A FIXED-POINT THEOREM

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Communicated by F. SMITHIES

Received 30 August 1960

The object of this note is to prove a result intermediate between those fixed-point theorems (Brouwer, Schauder, Lefschetz) in which conditions are placed mainly on the space, and the following theorem, in which the strongest condition is on the mapping.

CONTRACTION MAPPING THEOREM. *In a complete metric space, every contraction mapping has just one fixed point.*

(A *contraction mapping* T is one satisfying

$$\rho(Tx, Ty) \leqslant K\rho(x, y) \tag{1}$$

for some $K < 1$ and all x, y.)

The above theorem is well known and easily proved.

If (1) holds with $K = 1$, I will call T *subisometric*. My result is:

THEOREM. *In a compact metric space in which the identity mapping can be uniformly approximated by contraction mappings, every subisometric mapping has a fixed point.*

Proof. Let T be a subisometric mapping. Let S be a contraction mapping with

$$\rho(x, Sx) < \epsilon$$

for all x. ST will be a contraction mapping. Thus ST will have a fixed point x. Therefore

$$\rho(x, Tx) = \rho(STx, Tx) < \epsilon.$$

If ϵ approaches O through a suitable sequence of values, x will approach a limit y (by the compactness of the space). Thus y will be a fixed point for T.

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A CHARACTERIZATION OF COMPACT REGULAR SPACES

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Communicated by J. H. WILLIAMSON

Received 24 February 1959

Let $\mathscr{B}$ be a family of finite character contained in 2^α where α is the collection of open sets in a topological space (J. L. Kelley, *General topology*, p. 32). We consider the following conditions where $M(\mathscr{B})$ denotes the set of maximal elements of $\mathscr{B}$.

https://doi.org/10.1017/S0305004100035404 Published online by Cambridge University Press