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# A note on the electrical potential distribution around a test charge in anisotropic collisional plasmas

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**Abstract.** In this paper, we discuss recent results concerning the electrical potential distribution around a small test charged particle in weakly ionized collisional plasmas in the presence of an ion flow. The effect of plasma absorption on the particle surface is taken into account and the results are compared with those from earlier models neglecting absorption. In the considered regime, the effect of plasma absorption is quite important and completely determines the long-range asymptote of the electrical potential.

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The character of the interactions between an object (particle) and surrounding plasmas is one of the most fundamental problems in plasma physics with applications such as to complex (dusty) plasmas [1–5], astrophysical plasmas [6], plasma of fusion devices [7–9] and plasma processing [10,11]. Of particular importance are basic processes such as particle charging, electrical potential distribution around the particle, inter-particle interactions, momentum and energy exchange between plasma and particle [12].

In the context of complex (dusty) plasmas, it is important to remind that very often they are subjected to external electric fields (e.g. RF sheaths, positive column and striations in DC discharges and ambipolar electric fields in the bulk plasma). In such cases, plasmas are anisotropic. Depending on the strength of the electric field, the ion drifts relative to the (often stationary) dust particles can be subthermal ($M_T \leqslant 1$) – weak anisotropy or suprathermal ($M_T \gg 1$) – strong anisotropy. Here, $M_T = u/v_{\mathrm{Ti}}$ is the thermal mach number for ions which is the ratio of the ion drift velocity $u$ to the ion thermal velocity $v_{\mathrm{Ti}}$. One can apply linear dielectric response formalism to calculate the electrical potential created by a point-like charge at rest (or, similarly, around a charge moving with respect to stationary plasma). The result is [13]

$$\phi(\mathbf{r}) = \frac{Q}{2\pi^2} \int \frac{e^{i\mathbf{kr}} d\mathbf{k}}{k^2 \varepsilon(\mathbf{ku}, k)}, \tag{1}$$

where $Q$ is the particle charge and $\varepsilon(\mathbf{ku}, k)$ is the plasma dielectric function. Using a certain model for the dielectric function, the distribution of the electrical potential has been calculated in numerous works both using analytical approaches and numerical simulations (see [1,3] and references therein).

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Let us mention some of the relevant results. Using collisionless kinetic approach, it was shown that the far-field potential of a slowly moving (subthermal) test charge decreases as the inverse cube of the distance ($\propto r^{-3}$) [14–16] in conventional electron–ion plasmas. On the other hand, the far-field potential around a slowly moving charge in collisional plasma scales as $\propto r^{-2}$ if the collision frequency $\nu$ is larger than the plasma frequency $\omega_p$, and if $M_T \ll \omega_p/\nu$ [17, 18]. The far-field potential around a test charge slowly moving in a dusty plasma was calculated by Shukla et. al. [19] taking into account the dust charge fluctuation. It was shown that the long-range asymptote of the potential behind the particle scales as $\propto r^{-2}$. Another example is related to the theory of wakes behind an object in flowing plasmas that has been developed in connection to inter-particle interactions in complex plasmas [20–27]. Both collisionless and collisional situations were studied, but in all these cases the plasma absorption on the particle was usually neglected. The purpose of this paper is to briefly discuss the importance of plasma absorption for the electrical potential around a charged particle in anisotropic collisional plasmas.

Recently, a hydrodynamic model has been used, where a stationary negatively charged small spherical particle is immersed in a quasi-neutral highly collisional plasma [28]. A simple kinetic model accounting for collisions has also been developed recently but so far applied only to the isotropic plasma regime [29]. In the considered anisotropic regime the electric field is sufficiently weak so that the ions are drifting with subthermal velocity while electrons form stationary background. Plasma absorption occurs on the grain surface, i.e. it acts as a plasma sink. Ionization/recombination processes in the vicinity of the particle are neglected. The potential distribution downstream from the particle is obtained by solving Poisson equation coupled to the hydrodynamic equations for ions and Boltzmann equation for electrons using the standard linearization technique under the assumption $M_T \ll k_{\rm D}\ell_{\rm i} \ll 1$ [30]. The result can be written as

$$\phi(\mathrm{r}) = \frac{Q\exp(-rk_{\rm D})}{r} - \left(\frac{Qu\nu k_{\rm Di}^2}{2v_{\rm Ti}^2 k_{\rm D}^4}\right)\frac{2-(r^2k_{\rm D}^2+2rk_{\rm D}+2)\exp(-rk_{\rm D})}{r^2}$$
$$- \left(\frac{eJ_{\rm i}\nu}{v_{\rm Ti}^2 k_{\rm D}^2}\right)\frac{1-\exp(-rk_{\rm D})}{r} - \left(\frac{eJ_{\rm i}u\nu^2}{2v_{\rm Ti}^4 k_{\rm D}^4}\right)\left(1-\frac{2k_{\rm De}^2}{k_{\rm D}^2}\right)$$
$$\times \frac{2-[(1-k_{\rm De}^2/k_{\rm Di}^2)^{-1}r^2k_{\rm D}^2+2rk_{\rm D}+2]\exp(-rk_{\rm D})}{r^2}. \qquad (2)$$

Here, $\ell_{\rm i}$ is the ion mean-free path, $k_{\rm Di(e)} = \lambda_{\rm Di(e)}^{-1} = \sqrt{4\pi e^2 n_{\rm i(e)}/T_{\rm i(e)}}$ is the inverse ion (electron) Debye radius, $k_{\rm D} = \sqrt{k_{\rm De}^2 + k_{\rm Di}^2}$ is the inverse linearized Debye radius, $T_{\rm i(e)}$ is the ion (electron) temperature, $n_{\rm i} \approx n_{\rm e} \approx n_0$ is the unperturbed plasma density, $J_{\rm i}$ is the ion flux to the particle surface, and $e$ is the elementary charge. Let us briefly analyse the structure of (2). The first term is the usual isotropic Debye–Hückel potential and the second term represents the anisotropic part of the potential behind a non-absorbing particle. The third and fourth terms represent, respectively, the isotropic and anisotropic parts of the electrical potential associated with the effect of ion absorption. They disappear when the ion flux is set equal to zero, $J_{\rm i} = 0$. Making use of the known asymptotic expression for the ion flux to an infinitesimally small particle in the continuum limit, $J_{\rm i} \approx 4\pi a z \tau n_0 \ell_{\rm i} v_{T_{\rm i}}$ [31,32], the

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above expression for the potential can be simplified as

$$\phi(\mathrm{r}) = Q\Bigg[\frac{\exp(-rk_{\mathrm{D}})}{r} - \left(\frac{M_T k_{\mathrm{Di}}^2}{2\ell_{\mathrm{i}} k_{\mathrm{D}}^4}\right)\frac{2-(r^2k_{\mathrm{D}}^2+2rk_{\mathrm{D}}+2)\exp(-rk_{\mathrm{D}})}{r^2}$$
$$+\left(\frac{k_{\mathrm{Di}}}{k_{\mathrm{D}}}\right)^2\frac{1-\exp(-rk_{\mathrm{D}})}{r}+\left(1-\frac{2k_{\mathrm{De}}^2}{k_{\mathrm{D}}^2}\right)\left(\frac{M_T k_{\mathrm{Di}}^2}{2\ell_{\mathrm{i}}k_{\mathrm{D}}^4}\right)$$
$$\times\frac{2-[(1-k_{\mathrm{De}}^2/k_{\mathrm{Di}}^2)^{-1}r^2k_{\mathrm{D}}^2+2rk_{\mathrm{D}}+2]\exp(-rk_{\mathrm{D}})}{r^2}\Bigg]. \tag{3}$$

Here, $a$ is the particle radius, $z = |Q|e/aT_{\mathrm{e}}$ is the normalized particle charge, and $\tau = T_{\mathrm{e}}/T_{\mathrm{i}}$ is the electron-to-ion temperature ratio. The long-range asymptote of the anisotropic part of the electric potential for a non-absorbing particle is

$$\phi \approx -(Qu\nu/\omega_{\mathrm{pi}}^2 r^2)(k_{\mathrm{Di}}/k_{\mathrm{D}})^4. \tag{4}$$

Except of the factor $(k_{\mathrm{Di}}/k_{\mathrm{D}})^4$ this expression is identical to that obtained in [17] for the potential behind a slowly moving non-absorbing test charge in collisional plasma. The difference appears because in [17] only electron screening was taken into account. Thus, in contrast to the $\sim r^{-3}$ decay in collisionless regime, the potential of a non-absorbing particle in collisional plasmas exhibits $\sim r^{-2}$ long-range decay. In the presence of absorption, the long-range asymptote of the anisotropic part of the potential can be written as

$$\phi \approx -2(Qu\nu/\omega_{\mathrm{pi}}^2 r^2)(k_{\mathrm{Di}}/k_{\mathrm{D}})^4(k_{\mathrm{De}}/k_{\mathrm{D}})^2. \tag{5}$$

The effect of absorption-induced ion rarefication behind the particle suppresses the effect of ion focusing and the amplitude of the anisotropic part of potential decreases by the factor $(k_{\mathrm{D}e}/k_{\mathrm{D}})^2$. However, the contribution from the isotropic part associated with absorption is more important. It is always negative (for a negatively charged particle) and can be written as

$$\phi \approx (Q/r)(k_{\mathrm{Di}}/k_{\mathrm{D}})^2. \tag{6}$$

This contribution is most often dominant and completely determines the long-range behaviour of the potential [30].

In conclusion, we emphasize that the effect of plasma absorption which was previously neglected determines the far-field potential of a test charge in an anisotropic collisional plasma. The results can be useful in understanding particle dynamics and transport, sheath structure around a test charge in low-temperature laboratory and space plasmas. The authors are happy to dedicate this paper to Padma Kant Shukla on the occasion of his 60$^{\mathrm{th}}$ birthday.

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