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RESEARCH NOTES

A further example of $A^4+B^4+C^4+D^4=E^4$

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Recently, several examples (1) of primitive solutions of the above equation have been reported, in addition to the old first solution exhibited by Norrie ((2), pages 87–89) in 1911. The purpose of the present note is to outline a search method aimed at finding other solutions and to report on a new solution, which has been found by this method.

It was already pointed out in (1) that

$$E^4-D^4=(E-D)(E^2+D^2)(E+D)=A^4+B^4+C^4=0 \mod 625.$$

Let us write $$E=\frac{x+5^4y}{2}, \quad D=\pm\frac{x-5^4y}{2}.$$

(In the present work we have not treated the case in which (E^2+D^2) is divisible by 625.)

Then it can be shown that

$$\frac{E^4-D^4}{10^4}=\frac{xy(x^2+5^8y^2)}{2}=\left(\frac{A}{10}\right)^4+\left(\frac{B}{10}\right)^4+\left(\frac{C}{10}\right)^4=i^4+j^4+k^4=H,$$

where $i,j,k=$ integers.

Since H is a sum of 3 fourth powers it follows that: $H=0,1,2,3 \mod 16$ and $=0,1,2,3 \mod 5$ (the case of $H=0 \mod 5$ has been excluded because it is outside our range of search). Thus, for each value of y only a restricted set of x is possible. If $y=1$, for example:

$$x=13,9,1 \mod 20 \quad \text{and} \quad x=58,154,26 \mod 160 \quad (H=1 \mod 16),$$
$$x=68,4,36 \mod 160 \quad (H=2 \mod 16),$$
$$x=78,14,46 \mod 160 \quad (H=3 \mod 16).$$

Similar tables can be constructed for each y.

The solution that we have found is

$$955^4+1770^4+2634^4+5400^4=5491^4.$$

This was found for $y=13$, $x=2857$. In the range of our search ($x \leqslant y.5^4$), no new solution was obtained for $y=3$, 9 and 17. It should be noted that for this particular solution we have $A+B+D=C+E=13.5^4$.

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The author is now at the Laboratory for Nuclear Science, Massachusetts Institute of Technology.

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On the finiteness of semigroups in which $x^r = x$

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In this note we present a new proof of the following theorem (see (1)).

THEOREM. *For each r, $r \geqslant 2$, every finitely generated semigroup in which $x^r = x$ holds identically is finite if and only if every finitely generated group of exponent $r-1$ is finite.*

Notation. Throughout, r is fixed and S_k denotes a semigroup on k generators in which $x^r = x$. The elements of S_k are regarded as equivalence classes of words in k symbols $X_1, \ldots, X_k$. Upper case letters will denote words, and lower case letters the elements of the semigroup S_k; thus if the word W represents the element w of S_k, we write $W \in w \in S_k$, and also say that W is a word of S_k. $W = W'$ signifies that W and W' are identical while $W \sim W'$ signifies that W and W' are equivalent, i.e. represent the same element of S_k. If W is a word of S_k, we write $|W|$ for the *length* of W in the symbols X_i; e.g. $|X_1X_1X_2| = 3$. If each X_i appears in W, $1 \leqslant i \leqslant k$, we say that W is a *complete* word of S_k; any word W is *minimal* if $W \sim W'$ implies $|W| \leqslant |W'|$.

Remark. Clearly S_k is finite if and only if the lengths of minimal words of S_k are bounded from above.

LEMMA 1. *$W \sim AXB$ implies $W \sim W(XW)^{r-1}$.*

Proof.

$$W(XW)^{r-1} \sim AXB(XAXB)^{r-1} \sim (AX)^{r-2}A[XAXB(XAXB)^{r-1}] \sim AXB \sim W.$$

LEMMA 2. *Let T be a complete word of S_k. Then for any word X of S_k, $T \sim T(XT)^{r-1}$.*

Proof. By Lemma 1 we need only show that $T \sim AXB$ for some A, B. We show by induction on $|X|$ that $T \sim TXB$ for a suitable B. If $|X| = 0$, i.e. X is the empty word,

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