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Switching and induced flow effects in chiral smectic C liquid crystals

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We consider a chiral smectic C liquid crystal confined between parallel plates in the bookshelf geometry. Using a recently proposed continuum theory for such materials the behaviour of the cell is discussed when an electric field is applied and subsequently removed from the cell. We examine both symmetric and asymmetric boundary conditions for the director.

1 Introduction

Thermotropic liquid crystals are an intermediate state of matter between solids and liquids and, as one would expect, they possess some of the properties of both. For example, liquid crystals can flow like liquids but their molecules have some degree of order, reminiscent of a solid. There are many types of liquid crystal (see, for example, de Gennes & Prost [1]), but we confine attention to smectic C and chiral smectic C or smectic C* liquid crystals. These have both positional and orientational ordering of their molecules. If we regard the molecules of a liquid crystal as being made up of rigid rods, then in a smectic C liquid crystal the molecules tend to align themselves into layers (positional order) as well as all roughly pointing in the same direction (orientational order).

The liquid crystals in which we are mainly interested here are smectic C* liquid crystals. Within one layer of these materials we regard all molecules as lying parallel to each other with tilt angle θ to the layer normal but as we move through the liquid crystal in the direction parallel to the layer normal the anisotropic axis sweeps out the surface of a cone. (See Figure 1 in the article by Brown, Dunn and Jones in this volume.) In the presence of a chiral agent, the direction of tilt precesses around this smectic cone, viewed from layer to layer, and therefore a helical configuration is obtained, called the C* phase. Usually, the helical pitch p is much larger than the integer layer distance. It is these liquid crystals which have become of major scientific and commercial interest over recent years owing to the fact that they are ferroelectric materials; that is they possess a spontaneous polarization which can be utilized to develop a device with very fast switching properties [2]. The spontaneous polarization lies in the plane of the layers but perpendicular to the director $\boldsymbol{n}$.

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In this article we examine the switching and induced flow effects in two types of surface stabilized ferroelectric liquid crystal (SSFLC) cell. A SSFLC is simply a smectic C* liquid crystal confined between parallel plates with its layer's perpendicular to the bounding surfaces and the gap between the plates small enough so that the inherent chirality of the material is removed. The liquid crystal thus resembles a smectic C except for the presence of the spontaneous polarization. Switching properties of these materials are important for applications and it is also of relevance to consider the effects of induced flow in such cells given the fact that nematic liquid crystal devices can exhibit some interesting behaviour due to such backflows [3–5]. To investigate these effects we use a continuum theory which we outline in the next section. In §3 we describe the problem and the governing equations and boundary conditions. Finally, in §4 and 5, switching with and without the effects of the induced flow is discussed.

2 Continuum theory

The main aim of any macroscopic theory which attempts to model liquid crystals is to predict the behaviour of the anisotropic axis and to do so three main influences on $\boldsymbol{n}$ must be taken into account, namely elastic, flow and external field effects. In the case of smectic C* liquid crystals the continuum theory of Leslie *et al*. [6] and Carlsson *et al*. [7] takes these three influences into account, and we use their equations throughout. We adopt the notation and continuum model introduced in the preceding article by Atkin and Stewart. For convenience, we recall the constraints

$$a_i a_i = c_i c_i = 1, \quad a_i c_i = 0, \quad e_{ijk} a_{k,j} = 0, \tag{1}$$

where a repeated subscript implies summation and a comma denotes differentiation with respect to the relevant spatial variable. Often, however, the orientation of the smectic layers is assumed and thus the layer normal $\boldsymbol{a}$ is defined *a priori*. This will be the case throughout this paper.

Unlike the continuum theory for isotropic fluids, the balance equations for both linear and angular momentum have a significant part to play. From the balance of linear momentum and the assumption of incompressibility of the liquid crystal, we have

$$\rho \dot{v}_i = \rho F_i + t_{ij,j}, \quad v_{i,i} = 0, \tag{2}$$

with $\boldsymbol{v}$ the fluid velocity, $\boldsymbol{F}$ and $\boldsymbol{t}$ representing the body forces and stress tensor, respectively, while ρ is the fluid density. The superposed dot denotes the material time derivative. One finds that the stress tensor takes the form

$$t_{ij} = -p\delta_{ij} + \beta_p e_{pjk} a_{k,i} - \frac{\partial W}{\partial a_{k,j}} a_{k,i} - \frac{\partial W}{\partial c_{k,j}} c_{k,i} + \tilde{t}_{ij}, \tag{3}$$

where the vector $\boldsymbol{\beta}$ and the pressure p arise from the constraints $(1)_4$ and $(2)_2$ and $\tilde{\boldsymbol{t}}$ is the dynamic part of the stress tensor. Various forms for the elastic energy density W of smectic C* liquid crystals have been formulated [7, 8]. We choose to write it as

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$$2W = K_1(a_{i,i})^2 + (K_2 - K_4)(c_{i,i})^2 + (K_3 - K_4)\, c_{i,j}\, c_j\, c_{i,k}\, c_k + K_4\, c_{i,j}\, c_{i,j} + (K_5 - K_3)(c_i\, a_{i,j}\, c_j)^2 + 2K_6\, a_{i,i}(c_j\, a_{j,k}\, c_k) - 2K_7\, c_{i,j}\, c_j\, c_{i,k}\, a_k + 2(K_8 - K_7)\, c_{i,i}(c_j\, a_{j,k}\, c_k) + 2K_9\, a_{i,i}\, c_{j,j} - \frac{2\pi K_4}{q} e_{ijk}\, a_j\, c_k\, c_{i,p}\, a_p, \tag{4}$$

the K's being constants and q being the pitch of the smectic C*. The K's are related to the constants introduced in the preceding article via the identities contained in Leslie *et al.* [8]. Since W is necessarily positive definite one finds that K_2 and K_3 are both positive, together with some other inequalities which are surplus to our requirements.

The viscous stress tensor $\tilde{\boldsymbol{t}}$ is the sum of a symmetric part

$$(\tilde{t}_{ij}) = \mu_0 D_{ij} + \mu_1 a_k D_{kp} a_p a_i a_j + \mu_2(D_{ik} a_k a_j + D_{jk} a_k a_i) + \mu_3 c_k D_{kp} c_p c_i c_j + \mu_4(D_{ik} c_k c_j + D_{jk} c_k c_i) + \mu_5 c_k D_{kp} a_p(a_i c_j + a_j c_i) + \lambda_1(A_i a_j + A_j a_i) + \lambda_2(C_i c_j + C_j c_i) + \lambda_3 A_k c_k(a_i c_j + a_j c_i) + \kappa_1(D_{ik} a_k c_j + D_{jk} a_k c_i + D_{ik} c_k a_j + D_{jk} c_k a_i) + \kappa_2(a_k D_{kp} a_p(a_i c_j + a_j c_i) + 2a_k D_{kp} c_p a_i a_j) + \kappa_3(c_k D_{kp} c_p(a_i c_j + a_j c_i) + 2a_k D_{kp} c_p c_i c_j) + \tau_1(C_i a_j + C_j a_i) + \tau_2(A_i c_j + A_j c_i) + 2\tau_3 A_k c_k a_i a_j + 2\tau_4 A_k c_k c_i c_j, \tag{5}$$

and a skew-symmetric part

$$[\tilde{t}_{ij}] = \lambda_1(D_{jk} a_k a_i - D_{ik} a_k a_j) + \lambda_2(D_{jk} c_k c_i - D_{ik} c_k c_j) + \lambda_3 c_k D_{kp} a_p(a_i c_j - a_j c_i) + \lambda_4(A_j a_i - A_i a_j) + \lambda_5(C_j c_i - C_i c_j) + \lambda_6 A_k c_k(a_i c_j - a_j c_i) + \tau_1(D_{jk} a_k c_i - D_{ik} a_k c_j) + \tau_2(D_{jk} c_k a_i - D_{ik} c_k a_j) + \tau_3 a_k D_{kp} a_p(a_i c_j - a_j c_i) + \tau_4 c_k D_{kp} c_p(a_i c_j - a_j c_i) + \tau_5(A_j c_i - A_i c_j + C_j a_i - C_i a_j), \tag{6}$$

where

$$D_{ij} = \tfrac{1}{2}(v_{i,j} + v_{j,i}), \quad W_{ij} = \tfrac{1}{2}(v_{i,j} - v_{j,i}), \quad A_i = \dot{a}_i - W_{ij} a_j, \quad C_i = \dot{c}_i - W_{ij} c_j, \tag{7}$$

and the coefficients are constants. The above expression for $\tilde{\boldsymbol{t}}$ incorporates the Onsager relations but restrictions are placed on the viscosity coefficients through the entropy inequality

$$(\tilde{t}_{ij})\, D_{ij} - \tilde{g}^a_i\, A_i - \tilde{g}^c_i\, C_i \geqslant 0. \tag{8}$$

After some manipulation, one finds that balance of angular momentum yields

$$\left(\frac{\partial W}{\partial a_{i,j}}\right)_{,j} - \frac{\partial W}{\partial a_i} + G^a_i + \tilde{g}^a_i + \gamma a_i + \mu c_i + e_{ijk}\beta_{k,j} = 0, \tag{9}$$

$$\left(\frac{\partial W}{\partial c_{i,j}}\right)_{,j} - \frac{\partial W}{\partial c_i} + G^c_i + \tilde{g}^c_i + \tau c_i + \mu a_i = 0, \tag{10}$$

where the scalars γ, μ and τ are Lagrange multipliers arising from the constraints $(1)_{1,2,3}$ and

$$\tilde{g}^a_i = -2(\lambda_1 D_{ij} a_j + \lambda_3 c_j D_{jk} a_k c_i + \lambda_4 A_i + \lambda_6 A_j c_j c_i + \tau_2 D_{ij} c_j + \tau_3 a_j D_{jk} a_k c_i + \tau_4 c_j D_{jk} c_k c_i + \tau_5 C_i), \tag{11}$$

$$\tilde{g}^c_i = -2(\lambda_2 D_{ij} c_j + \lambda_5 C_i + \tau_1 D_{ij} a_j + \tau_5 A_i). \tag{12}$$

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There now only remains the question of writing down expressions for $\boldsymbol{G}^a$ and $\boldsymbol{G}^c$ which depends upon whether the applied field is magnetic or electric. In this paper we consider electric fields. However the behaviour of smectic C and C* liquid crystals in electric fields is complicated due to their biaxiality [9]. Further, we wish to illustrate the possible effects flow may have in certain smectic C* cells rather than model the electrical effects accurately. We therefore make a rather sweeping and naïve assumption and assume smectic C* liquid crystals are isotropic with respect to electric fields. We also neglect the transport of ions across the cell [10, 11]. In this case

$$G^a_i = -P(E_j a_j) b_i, \quad G^c_i = -P(E_j c_j) b_i, \quad b_i = e_{ijk} a_j c_k, \tag{13}$$

where P is the spontaneous polarization and $\boldsymbol{E}$ is the electric field. For non-chiral smectic C liquid crystals $\boldsymbol{G}^a$ and $\boldsymbol{G}^c$ are identically zero. The spontaneous polarization also has an effect on the electric displacement $\boldsymbol{D}$ [12], and one finds

$$D_i = \epsilon_0 \epsilon E_i + P b_i, \tag{14}$$

where ϵ_0 is the permitivity of free space, ϵ is the dielectric constant and $\boldsymbol{E}$ is the electric field. For time-independent fields, and assuming the charge free density of the liquid crystal is zero, we also have

$$e_{ijk} E_{k,j} = 0, \quad D_{i,i} = 0. \tag{15}$$

The former of these implies that there exists a function V, the voltage, such that

$$E_i = -V_{,i}. \tag{16}$$

3 Equations and boundary conditions

Throughout we consider a surface stabilized smectic C* liquid crystal confined between parallel plates a distance $2d$ apart with its layers everywhere perpendicular to the plates – the so-called 'bookshelf geometry'. We choose a Cartesian coordinate system such that the yz-plane is in the plane of the layers, the xy-plane is in the centre of the cell and the z-axis perpendicular to the plates. To this cell an electric field of the form

$$\boldsymbol{E} = (0, 0, E), \tag{17}$$

with E constant in time is applied and we assume that this field has no effect on the layers. We consider solutions of the form‡

$$\boldsymbol{a} = (1, 0, 0), \quad \boldsymbol{c} = (0, \cos\phi, \sin\phi), \quad \boldsymbol{v} = (u, v, 0), \tag{18}$$

with

$$\phi = \phi(z, t), \quad u = u(z, t), \quad v = v(z, t). \tag{19}$$

Since we have assumed that the chirality of the liquid crystal has been unwound by the boundaries ϕ need only have a spatial dependence through z. It turns out that the governing

‡ We have a flow component perpendicular to $\boldsymbol{a}$, which is impossible in the geometry of the preceding paper.

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equations are rather complicated and a numerical approach is necessary. We thus immediately introduce the substitutions

$$\tilde{z} = \frac{z}{d}, \quad \tilde{t} = \frac{kt}{d^2}, \quad \tilde{u} = \frac{du}{k}, \quad \tilde{v} = \frac{dv}{k}, \tag{20}$$

with $k = K_2/2\lambda_5$, to make our equations dimensionless. One can show from (8) that λ_5 is positive so k is also positive. From $(15)_2$

$$D_z = D(\tilde{t}), \tag{21}$$

so from $(13)_3$, (14), (17) and $(18)_{1,2}$

$$E = \frac{1}{\epsilon_0 \epsilon}(D - P\cos\phi), \tag{22}$$

which upon use of (16) implies

$$D = \frac{P}{2}\int_{-1}^{1} \cos\phi \, d\tilde{z} - \frac{V\epsilon_0 \epsilon}{2d}, \tag{23}$$

where V is the potential difference across the cell. With the choices (18) for $\boldsymbol{a}$, $\boldsymbol{c}$ and $\boldsymbol{v}$ one finds from the balance of angular momentum equation (10) that

$$\frac{\partial \phi}{\partial \tilde{t}} = \frac{\partial^2 \phi}{\partial \tilde{z}^2} - a(\phi)\frac{\partial \tilde{u}}{\partial \tilde{z}} - b(\phi)\frac{\partial \tilde{v}}{\partial \tilde{z}} + \frac{Pd^2}{K\epsilon_0 \epsilon}(P\cos\phi - D)\sin\phi, \tag{24}$$

where we have made the one constant approximation $K_2 = K_3 = K$, K being necessarily positive, and

$$a(\phi) = \frac{(\tau_1 + \tau_5)}{2\lambda_5}\cos\phi, \quad b(\phi) = \frac{(\lambda_2 \cos 2\phi + \lambda_5)}{2\lambda_5}. \tag{25}$$

Equations for the Lagrange multipliers $\boldsymbol{\beta}$, γ, μ and τ may also be written down, but it is sufficient to know that these variables can be determined. From the linear momentum equation $(2)_1$

$$\xi_1(\phi)\frac{\partial \tilde{u}}{\partial \tilde{z}} + \xi_2(\phi)\frac{\partial \tilde{v}}{\partial \tilde{z}} + 2\lambda_5 a(\phi)\frac{\partial \phi}{\partial \tilde{t}} = f(\tilde{t}), \tag{26}$$

$$\xi_2(\phi)\frac{\partial \tilde{u}}{\partial \tilde{z}} + \xi_3(\phi)\frac{\partial \tilde{v}}{\partial \tilde{z}} + 2\lambda_5 b(\phi)\frac{\partial \phi}{\partial \tilde{t}} = g(\tilde{t}), \tag{27}$$

where inertial terms have been neglected, f and g are functions of reduced time to be determined and

$$\left.\begin{aligned} 2\xi_1(\phi) &= \mu_0 + \mu_2 + 2\lambda_1 + \lambda_4 + (\mu_4 + \mu_5 - 2\lambda_2 + 2\lambda_3 + \lambda_5 + \lambda_6)\sin^2\phi, \\ 2\xi_2(\phi) &= (\kappa_1 + \tau_1 + \tau_2 + \tau_5 + 2(\kappa_3 + \tau_4)\sin^2\phi)\cos\phi, \\ 2\xi_3(\phi) &= \mu_0 + \mu_4 + \lambda_5 + 2\mu_3 \sin^2\phi\cos^2\phi + 2\lambda_2 \cos 2\phi. \end{aligned}\right\} \tag{28}$$

The third equation of linear momentum balance yields an expression for the pressure p. We are thus left with the task of solving three nonlinear partial differential equations and one integral equation. Before proceeding, however, we introduce one final simplification to the problem and replace the functions $\xi_1(\phi)$, $\xi_2(\phi)$ and $\xi_3(\phi)$ by constants denoted by ξ_1, ξ_2 and ξ_3, respectively. Given that the viscosities are at present unknown constants, this

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simplification is justified, since the sinusoidal terms in (28) are less than one in modulus. This has the twofold advantage of making our equations simpler to solve, and we now only have to estimate three numbers rather than the seven groups of viscosities present in (28).

We examine two pairs of boundary conditions on ϕ. The first set are the symmetric boundary conditions on $\boldsymbol{c}$ introduced by Clark & Lagerwall [2] in their bistable device, namely

$$\phi(-1,\tilde{t}) = \phi(1,\tilde{t}) = 0, \quad \tilde{t} \geqslant 0, \tag{29}$$

while the second set are asymmetric with

$$\phi(-1,\tilde{t}) = 0, \quad \phi(1,\tilde{t}) = \pi, \quad t \geqslant 0, \tag{30}$$

and were utilized in a device proposed by Patel [13]. The conditions on $\tilde{u}$ and $\tilde{v}$ at the boundaries are the no-slip conditions

$$\tilde{u}(-1,\tilde{t}) = \tilde{u}(1,\tilde{t}) = \tilde{v}(-1,\tilde{t}) = \tilde{v}(1,\tilde{t}) = 0, \quad \tilde{t} \geqslant 0. \tag{31}$$

Our initial conditions are

$$\tilde{u}(\tilde{z},0) = \tilde{v}(\tilde{z},0) = 0, \quad \phi(\tilde{z},0) = \phi_0(\tilde{z}), \quad -1 < \tilde{z} < 1, \tag{32}$$

the latter of these depending upon which boundary conditions we assume. For the pair (29) ϕ is uniformly equal to zero across the cell, but for the other set it is not immediately obvious what the correct initial condition is.

In what follows, we always use the same values for the relevant material parameters, that is

$$\left.\begin{aligned} &K = 10^{-11}\ \text{N}, \quad P = 10^{-4}\ \text{cm}^{-2}, \quad \epsilon_0 = 8.85\times10^{-12}, \quad \epsilon = 3.0,\\ &d = 2\times10^{-6}\ \text{m}, \quad \xi_1 = 2.5\times10^{-2}\ \text{kgm}^{-1}\,\text{s}^{-1}, \quad \xi_2 = 4.0\times10^{-3}\ \text{kgm}^{-1}\,\text{s}^{-1},\\ &\xi_3 = 5.0\times10^{-2}\ \text{kgm}^{-1}\,\text{s}^{-1}, \quad \tau_1+\tau_5 = -2.5\times10^{-3}\ \text{kgm}^{-1}\,\text{s}^{-1},\\ &\lambda_3 = 3.0\times10^{-2}\ \text{kgm}^{-1}\,\text{s}^{-1}, \quad \lambda_2 = -1.0\times10^{-2}\ \text{kgm}^{-1}\,\text{s}^{-1}. \end{aligned}\right\} \tag{33}$$

4 Switching without flow

Before discussing the full set of equations, it is constructive to examine behaviour when we ignore the effects of flow. The governing equations then read

$$\frac{\partial\phi}{\partial\tilde{t}} = \frac{\partial^2\phi}{\partial\tilde{z}^2} + \frac{Pd^2}{K\epsilon_0\epsilon}(P\cos\phi - D)\sin\phi, \quad D = \frac{P}{2}\int_{-1}^{1}\cos\phi\,d\tilde{z} - \frac{V\epsilon_0\epsilon}{2d}. \tag{34}$$

Of course, the problem is now over-defined, yet we proceed, since these equations are similar to those generally used in the literature on switching. To solve these equations two NAG routines are used. Some profiles of ϕ for the symmetric boundary conditions (29) when a voltage of 5 volts is applied are shown below. Figure 1 illustrates the desire of the polarization vector to align itself as much as possible in the direction of the field. Upon removal of the field $\boldsymbol{c}$ relaxes back to its initial rest state. As one expects, a field applied in the opposite direction has no effect on the cell. For asymmetric boundary conditions the initial condition must first be calculated and this is achieved by using a linear profile for ϕ as a first guess. As one sees from Figure 2, the true initial profile for the azimuthal angle is nonlinear, with ϕ approximately $\frac{1}{2}\pi$ throughout most of the cell apart from in the vicinity of the cell walls where there is a rapid adjustment to conform to the boundary conditions. Figure 2 also shows the behaviour of such a cell when a potential difference of 5 V is applied. The $\boldsymbol{c}$-director switches so that ϕ is equal to π throughout most of the cell. When

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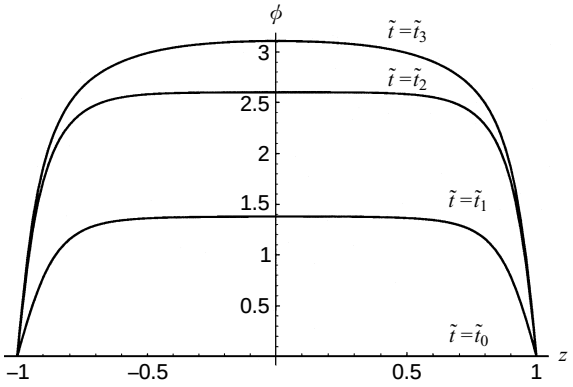


FIGURE 1. The azimuthal angle ϕ against $\tilde{z}$ at times $\tilde{t}=\tilde{t}_0-\tilde{t}_3$ during switching. The boundary conditions are symmetric.

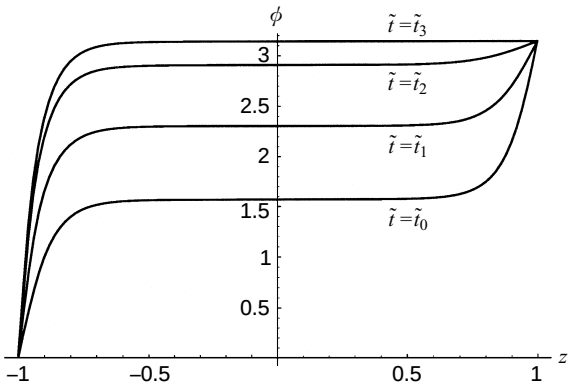


FIGURE 2. The initial condition ($\tilde{t}=\tilde{t}_0$) and switching profiles for asymmetric boundary conditions.

the field direction is reversed, the cell switches to another state in which ϕ is equal to zero throughout most of the cell. We do not discuss this case as it is straightforward to extroplate the results from the π-switched state. When the field is removed ϕ relaxes back to its initial profile. The shapes of the above profiles suggest that analyses employing boundary layer techniques may prove possible, but we do not investigate this here.

5 Switching with flow

The previous section has given us a basic understanding of what happens when an electric field is applied to a SSFLC cell with symmetric and asymmetric boundary conditions. Our task now is to include the induced flow in the model, which means we must solve (23), (24) and (26), (27) (with $\xi_1(\phi)$, $\xi_2(\phi)$ and $\xi_3(\phi)$ replaced by constants) subject to (29) or (30) and (31), (32). To do so, we use an algorithm due to Berreman [14] and NAG routines. We simply state our results.

The results for switching from the relaxed states are qualitatively the same as those obtained in the last section (see figures 3 and 4) for both the symmetric and asymmetric

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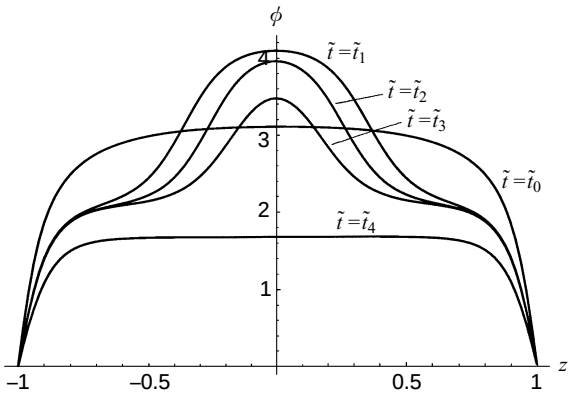


FIGURE 3. Profiles for ϕ when an electric field is removed – symmetric boundary conditions.

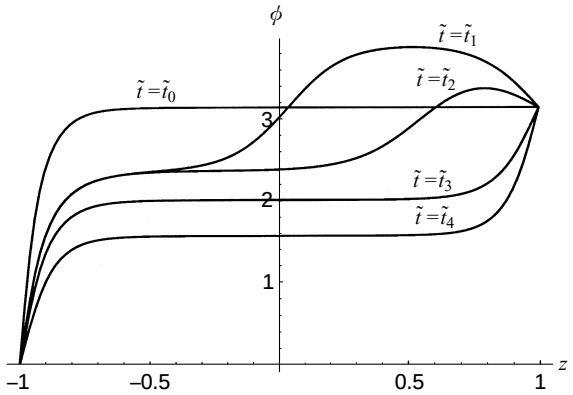


FIGURE 4. Profiles for ϕ when an electric field is removed – asymmetric boundary conditions.

boundary conditions. However, the times taken to reach the fully switched states are an order of magnitude higher than the switching times when flow is neglected.

When the field is subsequently switched off things are rather more interesting as shown by the graphs above. They illustrate that there is a significant backflow effect on the ***c***-director. When the boundary conditions on ϕ are symmetric ϕ attains values greater than π in the centre of the cell soon after the cell is switched off. For the asymmetric boundary conditions ϕ reaches values greater than π near the top plate. In both cases, the behaviour of ***c*** differs greatly from the situation when no flow is included in the model. Further, the times taken to reach the relaxed states are greater than the times taken when the effect of flow is ignored. The exact mechanism behind these effects is not quite so clear as it is in the case of nematics [4] due to there being two flow components, and we do not attempt an explanation here.

6 Conclusions

We have shown, using the continuum theory of Leslie, Stewart and Nakagawa, that backflow may have an effect on the switching behaviour of a SSFLC device. Our work indicates that backflow may increase switching times and, at least when a field is removed,

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can produce a so-called 'kickback' effect in the ***c***-director. Our results are qualitatively the same as those of Zou *et al.* [15], except their work suggests that backflow decreases switching times rather than increases them. However, a full comparison between the two pieces of work is not possible as Zou *et al.* neglect the macroscopic flow through the layers and differing values of material parameters, most notably viscosity coefficients, have been used. Work is currently underway to compare the two approaches. We also emphasize that no attempt has been made to model the phenomenon of ion transport across the cell.

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References

[1] de Gennes, P. G. & Prost, J. (1993) *The Physics of Liquid Crystals* (*Second Edition*) Clarendon Press, Oxford.

[2] Clark, N. A. & Lagerwall, S. T. (1980) Submicrosecond bistable electo-optic switching in liquid crystals. *Appl. Phys. Lett* **36** (11), 899–901.

[3] Gerritsma, C. J., van Doorn, C. Z. & van Zanten, P. (1974) Transient effects in the electrically controlled light transmission of a twisted nematic layer. *Phys. Lett.* **A48**, 263–264.

[4] van Doorn, C. Z. (1975) Transient behaviour of a twisted nematic liquid crystal layer in an electric field. *J. Phys.* (*Paris*) *Colloq.* **36** (C1), 261–263.

[5] Clark, M. G. & Leslie, F. M. (1978) A calculation of orientational relaxation in nematic liquid crystals. *Proc. R. Soc. Lond. A* **361**, 463–485.

[6] Leslie, F. M., Stewart, I. W. & Nakagawa, M. (1991) A continuum theory for smectic C liquid crystals. *Mol. Cryst. Liq. Cryst.* **198**, 443–454.

[7] Carlsson, T., Stewart, I. W. & Leslie, F. M. (1992) An elastic energy for the ferroelectric chiral smectic C* phase. *J. Phys. A* **25**, 2371–2374.

[8] Leslie, F. M., Stewart, I. W., Carlsson, T. & Nakagawa, M. (1991) Equivalent smectic C liquid crystal energies. *Cont. Mech. Therm.* **3**, 237–250.

[9] Jones, J. C., Raynes, E. P., Towler, M. J. & Sambles, J. R. (1990) Observation of dielectric biaxiality in a two-frequency smectic C and a reinterpretation of ferroelectric liquid crystal AC field effects. *Mol. Cryst. Liq. Cryst. Lett.* **7** (3), 91–99.

[10] Maximus, B., Deley, E., DeMeyere, A. & Pauwels, H. (1991) Ion transport in SSFLCD's. *Ferroelectrics* **121**, 103–112.

[11] Zou, Z., Clark, N. A. & Handschy, M. A. (1991) *Ferroelectrics* **121**, 147.

[12] Nakagawa, M. (1989) On the polarisation electric field in surface stabilised ferroelectric liquid crystals. *Mol. Cryst. Liq. Cryst.* **173**, 1–16.

[13] Patel, J. S. (1992) Ferroelectric liquid crystal modulator using twisted smectic structure. *Appl. Phys. Lett.* **60**, 280–282.

[14] Berreman, D. W. (1975) Liquid crystal twist cell dynamics with backflow. *J. Appl. Phys.* **46**, 3746–3751.

[15] Zou, Z., Clark, N. A. & Carlsson, T. (1994) Influence of backflow on the reorientation dynamics of ferroelectric liquid crystals. *Phys. Rev. E* **49**, 3021–3030.

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