TRANSIENT RESPONSE OF MAGNETOSTRICTIVE FUNCTIONALLY GRADED MATERIAL SQUARE PLATES UNDER RAPID HEATING

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ABSTRACT

We used the generalized differential quadrature (GDQ) method to compute the transient responses of thermal stresses and center deflection amplitude in the magnetostrictive functionally graded material (FGM) square plate under rapid heating acting at its lower surface. We obtained the GDQ solutions in the three-layer of magnetostrictive FGM plates subjected to four simply supported edges. We presented the transient responses of thermal stress and center deflection amplitude of magnetostrictive FGM plates with/without velocity feedback control, respectively, under the effects of the ratio of length to thickness, the power law index, the temperature of environment and the applied heat flux.

Keywords: GDQ, transient response, magnetostrictive, FGM, rapid heating.

1. INTRODUCTION

The functionally graded material (FGM) usually used as structural components in the environment of very high temperature. Many researchers studied the transient response of FGM plates in several interesting mechanics fields. In 1998, Praveen and Reddy used the finite element method to investigate the nonlinear thermo-elastic transient response of FGM plates [1]. Numerical results of stress and deflection were presented. In 2001, Yang and Shen studied the dynamic response of initially stressed FGM plate by using a semi-analytical approach method [2]. The plate with intermediate material properties were shown to have the intermediate natural frequency and transient response. In 2001, He *et al.* used the finite element method to examine the active control of FGM plates with integrated piezoelectric sensors and actuators [3]. The effects of volume fractions and feedback control gain on the dynamic responses of FGM plates are examined. In 2003, Vel and Batra studied the three-dimensional transient thermal stresses in FGM plates [4]. They presented the analytical solutions for the transient temperature, displacements, and thermal stresses at several critical locations. In 2005, Ootao and Tanigawa analyzed the transient thermal stresses of FGM rectangular plate due to non-uniform heat supply [5]. They obtained the three-dimensional numerical solution for the simple supported FGM plate. In 2006, Prakash and Ganapathi used the finite element method to investigate the supersonic flutter behavior of flat panels made of FGM [6]. Many different parameters were considered in the variation of critical aerodynamic pressure. In 2007, Yang and Huang studied the nonlinear transient response of FGM plates with general imperfections in thermal environments [7]. The numerical influences of imperfection location, temperature field, and volume fraction index were presented. In 2009, Xia and Shen presented the nonlinear vibration and dynamic response of FGM plates [8]. The numerical results of FGM plates with piezoelectric fiber reinforced composite actuators under thermal and electric loads were illustrated.

Some analyses of the magneto-electro-elastic materials had been presented. In 2010, Lee and Ma studied the anti-plane problem of functionally graded magneto-electro-elastic half-planes and provided the analytical results [9]. In 2010, Wang and Zhou provided a nonlinear transient constitutive model in the eddy current investigations and analyses for giant magnetostrictive materials [10]. In 2010, Wu *et al.* used the modified Pagano method to study and examine the static behavior of functionally graded magneto-electro-elastic plates [11]. In 2007, Lee and Ma presented the analytical full-field results for a magneto-electro-elastic half-plane [12]. In 1997, Huang and Kuo presented and provided the ellipsoidal inclusions analyses of piezoelectric/piezomagnetic materials [13]. Magnetostrictive materials had been used in the FGM for the mechanical engineering [14]. Terfenol-D was one of available magnetostrictive materials, could be used usually in the sensors and actuators to operate in the coupling actions of magnetism and mechanism. In 2008, Katranas *et al.* used the magnetostrictive layer in the bi-layer sensor strip to detect displacement [15]. The application of a sensor system can be used in en-

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gine exhaust pipes where the operating temperatures are high. In 2008, Grunwald and Olabi presented the advanced technologies for magnetostrictive actuator [16]. The actuator designed by magnetostrictive materials (Terfenol-D) can provide excellent features. In 2006, Ramirez *et al.* used the Ritz approach to study the free vibration of two-dimensional magneto-electro-elastic laminates [17]. In 2005, Pradhan got the analytical solutions for the FGM shells of embedded magnetostrictive layers under vibration [18]. In 2005, Lee and Reddy used the finite element method to make the non-linear response of laminated magnetostrictive plate under thermo- mechanical loads [19]. In 2004, Buchanan studied the candidate materials of multiphase magneto-electro- elastic composites [20]. In 2007, Hong used the GDQ method to study the laminated magnetostrictive plate under the thermal vibration [21]. In 2009, Hong used the GDQ method to compute the transient response of laminated magnetostrictive plate under thermal vibration [22]. The computational results of thermal stresses and center displacement with and without shear effect were presented, respectively in the thin and thick plate. It is interesting to study the transient responses for the laminated magnetostrictive FGM square plate under rapid heating acting at its lower surface with the GDQ method. The obtained results are interesting for engineering design. Comparison between results coming from different parameters is given and discussed. The numerical approach of the transient response is convincing.

2. THERMOELASTIC STRESS-STRAIN RELATIONS

The time dependent of displacements u, v and w in the x, y and z direction, respectively can be rewritten in the Yang-Norris-Stavsky method of first-order shear deformation form [30]. We consider the power law index R_n of FGM plate that mounted with magnetostrictive layers under uniformly distributed loading and thermal effect [23]. The in-plane stresses and the interlaminar shear stresses with magnetostrictive effect for the k^{th} layer can be rewritten as follows [19,21]:

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_{xy}\end{Bmatrix}_{(k)} = \begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{bmatrix}_{(k)} \begin{Bmatrix}\varepsilon_x - \alpha_x \Delta T\\ \varepsilon_y - \alpha_y \Delta T\\ \varepsilon_{xy} - \alpha_{xy} \Delta T\end{Bmatrix}_{(k)} - \begin{bmatrix}0 & 0 & \tilde{e}_{31}\\ 0 & 0 & \tilde{e}_{32}\\ 0 & 0 & \tilde{e}_{36}\end{bmatrix}_{(k)} \begin{Bmatrix}0\\ 0\\ \tilde{H}_z\end{Bmatrix}_{(k)} \tag{1}$$

$$\begin{Bmatrix}\sigma_{yz}\\ \sigma_{xz}\end{Bmatrix}_{(k)} = \begin{bmatrix}\bar{Q}_{44} & \bar{Q}_{45}\\ \bar{Q}_{45} & \bar{Q}_{55}\end{bmatrix}_{(k)} \begin{Bmatrix}\varepsilon_{yz}\\ \varepsilon_{xz}\end{Bmatrix}_{(k)} - \begin{bmatrix}\tilde{e}_{14} & \tilde{e}_{24} & 0\\ \tilde{e}_{15} & \tilde{e}_{25} & 0\end{bmatrix}_{(k)} \begin{Bmatrix}0\\ 0\\ \tilde{H}_z\end{Bmatrix}_{(k)} \tag{2}$$

where $\Delta T = T_{0rapid} + z/h^* T_1(x, y, t)$ is the temperature difference between the FGM laminate and curing area, in which T_{0rapid} is the temperature rise due to when the rapid heating flux starts acting at the lower surface of FGM plate, $T_1(x, y, t)$ is the general temperature distribution of FGM plate and functions of axis directions x, y and time t, h^* is the total plate thickness in the axis z direction. α_x and α_y are the coefficients of thermal expansion, α_{xy} is the coefficient of thermal shear. $\bar{Q}_{ij}$ is the so called transformed reduced stiffness [24]. ε_x, ε_y, ε_{xy} are in-plane strains and ε_{yz}, ε_{xz} are shear strains. $\tilde{e}_{ij}$ is the transformed magnetostrictive coupling moduli. $\tilde{H}_z$ is the magnetic field intensity, can be expressed as follows: $\tilde{H}_z(x, y, t) = k_c \tilde{I}(x, y, t)$ with velocity feedback control $\tilde{I}(x, y, t) = c(t)\,\partial w/\partial t$ where k_c is the coil constant, $\tilde{I}(x, y, t)$ is the coil current, $c(t)$ is the control gain.

3. DYNAMIC DISCRETIZED EQUATIONS

We use the two-dimensional GDQ method [21,25,30] to discrete the dynamic equilibrium differential equations included the magnetostrictive loads in terms of displacements and shear rotations [14,19] under the vibration of time sinusoidal displacement and temperature as follows:

$$u = [u^0(x, y) + z\psi_x(x, y)]\sin(\omega_{mn} t) \tag{3a}$$

$$v = [v^0(x, y) + z\psi_y(x, y)]\sin(\omega_{mn} t) \tag{3b}$$

$$w = w(x, y)\sin(\omega_{mn} t) \tag{3c}$$

$$T_{0rapid} = \bar{T}_{0rapid} \sin\left(\frac{\pi}{a}x\right)\sin\left(\frac{\pi}{b}y\right)\sin(\gamma t) \tag{4a}$$

$$T_1(x, y, t) = \bar{T}_1 \sin\left(\frac{\pi}{a}x\right)\sin\left(\frac{\pi}{b}y\right)\sin(\gamma t) \tag{4b}$$

$$\Delta T = \left(\bar{T}_{0rapid} + \frac{z}{h^*}\bar{T}_1\right)\sin\left(\frac{\pi}{a}x\right)\sin\left(\frac{\pi}{b}y\right)\sin(\gamma t) \tag{4c}$$

where u^0 and v^0 are tangential displacements, w is transverse displacement of the middle-plane, ψ_x and ψ_y are the shear rotations, t is time, ω_{mn} is natural frequency of the plate, γ is frequency of applied heat flux. a and b is the length in the x, y direction of the plate, respectively. $\bar{T}_{0rapid}$ is the rapid heating temperature amplitude functions of z and t, the following simply rapid heat temperature equation [28-29] is used when the FGM plate subjected to the rapid heating acting at its lower surface:

$$\bar{T}_{0rapid} = \frac{h^* q_0}{\kappa_{fgm}}\left[\frac{\beta t}{\pi^2} + \frac{1}{2}\left(\frac{z}{h^*} + \frac{1}{2}\right)^2 - \frac{1}{6} - \frac{2}{\pi^2}\sum_{j=1}^{\infty}\frac{(-1)^j}{j^2} e^{-j^2 \beta t}\cos j\pi\left(\frac{z}{h^*} + \frac{1}{2}\right)\right] \tag{5}$$

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where $\beta = \pi^2 \kappa_{fgm} / h^{*2}$, q_0 is the applied heat flux, κ_{fgm} is the FGM material conductivity, can be assumed in the averaged form $\kappa_{fgm} = (\kappa_1 + \kappa_2)/2$, in which κ_1 and κ_2 are the conductivities corresponding to the FGM material 1 and material 2. $\overline{T}_1$ is the general temperature amplitude of FGM plate

And the following non-dimensional parameters are used in the discretized formulations:

$$X = x/a, Y = y/b, U = u^0/a, V = v^0/b,$$
$$W = 10h^* w/(\alpha_1 \overline{T}_1 a^2) \quad (6)$$

where α_1 is the coefficient of thermal expansion of FGM material 1.

Thus the dynamic discretized equations in matrix form of GDQ method can be obtained.

4. NUMERICAL RESULTS

We consider the FGM plate consist of two constituent materials: SUS304 and Si_3N_4. The material properties E_1, E_2, ν_1, ν_2, ρ_1, ρ_2, α_1, α_2 and κ_1, κ_2 of materials FGM are calculated by the following individual properties P_i equation:

$$P_i = P_0(P_{-1}T^{-1} + 1 + P_1 T + P_2 T^2 + P_3 T^3) \quad (7)$$

where T is the temperature of environment, the temperature-dependent coefficients P_0, P_{-1}, P_1, P_2 and P_3 as listed in Table 1 [18,26-27]. The elastic modules of the Terfenol-D magnetostrictive material can be found in the paper [18,21]. The magnetostrictive coupling moduli are $e_{31} = e_{32} = E^m d^m$ and E^m = 26.5GPa, d^m = $1.67 \times 10^{-8}\,\mathrm{mA}^{-1}$ for Terfenol-D, where the superscript of m denotes magnetostrictive layer. We consider the three-layer $(0^{\circ m}/0^{\circ}/0^{\circ})$ magnetostrictive FGM plate including shear deformation under four sides simply supported and with SUS304 thickness equal to Si_3N_4 thickness. Before rapid heating, $T_{0rapid} = 0$, there are no heat generation, the heat conduction equation for the host laminate would be used [30]. Thus we can find the value of γ before the rapid heating acting at the FGM plate ($T_{0rapid} = 0$).

Firstly, we find the $N \times M = 17 \times 17$ grid points have the good center deflection amplitude (unit: mm) dynamic convergence results listed in Table 2 with $k_c c(t) = -10^9$, t = 6sec, rapid heat flux $q_0 = 2\ J/(s\ m^2)$, without applied pressure load $q = 0$, without in-plane distributed forces, $a/b = 1.0$, $h_3/h^* = 0.1$, h^* = 1mm, $m = n = 1$, $R_n = 1$, $T = 1000^{\circ}K$ (ideal temp.), $\overline{T}_1 = 1^{\circ}K$ and use further in the GDQ computations of transient responses for deflection and stress for the upper surface mounted magnetostrictive layer of the three-layer $(0^{\circ m}/0^{\circ}/0^{\circ})$ laminated FGM square plate, where the superscript of m denotes the magnetostrictive layer [30].

Secondly, for the transient responses analyses, we find the frequency $\gamma = 523.599/s$ of applied heat flux for three-layer $(0^{\circ m}/0^{\circ}/0^{\circ})$ magnetostrictive FGM square plate, then hold γ value constant at each time increment to find the corresponding time responses values of deflection and stress. We use the negative velocity feedback control values $k_c c(t)$ in time $t \le 0.2$ sec, time step is 0.001 sec to control and reduce the amplitude of center deflection $w(a/2, b/2)$. We like to investigate the effect of the ratio of length to thickness a/h^* on the transient values of deflection and stress. Figure 1 shows the transient response of the transverse center deflection amplitude $w(a/2, b/2)$ (unit: mm) with and without $k_c c(t)$ values at $a/h^* = 5$ and 100, respectively in magnetostrictive FGM square plate, power law index $R_n = 1$, $T = 653^{\circ}K$ (actual max. temp.), $h_3/h^* = 0.1$, $h^* = 1$ (unit: mm), $a/b = 1$, under rapid heat flux $q_0 = 2\ J/(s\ m^2)$ and shear effect. We find the transient values of the center deflection amplitude $w(a/2, b/2)$ with feedback control values ($k_c c(t) = -10^9$ controlled in time $t \le 0.2$ sec) are lower than that $w(a/2, b/2)$ without control loop

Table 1 Temperature-dependent coefficients of constituent FGM materials

Materials	P_i	P_0	P_{-1}	P_1	P_2	P_3
SUS304	$E_1(P_a)$	201.04E09	0	3.079E-04	-6.534E-07	0
	ν_1	0.3262	0	-2.002E-04	3.797E-07	0
	$\rho_1\,(Kg/m^3)$	8166	0	0	0	0
	$\alpha_1\,(^{\circ}K^{-1})$	12.33E-06	0	8.086E-04	0	0
	$\kappa_1\,(W/m^{\circ}K)$	15.379	0	0	0	0
	$C_{v1}(J/Kg^{\circ}K)$	496.56	0	-1.151E-03	1.636E-06	-5.863E-10
Si_3N_4	$E_2(P_a)$	348.43E09	0	-3.70E-04	2.16E-07	-8.946E-11
	ν_2	0.24	0	0	0	0
	$\rho_2\,(Kg/m^3)$	2370	0	0	0	0
	$\alpha_2\,(^{\circ}K^{-1})$	5.8723E-06	0	9.095E-04	0	0
	$\kappa_2\,(W/m^{\circ}K)$	13.723	0	0	0	0
	$C_{v2}\,(J/Kg^{\circ}K)$	555.11	0	1.016E-03	2.92E-07	-1.67E-10

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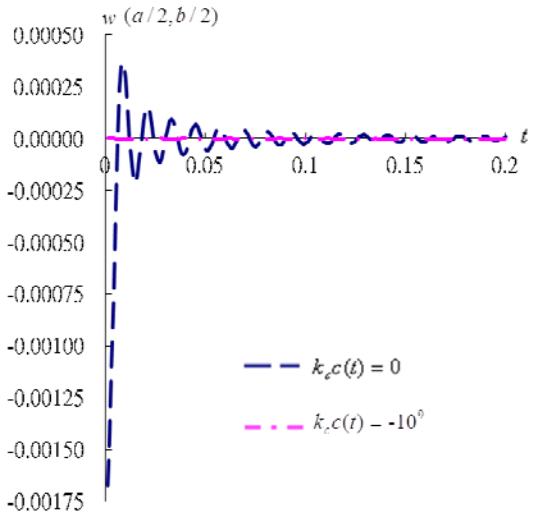
Table 2 Convergence of $(0^{\circ m}/0^{\circ}/0^{\circ})$ square plate

a/h^*	GDQ $N\times M$	Amplitude $w(a/2, b/2, 6)$
100	13 × 13	–0.600863E-06
	15 × 15	–0.600867E-06
	17 × 17	–0.600846E-06
50	13 × 13	–0.244701E-05
	15 × 15	–0.239809E-05
	17 × 17	–0.239835E-05
20	13 × 13	–0.141095E-04
	15 × 15	–0.142853E-04
	17 × 17	–0.139883E-04
10	13 × 13	–0.102590E-03
	15 × 15	–0.110403E-03
	17 × 17	–0.110769E-03
5	13 × 13	0.324146E-03
	15 × 15	0.323996E-03
	17 × 17	0.323917E-03

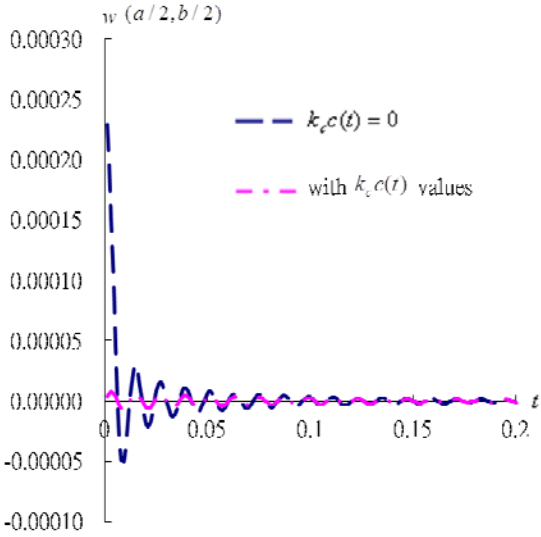
($k_c c(t) = 0$ uncontrolled) for thick ($a/h^* = 5$) magnetostrictive FGM plate. The transient values of $w(a/2, b/2)$ with feedback control values ($k_c c(t) = 10^9$ at time $t = 0.176$ sec and 0.177 sec, $k_c c(t) = -10^9$ in others time $t \le 0.2$ sec) are lower than that $w(a/2, b/2)$ without control loop ($k_c c(t) = 0$ uncontrolled) for thin ($a/h^* = 100$) magnetostrictive FGM plate by the GDQ method. The amplitude $w(a/2, b/2)$ of magnetostrictive FGM plate can converge by itself. The amplitude $w(a/2, b/2)$ can be controlled into smaller value. The amplitudes of uncontrolled $w(a/2, b/2)$ at thin magnetostrictive FGM plate are very much lower than that at thick magnetostrictive FGM plate. The amplitudes of controlled $w(a/2, b/2)$ at thin magnetostrictive FGM plate are greater than that at thick magnetostrictive FGM plate. The compared enlargement amplitudes of controlled $w(a/2, b/2)$ in $a/h^* = 100$ and $a/h^* = 5$ are shown in Fig. 1(c).

Figure 2 shows the corresponding transient response of the dominated stress σ_x (unit: GPa) at center position of upper surface $Z = 0.5h^*$ with and without $k_c c(t)$ values as the deflection analyses in Fig. 1 of thin and thick magnetostrictive FGM square plate. We find the amplitudes of the stress σ_x oscillate and have the smaller amplitude value 8.86E-06 GPa in the controlled $k_c c(t) = -10^9$ case than the amplitude value 1.06E-05 GPa in the $k_c c(t) = 0$ case for the thick plate ($a/h^* = 5$). The amplitudes of the stress σ_x oscillate in good harmonic and have the smaller values in controlled $k_c c(t)$ values case (amplitude value 9.57E-06 GPa) than in $k_c c(t) = 0$ case (amplitude value 9.69E-06 GPa) for the thin plate ($a/h^* = 100$). The amplitudes of stress σ_x become lower value when the magnetostrictive FGM plate becomes thinner ($a/h^* = 100$) for the uncontrolled case $k_c c(t) = 0$.

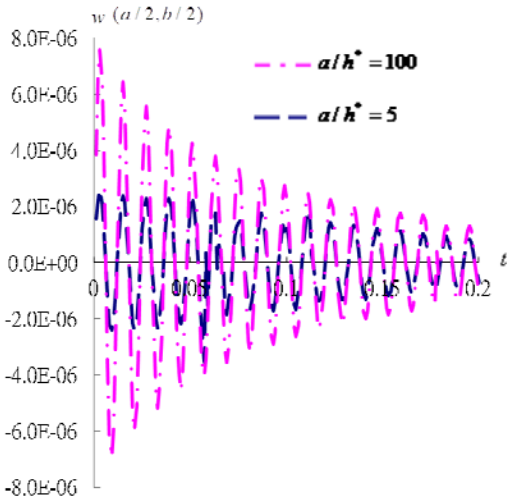
We like to investigate the effect of the power law index R_n on the transient values of deflection and stress. Figure 3 shows the transient response of the transverse center deflection amplitude $w(a/2, b/2)$ (unit: mm) with and without $k_c c(t)$ values at higher $R_n = 10$ in magneto-



(a) $w(a/2, b/2)$ vs. t for $a/h^* = 5$



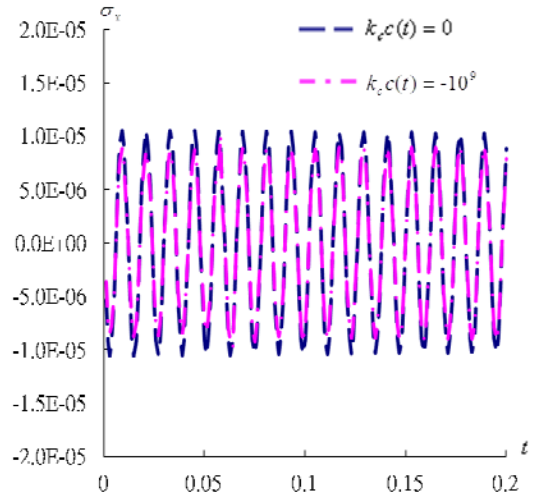
(b) $w(a/2, b/2)$ vs. t for $a/h^* = 100$



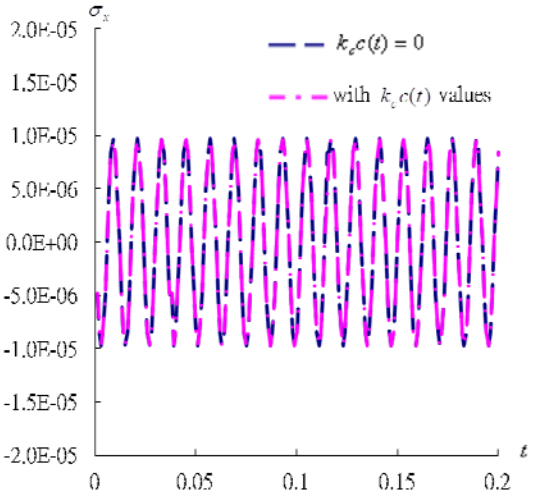
(c) Compared $w(a/2, b/2)$ vs. t as using $k_c c(t)$ controlled values

Fig. 1 Deflection $w(a/2, b/2)$ (unit: mm) versus transient response t for the ratio of length to the thickness $a/h^* = 5$ and 100.

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(a) σ_x vs. t for $a/h^* = 5$



(b) σ_x vs. t for $a/h^* = 100$

Fig. 2 Stress σ_x (unit: GPa) versus transient response t for the ratio of length to the thickness a/h^* = 5 and 100.

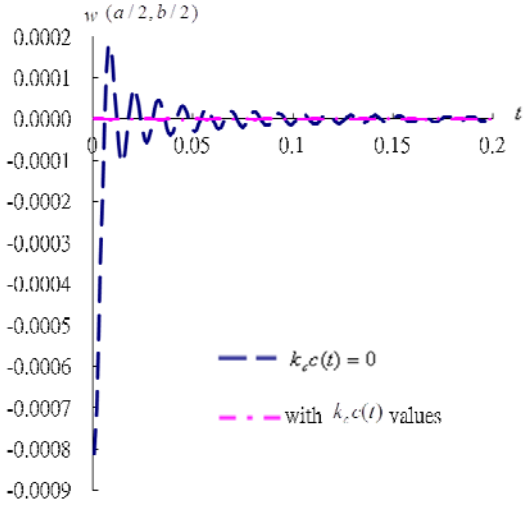


Fig. 3 Deflection $w(a/2, b/2)$ (unit: mm) versus transient response t for the power law index R_n = 10.

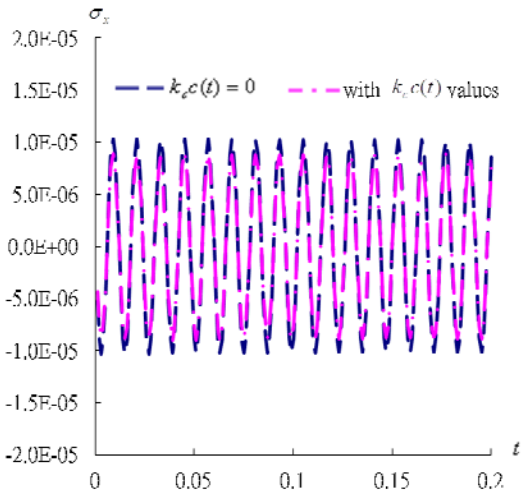


Fig. 4 Stress σ_x (unit: GPa) versus transient response t for the power law index R_n = 10.

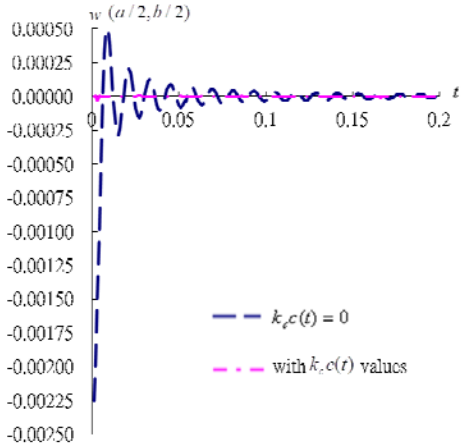
strictive FGM square plate, $a/h^* = 5$, $T = 653^\circ K$, $h_3/h^* = 0.1$, $h^* = 1$ (unit: mm), $a/b = 1$, under rapid heat flux $q_0 = 2\ J/(s\ m^2)$ and shear effect. We find the amplitude $w(a/2, b/2)$ of magnetostrictive FGM plate can converge by itself. The amplitude of $w(a/2, b/2)$ can be controlled into smaller value with feedback control values $k_c c(t) = 10^9$ at time $t = 0.170$ sec, $k_c c(t) = -10^9$ in others time $t \le 0.2$ sec. The uncontrolled amplitudes of $w(a/2, b/2)$ become very much lower value when compared with lower $R_n = 1$ case in Fig. 1(a).

Figure 4 shows the corresponding transient response of the dominated stress σ_x (unit: GPa) at center position of upper surface $Z = 0.5h^*$ with and without $k_c c(t)$ values as the deflection analyses in Fig. 3 of $R_n = 10$ in magnetostrictive FGM square plate. We find the amplitudes of the stress σ_x oscillate in good harmonic and have the smaller amplitude value 8.88E-06 GPa in controlled $k_c c(t)$ values case than the amplitude value 1.03E-05 GPa in the uncontrolled $k_c c(t) = 0$ case.

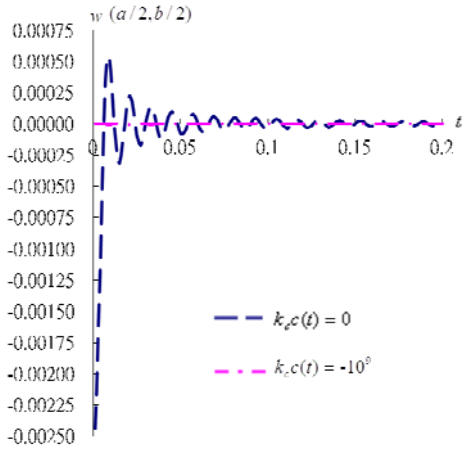
We like to investigate the effect of the temperature of environment T on the transient values of deflection and stress. The critical maximum temperature is $653^\circ K$ for the magnetostrictive material Terfenol-D can operate successively. The usual room temperature is $300^\circ K$ would be used to compare the effects of increasing temperatures with $400^\circ K$ and $653^\circ K$. Figure 5 shows the transient response of the transverse center deflection amplitude $w(a/2, b/2)$ (unit: mm) with and without $k_c c(t)$ values at environment temperature $T = 400^\circ K$ and $T = 300^\circ K$ in magnetostrictive FGM square plate, $a/h^* = 5$, $R_n = 1$, $h_3/h^* = 0.1$, $h^* = 1$ (unit: mm), $a/b = 1$, under rapid heat flux $q_0 = 2\ J/(s\ m^2)$ and shear effect. We find the amplitude $w(a/2, b/2)$ of magnetostrictive FGM plate can be controlled and converge by itself. The amplitude $w(a/2, b/2)$ can be controlled into smaller value with feedback control values $k_c c(t) = 10^8$ at time $t = 0.003$ sec, $k_c c(t) = 10^9$ at time $t = 0.087$ sec and $t = 0.163$ sec, $k_c c(t) = -10^9$ in others time $t \le 0.2$ sec, for $T = 400^\circ K$ case. The amplitudes of uncontrolled $w(a/2,$

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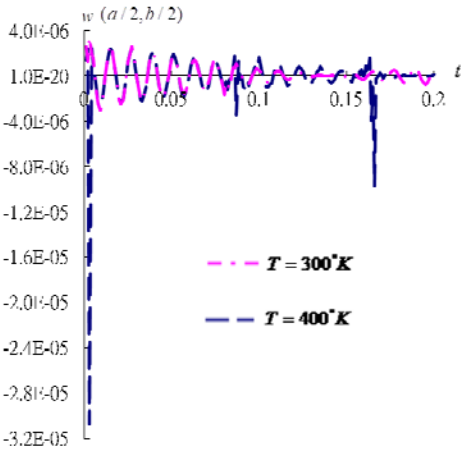
$b/2$) become greater value when compared with T = 653°K case in Fig. 1(a). The amplitudes of uncontrolled $w(a/2, b/2)$ in T = 300°K case are greater than that T = 400°K case. The compared enlargement amplitudes of controlled $w(a/2, b/2)$ in T = 300°K and T = 400°K are shown in Fig. 5(c).



(a) $w(a/2, b/2)$ vs. t for T = 400°K



(b) $w(a/2, b/2)$ vs. t for T = 300°K

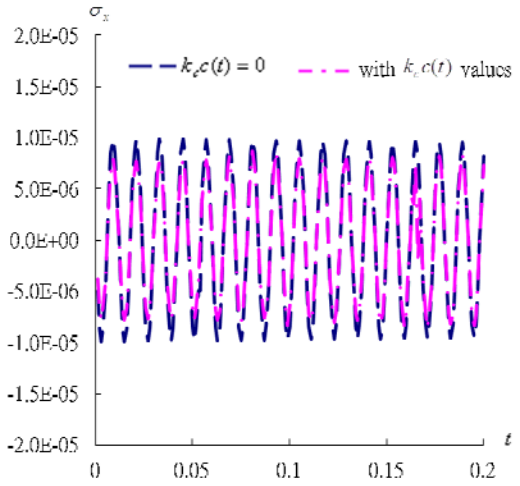


(c) Compared $w(a/2, b/2)$ vs. t as using $k_c c(t)$ controlled values

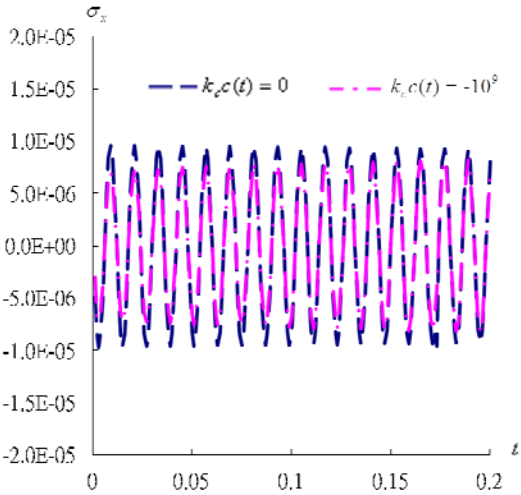
Fig. 5 Deflection $w(a/2, b/2)$ (unit: mm) versus transient response t for the temperature of environment T = 400°K and T = 300°K.

Figure 6 shows the corresponding transient response of the dominated stress σ_x (unit: GPa) at center position of upper surface $Z = 0.5h^*$ with and without $k_c c(t)$ values as the deflection analyses in Fig. 5 of T = 400°K and T = 300°K in magnetostrictive FGM square plate. We find the amplitudes of the stress σ_x oscillate in good harmonic and have the smaller amplitude value 7.94E-06 GPa in controlled $k_c c(t)$ values case than the amplitude value 9.92E-06 GPa in the uncontrolled $k_c c(t)$ = 0 case for T = 400°K. The amplitude value 7.10E-06 GPa in controlled $k_c c(t)$ values case is smaller than the amplitude value 9.66E-06 GPa in the uncontrolled $k_c c(t)$ = 0 case for T = 300°K. The amplitudes of stress σ_x have the smaller values for T = 400°K and T = 300°K cases when compared with T = 653°K case in Fig. 2(a).

We like to investigate the effect of the applied heat flux q_0 on the transient values of deflection and stress. Figure 7 shows the transient response of the transverse center deflection amplitude $w(a/2, b/2)$ (unit: mm) with



(a) σ_x vs. t for T = 400°K



(b) σ_x vs. t for T = 300°K

Fig. 6 Stress σ_x (unit: GPa) versus transient response t for the temperature of environment T = 400°K and T = 300°K.

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and without $k_c c(t)$ values under rapid heat flux q_0 = $20J/(s\ m^2)$ at environment temperature $T = 653^{\circ}K$ in magnetostrictive FGM square plate, $a/h^* = 5$, $R_n = 1$, $h_3/h^* = 0.1$, $h^* = 1$ (unit: mm), $a/b = 1$ and shear effect. We find the amplitude $w(a/2, b/2)$ of magnetostrictive FGM plate can be controlled into smaller value. The amplitudes of uncontrolled $w(a/2, b/2)$ converge from time period $t < 0.075$ sec and become larger value for $t \geq 0.075$ sec due to the effect of rapid heat flux. The total system still under control and converge, because the deflection can be controlled into the smaller values.

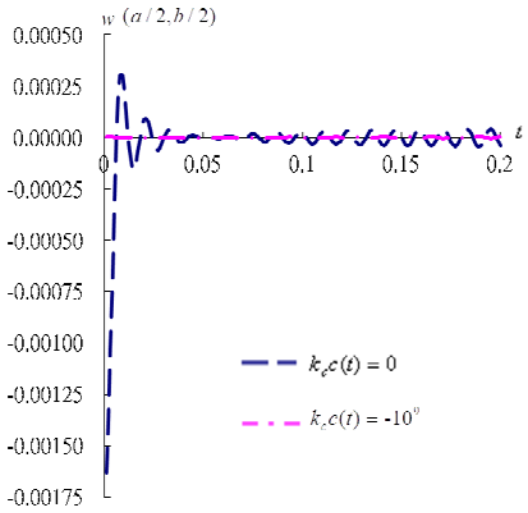


Fig. 7 Deflection $w(a/2, b/2)$ (unit: mm) versus transient response t for the applied heat flux q_0 = $20J/(s\ m^2)$.

Figure 8 shows the corresponding transient response of the dominated stress σ_x (unit: GPa) at center position of upper surface $Z = 0.5h^*$ with and without $k_c c(t)$ values as the deflection analyses in Fig. 7 of $q_0 = 20J/(s\ m^2)$ in magnetostrictive FGM square plate. We find the amplitudes of the controlled ($k_c c(t) = -10^9$) stress σ_x oscillate and then diverge for $t \geq 0.075$ sec, the uncontrolled ($k_c c(t) = 0$) stress σ_x oscillate and converge. The controlled amplitudes value 9.00E-06 GPa of stress σ_x have the greater value than the amplitudes value 8.86E-06 GPa of $q_0 = 2\ J/(s\ m^2)$ case in Fig. 2(a), for $t <$ 0.075 sec. Why does σ_x diverge for $t \geq 0.075$ sec in the controlled case $k_c c(t) = -10^9$? Because the values of the transverse center deflection amplitude $w(a/2, b/2)$ at the oscillated time period $t = 0.072$ sec to $t = 0.083$ sec are smaller than that corresponding amplitude $w(a/2, b/2)$ values at the next oscillated time period $t = 0.084$ sec to $t = 0.095$ sec in the case $q_0 = 20J/(s\ m^2)$, so it need more stresses to control the amplitudes of deflection into the smaller values for the next oscillated circles in the controlled case $k_c c(t) = -10^9$. The total system still under control and converge, because the deflection can be controlled into the smaller values by using suitable controlled value $k_c c(t) = -10^9$ as shown in Fig. 7, and the oscillated stress values are still operate in safe condition of materials.

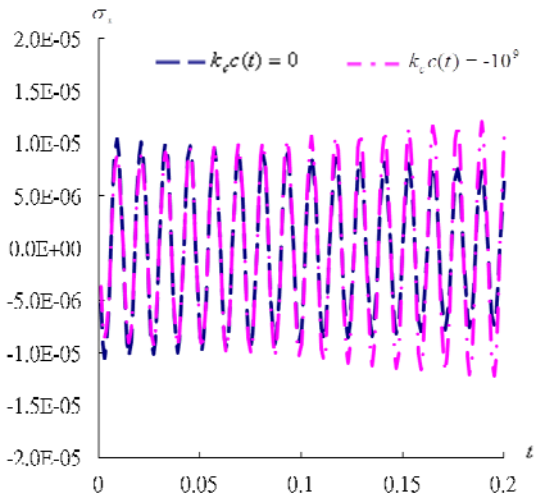


Fig. 8 Stress σ_x (unit: GPa) versus transient response t for the applied heat flux $q_0 = 20J/(s\ m^2)$.

5. CONCLUSIONS

The computational GDQ method can be used to calculate the transient responses of deflection and stress in the three-layer ($0^{\circ m}/0^{\circ}/0^{\circ}$) magnetostrictive FGM square plate under thermal vibration of sinusoidal temperature including rapid heating and shear deformation effect. We find the transient amplitude values of deflection are dependent totally on the parameters of a/h^*, R_n, T and $k_c c(t)$. We have the following results: (a) The transient amplitude of deflection in magnetostrictive FGM plate converges by itself and can be controlled into smaller amplitude by using suitable controlled gain $k_c c(t)$ value. (b) The transient amplitudes of uncontrolled deflection in magnetostrictive FGM plate: at thin plate ($a/h^* = 100$) are very much lower than that at thick plate ($a/h^* = 5$), with smaller power law index FGM ($R_n = 1$) are very much lower than that with higher power law index FGM ($R_n = 10$), in smaller environment temperature ($T = 400^{\circ}K$) are very much greater than that in higher environment temperature ($T = 653^{\circ}K$). (c) The transient amplitudes of stress σ_x become lower value when the magnetostrictive FGM plate becomes thinner ($a/h^* = 100$) for the uncontrolled case $k_c c(t) = 0$.

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