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Low cost automation using INS/GPS data fusion for accurate positioning

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SUMMARY

Low cost automation often requires accurate positioning. This happens whenever a vehicle or robotic manipulator is used to move materials, parts or minerals on the factory floor or outdoors. In last few years, such vehicles and devices are mostly autonomous. This paper presents the method of sensor fusion based on the Adaptive Fuzzy Kalman Filtering. This method has been applied to fuse position signals from the Global Positioning System (GPS) and Inertial Navigation System (INS) for the autonomous mobile vehicles. The presented method has been validated in 3-D environment and is of particular importance for guidance, navigation, and control of mobile, autonomous vehicles. The Extended Kalman Filter (EKF) and the noise characteristic have been modified using the Fuzzy Logic Adaptive System and compared with the performance of regular EKF. It has been demonstrated that the Fuzzy Adaptive Kalman Filter gives better results (more accurate) than the EKF. The presented method is suitable for real-time control and is relatively inexpensive. Also, it applies to fusion process with sensors different than INS or GPS.

KEYWORDS: Adaptive Kalman filtering, Fuzzy logic, Sensor fusion, INS, GPS.

1. INTRODUCTION

When navigating and guiding an autonomous vehicle, the position and velocity of the vehicle must be determined. The Global Positioning System (GPS) is a satellite-based navigation system that provides a user with the proper equipment access to useful and accurate positioning information anywhere on the globe, see Brown and Hwang.[1] However, several errors are associated with the GPS measurement. It has superior long-term error performance, but poor short-term accuracy. For many vehicle navigation systems, GPS is insufficient as a stand-alone position system. The integration of GPS and Inertial Navigation System (INS) is ideal for vehicle navigation systems. In general, the short-term accuracy of INS is good; the long-term accuracy is poor. The disadvantages of GPS/INS are ideally cancelled. If the signal of GPS is interrupted, the INS enables the navigation system to coast along until GPS signal is reestablished. The requirements for accuracy, availability and robustness are therefore achieved.

Kalman filtering is a form of optimal estimation characterized by recursive evaluation, and an internal model of the dynamics of the system being estimated. The dynamic weighting of incoming evidence with ongoing expectation produces estimates of the state of the observed system, see Abidi and Gonzalez.[2] An extended Kalman filter (EKF) can be used to fuse measurements from GPS and INS. In this EKF, the INS data are used as a reference trajectory, and GPS data are applied to update and estimate the error states of this trajectory. The Kalman filter requires that all the plant dynamics and noise processes are exactly known and the noise processes are zero mean white noise. If the theoretical behavior of a filter and its actual behavior do not agree, divergence problems will occur. There are two kinds of divergence: Apparent divergence and True divergence.[3] In the apparent divergence, the actual estimate error covariance remains bounded, but it approaches a larger bound than does predicted error covariance. In true divergence, the actual estimation covariance eventually becomes infinite. The divergence due to modeling errors is critical in Kalman filter application. If, the Kalman filter is fed information that the process behaved one way, whereas, in fact, it behaves another way, the filter will try to continually fit a wrong process. When the measurement situation does not provide enough information to estimate all the state variables of the system, in other words, the computed estimation error matrix becomes unrealistically small, and the filter disregards the measurement, then the problem is particularly severe. Thus, in order to solve the divergence due to modeling errors, we can estimate unmodeled states, but it adds complexity to the filter and one can never be sure that all of the suspected unstable states are, indeed, model states.[4] Another possibility is to add process noise. It makes sure that the Kalman filter is driven by white noise, and prevents the filter from disregarding new measurement. In this paper, a fuzzy logic adaptive system (FLAS) is used to adjust the exponential weighting of an weighted EKF and prevent the Kalman filter from divergence. The fuzzy logic adaptive controller (FLAC) will continually adjust the noise strengths in the filter's internal model, and tune the filter as well as possible. The FLAC performance is evaluated by simulation of the fuzzy adaptive extended Kalman filtering scheme of Figure 1.

2. WEIGHTED EKF

Because the processes of both GPS and INS are nonlinear, a linearization is necessary. An extended Kalman filter is used to fuse the measurements from the GPS and INS. To prevent divergence by keeping the filter from discounting

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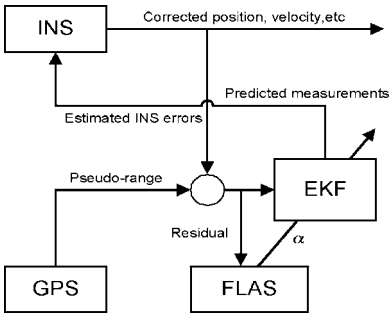


Fig. 1. Fuzzy adaptive extended Kalman filter.

measurements for large k, the exponential data weighting[4] is used.

The models and implementation equations for the weighted extended Kalman filter are:

Nonlinear dynamic model

$$\begin{aligned}\mathbf{x}_{k+1} &= f(\mathbf{x}_k, k) + \mathbf{w}_k \\ \mathbf{w}_k &\sim N(0, \mathbf{Q})\end{aligned} \tag{1}$$

Nonlinear measurement model

$$\begin{aligned}\mathbf{z}_k &= h(\mathbf{x}_k, k) + \mathbf{v}_k \\ \mathbf{v}_k &\sim N(0, \mathbf{R})\end{aligned} \tag{2}$$

Let us set the model covariance matrices equal to

$$\mathbf{R}_k = \mathbf{R}\alpha^{-2(k+1)} \tag{3}$$

$$\mathbf{Q}_k = \mathbf{Q}\alpha^{-2(k+1)} \tag{4}$$

where, $\alpha \geq 1$, and constant matrices $\mathbf{Q}$ and $\mathbf{R}$. For $\alpha > 1$, as time k increases, the $\mathbf{R}$ and $\mathbf{Q}$ decrease, so that the most recent measurement is given higher weighting. If $\alpha = 1$, it is a regular EKF.

By defining the weighted covariance

$$\mathbf{P}_k^{\alpha-} = \mathbf{P}_k^- \alpha^{2k} \tag{5}$$

The Kalman gain can be computed:

$$\begin{aligned}\mathbf{K}_k &= \mathbf{P}_k^- \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^T + \mathbf{R}\alpha^{-2(k+1)})^{-1} \\ &= \mathbf{P}_k^{\alpha-} \mathbf{H}_k^T \left(\mathbf{H}_k \mathbf{P}_k^{\alpha-} \mathbf{H}_k^T + \frac{\mathbf{R}}{\alpha^2} \right)^{-1}\end{aligned} \tag{6}$$

The predicted state estimate is:

$$\hat{\mathbf{x}}_{k+1}^- = f(\hat{\mathbf{x}}_k, k) \tag{7}$$

The predicted measurement is:

$$\hat{\mathbf{z}}_k = h(\hat{\mathbf{x}}_k^-, k) \tag{8}$$

The linear approximation equations can be presented in form:

$$\Phi_k \approx \left. \frac{\partial f(x,k)}{\partial x} \right|_{x=\hat{x}_k^-} \tag{9}$$

The predicted estimate on the measurement can be computed:

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^- + \mathbf{K}_k (\mathbf{z}_k - \hat{\mathbf{z}}_k) \tag{10}$$

$$\mathbf{H}_k \approx \left. \frac{\partial h(x,k)}{\partial x} \right|_{x=x_k^-} \tag{11}$$

Computing the *a priori* covariance matrix:

$$\mathbf{P}_{k+1}^- = \Phi_k \mathbf{P}_k \Phi_k^T + \mathbf{Q}\alpha^{-2(k+1)} \tag{12}$$

Re-writing (12) gives:

$$\mathbf{P}_{k+1}^{\alpha-} = \alpha^2 \Phi_k \mathbf{P}_k^{\alpha} \Phi^T + \mathbf{Q} \tag{13}$$

Computing the *a posteriori* covariance matrix gives:

$$\mathbf{P}_k^{\alpha} = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_k^{\alpha-} \tag{14}$$

The initial condition is:

$$\mathbf{P}_0^{\alpha-} = \mathbf{P}_0$$

In equation (10), the term $\mathbf{z}_k - \hat{\mathbf{z}}_k$ is called residuals or innovations. It reflects the degree to which the model fits the data.

3. INS AND GPS

The inertial navigation system (INS) consists of a sensor package, which includes accelerometers and gyros to measure accelerations and angular rates. By using these signals as input, the attitude angle and three-dimensional vectors of velocity and position are computed.[5] The errors in the measurements of force made by the accelerometers and the errors in the measurement of angular change in orientation with respect to inertial space made by gyroscopes are two fundamental error sources, which affect the error behavior of an inertial system. The inertial system error response, related to position, velocity, and orientation is divergent with time due to noise input.[6] There are biases associated with the accelerometers and gyros. In order to correct the errors of INS, the GPS measurements are used to estimate the inertial system errors, subtract them from the INS outputs, and then obtain the corrected INS outputs. There is number of errors in GPS, such as ephemeris errors, propagation errors, selective availability, multi-path, and receiver noise, etc. By using differential GPS (DGPS), most of the errors can be corrected, but the multi-path and receiver noise cannot be eliminated.

4. FUZZY LOGIC ADAPTIVE SYSTEM

It is assumed that both, the process noise $\mathbf{w}_k$, and the measurement noise $\mathbf{v}_k$ are zero-mean white sequences with known covariance $\mathbf{Q}$ and $\mathbf{R}$ in the Kalman filter. If the Kalman filter is based on a complete and perfectly tuned model, the residuals or innovations should be a zero-mean white noise process. If the residuals are not white noise, there is something wrong with the design and the filter is not performing optimally.[6] The Kalman filters will diverge or coverage to a large bound. In practice, it is difficult to know the exact values for $\mathbf{Q}$ and $\mathbf{R}$. In order to reduce computation, we have to ignore some errors, but sometimes those unmodeled errors will become significant. These are the instrument bias errors of INS. Sometimes the $\mathbf{w}_k$ may be different than zero mean. In those cases, the residuals can be used to adapt the filter. In fact, the residuals are the differences between actual measurements and best measure-

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ment predictions based on the filter's internal model. A well-tuned filter is that where the 95% of the autocorrelation function of innovation series should fall within the $\pm 2\sigma$ boundary.[7] If the filter diverges, the residuals will not be zero mean and become larger.

There are very few papers on application of fuzzy logic to adapt the Kalman filter. In reference [8] fuzzy logic is used to the on-line detection and correction of divergence in a single state Kalman filter. There were three inputs and two outputs to fuzzy logic controller (FLC), and 24 rules were used. In our works[9,10] the basic adaptive fuzzy logic controller has been introduced and designed. In this paper the new FLAC is proposed. The purpose of the fuzzy logic adaptive controller (FLAC) is to detect the bias of measurements and prevent divergence of the extended Kalman filter. It has been applied in three axes – East (x), North (y), and Altitude (z). The covariance of the residuals and mean values of residuals are used to decide the degree of divergence. The value of covariance relates to **R**. If the residual has zero mean, the equation for covariance of the residual is:

$$\bar{\mathbf{P}}_z = \mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^T + \mathbf{R} \tag{15}$$

The fuzzy adaptive Kalman filtering has been used for guidance and navigation of mobile robots, especially for 3-D environment. The navigation of flying robots requires a fast, and accurate on-line control algorithms. The "regular" Extended Kalman Filter requires a high number of states for accurate navigation and positioning and is unable to monitor the parameters changing. The FLAC requires smaller number of states for the same accuracy and therefore it would need less computational effort. Alternatively, the same number of states (as in "regular" filter) would allow for more accurate navigation.

4.1. Fuzzy adaptive Kalman filtering for parameter uncertainties

Sometimes, uncertain or time varying parameters are considered to exist in the **Q** and **R** matrices. The fuzzy adaptive Kalman filtering is used to detect and then adapt the filter on-line. There are two groups of fuzzy controllers. In those two fuzzy controllers, the covariance of the residuals and the mean of residuals are used as the inputs to both controllers for all three fuzzy inference engines. The exponential weighting a for first group controller and the scales for second group controller of three axes are the outputs.

The first group, which output is a**,** was used to detect the filter divergence and adapt the EKF. Generally, when the covariance is becoming large, and mean value is moving away from zero, the Kalman filter is becoming unstable. In this case, a large a will be applied. A large a means that process noises are added. It can ensure that in the model all states are sufficiently excited by the process noise. When the covariance is extremely large, there are some problems with the GPS measurements, so the filter cannot depend on these measurements anymore, and a smaller a will be used. By selecting appropriate α, the fuzzy logic controller will adapt

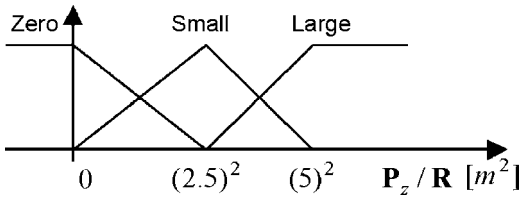


Fig. 2. Covariance membership functions.

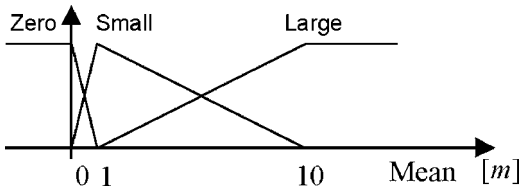


Fig. 3. Mean value membership functions.

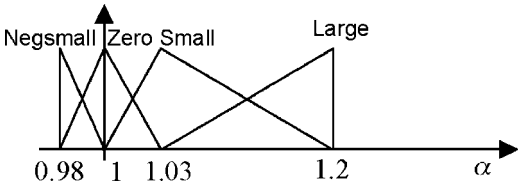


Fig. 4. α membership functions.

the Kalman filter optimally and try to keep the innovation sequence acting as zero-mean white noise. Some membership functions are shown at Figures 2, 3 and 4.
The fuzzy logic controller uses 9 rules, such as:

If *the covariance of residuals is large* ***and*** *the mean value is zero* ***Then*** α *is zero.*
If *the covariance of residuals is zero* ***and*** *the mean value is large* ***Then*** α *is small.*

The second group, which output is scale, was used to detect the change of measurement noise covariance **R**. From equation (15), the **R** is related to the covariance of residual, the larger the covariance of residual, the more the measurement noise. When the fuzzy logic controller finds that the covariance of residual is larger than that expected, it applies a large scale to adjust the α. A sample rule is:

If *the covariance of residuals is small* ***and*** *the mean values is small* ***then*** *the scale is large.*

Tables I and II are the rule table for those two groups of fuzzy controllers.

Table I. Rule table for α.

α		Mean Value		
		Z	**S**	**L**
P	**Z**	**Z**	**S**	**S**
	S	**S**	**L**	**S**
	L	**Z**	**NS**	**NS**

Note: S – Small; L – Large; Z – Zero; NS – Negative Small.

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Table II. Rule table for scale.

Scale		Mean Value		
		Z	**S**	**L**
P	**Z**	**Z**	**Z**	**Z**
	S	**S**	**S**	**S**
	L	**L**	**S**	**Z**

4.2. Fuzzy adaptive Kalman filtering for non-white process noise

It is assumed that the process noise $\mathbf{w}_k$ is white noise for Kalman filtering. But sometime the process noise could be correlated with itself, non-white. In this case, we can add a shaping filtering that manufactures colored noise $\mathbf{w}_k$ with a given spectral density from white noise, but it will increase the state variables. In some real-time situation, the computing time have a restriction for increasing the state variables. We can use a fuzzy adaptive Kalman filtering to adaptive the process noise rather than add more state variables. There are 9 rules and therefore, little computational time is needed. The membership functions for this fuzzy control are shown as Figures 5, 6, and 7.

The FLAC uses 9 rules, such as:

If *the covariance of residuals is large* ***and*** *the mean values are zero* ***Then*** α *is large.*

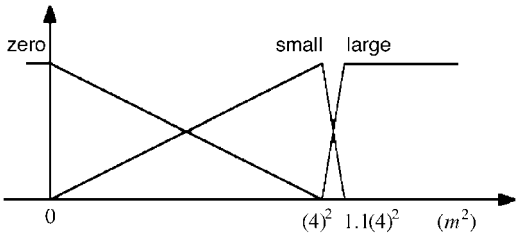


Fig. 5. Covariance membership functions.

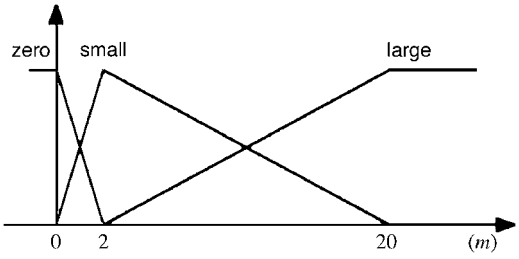


Fig. 6. Mean value membership functions.

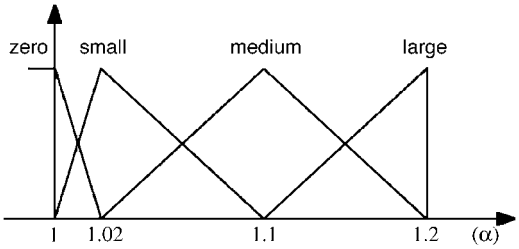


Fig. 7. α membership functions.

Table III. Rule table for FLAS.

α		Mean Value		
		Z	**S**	**L**
P	**Z**	**S**	**Z**	**Z**
	S	**Z**	**L**	**M**
	L	**L**	**M**	**Z**

Note: S – Small; M – Medium; L – Large; Z – Zero.

If *the covariance of residuals is zero* ***and*** *the mean values are large* ***Then*** α *is zero.*

5. SIMULATION

MATLAB codes developed by authors have been used to simulate and test the proposed method.

The state variables used in simulation are:

$$\mathbf{x}_k = [x_k, \dot{x}_k, y_k, \dot{y}_k, z_k, \dot{z}_k, c\Delta t, c\dot{\Delta} t] \tag{16}$$

The states are position, and velocity errors of the INS East, North, Altitude, GPS range bias and range drift.

5.1. Simulation experiment 1

The first part of simulation uses the fuzzy adaptive Kalman filtering for parameter uncertainties.

The designed standard deviation of GPS measurement **R** is 5 [m]. The designed standard deviations of **Q** for INS are 0.0012 meter, 0.0012 meter, and 0.0012 meter for the East (x), North (y), and Altitude (z), respectively.

The simulations (Tables IV, V and VI and Figures 8 and 9) show that after the filter is stabilized, the actual error

Table IV. Comparison of theoretical and actual error variance (X-axis).

Q	R	Theory	Actual
5Q	R	3.1711	3.3912
5Q	2R	5.3293	5.3121
3Q	2R	4.6896	4.8469
5Q	4R	8.9612	8.3122

Table V. Comparison of theoretical and actual error variance (Y-axis).

Q	R	Theory	Actual
5Q	R	2.5540	2.7227
5Q	2R	4.2877	4.1030
3Q	2R	3.7694	4.0864
5Q	4R	7.2002	7.7340

Table VI. Comparison of theoretical and actual error variance (Z-axis).

Q	R	Theory	Actual
5Q	R	0.8344	0.8072
5Q	2R	1.3979	1.1796
3Q	2R	1.2268	1.2989
5Q	4R	2.3417	2.5005

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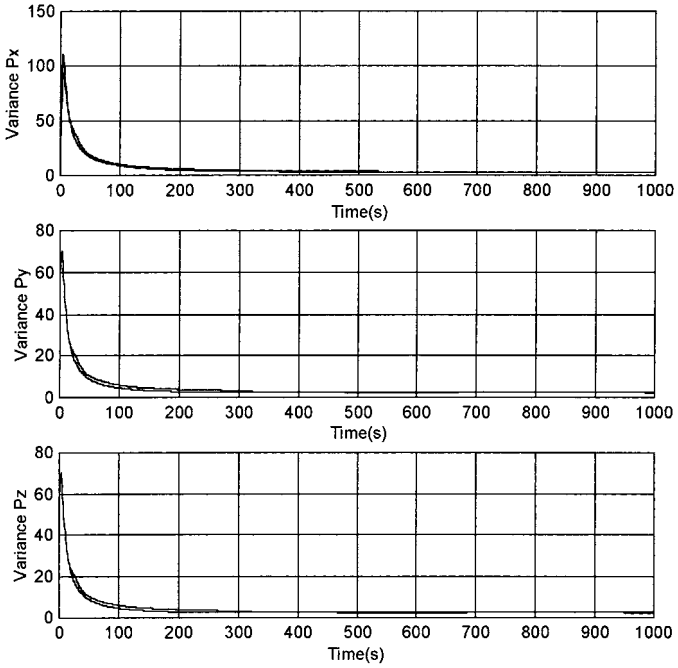


Fig. 8. Actual and theory covariance for 5Q and R.

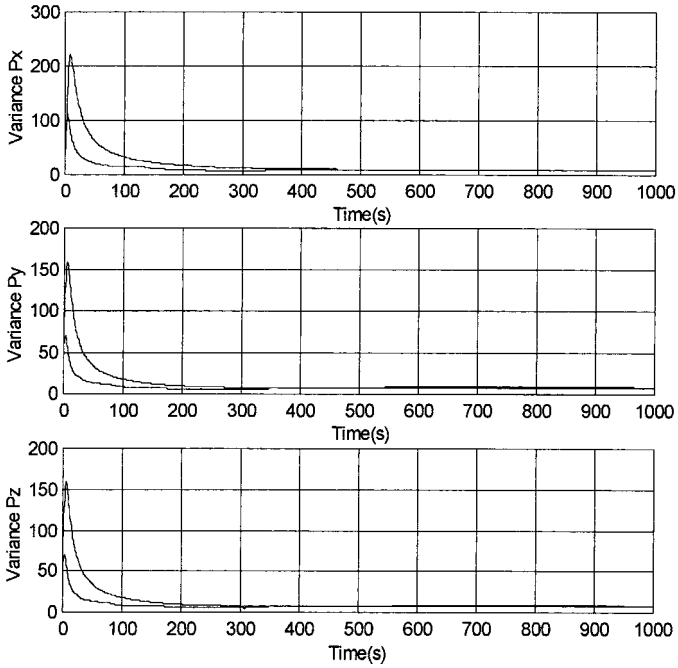


Fig. 9. Actual and theory covariance for 5Q and 4R.

covariance of fuzzy logic adaptive EKF almost agrees with the theory error covariance. In Tables IV, V and VI, the designed parameters are **Q** and **R**. The 5**Q**, 2**R** etc. mean that the real time parameters are 5 and 2 times as large as the designed **Q** and **R**. In Figure 8, and 9, The dashed lines are the covariance of theory EKF, and the solid lines are the covariance of fuzzy adaptive EKF.

5.2. Simulation experiment 2

In the second set of simulations, we simulate the inputs of non-white process noise. The covariance of GPS measurement R is 25 [m^2]. It is assumed that the measurements of INS have some biases. In the first part of this simulation (Figure 10), the mean values of INS are 0.0014 meter, 0.00035 meter, and 0.0007 meter for the East (x), North (y), and Altitude (z) respectively. A white noise with a standard deviation of 3 meter is added to GPS measurements. The sample period is 1 second. The first row in Figure 10 is the innovations of fuzzy adaptive EKF and EKF in East (x). The innovation of EKF had a large drift, and was stable at a high mean value. The fuzzy adaptive EKF clearly improved the performance of EKF, and the mean value was much smaller than that of EKF. Other figures present the corrected position (first column) and velocity (second column) errors. The corrected error is the current INS error minus estimated INS error. The dashed lines are the corrected errors of EKF, and the solid lines are the corrected errors of fuzzy adaptive EKF. The fuzzy adaptive EKF significantly reduced the corrected position and velocity errors. In the second part of this simulation (Figure 11), the same measurements as in

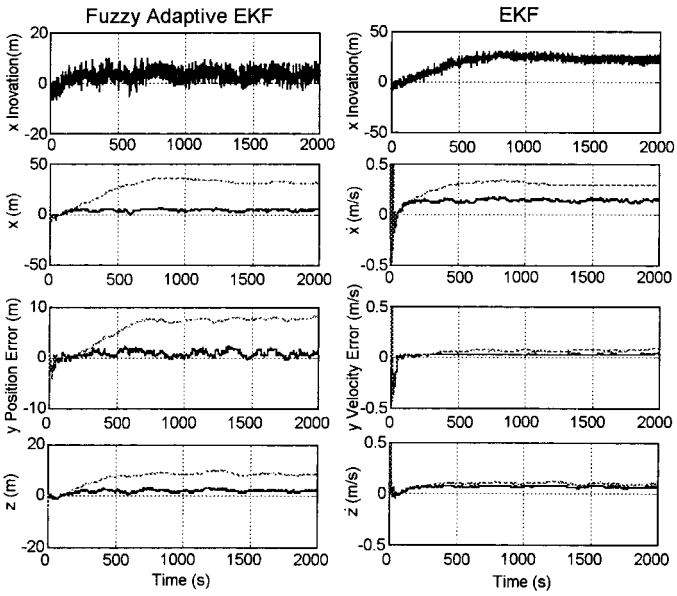


Fig. 10. Simulation A.

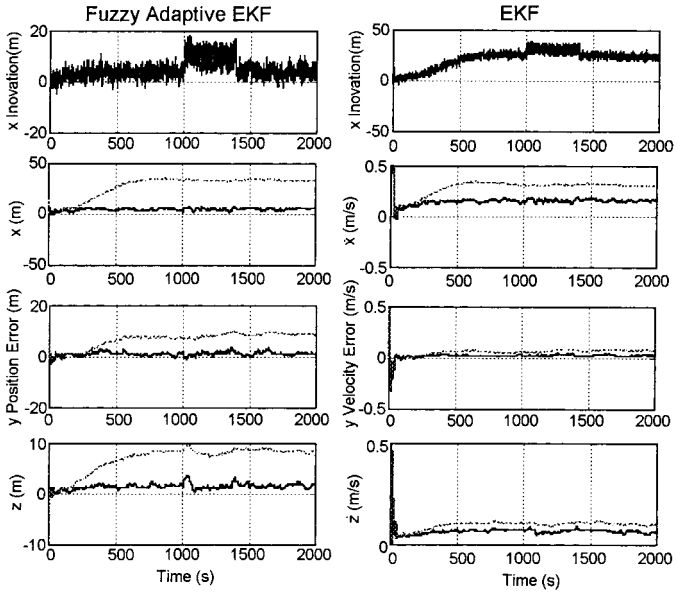


Fig. 11. Simulation B.

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the first part of this simulation for INS were used. A white noise with a standard deviation of 2 meter from 0 s to 1000 s and 1500 s to 2000 s was applied to GPS measurements. From 1000 s to 1500 s, the standard deviation of 6 meter with mean value of 6 meter was added to GPS measurements. Although, the GPS measurement noises features were changed, the fuzzy adaptive EKF still worked well. Those simulations also showed that the corrected errors of EKF were proportional to the mean values of INS measurements. In other word, the more errors are not modeled, the worse the EKF performs.

6. CONCLUSIONS

In this paper, a fuzzy adaptive extended Kalman filter has been developed to detect and prevent the EKF from divergence. By monitoring the innovations sequences, the FLAS can evaluate the performance of an EKF. If the filter does not perform well, it would apply an appropriate weighting factor **a** to improve the accuracy of an EKF.

The FLAS can use a lower order state-model without compromising accuracy significantly. In other words, for any given accuracy, the fuzzy adaptive Kalman filter may be able to use a lower order state model. The FLAS makes the necessary trade-off between accuracy and computational burden due to the increased dimension of the error state vector and associated matrices. When a designer lacks sufficient information to develop complete models or the parameters will slowly change with time, the fuzzy controller can be used to adjust the performance of EKF on-line, and it will remain sensitive to parameter variations by "remembering" most recent N data samples. It can be used to navigate and guide autonomous vehicles or robots and achieved a relatively accurate performance. Also, the presented method is suitable for low cost real-time control of autonomous vehicles.

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