J. Plasma Physics (2011), *vol.* 77, *part* 2, *pp.* 265–270. © Cambridge University Press 2010
doi:10.1017/S0022377810000048

On ion-dust streaming instability in a collisional magnetized plasma with warm dust

M. ROSENBERG

Department of Electrical and Computer Engineering, University of California, San Diego, La Jolla, CA 92093, USA
(rosenber@ece.ucsd.edu)

(Received 18 January 2010 and accepted 19 January 2010, first published online 26 February 2010)

Abstract. This brief communication discusses theoretically a resistive ion-dust streaming instability in a collisional dusty plasma, where the ions and electrons are magnetized, and the dust is unmagnetized. The instability is driven by ions streaming along the magnetic field. The emphasis is on the case where the dust has large thermal speed, and where the ion drift speed is $\lesssim$ the ion thermal speed. Application to possible laboratory experimental parameters is considered.

1. Introduction

The excitation of low-frequency dust acoustic waves [1] in collisional plasmas by streaming magnetized ions has been considered theoretically in several papers (*e.g.* [2–5]), motivated by the possibility of future dusty plasma experiments with large magnetic fields (see *e.g.* [6]). This includes instabilities driven by ions streaming across the magnetic field [2–4] or along the magnetic field [5]. This brief communication extends [5] to consider a resistive instability at long wavelengths, with emphasis on the case where the ion drift speed $\lesssim$ the ion thermal speed, and where the dust thermal speed is relatively large. The consideration of warm dust is motivated by recent experimental measurements of the dispersion relation of dust acoustic waves in plasmas with unmagnetized ions [7] and measurements of the velocity distribution of dust [8] that indicate that the dust kinetic temperature may be warm in some experiments (see also [9]). The analysis is given in Sec. 2, and the numerical results are in Sec. 3. Section 4 gives a brief summary.

2. Analysis

The model plasma is a weakly ionized dusty plasma composed of electrons, singly charged ions, negatively charged dust grains and neutrals. The mass, number density, temperature and thermal speed of each charged species are m_α, n_α, T_α and $v_\alpha = (T_\alpha/m_\alpha)^{1/2}$, respectively, where $\alpha = e, i, d$ refers to electrons, ions and charged dust, respectively. The electrons and ions are magnetized, while the dust is unmagnetized. Overall charge neutrality at equilibrium is given by $n_i = n_e + Z_d n_d$, where Z_d is the charge state of the dust grains that are assumed to have uniform radius a and mass m_d. There is an electric field $\mathbf{E}_0 = E_0\hat{\mathbf{z}}$ along the direction of an external magnetic field $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$. Each charged particle species acquires a drift of magnitude $u_{0\alpha} = eE_0/m_\alpha\nu_\alpha$, where ν_α is the collision frequency of species α, primarily assumed

https://doi.org/10.1017/S0022377810000048 Published online by Cambridge University Press

to be due to collisions with neutrals. The ions drift in the positive z-direction, while the electrons and dust grains drift in the negative z-direction. Here, we will consider the case where $u_{0i} \lesssim v_i$ (and $u_{0e} < v_e$), so in the absence of collisions, we would be considering a dust acoustic-type instability. In the following, we consider the frame where the dust drift $u_{0d} = 0$.

Electrostatic waves with frequency ω, much larger than the dust gyrofrequency, but much smaller than the ion gyrofrequency, are considered. Using a Maxwellian for the dust velocity distribution, and shifted Maxwellians for the ion and electron velocity distributions, the linear kinetic dispersion relation is (*e.g.* see [4, 10])

$$1 + \chi_e + \chi_i + \chi_d = 0\,, \tag{1}$$

where

$$\chi_j = \frac{1}{k^2\lambda_{Dj}^2}\,\left[1 + \zeta_j\, Z(\zeta_j)\Gamma_0(b_j)\right]\,\left[1 + \frac{iv_j}{\sqrt{2}k_z v_j}\Gamma_0(b_j)Z(\zeta_j)\right]^{-1}\,, \tag{2}$$

with $j = e, i$, and

$$\chi_d = \frac{1}{k^2\lambda_{Dd}^2}\,\left[1 + \zeta_d Z(\zeta_d)\right]\,\left[1 + \frac{iv_d}{\sqrt{2}kv_d}Z(\zeta_d)\right]^{-1}\,. \tag{3}$$

Here, Z is the plasma dispersion function [11] with the arguments

$$\zeta_j = \frac{\omega - \mathbf{k}\cdot\mathbf{u}_{0j} + iv_j}{\sqrt{2}k_z v_j}\,,$$

and

$$\zeta_d = \frac{\omega + iv_d}{\sqrt{2}kv_d}\,.$$

In addition, here $\lambda_{D\alpha} = (T_\alpha/4\pi n_\alpha Z_\alpha^2 e^2)^{1/2}$ is the Debye radius, $\rho_j = v_j/\omega_{cj}$ is the thermal gyroradius and ω_{cj} is the gyrofrequency of particle species j, k_z and $k_\perp$ are, the wave numbers parallel and perpendicular to $\mathbf{B}_0$, respectively, $b_j = k_\perp^2\rho_j^2$ and $\Gamma_0(b_j) = I_0(b_j)\exp(-b_j)$, with I_0 being the modified Bessel function of zero order.

2.1. Analytical results

We consider a resistive instability at long wavelengths, where $v_i/k_z v_i \gg 1$. In laboratory dusty argon plasmas, where T_e is typically $\sim$2 eV, the electron-neutral collision cross section can be much smaller than the ion-neutral collision cross section, so we can have simultaneously $v_e/k_z v_e < 1$ (see [12]). Further, assuming the electron drift speed $u_{0e} \ll v_e$, we may consider the regime $\zeta_e \ll 1$ while $\zeta_i \gg 1$. We assume the small Larmor radius limit holds for both electrons and ions, *viz.* $b_e \ll 1$ and $b_i \ll 1$, and also $b_i \ll 1/|2\zeta_i^2|$. For the dust, we consider the non-resonant regime, with $\zeta_d > 1$. Then (1) becomes roughly

$$1 + \frac{1}{k^2\lambda_{De}^2} - \frac{\omega_{pi}^2}{\bar{\omega}(\bar{\omega} + iv_i)}\frac{k_z^2}{k^2} - \frac{\omega_{pd}^2}{\omega^2}\left(1 + \frac{iv_d}{\omega}\right) + iF \approx 0\,, \tag{4}$$

where $\bar{\omega} = \omega - k_z u_{0i}$. In the last term on the left-hand side of (4), which represents dust Landau damping,

$$F \sim \sqrt{\frac{\pi}{2}}\,\frac{\omega\omega_{pd}^2}{k^3 v_d^3}\exp\left(-\frac{\omega^2}{2k^2v_d^2}\right)\,.$$

https://doi.org/10.1017/S0022377810000048 Published online by Cambridge University Press

Making the reasonable assumption that $k_z u_{0i} \gg \omega$ since we are considering ion drift speeds $u_{0i} \lesssim v_i$, and taking $v_i \gg k_z u_{0i}$, (4) becomes

$$1 + \frac{1}{k^2 \lambda_{De}^2} - i\frac{k_z^2}{k^2}\frac{\omega_{pi}^2}{v_i^2}\frac{v_i}{k_z u_{0i}} - \frac{\omega_{pd}^2}{\omega(\omega + iv_d)} + iF \approx 0\,. \tag{5}$$

We neglect dust Landau damping initially. We assume that $v_d < \omega$ (*i.e.* dust collisional effects are small), and $\omega_{pd}^2/\omega^2 \gg 1 + (k\lambda_{De})^{-2}$ (*i.e.* the contribution of the electron susceptibility is negligible, and $\omega \ll \omega_{pd}$). We can then solve (5) for $\omega = \omega_r + i\gamma$, where ω_r and γ are the real and imaginary parts of ω, respectively:

$$\frac{\omega_r + i\gamma}{\omega_{pd}} \sim \frac{1+i}{\sqrt{2}}\left(\frac{v_i}{\omega_{pi}}\frac{k}{k_z}\frac{ku_{0i}}{\omega_{pi}}\right)^{1/2} - \frac{iv_d}{2\omega_{pd}}\,. \tag{6}$$

The growth of this resistive instability is favored by smaller k_z, which also facilitates the condition $v_i/k_z v_i \gg 1$. Note, however, there is a minimum $k_z u_{0i}$ for instability. This can be seen by including the dust Landau damping term in a perturbative manner. Assuming $\omega_r > \gamma$ and using ω_r from (6), we find that the following approximate Landau damping term should be added to the right-hand side of (6):

$$\sim -\frac{i\omega_r}{\omega_{pd}}\sqrt{\frac{\pi}{8}}\frac{\omega_r^3}{k^3 v_d^3}\exp\left(-\frac{\omega_r^2}{2k^2 v_d^2}\right)\,. \tag{7}$$

The frequency and the growth rate of the long wavelength-resistive instability can be compared with the dust acoustic instability in the shorter wavelength range where $v_i/k_z v_i \ll 1$. We solve (1) in the limits $\zeta_i \lesssim 1$, $\zeta_e \ll 1$ and $\zeta_d \gg 1$, setting $\omega = \omega_r + i\gamma$, with $|\gamma| \ll \omega_r$. This yields for the dust acoustic instability (see also [12]):

$$\omega_r \approx \frac{k\lambda_D \omega_{pd}}{(1 + k^2\lambda_D^2)^{1/2}}, \tag{8a}$$

$$\frac{\gamma}{\omega_r} \sim \frac{1}{k^2\lambda_{Di}^2}\frac{\omega_r^2}{\omega_{pd}^2}\sqrt{\frac{\pi}{8}}\left(\Gamma_0(b_i)\frac{u_{0i}}{v_i} - \frac{T_i}{\delta T_e}\frac{u_{0e}}{v_e}\right) - \frac{v_d}{2\omega_r}. \tag{8b}$$

Here λ_D is the linearized Debye length, which is $\sim\lambda_{Di}$ in a laboratory dusty plasma where generally $T_e \gg T_i$, and $\delta = n_i/n_e$. To obtain (8b), it was assumed that $b_e \ll 1$, $k_z u_{0i}$, $k_z u_{0e} \gg \omega$ and $v_d < \omega$. It can be surmised from (8b) that as k_z/k decreases and $k_\perp/k$ increases, the growth rate of this instability for fixed u_{0i}/v_i can decrease owing to the decrease in $\Gamma_0(b_i)$.

3. Numerical results

Here we show numerical solutions of (1) for possible parameters of laboratory dusty plasmas with magnetized ions (and electrons) and unmagnetized dust. We consider a weakly ionized argon plasma with the following parameters: $B_0 \sim 1 \times 10^4$ G, neutral density $n_n \sim 10^{15}$ cm^{-3} (pressure $\sim$4 Pa), ion density $n_i \sim 10^8$ cm^{-3}, $T_e \sim 2$ eV and $T_i \sim 0.05$ eV. With these parameters, the ion gyrofrequency is $\omega_{ci} \sim 2.4 \times 10^6$ rad/s, the ratio of ion plasma frequency to ion gyrofrequency is $\omega_{pi}/\omega_{ci} \sim 0.9$ and the ion Debye length $\lambda_{Di} \sim 170$ μm. The collision frequencies of electrons and ions with neutrals are estimated using $\nu_{e,i} \sim \sigma_{e,i} n_n v_{e,i}$ (with collision cross sections $\sigma_e \sim 5 \times 10^{-16}$ cm^2 and $\sigma_i \sim 5 \times 10^{-15}$ cm^2 [13]). Thus, the ratio of the ion collision frequency to the ion gyrofrequency is $\nu_i/\omega_{ci} \sim 0.08$ (and $\nu_e/\omega_{ce} \ll 1$). Furthermore, the ratio of

https://doi.org/10.1017/S0022377810000048 Published online by Cambridge University Press

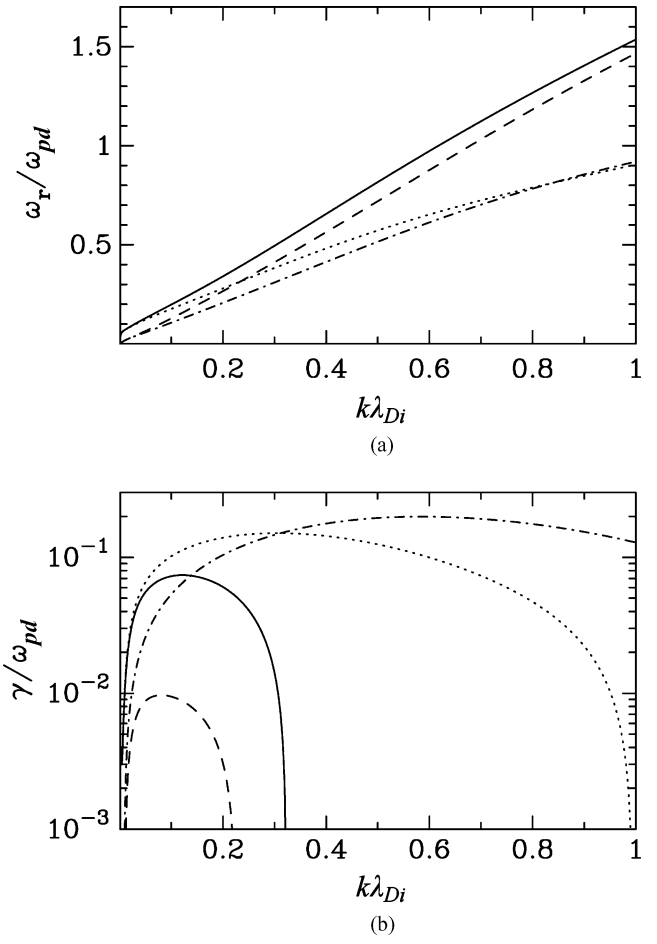


Figure 1. (a) Real ω_r and (b) imaginary γ parts of ω (normalized to ω_{pd}) versus $k\lambda_{Di}$ obtained by solving (1). Parameters are $m_i/m_p = 40$, $m_d/m_p = 6 \times 10^{11}$, $T_e/T_i = 40$, $\omega_{pi}/\omega_{ci} = 0.9$, negatively charged dust with $Z_d = 1.7 \times 10^3$, $n_d/n_i = 1 \times 10^{-4}$, $v_i/\omega_{pi} = 0.1$, $v_e/\omega_{pi} = 14$, $v_d/\omega_{pd} = 0.06$ and $u_{0i}/v_i = 1.0$, $u_{0e}/v_e = 0.25$. The curves correspond to different values of T_e/T_d and θ, where $\cos\theta = k_z/k$: $T_e/T_d = 2$, $\theta = 0$ (dot-dash curve); $T_e/T_d = 2$, $\theta = 80^\circ$ (dotted curve); $T_e/T_d = 0.2$, $\theta = 0$ (dashed curve); $T_e/T_d = 0.2$, $\theta = 80^\circ$ (solid curve).

the electron to ion drift speeds (normalized by their respective thermal speeds) is $(u_{0i}/v_i)(v_e/u_{0e}) \sim 4$.

We assume that the dust has uniform radius $a \sim 0.5$ μm, the dust mass is $m_d \sim 6 \times 10^{11}\, m_p$ (where m_p is the proton mass) and the dust density is $n_d \sim 1 \times 10^4$ cm^{-3} (corresponding to an inter-grain spacing $d \sim 300$ μm). We estimate the dust charge state using $Z_d \sim a\phi/e \sim 2.5\, aT_e/e^2 \sim 1700$ [14]. This yields a dust plasma frequency of $\omega_{pd} \sim 300$ rad/s. The dust-neutral collision frequency is estimated using the hard-sphere collision frequency in the free-molecular regime [15], given by $v_d \approx 4a^2(T_n m_n)^{1/2} n_n/m_d \sim 17$ s^{-1} (using $T_n \sim$ room temperature and the neutral mass $m_n \sim 40 m_p$). With these parameters, we have $v_d/\omega_{pd} \sim 0.06$, and $v_d/\omega_{cd} \sim 70$. Thus, the dust is unmagnetized while the ions and electrons are magnetized. We will

https://doi.org/10.1017/S0022377810000048 Published online by Cambridge University Press

consider the case where the dust thermal (kinetic) energy T_d is significantly larger than T_i. We note that with these parameters, the dust would be either weakly correlated or in the strongly coupled liquid phase, depending on the dust thermal (kinetic) energy T_d. However, it was shown that the ion-dust streaming instability in an unmagnetized dusty plasma is not affected much by strong coupling in the liquid phase [16].

Figure 1 shows solutions of (1) for the real ω_r and and the imaginary γ parts of the frequency (normalized by the dust plasma frequency) as a function of $k\lambda_{Di}$, for a set of dimensionless parameters corresponding to the parameters given above, and with fixed $u_0/v_i = 1.0$ and $u_{0e}/v_e = 0.25$. The curves are for different values of T_e/T_d and $k_z/k = \cos\theta$, where θ is the angle between **k** and **B**. In the case of relatively cooler dust, with $T_e/T_d = 2$ (*i.e.* $T_d = 1$ eV), the dust acoustic speed, $c_{sd} \sim \lambda_{Di}\omega_{pd}$, is significantly larger than the dust thermal speed, with $c_{sd} \sim 3.8v_d$. As can be seen from Fig. 1(b), growth of the dust acoustic instability tends to decrease as k_z/k decreases. This can be attributed to the decrease in the growth rate (8b) of the shorter wavelength dust acoustic instability as $\Gamma_0(b_i)$ decreases. In the case of warm dust, with $T_e/T_d = 0.2$ (*i.e.* $T_d = 10$ eV), the dust acoustic speed is only slightly larger than the dust thermal speed, $c_{sd} \sim 1.2v_d$. Only the long wavelength-resistive instability persists, and its growth rate tends to increase as k_z/k decreases, as discussed with relation to (6). For the warm dust case, when $\theta = 80^\circ$ ($k_z/k \sim 0.17$), at maximum growth the perpendicular and parallel wavelengths are $\sim$0.7 cm and $\sim$4 cm, respectively.

4. Summary

We considered theoretically a resistive ion dust streaming instability in a dusty plasma, in which the ions and electrons are magnetized and the dust is unmagnetized. The instability is driven by ions streaming along the magnetic field. The emphasis was on the case where the dust kinetic temperature is warm, and where the ion drift speed along $\mathbf{B}_0$ is $\lesssim$ the ion thermal speed. The long wavelength-resistive instability may persist, with larger growth at smaller k_z/k, when the dust is warm. Applications to possible laboratory experimental parameters were considered.

Acknowledgements

This work was partially supported by NSF PHY0903808. The author would like to dedicate this paper to her esteemed colleague Padma Kant Shukla on the occasion of his 60th birthday.

References

[1] Rao, N. N., Shukla, P. K. and Yu, M. Y. 1990 *Planet. Space Sci.* **38**, 543.
[2] Shukla, P. K., Salimullah, M. and Sorasio, G. 2002 *Phys. Plasmas* **9**, 5121.
[3] D'Angelo, N. 2003 *Planet. Space Sci.* **51**, 393.
[4] Rosenberg, M. and Shukla, P. K. 2007 *J. Plasma Phys.* **73**, 189.
[5] Rosenberg, M. and Shukla, P. K. 2004 *J. Plasma Phys.* **70**, 317.
[6] Fortov, V. E., Ivlev, A. V., Khrapak, S. A., Khrapak, A. G. and Morfill, G. E. 2005 *Phys. Reports* **421**, 1.
[7] Thomas Jr., E., Fisher, R. and Merlino, R. L. 2007 *Phys. Plasmas* **14**, 123701.

https://doi.org/10.1017/S0022377810000048 Published online by Cambridge University Press

[8] Williams, J. D. and Thomas Jr., E. 2006 *Phys. Plasmas* **13**, 063509.
[9] Rosenberg, M., Thomas Jr., E. and Merlino, R. L. 2008 *Phys. Plasmas* **15**, 073701.
[10] Satyanarayana, P., Chaturvedi, P. K., Keskinen, M. J., Huba, J. B. and Ossakow, S. L. 1985 *J. Geophys. Res.* **90**, 12209.
[11] Fried, B. D. and Conte, S. D. 1961 *The Plasma Dispersion Function*. New York: Academic.
[12] Rosenberg, M. 2002 *J. Plasma Phys.* **67**, 235.
[13] Mitchner, M. and Kruger, Jr., C. H. 1973 *Partially Ionized Gases*. New York: Wiley.
[14] Shukla, P. K. and Mamun, A. A. 2002 *Introduction to Dusty Plasma Physics*. Bristol, UK: IOP Publ.
[15] Epstein, P. S. 1924 *Phys. Rev.* **23**, 710.
[16] Rosenberg, M. and Kalman, G. 1998 CP446, In: *Physics of Dusty Plasmas: 7th Workshop* (ed. M. Horanyi et al.). New York: AIP , pp. 135–141.

https://doi.org/10.1017/S0022377810000048 Published online by Cambridge University Press