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Visual tolerance analysis for engineering optimization

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Abstract. Classic methodologies of DOE are widely applied in design, manufacture, quality management and related fields. The resulting data can be analysed with linear modeling methods such as multiple regression which generates a set of equations, $Y = F(X)$, that enable us to understand how varying the mean of one or more inputs changes the mean of one of more responses. To develop, scale-up and transfer robust processes to manufacturing we also need to set the control tolerances of each critical X and understand the extent to which variation in the critical X's propagate through to variation in the Y's and how this may impact performance relative to requirements (or specifications). Visual tolerance analysis provides a simple way to understand and reduce propagation of variation from X's to Y's using models developed from DOE's or historical data. This paper briefly introduces the concept of tolerance analysis and extents this to visual tolerance analysis through defect profiles and defect parametric profiles. With the help of visual tolerance analysis, engineering and statistical analysts can work together to find the key factors responsible for propagating undesired variation into responses and how to reduce these effects to deliver a robust and cost effective process. A case study approach is used to aid explanation and understanding.

Keywords: Engineering optimization; tolerance analysis; visualization; defect profile; defect parametric profile

1 Introduction

With the help of screening DOE's, such as fractional or full factorials, D-optimal, or similar designs we can screen out the critical few factors by modeling main (and possibly interaction) effects. RSM designs then allow us to fine tune our model and predict the settings of the key factors required to optimize the mean of our responses.

Some DOE approaches such as Taguchi methods try to optimize response for mean and variance to deliver robust processes. Because they cross an inner array design in the control factors with an outer array design in the noise factors the total size of the design gets large very quickly and the inner array design tends to be limited to fractional factorial type designs which limit us to modeling linear effects which are not so useful when we need to explore wider factor ranges. The approach presented here requires significantly fewer total runs than Taguchi experiments (because it does not require the crossing of an inner array with an outer array) and delivers more information about how to optimize processes for robustness because it allows the modeling of more than main effects.

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2 Propagation of error

Conventional DOE and modeling methods help design and develop new products and processes and transfer them to production. As part of the transfer process it is often difficult to gain an understanding of how consistently the product or process will perform under actual use or manufacturing conditions. Figure 1 illustrates this dilemma. The graph on the left-hand side exemplifies development conditions where it is possible to control process inputs with little or no variation resulting in little or no run-to-run variation being transferred into the Y's when controlling the X's at their optimum settings. The real use case or manufacturing situation is illustrated in the right-hand graph where there is a certain amount of uncontrolled variation in the X's that gets transmitted to the Y's. Unfortunately, when mass production starts, we often find the variation is much bigger than expected. The actual process capability is lower than predicted from R&D trials, which results in additional engineering investigations and learning during manufacturing to incrementally improve capability.

The principle causes of this batch to batch variation are many and include uncontrollable batch to batch variation in critical X's about their optimum settings and random batch to batch variation (which may include

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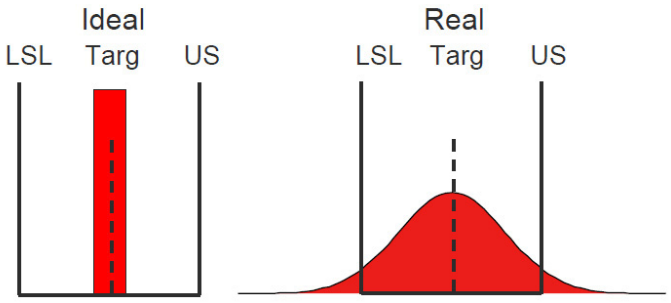


Fig. 1. Example of process capability.

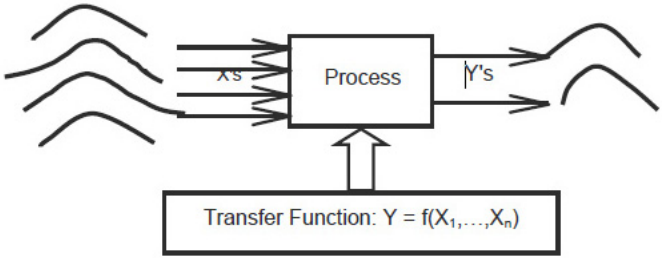


Fig. 2. General model of engineering process.

Table 1. Terms definition.

Terms	Definition
σ_Y^2	Variance of output variable Y
$\sigma_{X_n}^2$	Variance of input variable X_n
$\left(\frac{\partial f}{\partial X_n}\right)$	Sensitivity coefficient of input variable X_n
$\left(\frac{\sigma f}{\sigma X_n}\right)\sigma_{X_n}^2$	Contribution of input variable X_n to the variance of output variable

contributions from unidentified critical X's). Figure 2 represents the transfer function from X's (factors) to Y's (responses) with the transfer function $Y = F(X)$ being approximated by linear modeling methods such as multiple regression.

Regression methods are used to model the way changing the mean of one or more X's changes the mean of each Y. Such analysis may not adequately deal with the situation when it is not possible to control exactly the critical X's. The uncontrollable variation in the critical X's may get propagated through to Y as defined by equation (1) and Table 1.

$$\sigma_Y^2 = \left(\frac{\partial f}{\partial X_1}\right)^2 \sigma_{X_1}^2 + \ldots + \left(\frac{\partial f}{\partial X_n}\right)^2 \sigma_{X_n}^2. \quad (1)$$

3 Tolerance analysis

Tolerance analysis is an approach to model propagation of errors, and helps define cost effective control tolerances (not just target settings) of the critical input variables during R&D phases, to ensure that our responses are insensitive to variability of input variables, so that it can remain within customer requirement or engineering specification in production. The emphasis needs to be on cost effective control tolerances for the critical X's and not the definition of unreasonably tight tolerances that would be costly to maintain in production, i.e. what's the biggest control tolerance the process can sustain while transferring acceptable levels of variation into responses relative to specifications.

Competition and higher awareness of methods for quality improvement have driven the study and application of advanced techniques such as tolerance analysis. Visual tolerance analysis is one variant that exploits a wide range of different visual forms, such as graphs and profilers with animation controls to aid understanding and communication of propagation of variance. Developing information technology is simplifying the application of visual tolerance analysis, making it easier for engineering groups to reach more informed decisions about process tolerances and controls.

Table 2. Response specification.

Response	LSL	USL
Abrasion	100	200
Elasticity		2000
Elongation	400	550
Hardness	65	75

Table 3. Factor tolerances.

Factor	LL	UL
Silica	0.95	1.55
Silane	44	56
Sulfur	1.8	2.7

4 Case study

The following case study will demonstrate the application of visual tolerance analysis in engineering optimization.

4.1 Background

Visual tolerance analysis will be illustrated with data from Derringer and Suich [1]. There are four output responses as described in Table 2.

Earlier work established three critical factors with experimental ranges described in Table 3: (LL: Lower Limit, UL: Upper Limit)

Using the factor ranges in Table 3, a 20 run response surface design was conducted, the results of which are presented in Table 4.

4.2 Monte Carlo simulation process

After estimation and selection of a "best" regression model by standard least square method, the desirability function based on the requirements defined in Table 2 were used to optimize the process for average performance and gave the suggested factor settings of Silica = 0.99, Silane = 52,

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Table 4. Experiment schedule & result.

	Abrasion	Elasticity	Elongation	Hardness	Silica	Silane	Sulfur
1	102	900	470	67.5	0.7	40	2.8
2	120	860	410	65	1.7	40	1.8
3	117	800	570	77.5	0.7	60	1.8
4	198	2294	240	74.5	1.7	60	2.8
5	103	490	640	62.5	0.7	40	1.8
6	132	1289	270	67	1.7	40	2.8
7	132	1270	410	78	0.7	60	2.8
8	139	1090	380	70	1.7	60	1.8
9	102	770	590	76	0.3835	50	2.3
10	154	1690	260	70	2.0165	50	2.3
11	96	700	520	63	1.2	33.67	2.3
12	163	1540	380	75	1.2	66.33	2.3
13	116	2184	520	65	1.2	50	1.4835
14	153	1784	290	71	1.2	50	3.1165
15	133	1300	380	70	1.2	50	2.3
16	133	1300	380	68.5	1.2	50	2.3
17	140	1145	430	68	1.2	50	2.3
18	142	1090	430	68	1.2	50	2.3
19	145	1260	390	69	1.2	50	2.3
20	142	1344	390	70	1.2	50	2.3

and Sulfur = 2.1. As indicated in Figure 3 our predicted average responses of 131 for Abrasion, 1164 for Elasticity, 469 for elongation and 70 for hardness compare well with the requirements specified in Table 2.

With this model, Monte Carlo simulation can be used to assess the extent to which variation in inputs propagates through to variation in the outputs. For example discussions with engineers responsible for the process might suggest control tolerances for each of the three factors with a normal distribution about the targets defined in Figure 3 with standard deviations of 0.2, 4, and 0.2 respectively. A Monte Carlo simulation then randomly selects a value for each input factor from its proposed control distribution substitutes these values into the multiple regression model to give a predicted value for each of the four responses to which random error is added to represent the uncontrolled variation (estimated by the variation in the residuals from the regression model which represents variation due to uncontrollable factors or factors yet to be identified). For each response the random error is simulated from a normal distribution with a mean of zero and a standard deviation estimated from the residuals of the regression model. Running a large number of such simulations (in our case 5000) gives us an estimate of the defect rate (out of specification rate) for each response and the overall defect rate where one or more responses are out of specification. Figure 4 shows the results of 5000 Monte Carlo simulations for the above scenario and predicts an overall defect rate of 27.38%.

After applying the proposed control tolerances for the three critical factors and uncontrolled variation in each response, we get a very different view of the process. While on average we obtain good performance, in the long-term operation of the process we expect one or more of the responses to be out of specification 26% of the time. Using a process of trial and error we could change the mean and/or standard deviation of each the input distributions to try and find alternative solutions that give a lower overall defect rate. However this process of trial and error gets more time consuming and error prone as the number of factors and responses increase.

Two concepts of the JMP statistical discovery software – the defect profile and defect parametric profile – are used instead to provide objective functions to minimize the overall defect rate. The defect profiler illustrated in Figure 5 shows the defect rate as an isolated function of each factor. It graphs the overall defect rate (cubic root scale) for each response. The curves with different colors, e.g. red and blue (which is not available in the printed version), represent the relationship between the factor setting and defect rate relative to different specifications for each response, e.g. lower specification in red for the response and upper specification in blue for the response. The black curve represents the overall defect rate for the response. The value of the factor that minimises the overall defect rate for a particular factor is indicated at the turning point.

This graph shows the defect rate as a function of each factor as if it were held constant, but all the other factors are varied according to their random specification. If there are multiple outputs with Spec Limits, then there is a defect rate curve color-coded for each output and a black

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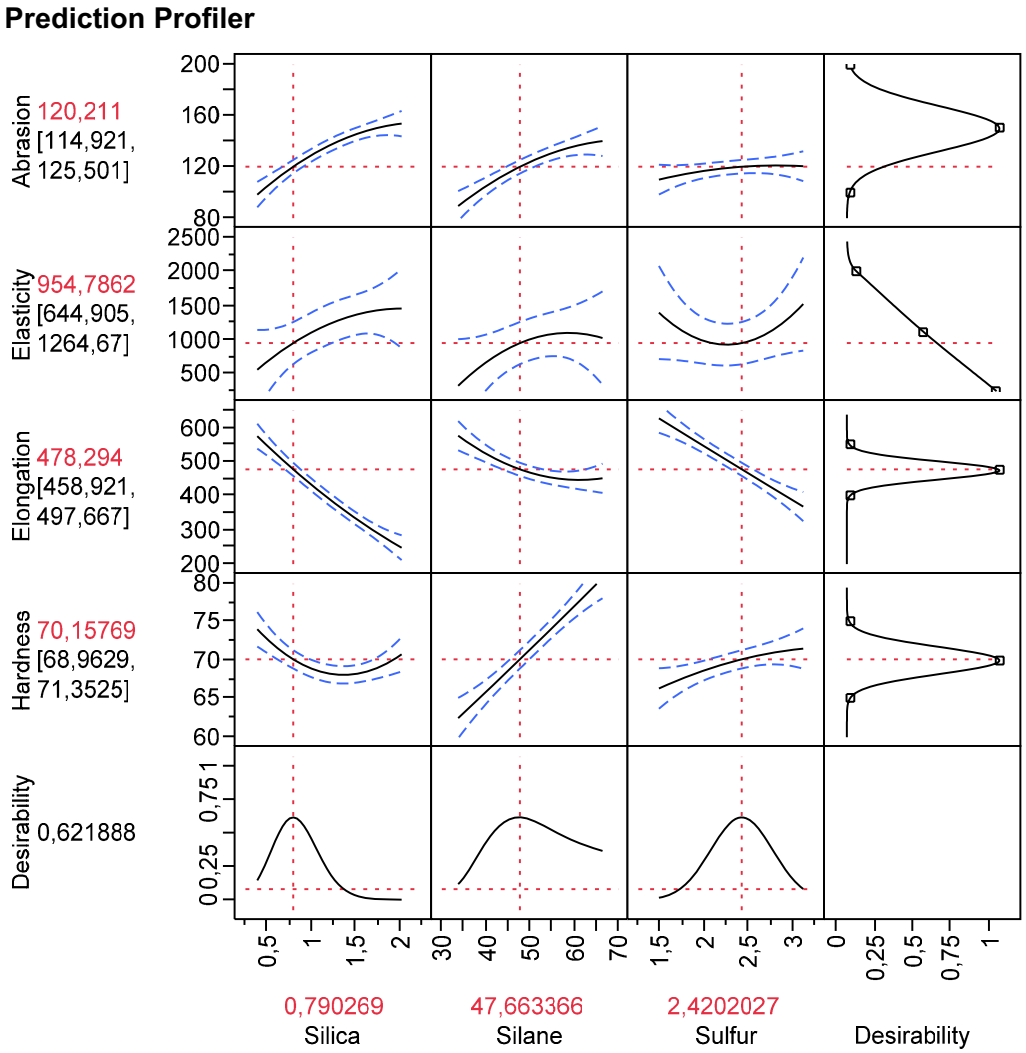


Fig. 3. Solution with maximum desirability.

curve shows the overall defect rate – this curve is above all the colored curves (as illustrated in Fig. 7).

Reported below each defect profile plot is the mean and standard deviation (SD). The mean is the overall defect rate, calculated by integrating the defect profile curve with the specified factor distribution. The standard deviation is a good measure of the sensitivity of the defect rates to the factor. It is quite small if either the factor profile is flat, or the factor distribution has a very small variance. Comparing SD's across factors is a good way to know which factor should get more attention to reducing variation.

Secondly, the defect parametric profile shown in Figure 6, shows how single changes in the factor distribution parameters affect the defect rate. We now have four curves with different colors (which is not available in the printed version) representing the effects of four different control strategies around the factor:

- mean Shift represents the change in overall defect rate by changing the mean of the factor;
- Std. Narrow represents the change in defect rate by reducing the standard deviation of the factor;
- LSL Chop, represents the change in defect rate by rejecting any values of the factor below the LSL through inspection;
- USL Chop, represents the change in defect rate by rejecting any values of the factor above the USL through inspection.

In addition, the red (which is not available in the printed version) dotted line represents the value of current mean of processing factor, while the two blue (which is not available in the printed version) dotted lines represent the value which equals current mean of processing factor plus or minus one current standard deviation respectively. The curve with the lowest minimum defect indicates the control strategy for the factor that results is biggest reduction in defect rate. In the case illustrated in Figure 6, the blue (which is not available in the printed version) curve corresponds to the lowest minimum defect rate, which indicates we can get biggest reduction in overall defect rate by reducing the standard deviation of the factor. This opportunity could now be explored to assess the cost benefit of this change.

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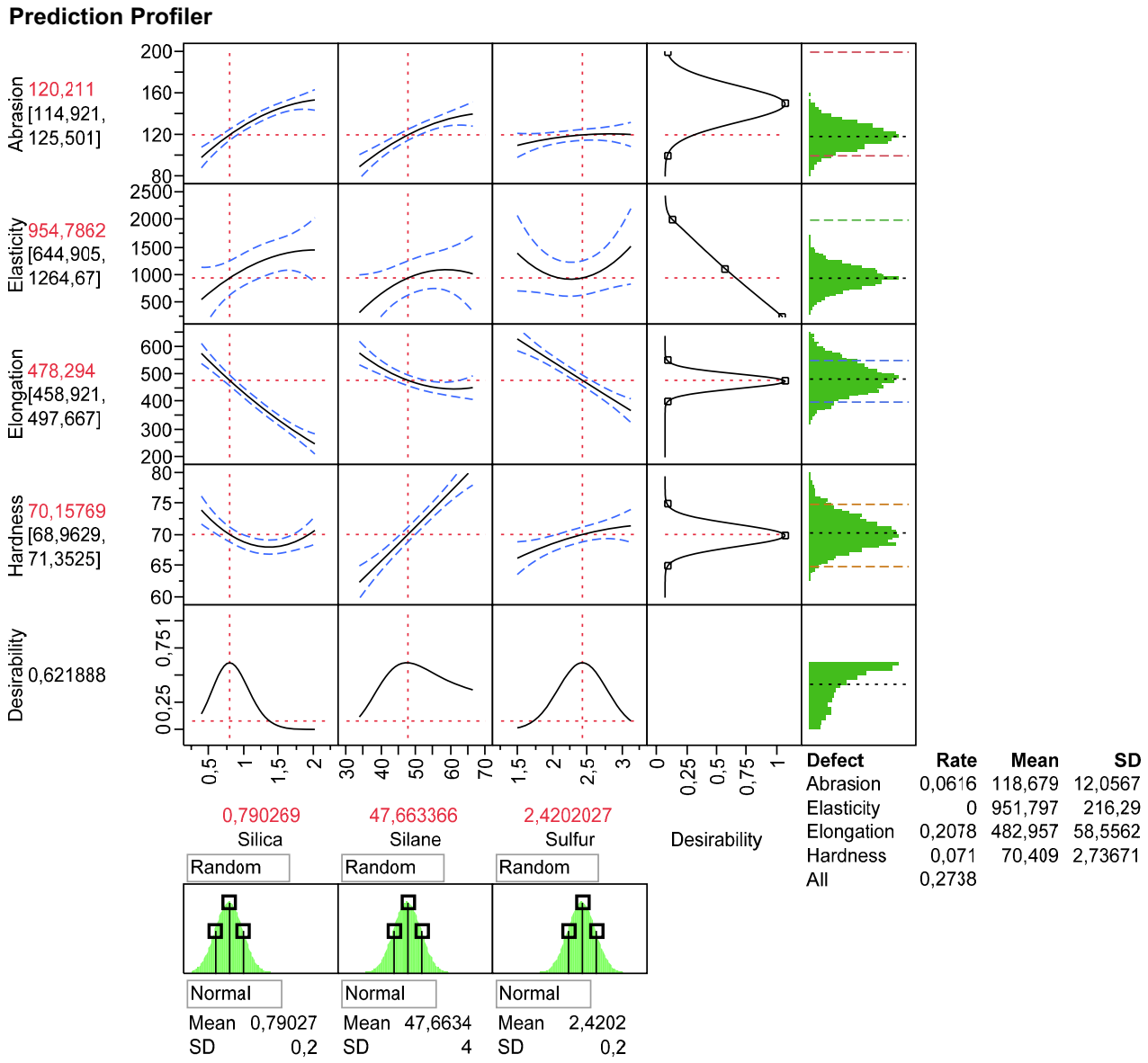


Fig. 4. Simulated defect rate for starting scenario.

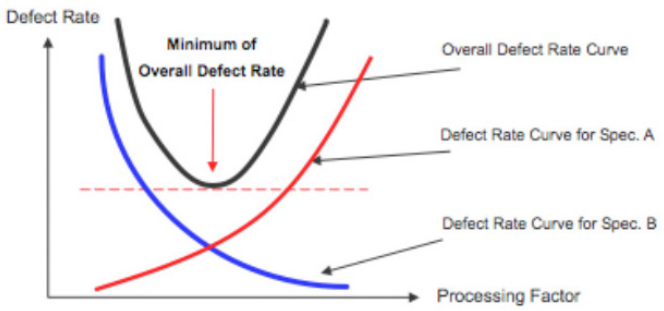


Fig. 5. Example of defect profile.

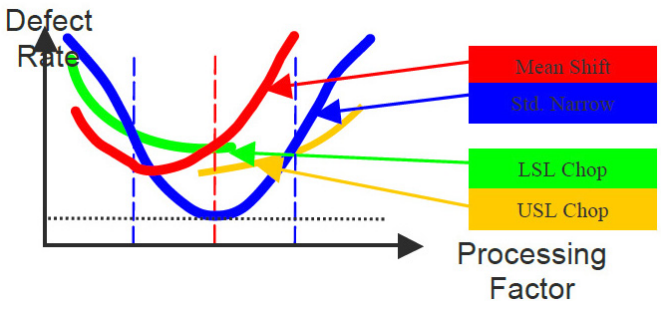


Fig. 6. Example of defect parametric profile.

Examining the defect parametric profiler for our situation we see that reducing the standard deviation of silica will have the greatest impact on defect rate. Indeed, if we could reduce the standard deviation to close to zero we would expect the overall defect rate to be close to 0.1 or 10%.

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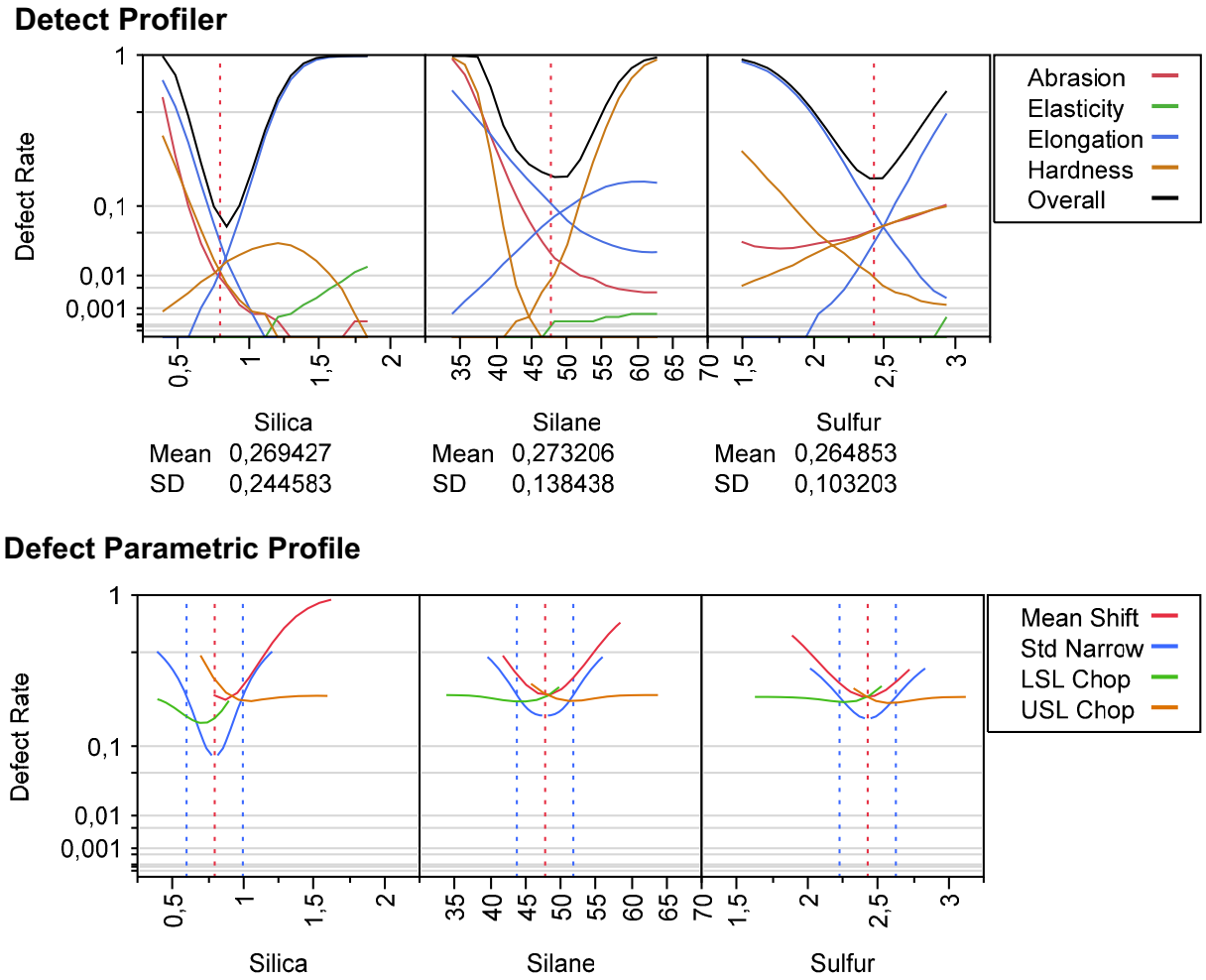


Fig. 7. Simulated defect profile and defect parametric profile before control tolerance improvement.

4.3 Process improvement

We decide that a control standard deviation for silica of 0 is impractical and instead try a 50% reduction to a standard deviation of 0.1. Running the Monte Carlo simulation with this change we get an estimated overall defect rate of 11.44%, which is a very useful improvement.

The updated defect parametric profile now shows the next biggest reduction in overall defect rate can be obtained by reducing the standard deviation of sulfur. Reducing this by 50% from 0.2 to 0.1 gives a predicted overall defect rate from Monte Carlo simulation of 6.22% as indicated in Figure 9.

The revised defect parametric profile in Figure 9 shows the next biggest reduction in overall defect rate can be obtained by reducing the standard deviation of silane. Reducing this by 50% from 4 to 2 gives a predicted overall defect rate from Monte Carlo simulation of 1.06% as indicated in Figure 10.

Of course, this is not the best we can get from the process. The defect parametric profile in Figure 10 indicates that further reduction in overall defect rate is possible by reducing the standard deviation of silica. We could continue iterating this way until we achieve the desired balance between the cost of increased factor control vs. defect reduction benefit. Additional reductions in defect rates could be obtained by targeting the random variation of 20.5 in elongation. If we could identify additional factors that are responsible for a large proportion of this variation then controlling these so far unidentified factors in a similar way to Silica, Silane, and Sulfur might provide a more cost effective way of further defect reduction.

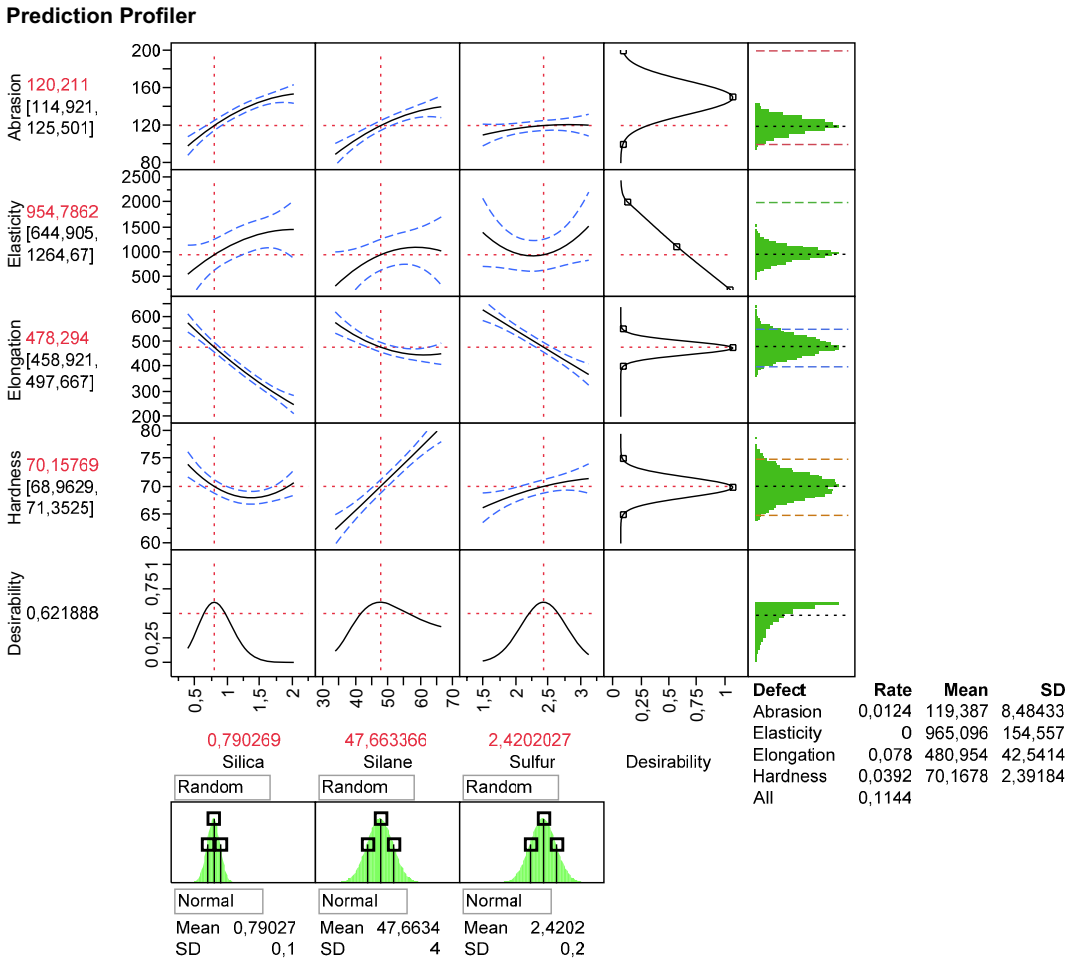
Table 5. Results comparison.

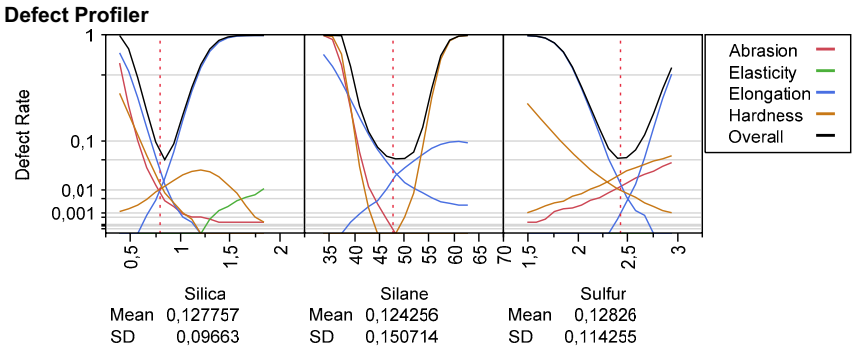
	Before	After
Silica SD	0.2	0.1
Silane SD	4	2
Sulfur SD	0.2	0.1
All defect rate	27.38%	1.06%

4.4 Complete results

The different control tolerances for each factor are summarised in Table 5, with the total estimated defect rate

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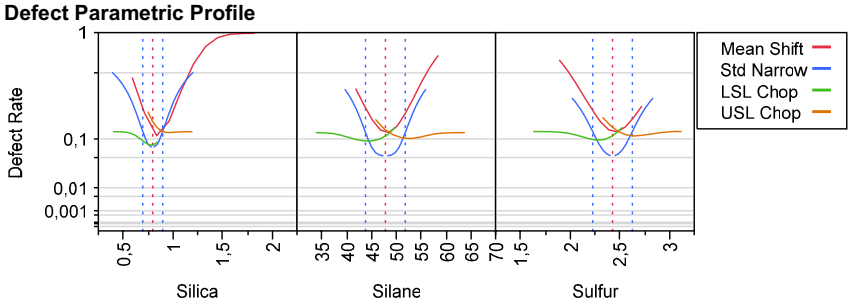


Fig. 8. Simulated prediction profiler after first iteration of control tolerance improvement.

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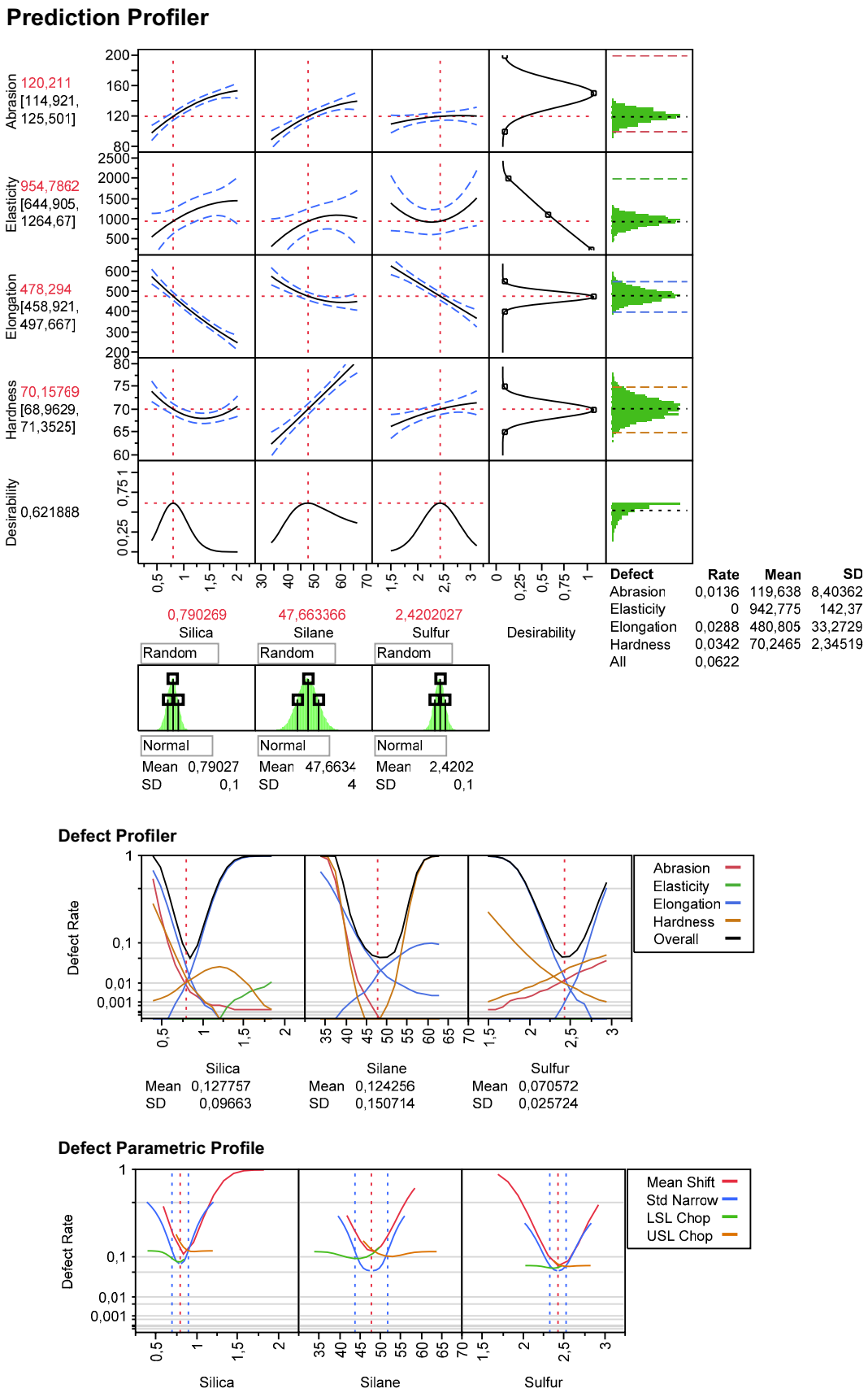


Fig. 9. Simulated prediction profiler after second iteration of control tolerance improvement.

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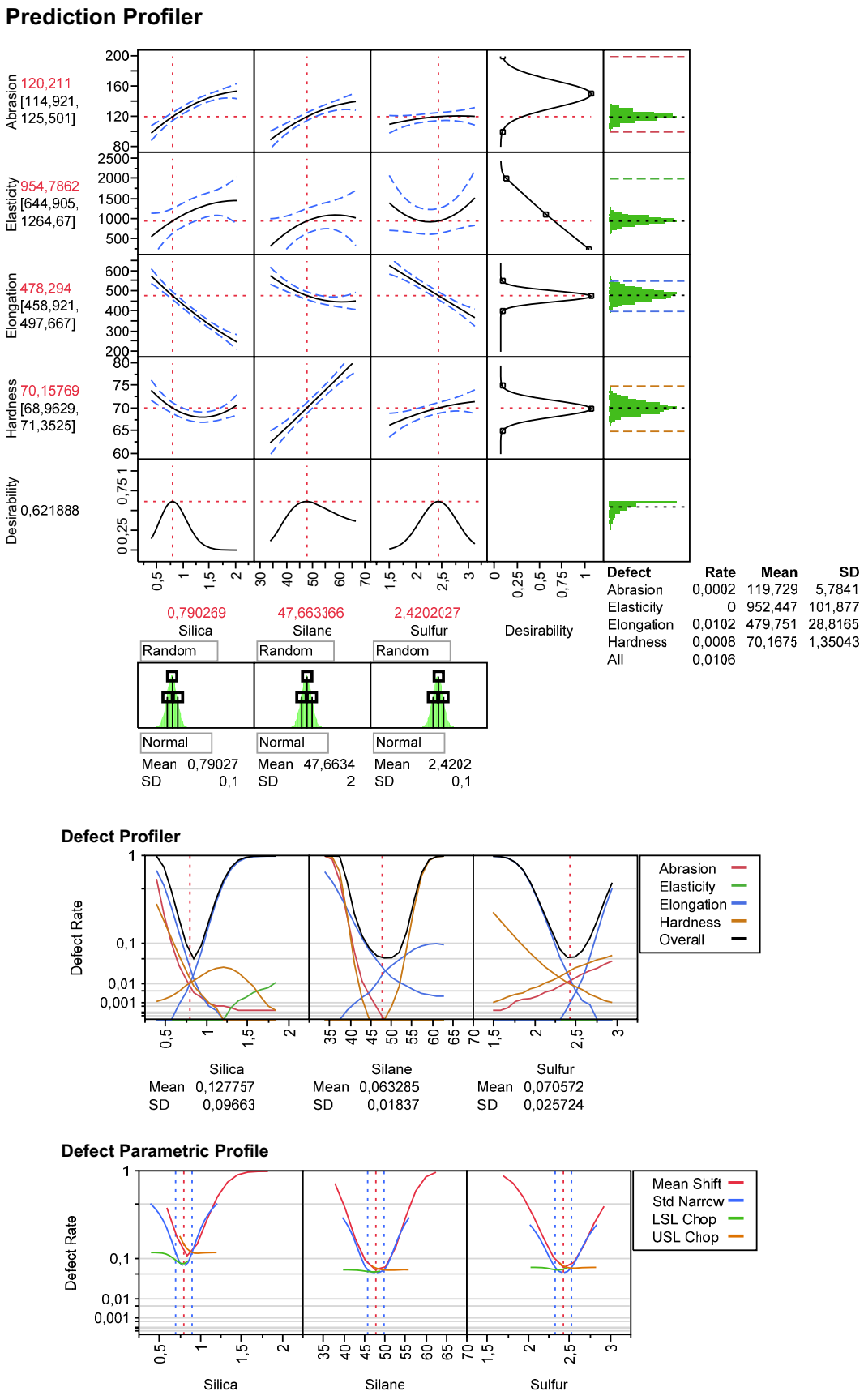


Fig. 10. Simulated prediction profiler after third iteration of control tolerance improvement.

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before and after control tolerance improvement in the key factors.

5 Conclusion

As a whole, classic DOE is the investigation of how the centrality of process performance can be impacted by controlling the target of each input variable. Tolerance analysis is the investigation of how the dispersion of process performance can be impacted by controlling both target and variability of each input variable. Essentially, these two methodologies are not contradictory, and can be highly complementary with visual approaches to tolerance analysis. In most cases, tolerance analysis is implemented in the phase of optimization during a classic DOE project. Visual tolerance analysis, helps build a communication bridge between two distinct domains – statistics and engineering by visualization – which is used to aid the understanding of engineering issues without the need for an extensive background in statistical methods. We believe such an approach speeds informed engineering decisions to help deliver higher quality at potentially lower cost.

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