105.44 A segment inside a square that is equal to the side of the square

Given a square $ABCD$, α is the length of a side. M is the midpoint of AB. CE is an altitude from the point C to MD (see Figure 1). Find the length of segment BE.

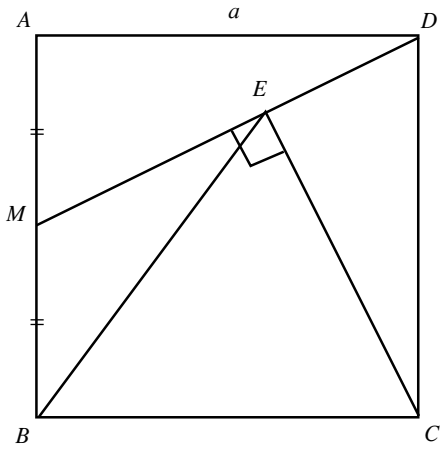


FIGURE 1

Two proofs without words:

Using Figures 2 and 3, one can discover that $EB = a$.

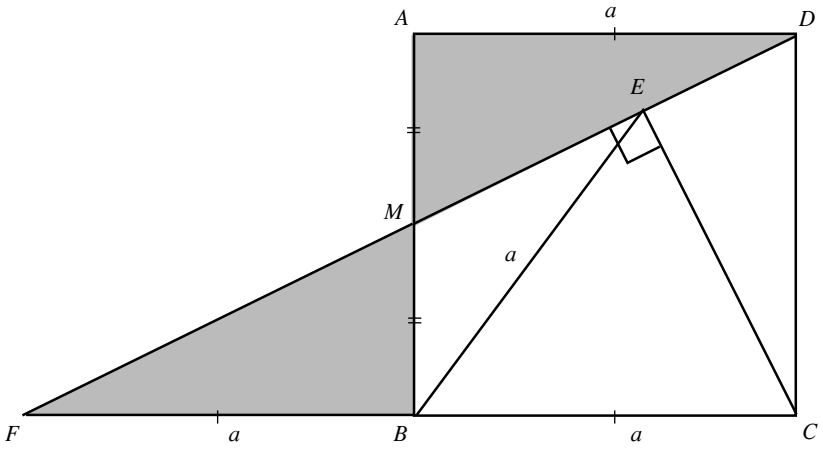


FIGURE 2

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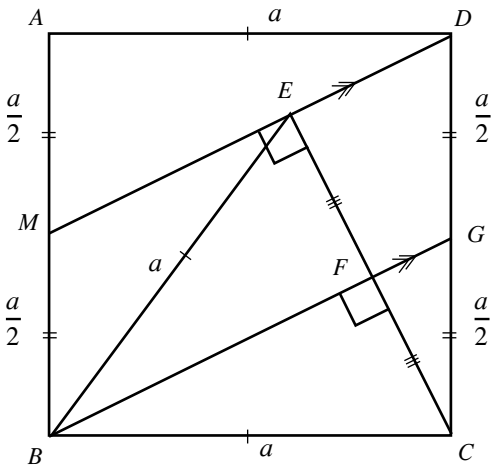


FIGURE 3

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10.1017/mag.2021.126
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105.45 British flag theorem for isosceles trapezia

The British flag theorem [1, 2] is an interesting theorem on a rectangle with an arbitrary point. Lardner [1] calls this theorem 'the most useful and remarkable theorem which may be inferred' from the results in Book II of Euclid's *Elements*.

We can consider a rectangle as a particular case of an isosceles trapezium. In this Note, we shall give a generalisation of the British flag theorem for an isosceles trapezium with proof using Stewart's theorem [3]. We also give a further extension for this generalisation with proof using the dot product of two vectors (see [4]).

Here we denote by XY the Euclidean distance between two points X and Y; $\overrightarrow{XY}$ the Euclidean vector connecting an initial point X with a terminal point Y [5].

Theorem 1 (British flag theorem): Let $ABCD$ be a rectangle and P be any point in its plane. Then we have the equation

$$PA^2 + PC^2 = PB^2 + PD^2. \tag{1}$$

https://doi.org/10.1017/mag.2021.126 Published online by Cambridge University Press