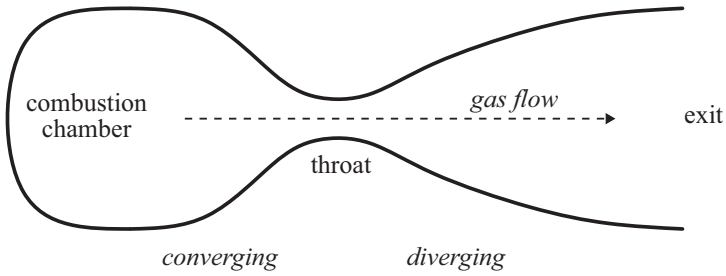


Fig. 5.4 A convergent–divergent nozzle, also known as a de Laval nozzle.

the gas continues to decrease, and the velocity increases further (figure 5.2 again). Ideally the exit of the nozzle would be at the point where its cross-sectional area corresponds to the gas pressure equalling the ambient pressure.

Thus to achieve the largest drop in pressure (and temperature) of the gas, and therefore achieve the largest exit velocity of the gas, we should use a convergent–divergent nozzle together with a sufficiently large combustion chamber to exit pressure ratio. If we use only a convergent nozzle, then the gas will not expand by a large amount within the nozzle – only up to the maximum in figure 5.3, even if the ambient pressure is zero – and so the exit velocity will be limited.

I have not said anything about how the nozzle area should vary with physical distance along the axis. Determining that requires a more detailed analysis, and a design which ensures that the flow remains steady and isentropic without turbulence. The key thing for us is that it converges then diverges.

Let us explore in more detail how the nozzle shape is producing a high velocity gas. The narrowest part of the nozzle, where the mass flow rate per unit area is largest, is called the *throat*. Differentiating equation 5.17 with respect to the pressure ratio we find that the pressure at the throat p_t is

$$\frac{p_t}{p_c} = \left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)}. \tag{5.18}$$

This ratio only depends on γ. The ratio does not vary much: for $\gamma = 1.25$ (polyatomic gas) it is 0.555 and for $\gamma = 1.4$ (diatomic gas) it is 0.528. The corresponding mass flow rate per unit area at the throat is

$$\frac{\dot{m}}{A_t} = p_c\sqrt{\gamma\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}\frac{\mathcal{M}}{RT_c}}. \tag{5.19}$$

It is instructive to compute also the velocity of the gas at the throat. Putting the expression for c_p (equation 5.8) into equation 5.6, the squared gas velocity at any point along the nozzle is

$$u^2 = \frac{2\gamma}{\gamma-1}\frac{R}{\mathcal{M}}T\left(\frac{T_c}{T}-1\right). \tag{5.20}$$

Using equation 5.7 we can convert the pressure ratio at the throat in equation 5.18 into a temperature ratio:

$$\frac{T_c}{T_t} = \frac{\gamma+1}{2}. \tag{5.21}$$

Putting this into equation 5.20 with $T = T_t$, we find that the gas velocity in the throat is

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$$u_t = \sqrt{\frac{\gamma R T_t}{\mathcal{M}}}. \tag{5.22}$$

This is equal to the local speed of sound in the gas, given in general by

$$a = \sqrt{\frac{\gamma R T}{\mathcal{M}}}. \tag{5.23}$$

This means that if the pressure ratio in the gas drops low enough such that $\dot{m}/A$ reaches the maximum in figure 5.3, then the gas velocity in the throat will reach the speed of sound. This in turn can only occur if the throat is narrow enough. If the maximum of the curve is not reached, then the gas will remain subsonic at the throat, and equations 5.18, 5.19, 5.21, and 5.22 do not apply.

Whether or not the gas continues to accelerate after the throat depends on whether it reaches the speed of sound at the throat. To see this, we need to relate changes in the velocity of the gas to changes in the area of the nozzle. As the mass flow rate $\dot{m}$ is constant, the derivative of equation 5.14 is

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{du}{u} = 0. \tag{5.24}$$

To eliminate the density from this we first combine equations 5.11 and 5.13 to get

$$\frac{dT}{T} = (\gamma - 1)\frac{d\rho}{\rho}. \tag{5.25}$$

Differentiating equation 5.6 gives $udu = -c_p dT$, which we then use together with equation 5.8 to eliminate dT in the above equation to get

$$\frac{d\rho}{\rho} = -\frac{u^2}{(\gamma - 1)c_p T}\frac{du}{u} = -\frac{u^2}{a^2}\frac{du}{u} = -M^2\frac{du}{u}, \tag{5.26}$$

where $M = u/a$ is the *Mach number*, the ratio of speed of the gas to the speed of sound in the gas. Substituting this into equation 5.24 finally gives an expression relating velocity to area anywhere along the nozzle in terms of just the Mach number,

$$\frac{du}{u} = \frac{1}{M^2 - 1}\frac{dA}{A}. \tag{5.27}$$

Equation 5.27 shows that when $M < 1$ (subsonic flow) then $M^2 - 1$ is negative, so an increase in the area causes a decrease in the velocity, and a decrease in area causes an increase in the velocity. This is characteristic also of incompressible fluids (as can be seen from equation 5.24 when $d\rho = 0$). In contrast, when $M > 1$ (supersonic flow) then $M^2 - 1$ is positive, so an increase in the area causes an increase in the velocity, and a decrease in area causes a decrease in the velocity. Recall that the velocity always has the opposite trend from the pressure (figure 5.2).

In the converging part of the nozzle, corresponding to the left of the maximum in figure 5.3, the velocity is always subsonic and so the velocity increases as the gas moves down the nozzle towards the throat. There are then two alternative outcomes arising from equation 5.27:

1. If the pressure at the throat is low enough that the gas attains the maximum in figure 5.3, then the gas reaches $M = 1$. When it infinitesimally exceeds this speed, the flow is supersonic, and so now an increase in the area of the nozzle (the diverging part) will result in a further increase in the velocity. This corresponds to the part of figure 5.3 to the right of the maximum.

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2. If instead the pressure at the throat is too high, such that the gas remains subsonic, then in the diverging part of the nozzle the gas velocity decreases again (and the pressure increases). This corresponds to the part of figure 5.3 to the left of the maximum again.

Which of these two conditions occurs for a given nozzle depends on the pressure ratio at the throat.

Thus a convergent–divergent nozzle will accelerate the gas to its highest velocity only if the gas reaches the speed of sound at the throat. This in turn occurs only if the nozzle is narrow enough and if the ambient pressure is low enough that the maximum in figure 5.3 can be attained. As we saw from equation 5.18, this requires the pressure at the throat to drop to about half the pressure in the combustion chamber.

Provided the gas hits the speed of sound at the throat, it continues to accelerate in the diverging part of the nozzle. The temperature of the gas continues to drop – this is the source of the increasing velocity of the gas – so the speed of sound also drops, and thus the gas becomes increasing supersonic.

When the system is configured such that the gas reaches $M = 1$ at the throat, the gas will not exceed $M = 1$ at the throat, even if the combustion chamber pressure is increased or the exit pressure decreased. If the chamber pressure is increased, then the pressure at the throat will increase and so the actual velocity at the throat will increase, but this is still the local speed of sound (equations 5.21 and 5.22). In this condition the flow is said to be *choked*. The velocity of the gas at the exit can still be increased by lowering the exit pressure (from equation 5.9). But once the flow is choked, the mass flow rate cannot be increased by lowering the exit pressure, as we see from equation 5.19. Because the flow is choked, changes in the nozzle downstream of the throat do not affect anything upstream of it. The exit velocity and mass flow rate can still be raised by increasing the pressure in the combustion chamber.

To help illustrate the above concepts, figure 5.5 summarizes various scenarios for gas flow along the nozzle. The figure shows how the pressure ratio p/p_c varies along the nozzle for several values of the ambient pressure p_a, which is lowered monotonically as we proceed from scenario (a) to scenario (g).

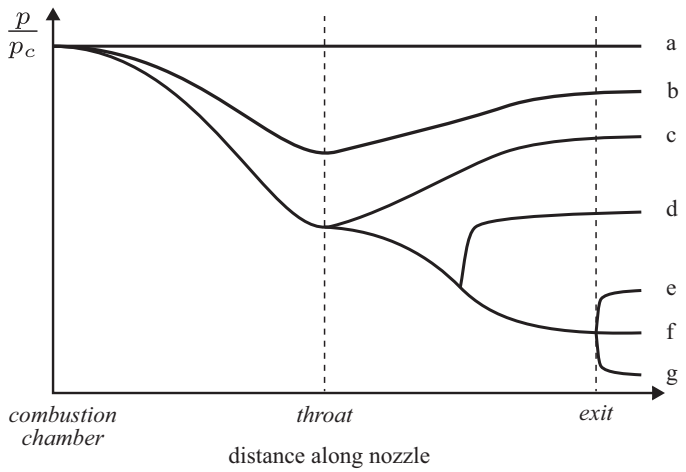


Fig. 5.5 A sketch of the variation of the gas pressure p along a nozzle relative to the pressure in the combustion chamber p_c for various scenarios (a) to (g) described in the text. In all scenarios the flow upstream of the throat is subsonic.

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(a) $p_a = p_c$. The pressure is constant everywhere. There is no flow.
(b) $p_a < p_c$. The gas now flows. The velocity increases in the converging part of the nozzle, but it remains subsonic throughout, so the pressure increases (and velocity decreases) again in the diverging part of the nozzle in accordance with equation 5.27.
(c) p_a is lowered further to the critical value that attains $M = 1$ in the throat, the maximum in figure 5.3. The flow is choked. Downstream of the throat, the flow is still subsonic and the pressure rises. In all following scenarios, the flow in the converging part of the nozzle remains the same as for this choked flow.
(d) Lowering p_a further puts the downstream flow into the supersonic condition. The pressure continues to decrease initially in the diverging part of the nozzle. But the ambient pressure is too large for the isentropic conditions to hold to the end of the nozzle, so at some point a shock wave forms, after which there is a sharp increase in the gas pressure and a decrease to subsonic velocities.
(e) p_a is now low enough that isentropic conditions hold throughout the nozzle, so the gas remains supersonic up until the exit (this is also the case for the next two scenarios). But the ambient gas pressure is still larger than the gas exit pressure – the flow is said to be 'over-expanded' – so undergoes a sudden increase in pressure at the exit, causing a sharp contraction of the gas.
(f) The gas exit pressure matches the ambient pressure and so flows smoothly out with the width of the nozzle at a supersonic speed.
(g) The ambient pressure is lower than the emerging gas pressure, so the gas will continue to expand outside the nozzle. The flow is said to be 'under-expanded'.

A rocket nozzle is designed to operate in condition (f). When the ambient pressure is negligible, the exhaust velocity is given by equation 5.10 and the mass flow rate by equation 5.19. These are the two key parameters for the rocket and thrust equations discussed in chapter 3.

Rockets launched from the Earth's surface experience a large ambient pressure at first. A nozzle used here should have a relatively large exit pressure and thus be relatively short in order to achieve condition (f). Higher in the atmosphere where the pressure is lower, the exit pressure should be lower and thus a longer nozzle is optimal. As a nozzle cannot be continuously adjusted during flight, it may be too wide and long at launch, in which case the exit gas pressure is too low. In this case we see the exhaust emerging in a column that is narrower than the nozzle exit, due to the over-expanded condition (e). In contrast, when the rocket is at high altitudes where the atmospheric pressure is lower, the nozzle may be too narrow and short, so the exit gas pressure is too high. We now see the exhaust expanding into a broad bell-shaped plume that is wider than the nozzle exit due to the under-expanded condition (g). This is another reason why rocket staging increases the efficiency of the rocket: the nozzle for each stage can be optimized for the average ambient pressure in which it operates.

5.3 The rocket as a heat engine

As a rocket converts thermal energy into mechanical energy it is a type of heat engine. Figure 5.6 shows the idealized pressure–volume diagram for a thermal rocket. Point 1 indicates where the fuel enters the combustion chamber. The fuel ignites, releasing heat and

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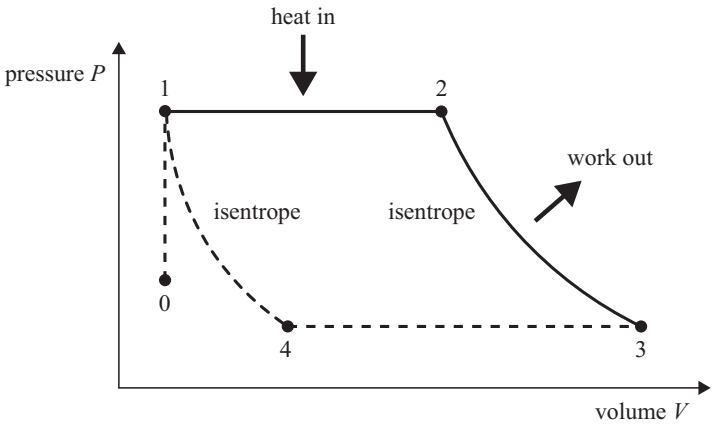


Fig. 5.6 The pressure–volume diagram of a thermal rocket engine. The engine performs the open sequence 1–2–3. A gas turbine (jet engine) performs the open sequence 4–1–2–3. If this is closed by returning isentropically to 4, then we have the Brayton cycle.

expanding at the constant chamber pressure to point 2. From points 2 to 3 the gas is expelled via the nozzle at constant entropy, doing work to propel the rocket and cooling in the process.

The rocket engine has an open cycle, in contrast to the closed cycles we may be more familiar with for internal combustion engines, such as the Otto or Carnot cycles. If, at point 3, the gas were compressed at constant pressure to point 4 (removing heat from the engine in the process), and then compressed at constant entropy back to point 1, we would have the closed Brayton cycle. An important difference between the open cycle of a rocket and the closed cycle of a four-stroke internal combustion engine is that in the rocket the PV diagram sequences occur simultaneously, whereas in the internal combustion engine they occur sequentially.

The open sequence 4–1–2–3 in figure 5.6 is that of an ideal gas turbine jet engine, where 4–1 corresponds to the intake and compression of ambient air. In a rocket, the step 0–1 can be considered as the compression of the fuel in the pumps before it reaches the combustion chamber, which is done at almost constant volume.

5.4 Chemical rockets

In discussing the principle of a thermal rocket above, we started with a hot gas in the combustion chamber. How this gas is heated is not relevant to the behaviour of the nozzle. All significant rockets launched from the Earth to date have used chemical engines, in which chemicals are combusted to produce a hot propellant. This requires that an oxidant be carried in addition to the fuel.

A common fuel is hydrogen, which when mixed with oxygen and combusted produces water. This water vapour is the exhaust that is then accelerated by the nozzle. In chemical rockets it is usual that the fuel (and oxidizer) is the source of both the energy and the mass that propels the rocket. This does not have to be the case, as we shall see with ion engines

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in chapter 6. In some rockets, additional hydrogen is mixed into the propellant after the combustion. This hydrogen is not burned, but rather acts to lower the mean molar mass of the propellant. Because of equation 5.9, this increases the exhaust velocity (assuming T_c is kept constant).

A brief note on terminology. With chemical thermal rockets it is common to refer to the fuel and oxidizer together (prior to combustion) as the propellant, and to refer to the product of the reaction that is exhausted through the nozzle as the exhaust. In more general contexts it is common to refer to the exhausted product as the propellant (for example in the context of nuclear rockets). I mostly use the latter definition, but it should be clear from the context what is meant.

There are various types of chemical rocket fuel, best divided into liquid and solid fuels. One of the most important criteria of a rocket fuel is its energy per unit mass, in order to achieve the largest exhaust velocity for the lowest possible mass. Also important is the mean molar mass. Although a larger mass lowers the exhaust velocity, it increases the mass flow rate, and so can increase the thrust overall (see equation 3.10). This is particularly important for launch when the gravity of the Earth must be overcome (see section 3.10). Other criteria are important too. These include: a high density so that the fuel does not take up too much volume, because large containers require more mass; complexity and cost of production; ease and safety of handling and storage.

5.4.1 Liquid fuels

In liquid engines, the fuel and oxidizer are stored in separate tanks. They are pumped into the combustion chamber where they are mixed and ignited. The rate of pumping can be used to control the mass flow rate (throttling) and thus the thrust of the rocket.

A common fuel–oxidizer combination is liquid hydrogen and liquid oxygen (LOX). The temperature achieved by their combustion depends on their mixture ratio. With an oxygen-to-hydrogen mass mixture ratio of 4.8, the reaction achieves $T_c = 3250\,\mathrm{K}$.[1] If the exhausted propellent were entirely water vapour, with molar mass $\mathcal{M} = 0.018\,\mathrm{kg\,mol^{-1}}$ and ratio of specific heats $\gamma = 1.31$, then equation 5.9 tells us that the exhaust velocity into a vacuum ($p/p_c = 0$) would be $3.56\,\mathrm{km\,s^{-1}}$. However, at this mixture ratio, some of the hydrogen remains uncombusted.[2] This lowers $\mathcal{M}$ and raises γ. For real gases, values of $\mathcal{M} = 0.0113\,\mathrm{kg\,mol^{-1}}$ and $\gamma = 1.25$ are more representative for imperfect combustion (there is also OH and atomic O and H in the exhaust). With these values and $T_c = 3250\,\mathrm{K}$, equation 5.9 then predicts an exhaust velocity into a vacuum of $4.89\,\mathrm{km\,s^{-1}}$, but in practice it is closer to $4.55\,\mathrm{km\,s^{-1}}$ (also because the uncombusted hydrogen has to be heated, which lowers the temperature). A mixture ratio of eight would increase the combustion temperature – to about 3750 K – but the change in γ and $\mathcal{M}$ results in a lower exhaust velocity. The value of $4.55\,\mathrm{km\,s^{-1}}$ is more or less the maximum exhaust velocity that can be achieved with a hydrogen–oxygen mixture.

[1] The combustion temperature is actually higher than the value given, but some of the water produced dissociates, lowering the temperature.

[2] The oxygen atom is 16 times more massive than the hydrogen atom, so an O_2/H_2 mass ratio of eight is needed for complete combustion via $2H_2 + O_2 \rightarrow 2H_2O$.

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Hydrogen is a favoured fuel because it has a very large energy per unit mass of 120 MJ kg^{-1}.[3] However, hydrogen and oxygen are gases at room temperature and pressure, with low densities, so would require large and massive fuel tanks. They are therefore liquefied by cooling, increasing their density, although this makes them more complicated to store and handle.

Another commonly used liquid fuel is the hydrocarbon kerosene, or rather its highly refined variant RP-1. This too is combusted with liquid oxygen. Although this achieves a higher stable combustion temperature than hydrogen, it provides a lower exhaust velocity on account of the larger molar mass of its exhaust, a significant component of which is carbon dioxide. Typical values are 2.6–3.4 km s^{-1}. This larger mass is useful for obtaining a larger thrust. Another advantage of RP-1 over hydrogen is that although also highly flammable, it does not need cooling, so is overall easier to handle. Although RP-1 has a lower energy per unit mass than hydrogen (43 MJ kg^{-1}) it has a much larger mass density, giving it a higher energy per unit volume than hydrogen (35 MJ per litre vs 8.5 MJ per litre.) This means that smaller tanks can be used, which both have less mass and cause less atmospheric drag.

A third widely used liquid propellant is hydrazine, N_2H_4. This dissociates exothermically in the presence of a catalyst, such as iridium, so does not need an oxidizer. Such a monopropellant makes the engine design simpler. Typical exhaust velocities are 2.2–2.4 km s^{-1}. Like RP-1, hydrazine is liquid at room temperature, so is easier to handle and store than liquid hydrogen and oxygen. It still needs to be handled carefully, however, as it is very toxic. Hydrazine can also be used as a bipropellant, typically with nitrogen tetroxide (N_2O_4) as the oxidizer (also toxic). This gives a higher exhaust velocity, up to about 2.9 km s^{-1}. The reaction is hypergolic, meaning that combustion takes places spontaneously on contact between the fuel and oxidizer.

One disadvantage of hydrazine is that it freezes at around 2 °C, and partly for this reason derivatives of hydrazine with lower freezing points are more commonly used. These include monomethylhydrazine (MMH) and unsymmetric dimethylhydrazine (UDMH), which when oxidized with N_2O_4 can produce exhaust velocities of around 3.4 km s^{-1}.

While hydrogen–oxygen produces the largest exhaust velocity of rocket engines in general use, higher exhaust velocities can be achieved. Replacing oxygen with fluorine produces a higher combustion temperature (around 4200 K) and an exhaust velocity of around 4.8 km s^{-1}. Tripropellants offer even more possibilities. A mixture of lithium, fluorine, and hydrogen produces an exhaust velocity of 5.3 km s^{-1}, for example. However, fluorine is expensive and toxic, and the exhaust contains highly toxic hydrogen fluoride, making it very difficult to use for launch from the ground.

Other chemical reactions can release even more energy. Energy is required to dissociate molecular hydrogen, so the recombination of atomic hydrogen into its molecular form releases this energy. In principle this can give an exhaust velocity of nearly 21 km s^{-1}. However, the hydrogen needs to be stored at extremely low temperatures (a few kelvins) and in a strong magnetic field (several teslas), to prevent premature recombination, and this renders it impractical as a fuel. A more speculative idea is the use of metallic hydrogen. Under extremely large pressures, probably several hundred gigapascals, hydrogen is expected to

[3] As we need eight times as much mass of oxygen to completely burn hydrogen, the energy per unit mass for such a hydrogen–oxygen mixture is nine times lower.

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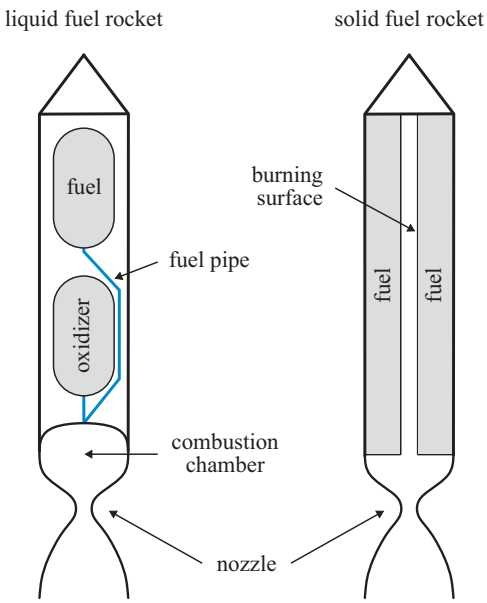
become metallic. When this pressure is released, the work energy put into the compression is also released, and is theoretically enough to achieve an exhaust velocity of $17\,\mathrm{km\,s^{-1}}$. Another idea is to store helium in an excited state. The energy released from the decay back to the ground state would theoretically achieve an exhaust velocity of $31\,\mathrm{km\,s^{-1}}$. However, even when stored at very low temperatures, the lifetime of the excited state is only a few hours, making this effectively unusable.

5.4.2 Solid fuels

Liquid fuels need to be pumped into the combustion chamber at the right rate to achieve the required combustion conditions. This requires not only separate fuel tanks but also a propellant delivery system. Some of the fuels are also quite complicated to handle, such as cryogenic hydrogen and oxygen.

Solid fuels overcome these problems. The fuel and oxidizer are mixed in solid form in a long tube and then ignited. The fuel burns from the inside out along the length of the fuel block, so at any one time the inside surface around the axis is undergoing combustion (see figure 5.7). Thermodynamically, solid fuels behave just like liquid ones, in the sense that they produce a hot gas that is expelled through a hole into the convergent–divergent nozzle at the bottom of the fuel block.

A typical solid propellant is a mixture of ammonium perchlorate, aluminium powder, and a polymeric binder, which together contains fuel and oxidant in one material. This is often referred to as ammonium perchlorate composite propellant (APCP). Aluminium has a slightly lower energy per unit mass than RP-1, but a higher mass density and so a larger energy per unit volume (84 MJ per litre vs 35 MJ per litre). APCP can achieve an exhaust velocity of around $2.7\,\mathrm{km\,s^{-1}}$. Other materials can also be used, such as zinc–sulphur, although this gives a lower exhaust velocity.



A comparison of a liquid rocket engine with a solid rocket engine. Fig. 5.7

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Solid-fuelled rockets predate liquid ones by many centuries. The earliest known solid-fuelled rockets were used in thirteenth-century China, propelled by gunpowder, which is a mixture of sulphur, charcoal, and potassium nitrate (the latter contains the oxidizer). The first liquid-fuelled rocket (using petrol and liquid oxygen), in contrast, was not launched until 1926.

As it does not need any pumps and pipelines, a solid-fuelled rocket engine is simpler than a liquid-fuelled one, making it cheaper and more reliable. One disadvantage of solid-fuelled rocket engines is that they normally cannot be throttled or switched off. One ignited, they burn until exhausted, meaning they also cannot be individually tested. The lack of pumps also means it is harder to achieve a stable thrust. For this reason the geometry of the fuel pellets and combustion surface must be carefully designed.

Prior to ignition the solid fuel is stable, so it can be stored for long periods. This makes solid fuels suitable for applications where the rocket may not be used for a long period, or where it needs to be launched quickly without having to fuel it. One such application is strategic nuclear weapons; the USA replaced its hydrazine-fuelled Titan II missiles with the solid-fuelled Minuteman partly for this reason.

5.5 Real rockets

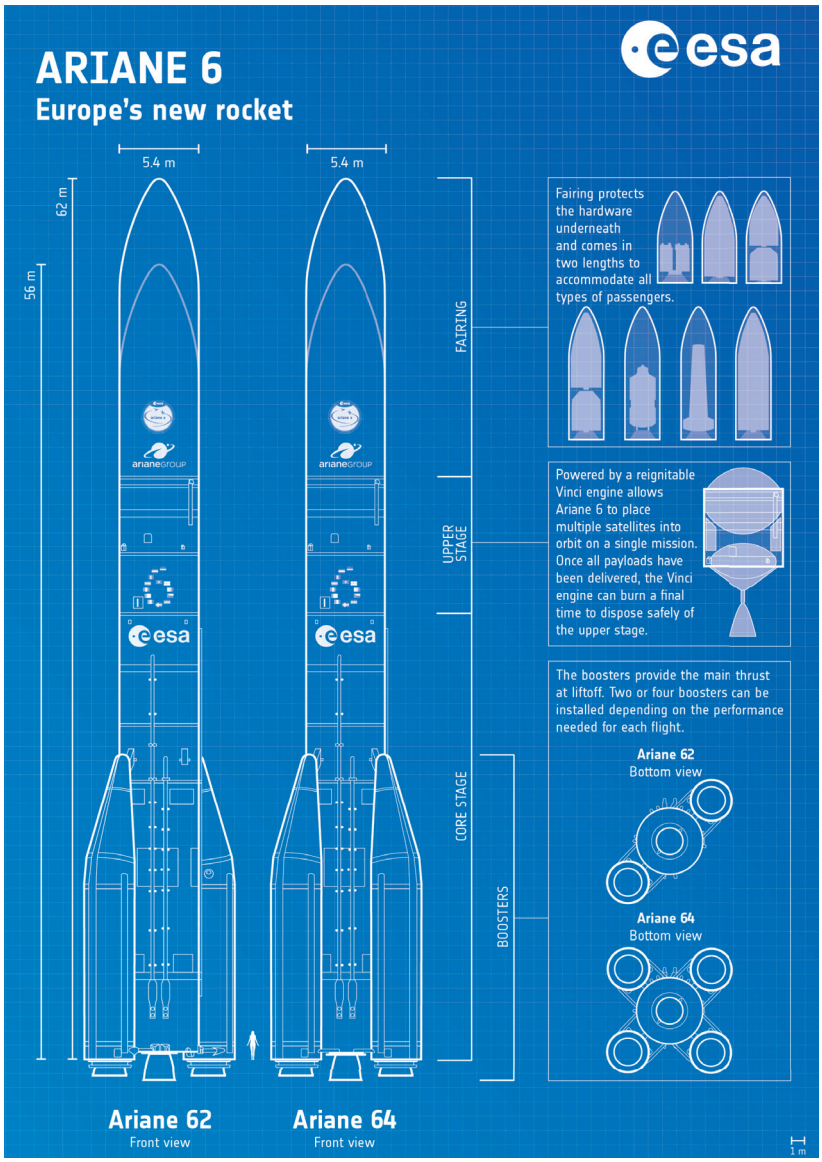
Let us now look briefly at two examples of real rockets used in spaceflight.

One of the most powerful rockets ever to launch was the Saturn V, which sent the Apollo missions to the Moon (figure 3.8). It had a height of 111 m (with payload), a maximum diameter of 10 m, a dry mass of 179 t plus 50 t payload, and a launch mass (including propellant) of around 2900 t. It comprised three stages, all powered by liquid fuels. The first stage used 650 t of RP-1 together with 1500 t of LOX. Five F-1 engines achieved a combined thrust of 34.5 MN. Each engine provided an effective exhaust velocity of 2.6–3.0 $\mathrm{km\,s^{-1}}$ (depending on altitude) and burned propellant – fuel and oxidizer – at a rate of 2.6 $\mathrm{t\,s^{-1}}$, operating at a combustion chamber pressure of 7 MPa. The burn time was around 165 s and took the rocket to an altitude of about 60 km. The second stage used five J-2 engines powered by liquid hydrogen. They provided a vacuum exhaust velocity of 4.1 $\mathrm{km\,s^{-1}}$, a combined thrust of 5.1 MN, and burned for six minutes. The third stage used a single J-2 liquid hydrogen engine. This used two burns: the first to put the rocket into Earth orbit, and the second to put it on a translunar injection orbit.

This phenomenal capacity of the Saturn V rocket was necessary to send 50 t of payload to the Moon. To get a sense of how powerful this was, the first stage had a power of around 45 GW (equation 3.23). This was about 0.6% of the rate of energy use across the entire world at the time.

The Ariane 6 rocket (figure 5.8) also has three stages, one of which is a set of two or four solid rocket boosters (SRBs). At launch, the rocket is lifted by both the core stage and the SRBs operating in parallel. The core stage is powered by a Vulcain 2.1 engine running on liquid hydrogen and LOX (140 t propellant). This operates at a combustion chamber pressure of 12 MPa, provides a maximum thrust of 1.4 MN, and delivers a vacuum exhaust velocity of 4.3 $\mathrm{km\,s^{-1}}$. The burn time is around 470 s. In parallel to this, two or four SRBs (depending on payload mass) provide significantly more thrust for

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Ariane 6. Credit: European Space Agency (ESA), ESA Standard Licence.

Fig. 5.8

the initial part of the launch. Each SRB provides a thrust of 3.5 MN and has 142 t of propellant. The exhaust velocity is 2.7 km s^{-1}. They deplete their propellant within 135 s, after which they are ejected, leaving the core stage to continue to propel the rocket. When this has burned out, it too is ejected and the upper stage kicks in. This is a single Vinci liquid hydrogen engine that provides an exhaust velocity of 4.5 km s^{-1} and a thrust of 0.18 MN. This stage can be turned off and restarted up to three times in order to perform orbital injections.

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5.6 Summary

A thermal rocket works by heating a gas to a high temperature and expelling it through a convergent–divergent nozzle. The nozzle maximizes the conversion of the internal energy of the hot gas (random motion) into kinetic energy (directed motion). When the throat of the nozzle has the right size, and the ratio of the pressure in the combustion chamber to the ambient pressure is large enough, the gas hits the speed of sound at the throat. The gas then continues to accelerate through the diverging part of the nozzle to supersonic speeds by converting more thermal energy into kinetic energy. If we only used a convergent nozzle, then the maximum exhaust velocity would be the local speed of sound of the gas.

The two critical terms in the description of rockets are the exhaust velocity and the mass flow rate. They are given by equations 5.9 and 5.19, respectively, for a thermal rocket. The exhaust velocity can be increased by raising the temperature in the combustion chamber or by using propellants (combustion products) that have a small molar mass $\mathcal{M}$ or a small ratio of specific heat capacities γ.

Chemical thermal rockets may use liquid or solid fuels. Liquid fuels deliver higher exhaust velocities: a liquid hydrogen–oxygen mix can provide up to $4.55\,\mathrm{km\,s^{-1}}$, about as high as can be practically achieved with chemical reactions. Solid fuels provide lower exhaust velocities (around $2.7\,\mathrm{km\,s^{-1}}$), but as solid-fuelled rocket engines do not need separate fuel tanks, pipes, or pumps, they are simpler and cheaper.

Because their effective exhaust velocities are low, chemical rockets cannot be used for fast interstellar travel. Even with an exhaust velocity of $5\,\mathrm{km\,s^{-1}}$ and an unfeasibly large mass ratio of 10 000, they could still only achieve $\Delta v = 46\,\mathrm{km\,s^{-1}}$.

5.7 Exercises

1. Once the choked condition has been achieved in a convergent–divergent nozzle, what happens if the throat area is (a) increased, or (b) decreased?
2. What mass of liquid hydrogen–oxygen propellant is required to get a 100 t spacecraft (dry mass) into orbit? Assume $\Delta v = 11\,\mathrm{km\,s^{-1}}$ is required, and that the entire chemical energy of the propellant is converted into the kinetic energy of the exhaust.
3. As the gas expands in the nozzle, some of its thermal energy is converted into kinetic energy. When this expansion is adiabatic, it does not transfer any energy to the rocket by definition. So how does the gas propel the rocket?

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