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Wake potentials in plasmas containing elongated dust rods

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Abstract. We study interaction potentials in unmagnetized as well as magnetized dusty plasmas composed of electrons, ions, and elongated charged dust rods. By using appropriate dielectric constants for coupled dipole oscillons and dust acoustic waves and the test particle approach, we determine the Debye–Hückel and dynamical oscillatory wake potentials. The relevance of our investigation to the alignment of dust rods in laboratory dusty plasmas is pointed out.

1. Introduction

In recent years, there has been a great deal of interest in studying the formation and dynamics of dust crystals in plasma environments with the hope of creating new materials (Ikezi 1986; Thomas et al. 1994; Chu and I 1994; Hayashi and Tachibana 1994; Melzer et al. 1994; Hayashi 1999; Zuzic et al. 2000; Verheest 2000; Piel and Melzer 2002; Shukla and Mamun 2002). Although most of the previous experiments (Chu and I 1994; Thomas et al. 1994; Hayashi and Tachibana 1994; Melzer et al. 1994; Takahashi et al. 1998; Hayashi 1999; Zuzic et al. 2000) have dealt with plasma crystals composed of microspheres, recent experiments (Mohideen et al. 1998; Molotkov et al. 2000; Rahman et al. 2001; Annaratone et al. 2001) have conclusively demonstrated the formation of organized structures consisting of elongated cylindrical rod-like grains having sizes several micrometers in diameter and a few hundred micrometers in length. The Coulomb crystallization of charged dust grains could be caused by the plasma wake field and ion focusing (Nambu et al. 1995; Shukla and Rao 1996) and dipole–dipole interactions (Shukla and Mamun 2002). Furthermore, in cosmic environments, dust particles of different sizes and shapes are quite common (Voshchinnikov and Semenov 2000; Mendis 2002). The observed infrared and submillimeter radiation are attributed to the thermal emission from dust clouds heated by shock waves, the universal ultra-violet

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radiation, or stellar radiation. Elongated dust grains are assumed to be formed by coagulation of smaller particulates in partially or fully ionized plasmas by some attractive forces (Bingham and Tsytovich 2001; Bingham et al. 2002), the details of which are not fully understood. However, it is thought that inelastic, adhesive, and collective interactions between micrometer-sized dust particles give rise to kilometer-sized bodies, which are known as planetesimals. Results from a microgravity aggregation experiment (Blum et al. 2000) flown onboard the space shuttle revealed the structure and growth of dust agglomeration. Specifically, Blum et al. (2000) reported that a thermally aggregating swarm of dust particles evolves very rapidly and forms unexpected open-structure agglomerates.

Obviously, the dust grains formed in laboratory and astrophysical environments by nucleosynthesis and coagulation may have any shape and size. Recently, many authors (Mahmoodi et al. 2000; Tskhakaya and Shukla 2001; Tskhakaya et al. 2001; Tskhakaya et al. 2002) have investigated the electrodynamics and dispersion properties of dusty plasmas whose constituents are electrons, ions, and elongated and rotating charged grains in the absence and or in the presence of external magnetic fields. There is a distribution of charges on a rod-like grain. The inhomogeneous charge distribution on a grain gives rise to formation of a finite dipole moment. It is shown that the oscillating dipoles of the charged dust grains give rise to a new wave mode, which is known as the 'dipole oscillon' (Shukla 2001a).

Recently, Shukla (2001a, b) has investigated the formation of wake potentials in an unmagnetized dusty plasma consisting of electrons, ions, and elongated dust grains, by neglecting the dust particle dynamics. Thus, dipole oscillons are decoupled from the dust acoustic wave (Rao et al. 1990; Shukla 2001c). In the present paper, we take into account the dust dynamics and dust dipole moments of an ensemble of elongated dust rods and study the Debye–Hückel and the dynamical wake potentials in dusty plasmas with and without an external magnetic field.

In Sec. 2, we consider the potential around a test dust rod in an unmagnetized dusty plasma composed of elongated dust grains that are mobile. We use the dielectric constant of the dusty plasma that accounts for the combined effects of the moment of inertia and the dynamics of dust rods. Explicit results for the short-range Debye–Hückel and long-range dynamical wake potentials are presented. In Sec. 3, we show that the presence of an external magnetic field in a dusty plasma modifies the Debye–Hückel and dynamical wake potentials. A brief discussion of our results is contained in Sec. 4.

2. Wake potential in unmagnetized dusty plasmas

We consider an unmagnetized dusty plasma composed of electrons, ions, and negatively charged elongated charged dust rods. These non-spherical rod-like grains may acquire opposite charges at their edges in the presence of some electric fields and will form electric dipoles which will cause the spinning and rotational motions of the dust rods in the plasma. The physics of such elongated and rotating charged dust grains in the presence of an external magnetic field has been extensively studied recently (Tskhakaya et al. 2001).

The oscillating dipoles of elongated dust grains support a low-frequency wave mode (the dipole oscillon) whose frequency is given by (Shukla 2001a)

$$\omega = k_{\perp}\lambda_{\mathrm{D}}\Omega_{\mathrm{el}}, \tag{1}$$

where $\lambda_{\mathrm{D}} = \lambda_{\mathrm{De}}\lambda_{\mathrm{Di}}/(\lambda_{\mathrm{De}}^2 + \lambda_{\mathrm{Di}}^2)^{1/2}$ is the effective dusty plasma Debye radius,

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$\lambda_{De}(\lambda_{Di})$ is the electron (ion) Debye radius, $\Omega_{el} = (4\pi d^2 n_{d0}/I)^{1/2}$, d is the magnitude of the dipole moment, I is the moment of inertia of elongated dust rods, n_{d0} is the dust number density, and $k_\perp$ is the component of the wave vector $\mathbf{k}$ transverse to the axial direction around which dust rods oscillate. We assume $\lambda_D \ll r_0 \ll L \ll \lambda_{mfp}$, where r_0 is the intergrain distance, L is the characteristic size of the dusty plasma system, and λ_{mfp} is the mean-free-path of electrons and ions. We note that (1) does not include the effect of dust charge fluctuations (Shukla and Stenflo 2001; Shukla and Mamun 2002). The latter are unimportant for $4\pi n_{d0} r_d \lambda_D^2 \nu_1/\nu_2 \ll 1$, where r_d is the dust radius and ν_1 and ν_2 are defined on p. 61 of Shukla and Mamun (2002).

However, when elongated oscillating dust rods move, one finds (Mahmoodi et al. 2000) that the dipole oscillons are coupled with the dust acoustic wave. For $kv_{td} \ll \omega \ll kv_{te}, kv_{ti}$, the linear dielectric constant of an unmagnetized dusty plasma in the presence of coupled dipole oscillations and dust acoustic waves is (Mahmoodi et al. 2000)

$$\epsilon(\omega, \mathbf{k}) = 1 + \frac{1}{k^2\lambda_D^2} - \frac{\omega_{pd}^2}{\omega^2} - \frac{k_\perp^2}{k^2}\frac{\Omega_{el}^2}{\omega^2}, \tag{2}$$

where v_{tj} is the thermal speed of the particle species j (j equals e for electrons, i for ions and d for dust grains), $\omega_{pd} = (4\pi n_{d0} Z_d^2 e^2/m_d)^{1/2}$ is the dust plasma frequency, Z_d and m_d are the number of the average electronic charge and mass of a typical non-spherical dust grain, respectively, e is the magnitude of the electron charge, $Z_d n_{d0} = n_{i0} - n_{i0}$, and $n_{e0}(n_{i0})$ is the electron (ion) number density.

The electrostatic potential around a test dust particulate in the presence of any electrostatic mode $(\omega, \mathbf{k})$ in a plasma with the dielectric response function $\epsilon(\omega, \underline{k})$ is (Krall and Trivelpiece 1973)

$$\Phi(\mathbf{x}, t) = \int \frac{q_t}{2\pi^2 k^2} \frac{\delta(\omega - \mathbf{k}\cdot\mathbf{v}_t)}{\epsilon(\omega, \mathbf{k})} \exp(i\mathbf{k}\cdot\mathbf{r})\, d\mathbf{k}\, d\omega, \tag{3}$$

where $\mathbf{r} = \mathbf{x} - \mathbf{v}_t t$, $\mathbf{v}_t$ is the velocity vector of the test dust particulate, and q_t is its charge. We note that (3) has been obtained by Fourier transforming Poisson's equation in the presence of a delta function source and with appropriate electron and ion number density perturbations, viz. $n_{e1} \approx n_{e0} e\Phi/T_e$ and $n_{i0} \approx -k^2\chi_i\Phi/4\pi e$, where χ_i is the ion susceptibility. Hence, the expression (3) is valid for magnetized plasmas also in which the ion susceptibility is anisotropic (Shukla and Mamun 2002).

Let us now consider that the elongated test dust particulate, located at the origin at $t = 0$, moves parallel to the z-axis with velocity $\mathbf{u}_0 = \hat{\mathbf{z}} u_0$ in resonance with the parallel phase velocity of the low-frequency dusty plasma wave $(\omega, \mathbf{k})$. Thus, the inverse of the dielectric function (2) becomes

$$\frac{1}{\epsilon(k_\parallel u_0, t)} = \frac{k^2\lambda_0^2}{1 + k^2\lambda_0^2}\left(1 + \frac{\omega_k^2}{k_\parallel^2 - \omega_k^2}\right), \tag{4}$$

where

$$\omega_k^2 = \frac{k^2}{M^2(1 + k^2\lambda_0^2)}, \tag{5}$$

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with

$$M = \frac{u_0}{\omega_{\rm p}\lambda_0},$$

$$\omega_{\rm p}^2 = \omega_{\rm pd}^2 + \Omega_{\rm el}^2, \tag{6}$$

$$\lambda_0 = \frac{\lambda_{\rm D}}{\left(1 + C_0^2/u_0^2\right)^{1/2}},$$

and

$$C_0 = \lambda_{\rm D}\Omega_{\rm el}.$$

Substituting (4) into (3) and carrying out the integrations in the usual manner (Nambu and Akama 1985), we obtain

$$\Phi(\mathbf{x}, t) = \Phi_{\rm D} + \Phi_{\rm w}, \tag{7}$$

where the Coulombian Debye–Hückel shielding potential is

$$\Phi_{\rm D}(r) = \frac{q_{\rm t}}{r}\exp\left(-\frac{r}{\lambda_0}\right), \tag{8}$$

and the non-Coulombian interaction potential, involving the collective interactions between the test particulate and the coupled dipole oscillons and dust acoustic waves, is

$$\Phi_{\rm w}(\rho, z) = \left(\frac{q_{\rm t}\lambda_0^2 M^{-2}}{\pi}\right)\int \frac{k^2 J_0(k_\perp\rho)e^{ik_\parallel z}}{\left(1 + k^2\lambda_0^2\right)\left[k_\parallel^2\left(1 + k^2\lambda_0^2\right) - k^2M^{-2}\right]}k_\perp\, dk_\perp\, dk_\parallel. \tag{9}$$

In (9), ρ and z are the cylindrical coordinates and J_0 is the zero-order cylindrical Bessel function of the argument $(k_\perp\rho)$.

Introducing the dimensionless notation $\mathbf{K} = \mathbf{k}\lambda_0$, (9) can be re-written as

$$\Phi_{\rm w}(\rho, z) = \left(\frac{q_{\rm t}M^{-2}}{\pi\lambda_0}\right)\int \frac{K^2}{1 + K^2}\frac{J_0(K_\perp\rho/\lambda_0)e^{iK_\parallel z/\lambda_0}}{\left(K_\parallel^2 + K_0^2\right)\left(K_\parallel^2 - K_1^2\right)}K_\perp\, dK_\perp\, dK_\parallel, \tag{10}$$

where

$$K_{0,1}^2 = \pm\frac{1 + K_\perp^2 - M^{-2}}{2} + \sqrt{K_\perp^2M^{-2} + \left(1 + K_\perp^2 - M^{-2}\right)^2/4}\,. \tag{11}$$

Case (i). Assuming $M \gg 1$ and $K_\perp \ll \sqrt{1 - M^{-2}}$, we obtain the dynamical wake potential from (10) as

$$\Phi_{\rm w}(\rho = 0, z) = \frac{2q_{\rm t}(1 - M^{-2})}{|z|}\cos\left(\frac{|z|}{L_s}\right), \tag{12}$$

where the effective attraction length of the oscillatory wake potential is denoted by $L_s = \sqrt{M^2 - 1}\,\lambda_0$.

Case (ii). Assuming $M \ll 1$ and $K_\perp \ll 1$, we obtain the dynamical wake potential as

$$\Phi_{\rm w}(\rho = 0, z) \approx \frac{q_{\rm t}}{M\lambda_0}\sin\left(\frac{|z|}{L_s'}\right), \tag{13}$$

where $L_{\rm s}' = M\lambda_0$.

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3. Wake potential in magnetized dusty plasmas

Here, we consider a dusty plasma containing electrons, ions, and negatively charged elongated dust rods in a uniform magnetic field $\mathbf{B}_0 \parallel \hat{\mathbf{z}} B_0$, where $\hat{\mathbf{z}}$ is the unit vector along the z axis and B_0 is the strength of the external magnetic field. We consider electrostatic waves in the approximations, $\omega_{\rm cd} \ll \omega \ll \omega_{\rm ci}, k_\parallel v_{\rm te}$. Hence, hot electrons rapidly thermalize along $\hat{\mathbf{z}}$ and establish a Boltzmann distribution, while magnetized ions execute the $\mathbf{E} \times \mathbf{B}_0$ motion and also suffer the polarization drift, where $\mathbf{E}$ is the wave electric field. Elongated dust rods remain unmagnetized.

The dielectric constant of a magnetized dusty plasma in the presence of low-frequency electrostatic waves is (Tskhakaya et al. 2001)

$$\epsilon(\omega, \mathbf{k}) = 1 + \frac{1}{k^2 \lambda_{\rm De}^2} + \frac{k_\perp^2}{k^2} \frac{\omega_{\rm pi}^2}{\omega_{\rm ci}^2} - \frac{k_\parallel^2}{k^2} \frac{\omega_{\rm pi}^2}{\omega^2} - \frac{\omega_{\rm pd}^2}{\omega^2} - \frac{k_\perp^2}{k^2} \frac{\Omega_{\rm el}^2}{\omega^2}. \tag{14}$$

For the resonant interaction ($\omega \simeq k_\parallel u_0$) and for nearly perpendicular propagation ($k_\parallel^2 \ll k^2$), the inverse of (14) can be written as

$$\frac{1}{\epsilon(k_\parallel u_0, \mathbf{k})} = \frac{k^2 \lambda_0^2}{1 + (1+f) k^2 \lambda_0^2} \left(1 + \frac{\omega_k'^2}{k_\parallel^2 u_0^2 - \omega_k'^2} \right), \tag{15}$$

where

$$\omega_k'^2 = \frac{k^2 C_0^2}{1 + (1+f) k^2 \lambda_0^2}, \tag{16}$$

$f = \omega_{\rm pi}^2/\omega_{\rm ci}^2$, $\lambda_0 = \lambda_{\rm De}/[1 - \lambda_{\rm De}^2(\omega_{\rm pi}^2 - \Omega_{\rm el}^2)/u_0^2]^{1/2}$, and $C_0 = \lambda_0 \sqrt{\omega_{\rm pd}^2 + \Omega_{\rm el}^2}$.

Substituting (15) into (3), writing $\Phi = \Phi_{\rm I} + \Phi_{\rm II}$, and following the mathematical steps in Sec. 2, we obtain the modified Debye–Hückel interaction potential in the presence of the external magnetic field as

$$\Phi_{\rm I}(r) = \frac{q_{\rm t}}{(1+f) r} \exp\left(- \frac{r}{\sqrt{1+f}\, \lambda_0} \right). \tag{17}$$

The second part of (15) when substituted in (3) yields

$$\Phi_{\rm II}(\rho, z) = \left(\frac{q_{\rm t} \lambda_0^2 M^{-2}}{\pi} \right) \int \frac{k^2 J_0(k_\perp \rho) e^{i k_\parallel z} k_\perp \, dk_\perp \, dk_\parallel}{\{1 + (1+f) k^2 \lambda_0^2\} \left[k_\parallel^2 \{1 + (1+f) k^2 \lambda_0^2\} - k^2 M^{-2} \right]}, \tag{18}$$

where $M = u_0/C_0$.

Now, introducing the dimensionless notation, $\mathbf{K} = \sqrt{1+f}\, \lambda_0 \mathbf{k}$, we obtain from (18)

$$\Phi_{\rm w}(\rho, z) = \left\{ \frac{q_{\rm t} M^{-2}}{\pi (1+f)^{3/2} \lambda_0} \right\} \times \int \frac{K^2}{1+K^2} \frac{J_0(K_\perp \rho / \sqrt{1+f}\, \lambda_0)}{(K_\parallel^2 + K_0^2)(K_\parallel^2 - K_1^2)} e^{i K_\parallel z / \sqrt{1+f}\, \lambda_0} K_\perp \, dK_\perp \, dK_\parallel, \tag{19}$$

where

$$K_{0,1}^2 = \pm \frac{1 + K_\perp^2 - M^{-2}}{2} + \sqrt{K_\perp^2 M^{-2} + (1 + K_\perp^2 - M^{-2})^2/4}.$$

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For $M \gg 1$, $K_\perp < 1$, we have

$$K_0^2 \approx 1, \quad K_1^2 \approx \frac{K_\perp^2}{M^2 - 1}. \tag{20}$$

Following Shukla (2001a, b), we finally obtain

$$\Phi_{\rm II}(\rho = 0, z) = \frac{q_{\rm t}(M^2 - 1)}{M^2(1 + f)} \frac{\cos\left(|z|/L_\parallel\right)}{|z|}, \tag{21}$$

where

$$L_\parallel = \sqrt{M^2 - 1}\sqrt{1 + f}\,\lambda_0. \tag{22}$$

For $M < 1$, $K_\perp < 1$, we similarly obtain the dynamical wake potential in the magnetized dusty plasma with elongated dust grains as

$$\Phi_{\rm II}(\rho = 0, z) \approx \frac{q_{\rm t}}{(1 + f)^{3/2}\lambda_0} \sin\left(\frac{|z|}{L'_\parallel}\right), \tag{23}$$

where $L'_\parallel \approx M\sqrt{1 + f}\,\lambda_0$. Thus, the external magnetic field may be applied to control the strength and lattice spacing of the oscillatory wake potential. The latter may be responsible for the dust rod alignment in dusty plasma crystals composed of non-spherical rod-like grains.

4. Discussion

In this paper, we have investigated the potential around a test dust rod in a streaming dusty plasma whose constituents are electrons, ions, and elongated charged dust rods. We have accounted for the dipole moment and the dust rod dynamics and have used the appropriate dielectric constants for the coupled dipole oscillons and dust acoustic waves to study the short-range Debye–Hückel and long-range oscillating wake potentials in plasmas without and with an external magnetic field. It is found that the profile of the oscillating wake potential in an unmagnetized plasma assumes a form that is significantly different from that reported by Shukla (2001b). The difference is attributed to the consideration of the dust rod dynamics, which modifies the frequency of the dipole oscillons that resonate with a test dust rod to produce the wake potential. For unmagnetized dusty plasmas, the effective attraction length of the wake field depends on the characteristic shape of the elongated dust rods ($\Omega_{\rm el}$). On the other hand, in a magnetized dusty plasma, we find that the Debye–Hückel shielding potential is modified and the effective length of the wake potential is increased by a factor $\omega_{\rm pi}/\omega_{\rm ci} > 1$ (cf. (21) and (23)). In the negative regions of the wake potential, the ions can be focused providing an attraction for like-elongated grains. Consequently, a chain of macroscopic dust rods is formed (Mohideen et al. 1998; Molotkov et al. 2000; Rahman et al. 2001; Annaratone et al. 2001) due to their vertical alignment. With the application of an external magnetic field, dust crystals of elongated dust grains may be fragile as the strength of the wake potential is reduced by a factor f for $M > 1$, and $f^{3/2}$ for $M < 1$ and a larger effective attraction length. Future experiments should be designed and conducted to verify the theoretical predictions made herein.

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