J. Plasma Physics (2007), *vol.* 73, *part* 6, *pp.* 911–920. © 2007 Cambridge University Press
doi:10.1017/S0022377807006381 First published online 9 February 2007 Printed in the United Kingdom

Effect of the general loss-cone distribution function on kinetic Alfvén waves—a kinetic approach

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(Received 25 September 2006, accepted 27 November 2006)

Abstract. Kinetic Alfvén waves are investigated in the presence of a general loss-cone distribution function including finite electron pressure and ion-gyroradius effects. The dispersion relation and damping/growth rate are evaluated for different electron to ion temperature ratios, T_e/T_i, using a kinetic approach. The wave frequency ω and damping/growth rate γ_L are evaluated for two regimes of propagation, $k_\perp\rho_i < 1$ and $k_\perp\rho_i > 1$, where $k_\perp$ is the perpendicular wave number and ρ_i is the ion-gyroradius. An enhancement of the wave frequency and a reduction in the damping rate are predicted by steep loss-cone distribution indices and T_e/T_i. The growth of the wave is also noticed at higher values of the distribution index and lower T_e/T_i. Plasma parameters appropriate to the plasma sheet boundary layer (PSBL) are used to discuss the propagation of kinetic Alfvén waves from the PSBL to the auroral ionosphere.

1. Introduction

In the past, the general loss-cone distribution function, also known as the Dory–Guest–Harris (DGH) distribution [1] function, has been widely used for the analysis of electrostatic and electromagnetic instabilities [2–7]. More recently, Mishra and Tiwari [8] and Ahirwar et al. [9,10] adopted this distribution function for the study of electrostatic and electromagnetic ion-cyclotron waves, respectively, pertaining to the auroral ionospheric region. Also, kinetic Alfvén waves (KAWs) have been investigated in the presence of a general loss-cone distribution, using a particle aspect approach, by Baronia and Tiwari [11], Dwivedi et al. [12] and Duan et al. [13] for space plasma applications. Drift waves have also been analysed by Tiwari and Varma [14, 15] and Varma and Tiwari [16, 17] based on a particle aspect approach using general loss-cone distribution functions, and the importance of this distribution function has been emphasized with regards to various applications in laboratory and space plasmas. However, in most of these studies, the analysis was limited to $k_\perp\rho_i < 1$, where $k_\perp$ is the perpendicular wave number and ρ_i is the ion-gyroradius, and full treatment of perpendicular integrals are less emphasized.

KAWs have always been applied to a variety of research areas [18] including space plasmas [19, 20], astrophysical plasmas [21] as well as laboratory plasmas [22, 23]. These waves also encounter mirror-like geometries in a variety of situations during their propagation, and the general loss-cone distribution may be appropriate for

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discussing the wave dynamics. Thus, we have analysed the effect of the general loss-cone distribution on the propagation of KAWs using a kinetic approach for two regimes, $k_\perp \rho_\mathrm{i} < 1$ and $k_\perp \rho_\mathrm{i} > 1$. The full treatment of perpendicular integrals is given and the propagation of KAWs from the plasma sheet boundary layer (PSBL) to the auroral acceleration region is discussed as an example.

The most important aspect of KAWs as observed in the auroral oval, and more recently in the high-latitude magnetosphere [19], is their ability to accelerate electrons to energies and in fluxes sufficient to cause visible aurora [24]. Hasegawa [18] developed the idea that an Alfvén wavelength projected perpendicular to the geomagnetic field (B_0) is of the order of an ion-acoustic gyroradius or less and carries a parallel electric field that is sufficient to accelerate electrons to form aurora [25]. It should be noted that discrete aurora and diffuse aurora are two different phenomena. Diffuse aurora are also caused by electron precipitation, resulting from the emptying of the loss-cone of mirror bouncing plasma sheet electron distributions, which give rise to a broader, less intense aurora. The refilling of the loss-cone is belived to be caused by wave–particle pitch angle diffusion at the magnetotail equator [26].

In the majority of theoretical work carried out so far in the analysis of KAWs the velocity distribution functions have been assumed as either ideal Maxwellian or bi-Maxwellian, ignoring the steepness of the loss-cone feature. Plasma in mirror-like devices and in the auroral region with curved and converging magnetic field lines may considerably depart from a Maxwellian distribution and become anisotropic, provided there is a relatively low degree of plasma collisionality. Thus KAWs generated in a highly anisotropic PSBL [19, 20] and propagating towards the auroral ionosphere may be expressed using a general loss-cone distribution function rather than with an ideal Maxwellian distribution.

This paper contains a mathematical derivation of the dispersion relation and the damping/growth rate of the KAWs in the presence of a general loss-cone distribution function with finite electron pressure (terms involving $T_\mathrm{e}/T_\mathrm{i}$) in an anisotropic plasma.

2. Mathematical model

We consider a collisionless plasma in which the evolution of the distribution function of the jth species is described by the Vlasov–Maxwell equation

$$\left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial x} + \frac{e_j}{m_j}\left\{\mathbf{E}(x,t) + \frac{\mathbf{v}\times\mathbf{B}(x,t)}{c}\right\}\cdot\frac{\partial}{\partial v}\right]F_j(x,v,t) = 0, \tag{1}$$

where $\mathbf{E}(x,t)$ and $\mathbf{B}(x,t)$ are the electric and magnetic fields, respectively, and e_j and m_j are the charge and mass, respectively, of the particle j. Here, $F_j(x,v,t)$ is the density of particles of species j in six-dimensional phase space. Moreover, the electric and magnetic fields are determined self-consistently from Maxwell's equations in terms of the plasma current density and charge density

$$\rho(x,t) = \sum_j e_j \int d^3v\, F_j(x,v,t),$$

$$J(x,t) = \sum_j e_j \int d^3v\, v F_j(x,v,t),$$

as well as external charge and current sources.

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Throughout the present analysis it is assumed that the plasma is immersed in a uniform, externally applied magnetic field $\mathbf{B}_0 = B_0\hat{e}_z$, and that the plasma equilibrium state ($\partial/\partial t = 0$) is characterized by neutrality $\sum \hat{n}_j e_j = 0$ and zero electric field E_0. It is also assumed that the equilibrium plasma current (if any) is sufficiently weak that the corresponding equilibrium self-magnetic field has a negligible effect compared with the applied magnetic field $B_0\hat{e}_z$.

2.1. Basic assumptions

To determine the dispersion relation and damping/growth rate, we use the general loss-cone distribution of the form [1, 12]

$$F_j(\mathbf{v}_\perp, v_\parallel) = n_0 F_\perp(\mathbf{v}_\perp) F_\parallel(v_\parallel) \tag{1}$$

where n_0 is the zeroth-order density and

$$F_\perp(\mathbf{v}_\perp) = \frac{v_\perp^{2J}}{\pi v_{T\perp}^{2(J+1)} J!} \exp\left[-\frac{v_\perp^2}{v_{T\perp}^2}\right], \tag{1a}$$

$$F_\parallel(v_\parallel) = \frac{1}{\pi^{1/2} v_{T\parallel}} \exp\left[-\frac{v_\parallel^2}{v_{T\parallel}^2}\right], \tag{1b}$$

where $v_{T\parallel}^2 = 2T_\parallel/m$ and $v_{T\perp}^2 = (J+1)^{-1} \cdot (2T_\perp/m)$ are the square of parallel and perpendicular velocities, respectively, and J is an integer known as the loss-cone index [12]. The case $J = 0$ represents a bi-Maxwellian distribution and $J = \infty$ the Dirac delta function. Index J characterizes the width of the loss cone. Moreover, $F_\perp(\mathbf{v}_\perp)$ peaks at approximately $J^{1/2} v_{T\perp}$ and has a half-width $\Delta\mathbf{v}_\perp \sim J^{-1/2} v_{T\perp}$ [17, 27].

2.2. General dispersion relation for KAWs

We consider a plasma with an external magnetic field $\mathbf{B}$ in the z-direction and, since the KAW has its electric vector and, wave vector in the same plane, we can write [28, 29]

$$\begin{vmatrix} D_{xx} & D_{xz} \\ D_{zx} & D_{zz} \end{vmatrix} = 0, \tag{2}$$

where

$$D_{xx}(\mathbf{k}, \omega) = 1 - \frac{c^2 k_\parallel^2}{\omega^2} + \sum_j \frac{\omega_{pj}^2}{\omega} \sum_{n=-\infty}^{\infty} \int d^3v\, \mathbf{v}_\perp \frac{(n^2/b_j^2) J_n^2(b_j)}{(\omega - n\omega_{cj} - k_\parallel v_\parallel)}$$

$$\times \left\{ \frac{\partial F_j}{\partial \mathbf{v}_\perp} - \frac{k_\parallel v_\parallel}{\omega} \left(\frac{\partial F_j}{\partial \mathbf{v}_\perp} - \frac{\mathbf{v}_\perp}{v_\parallel} \frac{\partial F_j}{\partial v_\parallel} \right) \right\}, \tag{3}$$

$$D_{xz}(\mathbf{k}, \omega) = \frac{c^2 k_\parallel \mathbf{k}_\perp}{\omega^2} + \sum_j \frac{\omega_{pj}^2}{\omega} \sum_{n=-\infty}^{\infty} \int d^3v\, \mathbf{v}_\perp \frac{(n/b_j) J_n^2(b_j)}{(\omega - n\omega_{cj} - k_\parallel v_\parallel)}$$

$$\times \left\{ \frac{\partial F_j}{\partial v_\parallel} + \frac{n\omega_{cj}}{\omega} \left(\frac{v_\parallel}{\mathbf{v}_\perp} \frac{\partial F_j}{\partial \mathbf{v}_\perp} - \frac{\partial F_j}{\partial v_\parallel} \right) \right\}, \tag{4}$$

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$$D_{zx}(\mathbf{k},\omega) = \frac{c^2 k_\parallel k_\perp}{\omega^2} + \sum_j \frac{\omega_{pj}^2}{\omega} \sum_{n=-\infty}^{\infty} d^3 v v_\parallel \frac{(n/b_j) J_n^2(b_j)}{(\omega - n\omega_{cj} - k_\parallel v_\parallel)} \times \left\{ \frac{\partial F_j}{\partial \mathbf{v}_\perp} - \frac{k_\parallel v_\parallel}{\omega} \left(\frac{\partial F_j}{\partial \mathbf{v}_\perp} - \frac{\mathbf{v}_\perp}{v_\parallel} \frac{\partial F_j}{\partial v_\parallel} \right) \right\}, \tag{5}$$

$$D_{zz}(\mathbf{k},\omega) = 1 - \frac{c^2 k_\perp^2}{\omega^2} + \sum_j \frac{\omega_{pj}^2}{\omega} \sum_{n=-\infty}^{\infty} \int d^3 v v_\parallel \frac{J_n^2(b_j)}{(\omega - n\omega_{cj} - k_\parallel v_\parallel)} \times \left\{ \frac{\partial F_j}{\partial v_\parallel} + \frac{n\omega_{cj}}{\omega} \left(\frac{v_\parallel}{\mathbf{v}_\perp} \frac{\partial F_j}{\partial \mathbf{v}_\perp} - \frac{\partial F_j}{\partial v_\parallel} \right) \right\}, \tag{6}$$

where $\int d^3 v = 2\pi \int_0^\infty d\mathbf{v}_\perp \, \mathbf{v}_\perp \int_{-\infty}^\infty dv_\parallel$, $b_j = k_\perp v_\perp / \omega_{cj}$, $\omega_{pj}^2 = 4\pi n_j e_j^2 / m_j$ is the plasma frequency squared, $\omega_{cj} = e_j B_0 / m_j c$ is the cyclotron frequency and $\mathrm{Im}\,\omega > 0$ is assumed.

We use the general distribution function from (1a) and (1b),

$$\frac{\partial F_j}{\partial \mathbf{v}_\perp} = \left[\frac{2J}{\mathbf{v}_\perp} - \frac{2\mathbf{v}_\perp}{v_{T\perp}^2} \right] F_j(\mathbf{v}_\perp, v_\parallel),$$

$$\frac{\partial F_j}{\partial v_\parallel} = -\frac{2v_\parallel}{v_{T_\parallel}^2} \cdot F_j(\mathbf{v}_\perp, v_\parallel).$$

By substituting these values into (3) we obtain

$$D_{xx}(\mathbf{k},\omega) = 1 - \frac{c^2 k_\parallel^2}{\omega^2} + \sum_j \frac{\omega_{pj}^2}{\omega} \frac{n^2 \omega_{cj}^2}{k_\perp^2} \left[D_n^J(\lambda) \left(\frac{-2}{V_{T\perp}^2} \right) \left(\frac{\omega}{\omega - n\omega_{cj}} \right) - C_n^J(\lambda) \frac{2}{v_{T\parallel}^2 v_{T\parallel}} \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} k_\parallel v_\parallel \frac{\exp(-v_\parallel^2 / v_{T\parallel}^2)}{(\omega - n\omega_{cj} - k_\parallel v_\parallel)} dv_\parallel \right], \tag{7}$$

where

$$D_n^J(\lambda) = \int_0^\infty \left(1 - \frac{J v_{T\perp}^2}{v_\perp^2} \right) dv_\perp^2 \, J_n^2 \left(\frac{k_\perp v_\perp}{\omega_{cj}} \right) \frac{v_\perp^{2J}}{v_{T\perp}^{2(J+1)} J!} \exp\left(\frac{-v_\perp^2}{v_{T\perp}^2} \right), \tag{8}$$

$$C_n^J(\lambda) = \int_0^\infty 2\pi \mathbf{v}_\perp J_n^2 \left(\frac{k_\perp v_\perp}{\omega_{cj}} \right) \frac{v_\perp^{2J}}{\pi v_{T\perp}^{2(J+1)} J!} \exp\left[-\frac{v_\perp^2}{v_{T\perp}^2} \right]. \tag{9}$$

We have assumed the identities

$$\int_0^\infty dx^2 J_n^2(sx) \exp(-x^2) = \exp\left(\frac{-s^2}{2} \right) I_n(\lambda), \tag{10}$$

$$\int_0^\infty dx^2 x^2 J_n^2(sx) \exp(-x^2) = \exp\left(\frac{-s^2}{2} \right) \left[\frac{1}{2} s^2 (I_n'(\lambda) - I_n(\lambda)) + I_n(\lambda) \right], \tag{11}$$

and the plasma dispersion function

$$Z(\zeta) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp(-x^2)}{(x - \zeta)} dx, \quad Z(\zeta) \approx \frac{-1}{\zeta} - \frac{1}{2\zeta^3} + \cdots, \quad \zeta \gg 1$$

where for ions $\zeta = \omega - n\omega_{cj} / k_\parallel v_{T_\parallel \mathrm{i}}$, $v_{T_\parallel \mathrm{i}}$ is the ion thermal velocity and

$$\lambda = \tfrac{1}{2}(J+1) k_\perp^2 \rho_\mathrm{i}^2 = \tfrac{1}{2}(J+1) b_j.$$

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Thus from (7)

$$D_{xx}(\mathbf{k},\omega) \approx 1 - \frac{c^2 k_\parallel^2}{\omega^2} - \sum_j \frac{\omega_{\mathrm{pi}}^2}{\omega_{\mathrm{ci}}^2} \frac{D_n^J(\lambda)}{\lambda} \tag{12}$$

and, in the frequency range $\omega \ll \omega_{\mathrm{ci}}$ and for $n = 1$, we define

$$\varepsilon_\perp \cong -\frac{\omega_{\mathrm{pi}}^2}{\omega_{\mathrm{ci}}^2} \frac{D_1^J(\lambda)}{\lambda}, \tag{13}$$

$$D_{xx}(\mathbf{k},\omega) = \varepsilon_\perp - \frac{c^2 k_\parallel^2}{\omega^2}. \tag{14}$$

Similarly, from (4)–(6),

$$D_{xz}(\mathbf{k},\omega) \approx \frac{c^2 k_\parallel \mathbf{k}_\perp}{\omega^2}, \tag{15}$$

$$D_{zx}(\mathbf{k},\omega) \approx \frac{c^2 k_\parallel \mathbf{k}_\perp}{\omega^2} \tag{16}$$

and

$$D_{zz}(\mathbf{k},\omega) = 1 - \frac{c^2 k_\perp^2}{\omega^2} - \sum_j \frac{\omega_{\mathrm{p}j}^2}{\omega^2} C_0^J(\lambda) \zeta_{\mathrm{e}}^2 Z'(\zeta_{\mathrm{e}}) \tag{17}$$

where

$$C_0^J(\lambda) = \exp(-\lambda)[\lambda\{I_0'(\lambda) - I_0(\lambda)\} + I_0(\lambda)], \tag{18}$$

$$Z'(\zeta_{\mathrm{e}}) \approx -2\left(1 + i\sqrt{\frac{\pi}{2}} \frac{\omega}{k_\parallel v_{T_\parallel \mathrm{e}}}\right), \quad \zeta_{\mathrm{e}} = \frac{\omega}{\sqrt{2} k_\parallel v_{T_\parallel \mathrm{e}}}. \tag{19}$$

In the frequency range $\omega_{\mathrm{pe}}/\omega \gg 1$, where ω_{pe} is the electron plasma frequency, we define from (17)

$$\varepsilon_\parallel \approx C_0^J(\lambda) \sum_j \frac{\omega_{\mathrm{pe}}^2}{\omega^2} \zeta_{\mathrm{e}}^2 Z'(\zeta_{\mathrm{e}}), \tag{20}$$

$$\varepsilon_\parallel \approx C_0^J(\lambda) \frac{1}{\delta} \frac{m_{\mathrm{i}}}{m_{\mathrm{e}}} \frac{\omega_{\mathrm{pi}}^2}{k_\parallel^2 v_{T_\parallel \mathrm{e}}^2} \left(1 + i\sqrt{\frac{\pi}{2}} \frac{\omega}{k_\parallel v_{T_\parallel \mathrm{e}}}\right), \tag{21}$$

$$D_{zz}(\mathbf{k},\omega) \approx \varepsilon_\parallel - \frac{c^2 k_\perp^2}{\omega^2}. \tag{22}$$

Here, $v_{T_\parallel \mathrm{e}}$ is an electron thermal velocity. The presence of dust grains can also be incorporated through the term δ and $\delta = n_{i0}/n_{e0} > 1$ is a measure of the negative charge density carried by the dust [30]. However, we consider only $\delta = 1$ in our analysis as the presence of dust is of no relevance to the PSBL.

By substituting the values from (14)–(16) and (22) into (2) we obtain the general dispersion relation

$$\begin{bmatrix} \varepsilon_\perp - \dfrac{c^2 k_\parallel^2}{\omega^2} & \dfrac{c^2 k_\parallel k_\perp}{\omega^2} \\ \dfrac{c^2 k_\parallel k_\perp}{\omega^2} & \varepsilon_\parallel - \dfrac{c^2 k_\perp^2}{\omega^2} \end{bmatrix} = 0, \tag{23}$$

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where $k_\parallel$ and $k_\perp$ refer to wave vector components parallel or perpendicular to external magnetic field B, respectively. The terms $\varepsilon_\parallel$ and $\varepsilon_\perp$ refer to the dielectric tensor elements parallel or perpendicular to the external magnetic field, respectively. The analytical method is adopted to solve the dispersion relation to evaluate ω and the damping/growth rate $\gamma_{\rm L}$. By substituting the values for $\varepsilon_\perp$ and $\varepsilon_\parallel$ into (23) we obtain the general dispersion relation for a KAW,

$$\omega^2 = k_\parallel^2 V_{\rm A}^2 \left[\frac{2\lambda}{C_0^J(\lambda)}\frac{T_{\rm e}}{T_{\rm i}} - \frac{\lambda}{D_1^J(\lambda)}\right], \tag{24}$$

where $V_{\rm A}^2 = c^2.\omega_{\rm ci}^2/\omega_{\rm pi}^2$ is the square of the Alfvén velocity. The result is the same as that derived by Sallimullah and Rosenberg [30] if $C_0^J \approx 1$ and $D_1^J = \exp(-\lambda_{\rm i})I_1(\lambda_{\rm i})$.

2.3. Damping/growth rate

Assuming $\omega \to \omega_r + i\gamma_{\rm L}$, with $\gamma_{\rm L} \ll \omega$, we obtain an expression for the collisionless damping/growth rate of the KAW as

$$\gamma_{\rm L} = -\frac{1}{C_0^J(\lambda)}\left(\frac{m_{\rm e}}{m_{\rm i}}\right)\sqrt{\frac{\pi}{8}}\left(\frac{c^2 k_\perp^2}{\omega_{\rm pi}^2}\right)k_\parallel V_{T\parallel_{\rm e}}. \tag{25}$$

For $J = 0, 1, 2, \ldots$ the general dispersion relation and damping/growth rate are evaluated using the following recurrence relation [2]:

$$C_n^{(J+1)}(\lambda) = C_n^J(\lambda) + \frac{\lambda}{J+1}\frac{d}{d\lambda}C_n^J(\lambda), \tag{26}$$

$$D_n^{J+1}(\lambda) = \frac{J}{J+1}D_n^J(\lambda) + \frac{\lambda}{J+1}\frac{d}{d\lambda}D_n^J(\lambda). \tag{27}$$

Using the identities

$$\begin{aligned} I_{\nu-1}(\lambda) - I_{\nu+1}(\lambda) &= \frac{2\nu}{\lambda}I_\nu(\lambda),\\ I'_\nu(\lambda) &= I_{\nu-1}(\lambda) - \frac{\nu}{\lambda}I_\nu(\lambda),\\ I_{\nu-1}(\lambda) + I_{\nu+1}(\lambda) &= 2I'_\nu(\lambda),\\ I'_\nu(\lambda) &= I_{\nu+1}(\lambda) + \frac{\nu}{\lambda}I_\nu(\lambda), \end{aligned} \tag{28}$$

and with the help of (26) and (27) we evaluate

$$\begin{aligned} C_0^0(\lambda) &= e^{-\lambda}I_0(\lambda),\\ D_1^0 &= e^{-\lambda}I_1(\lambda) \end{aligned} \tag{29}$$

for $J = 0$. In addition,

$$\begin{aligned} C_0^1(\lambda) &= (1-\lambda)I_0(\lambda)e^{-\lambda} + \lambda I_1(\lambda)e^{-\lambda},\\ D_1^1(\lambda) &= \lambda I_0(\lambda)e^{-\lambda} - (1+\lambda)I_1(\lambda)e^{-\lambda} \end{aligned} \tag{30}$$

for $J = 1$. Similarly, for $J = 2$

$$\begin{aligned} C_0^2(\lambda) &= \frac{3}{2}\lambda I_1(\lambda)e^{-\lambda} + (1-2\lambda)I_0(\lambda)e^{-\lambda},\\ D_1^2(\lambda) &= \frac{\lambda}{2}I_0(\lambda)e^{-\lambda} + \lambda^2(1-\lambda)I_1(\lambda)e^{-\lambda}. \end{aligned} \tag{31}$$

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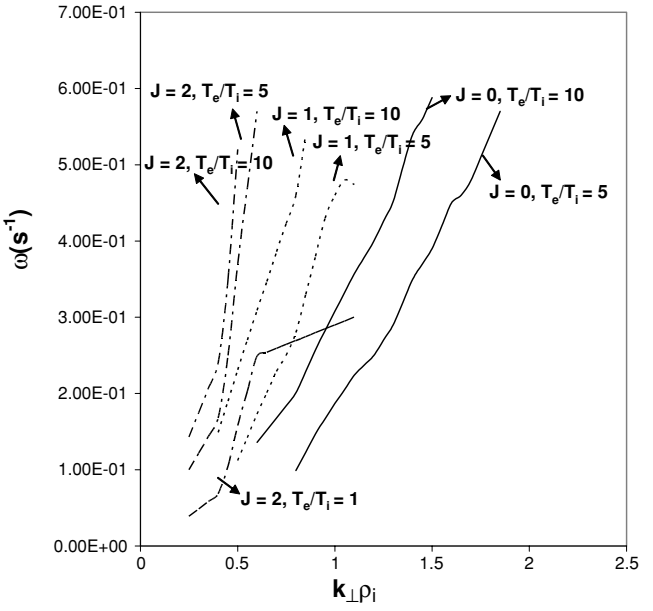


Figure 1. Variation of wave frequency ω (s^{-1}) versus $k_\perp \rho_i$ for different values of J and T_e/T_i.

By substituting the above values of $C_0^J(\lambda)$ and $D_1^J(\lambda)$ into the dispersion relation (24) and the damping/growth rate (25) we obtain the wave frequency and damping/growth rate for $J = 0, 1$ and 2.

3. Result and discussion

In the present analysis, the expressions for the dispersion relation and damping/growth rate are numerically evaluated for the KAW. We have used the following parameters appropriate to the PSBL [19,20]: $B_0 = 400$ nT, $V_A = 8.8 \times 10^8$ cm s^{-1}, $V_{T\|e} = 5.9 \times 10^9$ cm s^{-1}, $V_{T\perp i} = 8.8 \times 10^7$ cm s^{-1}, $k_\| = 10^{-10}$ cm^{-1}. The values of $C_0^J(\lambda)$ and $D_1^J(\lambda)$ affect the wave frequency and damping/growth rate significantly in the presence of a steep loss-cone distribution function. Equation (24) also shows that there exists a cut-off at $T_e/T_i = 2C_0^J(\lambda)/D_1^J(\lambda)$, and thus we can obtain a reasonable value of T_e/T_i above which the existence of waves is possible.

Figure 1 shows the variation of wave frequency $\omega(s^{-1})$ versus the perpendicular wave vector $k_\perp \rho_i$ for different values of the distribution indices J and temperature ratio T_e/T_i. Note that the wave frequency increases with $k_\perp \rho_i$ for all the distribution indices J; however, lower values of $k_\perp \rho_i$ are effective at higher J. Thus, the higher frequencies correspond to the lower $k_\perp \rho_i$ at steep loss-cone distributions. Similarly, higher values of T_e/T_i shift the spectrum towards lower $k_\perp \rho_i$. At a particular value of $k_\perp \rho_i$ the effect of altering T_e/T_i is to enhance the frequency, which is also true for altering the distribution index, J. Frequencies in the range 0.1–4 Hz, for $k_\perp \rho_i < 1$, correspond to the PSBL, whereas higher frequencies of KAWs may be the result of steep loss-cone distributions.

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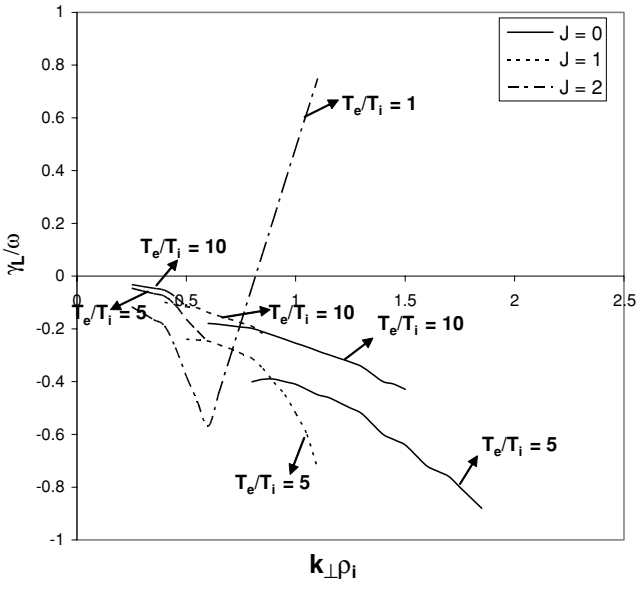


Figure 2. Variation of $\gamma_{\rm L}/\omega$ versus $\mathrm{k}_{\perp}\rho_{\rm i}$ for different values of J and $T_{\rm e}/T_{\rm i}$.

Figure 2 shows the variation of the damping/growth rate $\gamma_{\rm L}/\omega$ versus $k_{\perp}\rho_{\rm i}$ for different values of the distribution index J and temperature ratio $T_{\rm e}/T_{\rm i}$. It is seen that the damping rate is reduced at higher values of the distribution index and temperature ratio. The lower range of $k_{\perp}\rho_{\rm i}$ (<1) is sufficient to cause damping of a wave in a steep loss-cone distribution whereas damping at higher $k_{\perp}\rho_{\rm i}$ (>1) occurs for bi-Maxwellian distributions. Here, for $J = 2$ and $T_{\rm e}/T_{\rm i} = 1$, there is a sudden decrease of the damping rate at a particular value of $k_{\perp}\rho_{\rm i}$ and a growth rate is then observed. Thus, steep loss-cone distributions can generate KAWs at lower $T_{\rm e}/T_{\rm i}$ and higher distribution indices. The expression for the damping rate (25) shows no explicit dependence on wave frequency, ω; however, both are related via $k_{\perp}\rho_{\rm i}$. Therefore, we have normalized $\gamma_{\rm L}$ by the wave frequency. The plot of $\gamma_{\rm L}$ versus $k_{\perp}\rho_{\rm i}$ shows a shifting of $k_{\perp}\rho_{\rm i}$ towards the lower values at higher values of the distribution index.

Our findings may be applicable to the observed auroral arc formation in the near Earth space plasmas. The energy imparted to the KAW by magnetotail drivers propagates towards the auroral ionosphere [26], which is dissipated by accelerating auroral electrons. In the theoretical development in the literature the dynamics were described by considering either Maxwellian or bi-Maxwellian distribution functions. However, converging the magnetic field lines of the auroral region with the operative mirroring forces may considerably alter the particle distribution from Maxwellian to a steep loss-cone distribution and the previously described scenario of KAW propagation will be altered considerably. The KAW can be generated by a steep loss-cone distribution as our finding indicates. Thus, a steep loss-cone distribution may also be considered as one of the possible drivers of KAW in the distant magnetotail.

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4. Conclusion

We have evaluated the wave frequency and damping/growth rate for KAWs in the presence of a general loss-cone distribution function and electron–ion temperature ratio, T_e/T_i. It is predicted that lower values of $k_\perp \rho_i$ (<1) are much more effective for supporting the wave for a steep loss-cone distribution, whereas higher values of $k_\perp \rho_i$ (>1) are effective for the bi-Maxwellian distribution. The effect of T_e/T_i is similar to the effect of the distribution index, J. The damping rate is reduced in steep loss-cone distributions and wave emission is possible at higher values of the distribution indices and at lower values of the temperature ratio. It is also noticed in our example of PSBL that the waves emitted dissipate slowly and will travel further into the auroral ionosphere if the ions follow the steep loss-cone distribution function. The KAW is generated at lower T_e/T_i of PSBL at steep loss-cone regime.

Acknowledgement

The second author wishes to thank the Department of Science and Technology (DST), New Delhi for financial assistance.

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