For all p the first term of the inequality tends to zero; $\lambda(\alpha) \neq \nu$ and so, by the corollary to Lemma 1, there is a $q > 0$ such that

$$q^n(T-\lambda(\alpha)I)^n y \not\to 0.$$

Therefore, from (4) and Lemma 2, we have that, for all $\beta \in \tau$, $\beta \neq \alpha$,

$$q\,|\lambda(\alpha)-\lambda(\beta)| \geqslant 1.$$

Thus $\lambda(\alpha)$ is an isolated point of the set $\{\lambda(\alpha) : \alpha \in \tau\}$ which is in one-one correspondence with the set τ; both sets are therefore countable.

The main result now follows immediately. By the last lemma, if every element of the subspace $\mathfrak{M}_\sigma$ belongs to some point, they must all belong to the same point, and there is thus a λ such that for every element $x \in \mathfrak{M}_\sigma$

$$\|(T-\lambda I)^n x\|^{1/n} \to 0 \quad (n \to \infty).$$

It follows from this that $(T-\lambda I)$ regarded as an operator in $\mathfrak{M}_\sigma$ is quasi-nilpotent and σ contains the point λ only. Conversely, as noted in (2), if σ consists of the single point λ the operator $(T-\lambda I)$ is quasi-nilpotent in $\mathfrak{M}_\sigma$ which therefore consists of all those elements $x \in \mathfrak{B}$ which belong to the point λ.

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DISCONTINUOUS LINEAR FUNCTIONALS

By J. D. WESTON

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All norms for a finite-dimensional real vector space give the same topology; in particular, every linear functional on such a space is continuous with respect to any norm. The purpose of this note is to prove the following theorem, giving a characterization of finite-dimensional spaces:

Let f be a non-zero linear functional on a real vector space X, and suppose that f is not discontinuous with respect to any norm. Then X is finite-dimensional.

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Since $f \neq 0$, there is a point x such that $f(x) = 1$. Thus X has a non-empty subset B with the following property: B consists of linearly independent vectors, and $f(x) = 1$ for all $x \in B$. By Zorn's lemma, we may assume that B is maximal with respect to this property. With this assumption, let $x_0 \in B$ and let x be any point of X. Then x is in the subspace generated by B: for, if it were not we could enlarge B by adjoining $x + x_0$ if $f(x) = 0$ or $[f(x)]^{-1}x$ if $f(x) \neq 0$. Thus B is a basis of X.

Now let

$$\|x\| = \sup_{1 \leqslant i \leqslant n} |\alpha_i|,$$

where $x = \alpha_1 x_1 + \ldots + \alpha_n x_n$ and $x_1, \ldots, x_n$ are distinct elements of B. This defines a norm for X. By hypothesis, there is a positive number k such that $|f(x)| \leqslant k\|x\|$ for all $x \in X$. Taking

$$x = x_1 + \ldots + x_n,$$

with $x_1, \ldots, x_n$ in B, we find that $n \leqslant k$. Thus the number of elements in B cannot exceed k, so that X is finite-dimensional.

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A NOTE ON THE RELATION BETWEEN SCATTERING PHASE AND BOUND STATES

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ABSTRACT. The form of Levinson's relation is obtained for the case where the free Hamiltonian contains discrete eigenstates. The relevance of this to the ambiguous solutions of Low equations and dispersion relations is discussed.

1. Levinson (5) has shown that the scattering phase $\delta(k)$ for S-states in a spherically symmetric potential V satisfying a certain regularity condition is related to n, the number of bound S-states, by the equation

$$\delta(0) - \delta(\infty) = n\pi. \tag{1}$$

Jauch (4) has extended the result to elastic scattering by an arbitrary potential satisfying a given condition on the behaviour of the scattering wave functions at infinite energy. His argument uses the orthogonality and completeness relations for the eigenstates of the total Hamiltonian. Both authors consider the case where the total Hamiltonian, $H = H_0 + V$, and the free Hamiltonian, H_0, possess an identical continuous spectrum with eigenkets $|k^+\rangle$ and $|k\rangle$, respectively. In addition H possesses certain discrete bound states $|\alpha\rangle$. In this note we shall also consider the possibility that H_0 possesses discrete eigenstates $|\beta\rangle$.

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