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Simultaneous velocity, impact and force control

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SUMMARY

In this paper, we propose a control method to achieve three objectives simultaneously: velocity regulation during free motion, impact damping and finally force reference tracking. During impact, the parameters are switched in order to dissipate the energy of the system as fast as possible and the optimal switching criteria are deduced. The possibility of sliding regimes is analysed and the theoretical results are verified in simulations.

KEYWORDS: Robot; Force control; Impact Control, Switching.

1. Introduction

A major problem in robot force control is the abrupt change from free to constrained motion. The transition from one phase to the other, also called impact, is probably the most critical part of the task. The difference between the system dynamics in the two phases is considerable. This is emphasized by the fact that in the typical industrial applications of force control the environment is very stiff, making the system strongly underdamped with very high frequency of oscillations.

During the impact high peaks of force may occur and cause irreversible damage to the robot, the environment or the tool. Even if that does not happen, smaller peaks gradually damage the mechanisms of the robot. Another problem involved in impact is the possibility of bouncing.

All these drawbacks could easily be avoided by designing an overdamped controller if the characteristics of the environment are known. Unfortunately, this is often not the case. For this reason, the original parameters selected for the regulator may not be appropriate, and additional measures could be necessary if the system appears to be underdamped once contact is achieved. Another problem inherent in impact control is its extremely brief duration, which may last only for a few sampling periods.

As a consequence, an adaptive controller, for example, may be too slow to protect the system.

Some authors propose applying a controller whose only purpose is to soften the impact, which would only be applied during the transient phase. Its objective should be fast dissipation of energy rather than the tracking of a reference value. It should be replaced by another controller once the transition is finished in order to reach the force reference.

Impact control has been extensively researched and very diverse solutions have been proposed. The most complete review of different sources accompanied by an exhausting analysis of impact control has been made by Brogliato.[1] It should be noted that, as stated for example by Brach[2] and Brogliato[1] impact can be treated either by rigid or flexible models. The former method does not consider what is happening during the contact phase, it is only concerned with what happens before and after. It is assumed that the duration of the impact is infinitely short. The relation between the velocities at the time of contact and after the rebound is given by the coefficient of restitution. In the flexible model the impact is treated analytically, considering the robot and/or the environment as elastic bodies. This is the model used in this paper.

Regarding the methods for impact control, Brogliato *et al.*[3] proposed two methods to limit the number of rebounds and thus ensuring the stability of the system in rigid models. Nevertheless, most works are based on the flexible model. Volpe and Khosla[4] proposed three methods for impact control. All three are designed to avoid contact loss rather than protection against peaks of force. Hyde and Cutkosky[5] proposed modulation with feedforward pulses, which are computed to suppress the transitory harmonics. Ferretti *et al.*[6] applied an empirically determined feedforward during impact combined with the force regulator in order to avoid contact losses.

From the sources cited it can be deduced that impact techniques control vary widely and all of those do not use the same model. Some are designed to try to avoid contact losses, regardless of the possible peaks of force. Others are limited to guaranteeing the convergence of the system after a finite number of rebounds. Some consider that the characteristics of the environment are known.

It should be emphasized that the application of a regulator only for impact involves the necessity of another controller for tracking the reference force, so that control switching becomes necessary. This may create problems, such as limit cycles or sliding regimes, if switching criteria are not well established.

This paper proposes a unique controller valid both for force and impact control. During impact, the proportional constant and the feedforward term are switched in order to dissipate as fast as possible the energy of the system. The optimal switching criteria are deduced and verified in simulations. The switching of the feedforward has been described by Zotovic Stanisic and Valera.[7]

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Varying the parameters of the controller according to the state has been proposed previously. A compilation of several sources was made by Armstrong *et al.*[8] Possibly the first interesting contribution that should be mentioned was made by Franke.[9] Xu *et al.*[10,11] first introduced the application of parameter variation in force control and proposed a non-linear PD controller. Both the proportional and derivative constant varied between a minimum and a maximum value according to a non-linear law that took into account the signs of the force error and its derivative. Seraji[12,13] also provided some interesting contributions in this direction.

The switching of parameters was introduced in force control by Armstrong *et al.*,[8,14–16] in a study similar to the present work. The authors switch the gain matrix according to the state of the system. The essential idea is the same, but a different mathematical methodology was used. In the work of Armstrong *et al.*, LMI was used to demonstrate the validity of the method. The technique worked until the difference between the parameters exceeded a value that had to be found empirically. The main advantage of the method described in this paper is that it does not require any empirical adjustments. As will be explained later, it does not limit the value of the parameters and for extreme values the performance is better. Another difference between the work of Armstrong *et al.* and the present paper is that the former switches the feedback parameters and we switch the feedforward.

The controller described in this paper was designed for a one degree of freedom non-elastic robot, later generalized to six degrees of freedom.

The paper is organized as follows: the second section describes the system, i.e. the physical process model and the controller and some of the equations required for later use are deduced. The third section involves the deduction of the switching criteria and the simulation results. The fourth section generalizes the previous conclusions to six degrees of freedom. The final section summarizes the conclusions of the paper.

2. Description of the System

This section gives a description of the model used to deduce the equations, consisting of the physical process and the controller. The former has two elements: the robot and the environment. The case of a rigid (non-elastic) robot is considered. It can be modelled by a mass and a viscous damping:

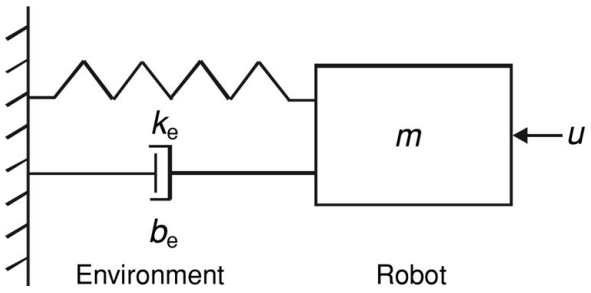


Fig. 1. Mass-spring-damper diagram.

$$u - f = ms^2x + bsx \tag{1}$$

where u is the motor force, s the Laplace operator, m the mass and x the position of the robot. The environment is represented by its interaction force with the robot. It can be expressed in the following way:

$$f = k_e(x - x_e) + b_e sx, \tag{2}$$

where f is the interaction force, k_e the stiffness, b_e the damping and x_e the position of the environment. This is shown in the mass-spring-damper model represented in Fig. 1.

A proportional regulator with feedforward and active damping was used in the controller. The corresponding control action is

$$u = ff + k_p(f_{ref} - f) - b_a v, \tag{3}$$

where k_p is the proportional constant, f_{ref} the force reference, ff the feedforward term, b_a the active damping, v the velocity and f the reaction force of the environment.

This controller has been used by several authors, for example Volpe and Khosla[17]. The purpose of the feedforward term is to ensure that the reference value of the force will be reached. This could also be achieved with an integrator, but the feedforward is better from the point of view of stability and response time. The value of the feedforward term that ensures reaching the reference will be deduced later. The objective of the velocity feedback is to damp the system and is used instead of derivative control action, since the force derivative cannot be deduced due to the high noise from the force sensor. A schema of the system is shown in Fig. 2.

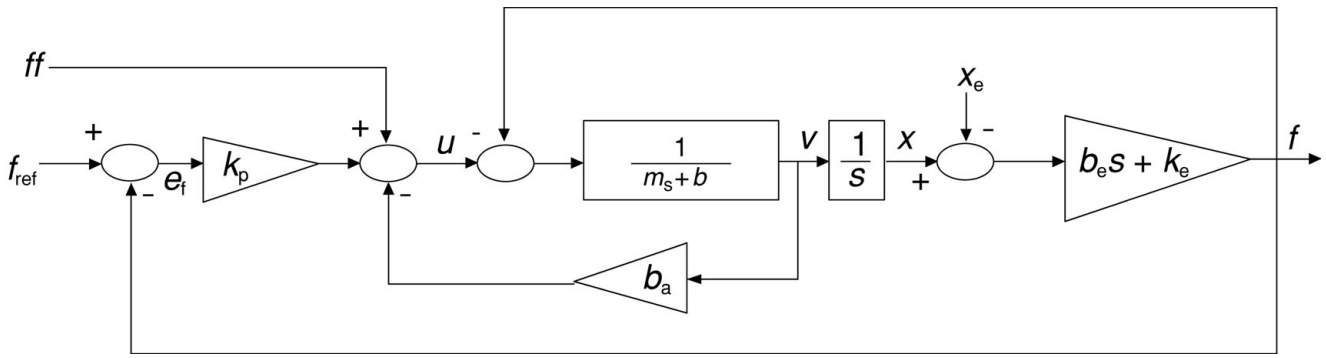


Fig. 2. Block diagram of the system.

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Substituting (2) and (3) in (1) it is easy to obtain

$$\ddot{x} = \frac{1}{m}(ff + k_p(f_{ref} - k_e(x - x_e) - b_e\dot{x}) - (b + b_a)v$$
$$- k_e(x - x_e) - b_e\dot{x}) = \frac{1}{m}(ff + k_p f_{ref} + (k_p + 1)k_e x_e$$
$$- (k_p + 1)k_e x - (k_p + 1)b_e v - (b + b_a)v) \quad (4)$$

It is a second-order system with positive coefficients, thus it is always stable. The final position may is obtained for the values of velocity and acceleration zero

$$x_\infty = \frac{k_p f_{ref} + (k_p + 1)k_e x_e + ff}{(k_p + 1)k_e}. \quad (5)$$

And the final force

$$f_\infty = k_e(x_\infty - x_e) = k_e \frac{k_p f_{ref} + (k_p + 1)k_e x_e + ff}{(k_p + 1)k_e}$$
$$-k_e x_e = \frac{k_p f_{ref} + ff}{(k_p + 1)}. \quad (6)$$

It seems logical to assign the value to the feedforward

$$ff = f_{ref}. \quad (7)$$

Then the reference force will be reached

$$f_\infty = f_{ref}. \quad (8)$$

In free motion the interaction force is zero. In this case, applying the Laplace transform to Eq. (4) the dynamics is the following:

$$s^2 x = \frac{1}{m}(ff + k_p f_{ref} - (b + b_a)sx). \quad (9)$$

The position of the robot will be

$$x(s) = \frac{k_p f_{ref} + ff}{ms^2 + (b + b_a)s}. \quad (10)$$

And the velocity is

$$v(s) = sx(s) = \frac{k_p f_{ref} + ff}{ms + b + b_a}. \quad (11)$$

The final values of the position and the velocity are

$$x_\infty = \infty$$
$$v_\infty = \frac{k_p f_{ref} + ff}{b + b_a}. \quad (12)$$

Therefore, in the stationary state in free motion the robot moves at constant speed. This is equivalent to a velocity control. It may be more appropriate than position control because in some cases the exact coordinate of the environment is unknown. Since k_p, f_{ref} and ff are used for force control in constrained motion, the adjustment of the velocity in free motion can be achieved by active damping.

For example, if the desired velocity is v_{ref}, it is easy to obtain b_a from (12)

$$b_a = \frac{k_p f_{ref} + ff}{v_{ref}} - b. \quad (13)$$

Thus, assigning the value obtained in (13) to the active damping, velocity control is achieved in free motion and assigning a feedforward according to (7) a force control is obtained in constrained motion.

3. Switching the Parameters

In the previous section it has been explained how to control velocity in free motion and force in constrained motion simultaneously. The most dangerous part of the task is, nevertheless, the transition from one phase to the other, i.e. the impact.

This section deals with impact control, which is achieved by means of switching the parameters of the controller (3). The optimal switching for each parameter is deduced and an analysis is made of the sliding regimes.

To deduce the switching criteria the following Lyapunov function is used:

$$V = \frac{1}{2}((x - x_\infty)^2 + \dot{x}^2)$$
$$= \frac{1}{2}\left(\left(x - \frac{k_p f_{ref} + (k_p + 1)k_e x_e + ff}{(k_p + 1)k_e}\right)^2 + \dot{x}^2\right). \quad (14)$$

This represents the Euclidian distance from the equilibrium point x_∞ in the phase plane. It is clear that faster convergence means faster energy dissipation. On the other hand, the term $(1/2)(x - x_\infty)^2$ is equivalent to the elastic potential energy of the environment, scaled by a factor that may depend on the units. In the same way, the term $(1/2)\dot{x}^2$ is proportional to the kinetic energy. Therefore, it may be stated that V represents the total energy of the system. Any quadratic function would have the same effect.

In the typical applications of Lyapunov functions, the origin of the coordinate system is located at the equilibrium point, and hence it is not taken into account. Nevertheless, when a parameter is switched, the equilibrium point may also change, what may influence the stability and general behaviour of the system. For this reason, x_∞ is included in the considerations.

The derivative of (14) is

$$\dot{V} = (x - x_\infty)\dot{x} + \dot{x}\ddot{x} = \left(x - \frac{k_p f_{ref} + (k_p + 1)k_e x_e + ff}{(k_p + 1)k_e}\right)\dot{x}$$
$$+ \dot{x}\ddot{x} = \left(x - \frac{k_p f_{ref} + (k_p + 1)k_e x_e + ff}{(k_p + 1)k_e}\right)\dot{x}$$
$$+ \frac{1}{m}(ff + k_p f_{ref} + (k_p + 1)k_e x_e - (k_p + 1)k_e x$$
$$- (k_p + 1)b_e\dot{x} - (b + b_a)\dot{x})\dot{x}. \quad (15)$$

During free motion the robot has kinetic energy. When contact is achieved it is transformed into elastic potential

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energy of the environment. Since the system is typically highly underdamped, it oscillates and the energy is transformed from one form to the other several times. This implies peaks of force, the first of which is the highest and thus the most critical. However, subsequent ones gradually damage the system mechanisms.

The function of the impact controller should be to dissipate the initial energy of the system as fast as possible. In this way, the oscillations are damped, the peaks of force are lower, the system is better protected and the possibility of bouncing is reduced.

Expression (15) represents the rate of change of the energy. Since the function of the impact controller is to dissipate the initial energy of the system as fast as possible, it should be designed to reduce $\dot{V}$. For this reason the criterion for optimal switching conditions will be that expression (15) must always be as small as possible. This will mean that the energy is dissipated faster and that better impact damping is achieved.

The following subsections will deal with the analysis of the switching for two of the three controller parameters and will be limited to the case of parameters switched between two values: the minimal and the maximal. The system performance would possibly be improved by using more than two values for the parameters, and this possibility will be considered in future research.

3.1. The proportional constant

To deduce the switching criteria the partial derivative of (15) with respect to the proportional constant is used

$$\frac{\delta \dot{V}}{\delta k_{\mathrm{p}}} = \frac{1}{m}(f_{\mathrm{ref}} + k_{\mathrm{e}}x_{\mathrm{e}} - k_{\mathrm{e}}x - b_{\mathrm{e}}\dot{x})\dot{x} = \frac{1}{m}(f_{\mathrm{ref}} - f)\dot{x} = \frac{1}{m}e_{\mathrm{f}}\dot{x}, \tag{16}$$

where e_{f} is the force error

$$e_{\mathrm{f}} = f_{\mathrm{ref}} - f. \tag{17}$$

This expression represents the rate of energy dissipation as a function of the proportional constant. When it is positive the energy is dissipated slower as k_{p} increases, and when it is negative energy dissipation is faster. In order to dissipate the energy as fast as possible it is logical to assume the following law for switching the proportional constant:

$$k_{\mathrm{p}} = \begin{cases} k_{\mathrm{pmin}} & \text{if} \quad e_{\mathrm{f}}\dot{x} > 0 \\ k_{\mathrm{pmax}} & \text{if} \quad e_{\mathrm{f}}\dot{x} < 0 \end{cases}. \tag{18}$$

The rate of energy dissipation is higher and thus the impact is softer as the k_{pmax} is increased and k_{pmin} is decreased. Extreme values of k_{p} improve the performance.

3.1.1. Sliding regime analysis. This subsection analyses the possibility of the appearance of sliding regimes as a consequence of the switching of the proportional constant. Some basic concepts of sliding modes and sliding regimes will be explained for the sake of clarity.

According to (18), k_{p} switches when $e_{\mathrm{f}}\dot{x}$ goes through zero. In this case it is said that the system is on the switching surface defined as

$$S = e_{\mathrm{f}}\dot{x} = 0. \tag{19}$$

It can be stated that if any of the following statements is true, the system is moving away from the surface:

$$S > 0 \quad \text{and} \quad \dot{S} > 0, \tag{20}$$

$$S < 0 \quad \text{and} \quad \dot{S} < 0. \tag{21}$$

In the first case, S is positive and increasing and in the second case it is negative and decreasing. In both cases the distance from the surface is increasing.

Conditions (20) and (21) may be summarized in only one condition as

$$S\dot{S} > 0. \tag{22}$$

In the contrary cases

$$S > 0 \quad \text{and} \quad \dot{S} < 0, \tag{23}$$

$$S < 0 \quad \text{and} \quad \dot{S} > 0. \tag{24}$$

The system is tending towards the surface, since the distance is decreasing.

Conditions (23) and (24) may be summarized in only one condition as

$$S\dot{S} < 0. \tag{25}$$

If no switching were performed the system would be smooth, oscillatory. In every period the surface would be crossed, usually more than once. First, the system would tend towards the surface (condition (23) or (24) is satisfied), intersect it ($S = 0$ on the surface) and move away from it (condition (20) or (21)). The system would cross the surface and leave it. These four cases (20), (21), (23) and (24) are represented in Fig. 3.

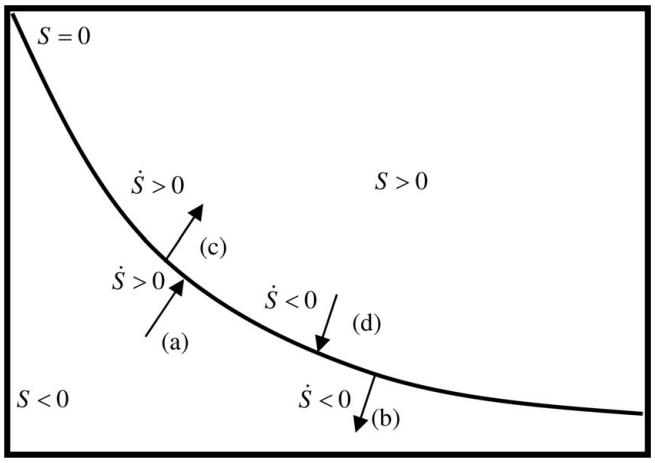


Fig. 3. The four possible cases. The thick line represents the switching surface $S = 0$. Above the surface $S > 0$, and below it $S < 0$. The following cases are possible: (a) $S < 0$ and $\dot{S} > 0$, the system tends towards the surface. (b) $S > 0$ and $\dot{S} < 0$, the system is moving away from the surface, (c) $S > 0$ and $\dot{S} > 0$, the system is moving away from the surface and (d) $S > 0$ and $\dot{S} < 0$, the system heads for the surface.

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Nevertheless, the fact that k_p is switched causes a change in the dynamics of the system that may push it back to the switching surface. In this case, k_p and the dynamics are such that the system always tends towards the surface, regardless of which side of the surface it is. This is known as a sliding regime and is harmful because the system is stuck on the switching surface instead of performing the task.

The sign of S changes when the system crosses the surface, thus neither (23) nor (24) may hold true either before or after switching. Therefore, in order to reach a sliding regime condition (23) must hold true before and (24) after switching or vice-versa. For these reasons, it will be considered that a sliding regime occurs if the sign of $\dot{S}$ changes when crossing the surface.

Briefly, the conclusions may be summarized in the following statements:

- A sliding regime may occur if $S\dot{S} < 0$ both before and after crossing the switching surface.
- $S\dot{S} < 0$ means that the system is tending towards the surface, thus it must always be satisfied before the system crosses it as otherwise the system would not reach the surface.
- The sign of S changes after crossing the surface. In order to reach a sliding regime, the sign of $\dot{S}$ must also change.
- Changing the sign of $\dot{S}$ does not happen naturally, but only when k_p is switched.
- It will therefore be considered that sliding regimes may happen if the switching of the parameter may cause the sign of $\dot{S}$ to change.

According to Eq. (18) the proportional constant switches when either the velocity or the force error change their signs. Thus, there are two switching (and potentially sliding) surfaces

$$\begin{aligned} S_1 &= e_f = 0, \\ S_2 &= \dot{x} = 0. \end{aligned} \tag{26}$$

The following section analyses the possibilities of the appearance of a sliding regime on each surface.

Sliding regime on S_1: For a sliding regime to occur on S_1, the switching of k_p must cause the sign of $\dot{S}_1$ to change. Since $e_f = 0$ on the surface, according to (3) the value of k_p does not have any influence on the control action and thus on the dynamics of the system either. Therefore there cannot be sliding regimes on S_1 for any values of k_{pmin} and k_{pmax}.

Sliding regime on S_2: Given that the system is oscillatory, S_2 is intersected twice in every period: when it goes from positive to negative and vice versa. The two cases will by analysed separately.

First, when velocity goes from positive to negative:

Assuming that velocity goes to zero at time t_k:

$$\dot{x}(t_k) = 0. \tag{27}$$

This may be stated

$$\begin{aligned} &\dot{x}(t_k - \varepsilon) > 0 \quad \text{and} \\ &\dot{x}(t_k + \varepsilon) < 0. \end{aligned} \tag{28}$$

Since the velocity is decreasing. The acceleration must be negative

$$\ddot{x} = \frac{d\dot{x}}{dt} < 0. \tag{29}$$

On the other hand, since velocity is a derivative of position, the position has an extreme point when the velocity is zero, i.e. on the surface. According to Eq. (28), the velocity is positive before and negative after reaching the extreme point, thus it must be a maximum

$$x = x_{max}. \tag{30}$$

According to (2)

$$f = k_e(x_{max} - x_e) - b_e\dot{x}. \tag{31}$$

Since on the surface $\dot{x} = 0$ and $k_e >> b_e$,

$$f = k_e(x_{max} - x_e) \approx f_{max}, \tag{32}$$

the force is near a maximum.

Since the equilibrium point is the force reference according to Eq. (8) the force oscillates around f_{ref}. The maximum will be higher than the equilibrium point. Thus, the force error will be

$$e_f = f_{ref} - f_{max} < 0. \tag{33}$$

The force error has a minimum and is negative.

As a consequence of (28) and (33), $e_f\dot{x}$ switches from negative to positive, and k_p from the maximal to the minimal value according to (18).

These conclusions may be summarized in Table I.

The derivative of S_2

$$\begin{aligned} \dot{S}_2 = \ddot{x} &= \frac{1}{m}(k_p f_{ref} + (k_p + 1)k_e x_e + f_{ref} \\ &\quad - (b + b_a + (k_p + 1)b_e)\dot{x} - (k_p + 1)k_e x) \\ &= \frac{1}{m}((k_p + 1)e_f - (b + b_a)\dot{x}). \end{aligned} \tag{34}$$

Since the acceleration is negative before the switching (Eq. 29), it must become positive afterwards in order to reach a sliding regime

$$\frac{1}{m}((k_{pmin} + 1)e_f - (b + b_a)\dot{x}) > 0. \tag{35}$$

Table I. The values of the relevant magnitudes when velocity goes from positive to negative.

Magnitude	Before crossing the surface	After crossing the surface
Velocity	>0	<0
Acceleration	<0	?
Force	Near maximum	Near maximum
Force error	<0	<0
$e_f\dot{x}$	<0	>0
Proportional constant	k_{pmax}	k_{pmin}

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Table II. The values of the relevant magnitudes when velocity goes from negative to positive.

Magnitude	Before crossing the surface	After crossing the surface
Velocity	<0	>0
Acceleration	>0	?
Force	Near minimum	Near minimum
Force error	>0	>0
$e_f\dot{x}$	<0	>0
Proportional constant	k_{pmax}	k_{pmin}

Given that the mass is always positive and that on the switching surface S_2 $\dot{x} = 0$ (Eq. (26)), this expression becomes

$$(k_{pmin} + 1)e_f > 0. \tag{36}$$

Since, as stated above, the force has a maximum

$$e_f < 0. \tag{37}$$

Thus the only possibility of a sliding regime according to (36) is

$$k_{pmin} + 1 < 0. \tag{38}$$

A sliding regime may therefore happen only if a value of k_{pmin} smaller than -1 is assigned.

It should be emphasized that a negative value of the proportional constant by itself makes the system unstable. Nevertheless, it has been demonstrated that when it is switched with a positive value according to Eq. (18), it increases energy dissipation and hence improves the stability.

The following section analyses the case of velocity going from negative to positive.

The reasoning is equivalent to the previous case but some values are opposite. Acceleration is positive, position has a minimum, force also has a minimum and the force error has a maximum and is positive. As a consequence, $e_f\dot{x}$ switches from negative to positive, and k_p from the maximal to the minimal value according to (18).

This is summarized in Table II.

Before switching, the acceleration must be positive. The sliding regime appears if it becomes negative after switching:

$$\frac{1}{m}((k_{pmin} + 1)e_f - (b + b_a)\dot{x}) < 0. \tag{39}$$

Given that the mass is always positive and that on the switching surface S_2 $\dot{x} = 0$ (Eq. (26)), this expression becomes

$$(k_{pmin} + 1)e_f < 0. \tag{40}$$

Since, as stated above, the force has a minimum

$$e_f > 0. \tag{41}$$

Thus the only possibility of a sliding regime is

$$k_{pmin} < -1. \tag{42}$$

Therefore, to avoid sliding regimes it is sufficient to assign k_{pmin} a value higher than -1.

As stated before, according to Eq. (18) the dissipation will be better if the values k_{pmin} and of k_{pmax} are extreme. Nevertheless, the sliding regimes limit the value of k_{pmin} to -1. k_{pmax} should be as high as possible.

3.1.2. Simulation results. The first step in the simulations was the selection of the system parameters. Regarding the characteristics of the environment, k_e and b_e, very different values may be found in the works of different authors. This is partially due to the fact that different materials were used in experiments and simulations. For example, Xu *et al.*[11] used the values $k_e = 11010$ N/m, $b_e = 10$ Ns/m. Seraji[12,13] used values of 25, 50, 75, 100, 150 and 200 lb/in. These correspond to 4390, 8780, 13,170, 17,560, 26,340 and 35,120 N/m, respectively. Regarding the damping, the value was 10 lb s/in, equivalent to 1756 Ns/m. Chiaverini *et al.*[18] adopted the value of 105 N/m for stiffness, neglecting damping.

To sum up, the values of the stiffness vary from 4390 to 10^5 N/m, and those of the damping from 0 to 1756 Ns/m. However, in all cases the stiffness is much greater than the damping. The values adopted in this work are $k_e = 10^6$ N/m, $b_e = 10$ Ns/m. The system obtained is thus highly underdamped. The stiffness is at least an order of magnitude higher than any of those used by aforementioned authors. This is an unfavourable case but the effect of impact control can be better appreciated.

Regarding the remaining simulation parameters, the following values were assumed: $m_d = 1$ kg, $f_{ref} = 100$ N, $k_p = 10$ (when not switched), $b_a = 0$. The value of the mass is in a realistic range, since the elements of real robots may have mass from a few hundred grams to a few hundred kilograms. The value of the reference force does not have any influence on the results of the simulations. The values of k_p and b_a are chosen to make the system highly underdamped.

It is assumed the robot impacts the environment at time zero. Also, for simplicity, it was assumed that the origin of the coordinate system is on the surface of the environment, i.e. $x_e = 0$.

The simulations were performed with Matlab 7.1.0.

The sampling period was 1 μs.

For the simulations, a state-space representation of the system was used

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\end{bmatrix} = Ax + Bu = \begin{bmatrix}0 & 1\\ -\frac{k_e}{m} & -\frac{b_e}{m}\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{m}\end{bmatrix}u \qquad F = Cx = \begin{bmatrix}k_e & b_e\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\end{bmatrix} \tag{43}$$

The control action u is computed in every sampling period according to Eq. (3).

It was assumed that the system is continuous, while the control action is discrete, i.e. it changes every sampling period.

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In order to implement this, an auxiliary matrix was used

$$A_{\text{aux}} = \begin{bmatrix} 0 & 1 & 0 \\ -\dfrac{k_e}{m} & -\dfrac{b_e}{m} & \dfrac{1}{m} \\ 0 & 0 & 0 \end{bmatrix}. \tag{44}$$

Assuming an expanded form of the state

$$z = \begin{bmatrix} x \\ \dot{x} \\ u \end{bmatrix}. \tag{45}$$

From Eqs. (43)–(45) it can be deduced

$$\dot{z} = A_{\text{aux}} z. \tag{46}$$

Since the control action is constant during a sampling period, the behaviour of the system is continuous and may be represented by the following equation:

$$z(t) = z_0 \exp(A_{\text{aux}} t), \tag{47}$$

where z_0 is the value of the expanded state at the beginning of the sampling period.

The value of the state at the end of the sampling period

$$z_{\text{final}} = z_0 \exp(A_{\text{aux}} t_s), \tag{48}$$

where t_s is the sampling period.

In this way, a simulation of the behaviour of the system between two sample times was achieved as continuous, as is really the case. Updating the control action was also simulated as discrete, as is normally the case.

The simulations were made at first assigning ever-smaller values to k_{pmin}, while keeping k_{pmax} constant. Next, the opposite case: k_{pmin} was kept constant, while the values of k_{pmax} were increased in several successive experiments. The effectiveness of the switching criteria is verified by testing the two cases separately, otherwise the positive results in one case could compensate for the negative in the other, giving a false appearance of the validity of the method.

The simulations results are represented in Figs. 4 and 5.

It can be observed in the graphics that, both increasing k_{pmax} and decreasing k_{pmin} improve the damping of the system, which confirms the theoretical results.

It can also be observed that the dynamics of the system are faster for higher values of k_{pmax} and slower for lower values of k_{pmin}.

Figures 4 and 5 represent the way the force overshoots are reduced, thus improving the protection of the system.

The behaviour of the position of the robot is also oscillatory, very similar to force, according to Eq. (2). With the dissipation of the energy, the extreme points of the position are also reduced. This means that the minima of the position are closer to the equilibrium point, and thus the possibility of contact loss is reduced.

Therefore, the switching of k_p improves both the peaks of force and the possibility of bouncing.

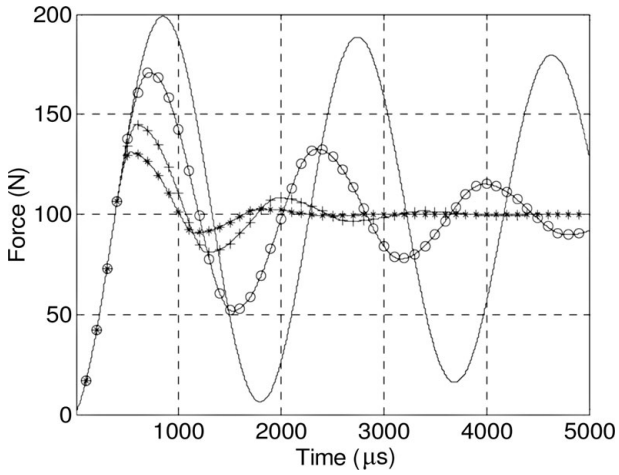


Fig. 4. The switching of k_{pmax}. $k_{\text{pmin}} = 10$ in all the cases. Full line: $k_{\text{pmax}} = 10$ (no switching); Circles: $k_{\text{pmax}} = 20$; Crosses: $k_{\text{pmax}} = 50$; Stars: $k_{\text{pmax}} = 100$.

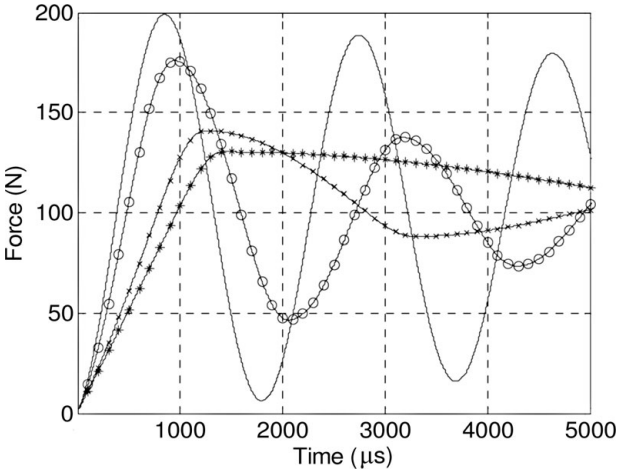


Fig. 5. The switching of k_{pmin}. $k_{\text{pmax}} = 10$ in all the cases. Full line: $k_{\text{pmin}} = 10$ (no switching); Circles: $k_{\text{pmin}} = 5$; Crosses: $k_{\text{pmin}} = 0$; Stars: $k_{\text{pmin}} = -0.9$.

3.2. Switching the feedforward

The partial derivative of expression (15) with respect to the feedforward is

$$\frac{\delta \ddot{V}}{\delta ff} = \dot{x}\left(1 + \frac{1}{m}\right). \tag{49}$$

This implies the following switching criteria:

$$ff = \begin{cases} ff_{\text{min}} & \text{if} \quad \dot{x} > 0 \\ ff_{\text{max}} & \text{if} \quad \dot{x} < 0 \end{cases}. \tag{50}$$

It should be reminded that switching is to be performed only during the transitory phase. Afterwards, feedforward should be set according to (7) to reach the reference value.

3.2.1. Sliding regime analysis. According to (50), the feedforward switches when the velocity changes its sign. Thus, the switching surface is

$$S = \dot{x} = 0. \tag{51}$$

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As in the case of k_p, a sliding regime occurs when switching the feedforward causes the change of $\dot{S}$.

The system switches twice every period. Both cases will be analysed separately.

According to (50) when velocity goes to negative from positive the feedforward is switched from ff_{max} to ff_{min}.

Obviously, this may happen only if velocity is increasing, i.e. the acceleration is positive. The system enters a sliding regime if the switching makes acceleration become negative. In order to avoid this, the following condition must hold:

$$\ddot{x} = \frac{1}{m}(k_p(f_{ref} - f) + ff_{min} - (b + b_a)\dot{x} - f) > 0. \quad (52)$$

This is equivalent to

$$ff_{min} > -k_p(f_{ref} - f) + (b + b_a)\dot{x} + f. \quad (53)$$

Since near the surface $\dot{x} \approx 0$, the condition for avoiding a sliding regime will be

$$ff_{min} > -k_p(f_{ref} - f) + f. \quad (54)$$

The case when velocity goes from positive to negative is completely symmetric. Feedforward switches from ff_{min} to ff_{max}. Acceleration is negative before and must also be negative after switching, to avoid sliding regimes

$$\ddot{x} = \frac{1}{m}(k_p(f_{ref} - f) + ff_{max} - (b + b_a)\dot{x} - f) < 0, \quad (55)$$

which is equivalent to

$$ff_{max} < -k_p(f_{ref} - f) + f. \quad (56)$$

Since all the elements are known or measurable the appearance of sliding regimes may be predicted. In this case switching is not to be performed. Another possibility is to assign to ff_{min} and ff_{max} values according to (54) and (56), respectively.

3.2.2. Simulation results. Simulation set-up is identical to the one discussed in Section 3.1.2.

Switching criteria (50) were verified by means of simulations. Similarly to the case of the proportional constant, first several simulations were carried out decreasing ff_{min} while keeping ff_{max} constant. Following, the opposite has been made in successive simulations.

The results of the simulations are represented in Fig. 6 and 7.

It can be appreciated in the simulations that both decreasing ff_{min} and increasing ff_{max} improve the damping of the system. The former reduces the maxima and the latter the minima, because ff_{min} is active when penetrating the environment, and ff_{max} when retiring. Thus, ff_{min} reduces the peaks of force and ff_{max} the possibility of bouncing.

It may also be appreciated that for the value $ff_{max} = 500$ the system enters a sliding regime after the second maximum.

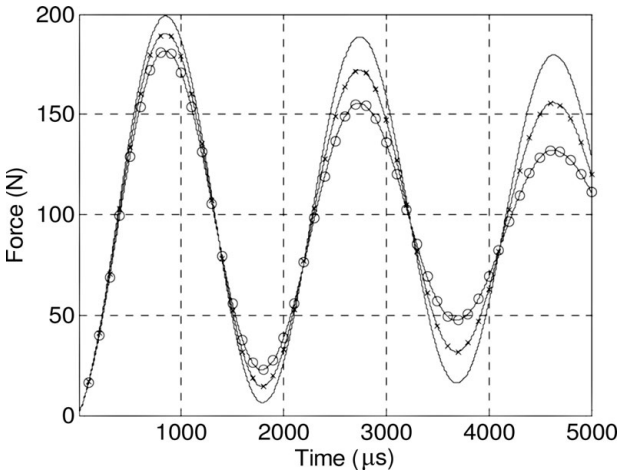


Fig. 6. Diagram of the force when switching ff_{min}. Full line: $ff_{min} = 100$ (no switching), crosses: $ff_{min} = 50$, circles: $ff_{min} = 0$.

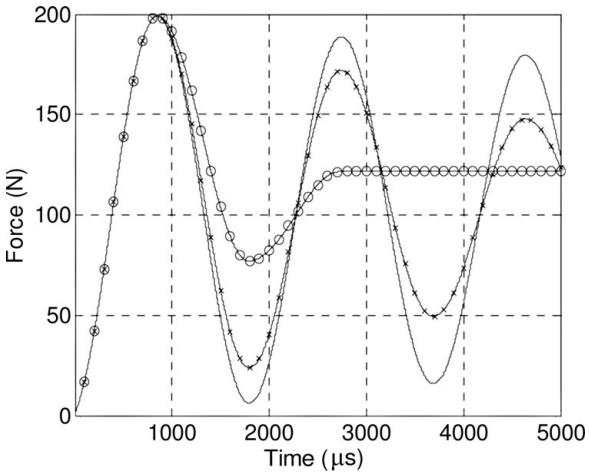


Fig. 7. Diagram of the force when switching ff_{max}. Full line: $ff_{max} = 100$, crosses: $ff_{max} = 200$, circles: $ff_{max} = 500$.

4. Generalization to Six Degrees of Freedom

This section describes the generalization of the previous conclusions to six degrees of freedom. As will be explained later, it is achieved by decoupling the different directions. Once the system is decoupled, the analysis made for one degree of freedom is valid for six degrees of freedom. Capital letters will be used for the magnitudes for six degrees of freedom (vectors and matrices), while lower case letters will be used for the case of one degree of freedom (scalars).

First, some basic facts necessary to understand the system are presented, such as the dynamics of a robot arm or the change from Cartesian to joint coordinates. This subject has been widely studied and is explained in detail, for example, by Sciavicco and Siciliano.[19] Nevertheless, we will include a brief description for the sake of clarity.

If an external force is acting on the tool, the dynamics of a robot arm may be described by the following equation:

$$T - J^T(Q)F = D(Q)\ddot{Q} + H(Q, \dot{Q}) + G(Q), \quad (57)$$

where T is the vector of joint torques, $J(Q)$ the Jacobian matrix of the robot, F is the vector of external forces, Q the vector of joint positions, $\dot{Q}$ the vector of joint velocities, $\ddot{Q}$

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the vector of joint accelerations, $D(Q)$ is the inertia matrix of the robot, $H(Q, \dot{Q})$ the matrix of centrifugal and Coriolis torques and $G(Q)$ the vector of gravity torques on the motors.

The dynamics of the system in the task space may be obtained

$$F_\tau - F = B(X)\ddot{X} + C(X, \dot{X}) + L(X), \qquad (58)$$

where X, $\dot{X}$ and $\ddot{X}$ are the Cartesian position. Velocities and accelerations vector of the final effector, F_τ is the vector of joint torques transformed to Cartesian coordinates, $B(X)$ the pseudo-inertia matrix in task space, $C(X, \dot{X})$ the vector of centrifugal and Coriolis torques in task space and $L(X)$ the vector of gravity torques on the motors in task space.

The terms of the equation are defined by the following expressions:

$$\begin{aligned} &\mathrm{T} = J^T(Q)F_\tau \\ &B(X) = (JD^{-1}(Q)J^T)^{-1} \\ &C\dot{X} = BJD^{-1}H\dot{Q} - B\dot{J}\dot{Q} \\ &L(X) = BJD^{-1}G \end{aligned} \qquad (59)$$

The purpose of the controller is to obtain an independent dynamics equivalent to Eq. (9) for each direction. The controller is composed of two parts. The first one is well known and has often been used for impedance control (for example, Volpe and Khosla[4]). It compensates the dynamics of the robot and decouples the system. The second one imposes the required dynamics and also contains the control loop that ensures force tracking.

$$F_\tau = B(X)\ddot{X}_d + C(X, \dot{X})\dot{X} + L(X) + F, \qquad (60)$$

where $\ddot{X}_\mathrm{a}$ is the desired vector of accelerations. It is used to impose the dynamics of the system.

In order to obtain a system equivalent to the case of one degree of freedom, i.e. a dynamics equivalent to the one in Eq. (9), the following value for the desired acceleration is assumed:

$$\ddot{X}_\mathrm{d} = M_\mathrm{d}^{-1}(FF + K_\mathrm{p}(F_\mathrm{ref} - F) - B_\mathrm{a}\dot{X} - F), \qquad (61)$$

where M_d is the desired inertia matrix. It must be diagonal to achieve decoupling. FF is the feedforward vector. K_p is the matrix of proportional constants (diagonal). F_ref is the reference forces vector and B_a is the active damping matrix (diagonal).

In this way, the obtained system is decoupled in Cartesian coordinates. This allows the force to be controlled in any direction independently of the others and means that the analysis made for one degree of freedom is valid.

5. Conclusions

A force control task consists of free motion and constrained motion phases. In the first phase, velocity or position is controlled. In the second, the magnitude to be controlled is the force. It thus seems logical to use different regulators. Some authors propose a third controller to soften the transition from one phase to the other, which is known as impact control.

Switching between controllers is not a trivial problem as erroneous switching may cause loss of stability even if every controller is stable by itself. Another drawback of switching is the possibility of sliding regimes.

To avoid the possible problems of switching among controllers, this paper concentrates on simultaneous velocity, force and impact control.

The emphasis is on impact control. This should guarantee the attenuation of the peaks of force and, if possible, reduce the possibility of bouncing.

If the characteristics of the environment are known, impact control is easy to design. We therefore focus on the case of unknown environment characteristics.

In this case it is impossible to design a controller that achieves exactly the desired performance, for example, limiting the force overshoot to a given value. Nevertheless, it is possible to guarantee that peaks will be lowered and that the performance will be better with the impact controller than without it, regardless of the characteristics of the environment.

The first contribution of this paper is adjusting the controller in order to achieve velocity control in free motion and force control in constrained motion, thus avoiding the need to switch controllers and its associated problems.

The second and most important contribution is the application of the switching of the proportional constant and the feedforward for the attenuation of the impact. This is achieved avoiding the possible drawbacks of switching. The stability of the system is improved since the energy dissipation is increased. The sliding regimes are predictable and may be avoided.

Switching criteria are deduced in order to improve energy dissipation, so that peaks are reduced as well as the possibility of bouncing. This method improves impact control regardless of the characteristics of the environment.

The proposed method is an improvement over the work of Volpe and Khosla[4] and Ferretti *et al.*[6] since they only treat the avoidance of contact loss, and not the reduction of the peaks of force.

It also has an advantage over the method that needs total or partial knowledge of the characteristics of the environment, as for example, in the work of Hyde and Cutkovsky,[5] who require the system to be identified within 10%.

It does not need an empiric adjustment, unlike the method of Ferretti *et al.*[6]

The work most similar to this paper is proposed by Armstrong *et al.*.[8,14–16] Their basic idea is the same: switching the parameters of the system in order to reduce the energy of the system as fast as possible. The difference lies in the mathematical methodology that has been used.

The main improvements of the method presented in this paper with respect to the work of Armstrong *et al.* are the following:

The parameters are treated separately. It has been demonstrated that the optimal switching criteria are not the same for all the parameters. The performance is better if every parameter is switched in at the right time. In Armstrong's work they are all switched at the same time.

The analysis of sliding regimes was performed.

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This paper proposes controlling the three phases, while Armstrong only deals with impact and force control.

On the other hand, Armstrong analyses the case of an elastic robot, while this paper is limited to the non-elastic case. Nevertheless, the authors intend to deal with elastic robots in the near future.

The theoretical conclusions of this paper have been verified by means of simulations. For impact control, switching was performed between two values. In most cases, the simulations have demonstrated the improvement in impact control when the minimal value of the parameters is decreased as well as when the maximal value is increased. However, for extreme values of the parameters, the system may enter a sliding regime. Fortunately, these cases are predictable and may be avoided.

The proposed method always guarantees an improvement of the damping of the system, unless the system enters a sliding regime, and only needs a few sampling periods to be effective. The values to be assigned to the parameters are straightforward: the maximum value should be as big and the minimal value as small as possible, regardless of the characteristics of the environment. The criteria for sliding regime detection and avoidance are also independent of the environment. The method is therefore robust to uncertainty.

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References

1. B. Brogliato, *Nonsmooth Mechanics* (Springer Verlag, London, 1999).
2. R. Brach, *Mechanical Impact Dynamics—Rigid Body Collisions* (John Wiley & Sons Publishers, New York, 1991).
3. B. Brogliato, S. I. Nicolescu and P. Orhant, "On the control of finite dimensional mechanical systems with unilateral constraints," *IEEE Trans. Automat. Control* **42**(2), 200–215 (1997).
4. R. Volpe and P. Khosla, "A theoretical and experimental investigation of impact control for manipulators," *Int. J. Rob. Res.* **12**(4), 351–365 (1993).
5. J. Hyde and M. Cutkosky, "Controlling contact transitions," *IEEE Control Syst. Mag.* **14**(1), 25–30 (1994).
6. G. Ferretti, G. Magnani and A. Zavala Río, "Impact modelling and control for industrial manipulators," *IEEE Control Syst. Mag.* **18(4)**, 65–71 (1998).
7. R. Zotovic Stanisic and A. Valera Fernández, "Robot Force and Impact Control with Feedforward Switching," *Proceedings of the 17th IFAC world Congress*, Seoul, Korea (2008).
8. B. Armstrong, J. Gutierrez, B. Wade and R. Joseph, "Stability of phase-based gain modulation with designer-chosen switch functions," *Int. J. Rob. Res.* **25**(8), 781–796 (2006).
9. D. Franke, "Entwurf robuster Regelungssysteme mittels zustandsabhängiger Strukturännderung," *Regelungstechnik* **29**(1), 263–268 (1981).
10. Y. Xu, J. M. Hollerbach and D. Ma, "Force and Contact Transient Control Using Nonlinear PD Control," *Proceedings of the 1994 International Conference on Robotics and Automation* (1994) pp. 924–930.
11. Y. Xu, J. M. Hollerbach and D. Ma, "A nonlinear PD controller for force and contact transient control," *IEEE Control Syst. Mag.* **15**(1), 15–21 (1995).
12. H. Seraji and R. Colbaugh, "Force tracking in impedance Control," *Int. J. Rob. Res.* **16**(1), 97–117 (1997).
13. H. Seraji, "Nonlinear and adaptive control of force and compliance in manipulators," *Int. J. Rob. Res.* **17**(5), 467–484 (1998).
14. B. Armstrong, J. McPherson and Y. Li, "Stability of nonlinear PD control," *Appl. Math. Comp. Sci.* **7**(2), 101–120 (1997).
15. B. Armstrong and B. Wade, "Nonlinear PID control with partial state knowledge: Damping without derivatives," *Int. J. Rob. Res.* **19**(8), 715–731 (2000).
16. B. Armstrong, D. Neevel and T. Kusik, "New results in NPID control: tracking, integral control, friction compensation and experimental results," *IEEE Trans. Control Syst. Technol.* **9**(2), 399–406 (2001).
17. R. Volpe and P. Khosla, "A theoretical and experimental investigation of explicit force control strategies for robot manipulators," *IEEE Trans. Automat. Control* **38**(11), 1634–1650 (1993).
18. S. Chiaverini, B. Siciliano and L. Villani, "The parallel approach to force/ position control of robotic manipulators," *IEEE Trans. Rob. Automat.* **9**(4), 361–373 (1993).
19. L. Sciavicco and B. Siciliano, *Modeling and Control of Robot Manipulators* (McGraw-Hill Company Inc., New York, 1996).

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